

Chapter 3

Random Variables and Probability Distributions

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Section 3.1

Concept of a Random Variable

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Definition 3.1



A **random variable** is a function that associates a real number with each element in the sample space.

Definition 3.2



If a sample space contains a finite number of possibilities or an unending sequence with as many elements as there are whole numbers, it is called a **discrete sample space**.

Definition 3.3



If a sample space contains an infinite number of possibilities equal to the number of points on a line segment, it is called a **continuous sample space**.

Section 3.2

Discrete Probability Distribution

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Definition 3.4



The set of ordered pairs $(x, f(x))$ is a **probability function**, **probability mass function**, or **probability distribution** of the discrete random variable X if, for each possible outcome x ,

1. $f(x) \geq 0$,

2. $\sum_x f(x) = 1$,

3. $P(X = x) = f(x)$.

Definition 3.5



The **cumulative distribution function** $F(x)$ of a discrete random variable X with probability distribution $f(x)$ is

$$F(x) = P(X \leq x) = \sum_{t \leq x} f(t), \quad \text{for } -\infty < x < \infty.$$

Figure 3.1 Probability mass function plot

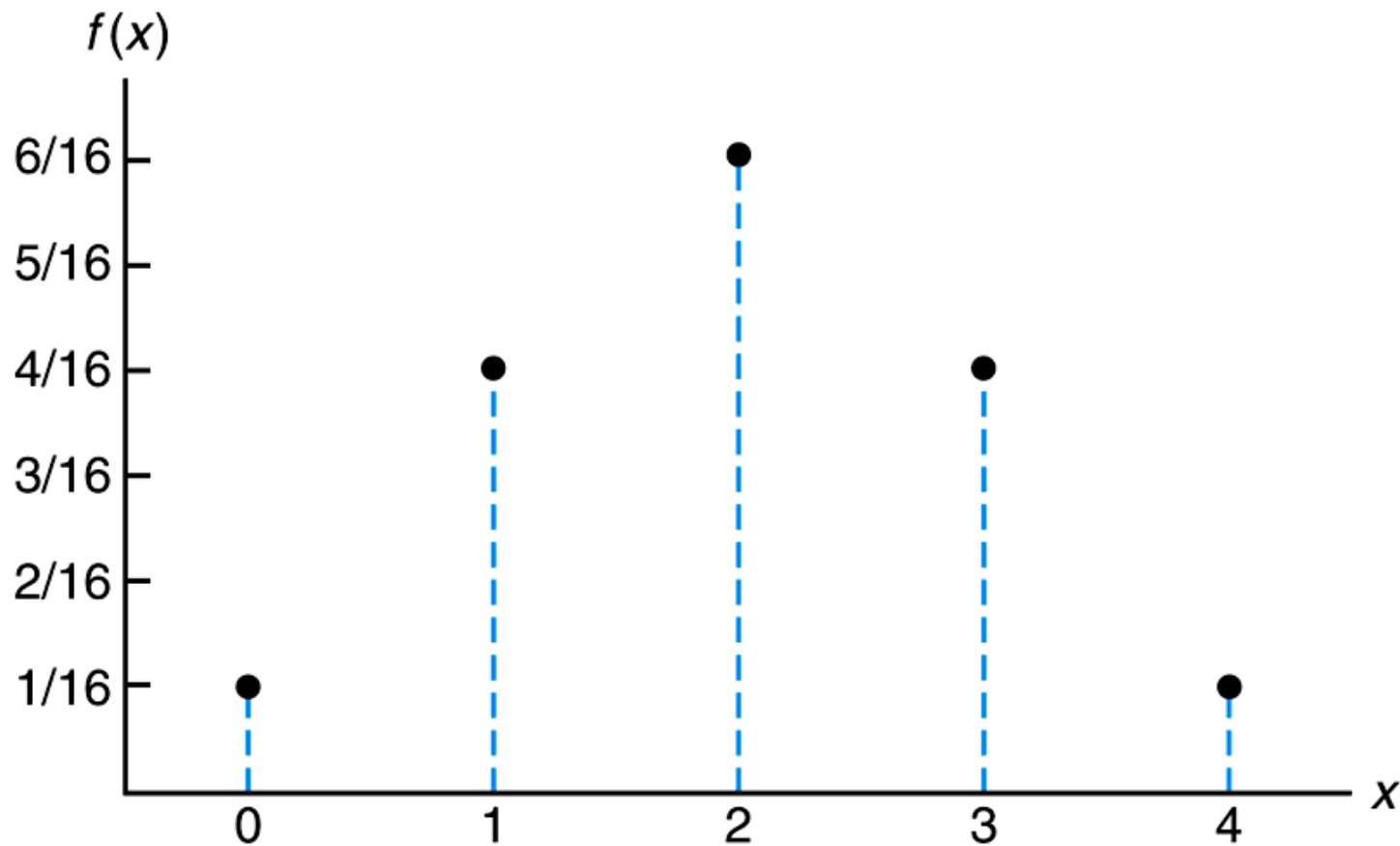


Figure 3.2 Probability histogram

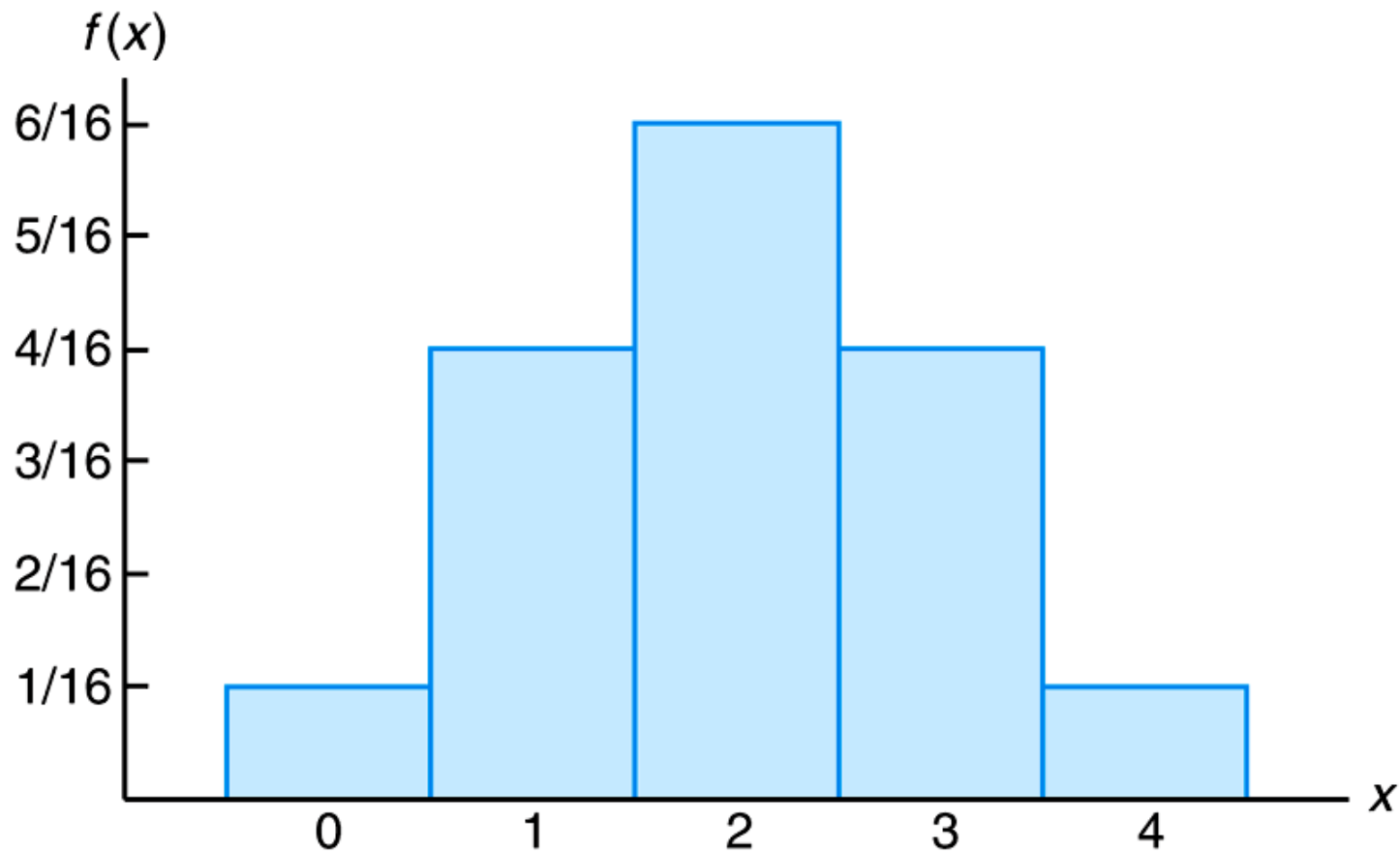
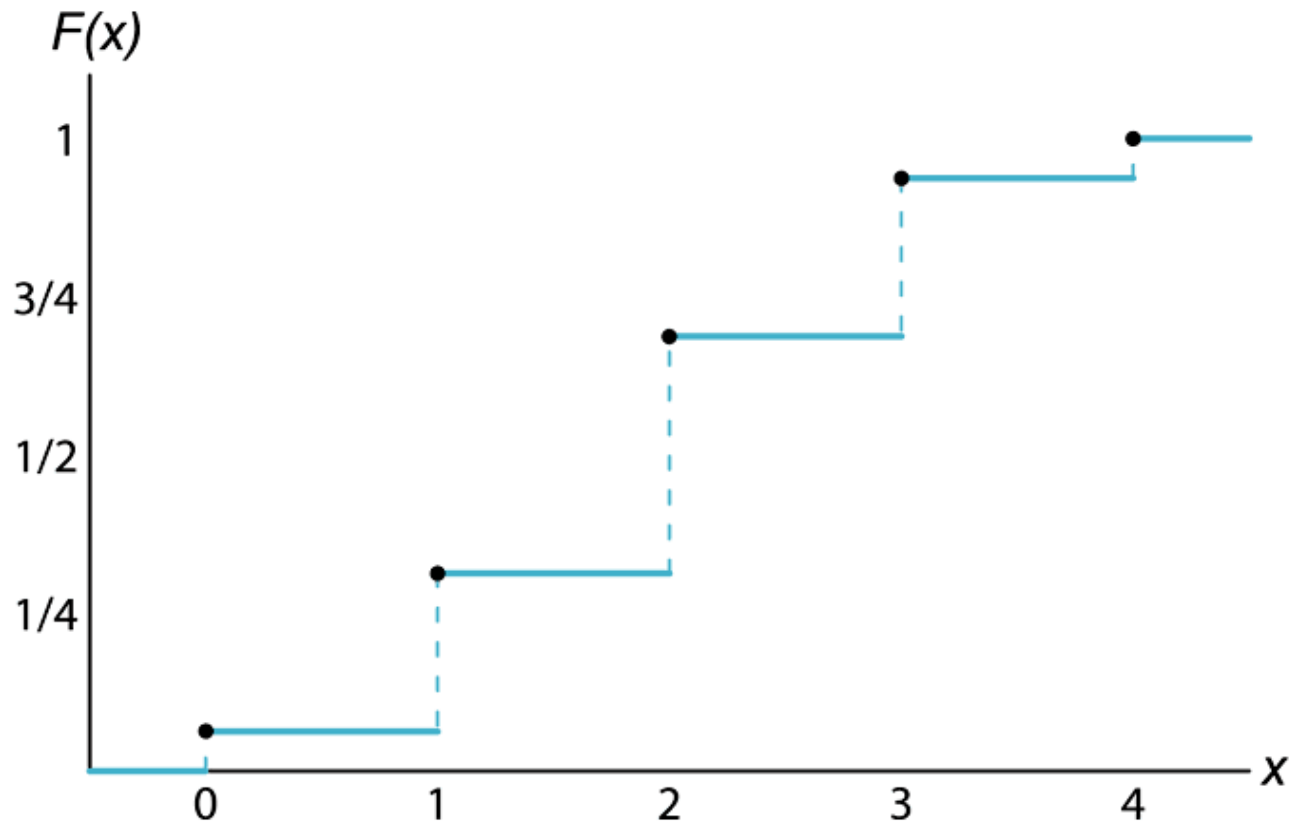


Figure 3.3 Discrete cumulative distribution function



Section 3.3

Continuous Probability Distributions

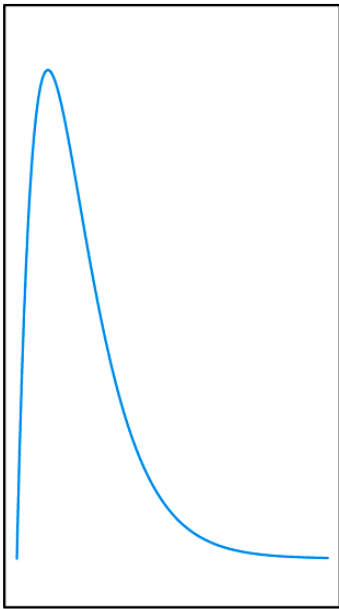
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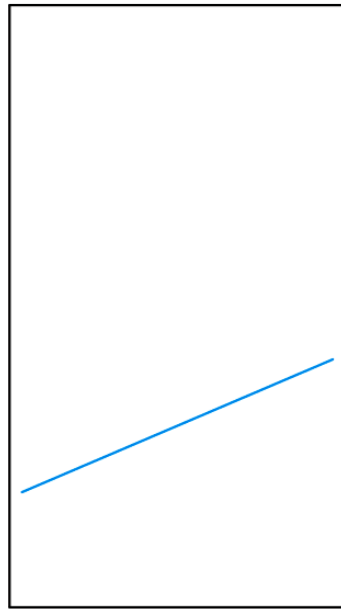


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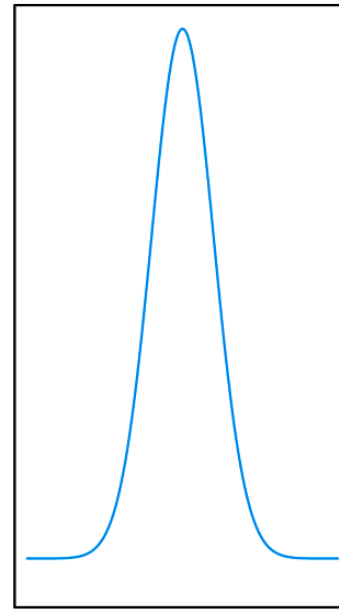
Figure 3.4 Typical density functions



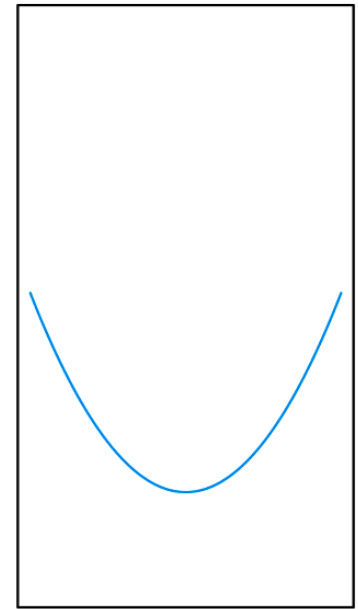
(a)



(b)

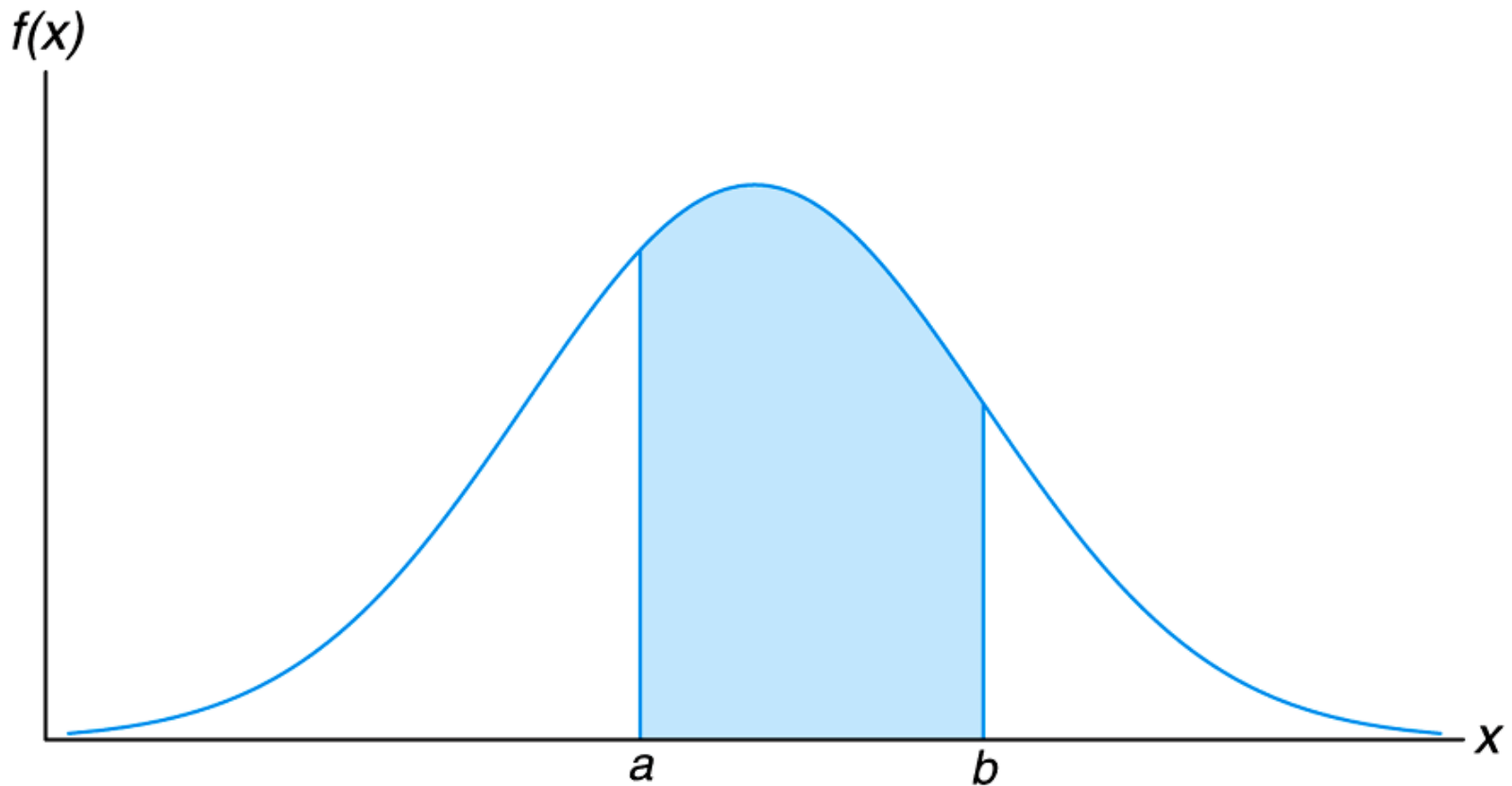


(c)



(d)

Figure 3.5 $P(a < X < b)$



Definition 3.6



The function $f(x)$ is a **probability density function** (pdf) for the continuous random variable X , defined over the set of real numbers, if

1. $f(x) \geq 0$, for all $x \in R$.
2. $\int_{-\infty}^{\infty} f(x) dx = 1$.
3. $P(a < X < b) = \int_a^b f(x) dx$.

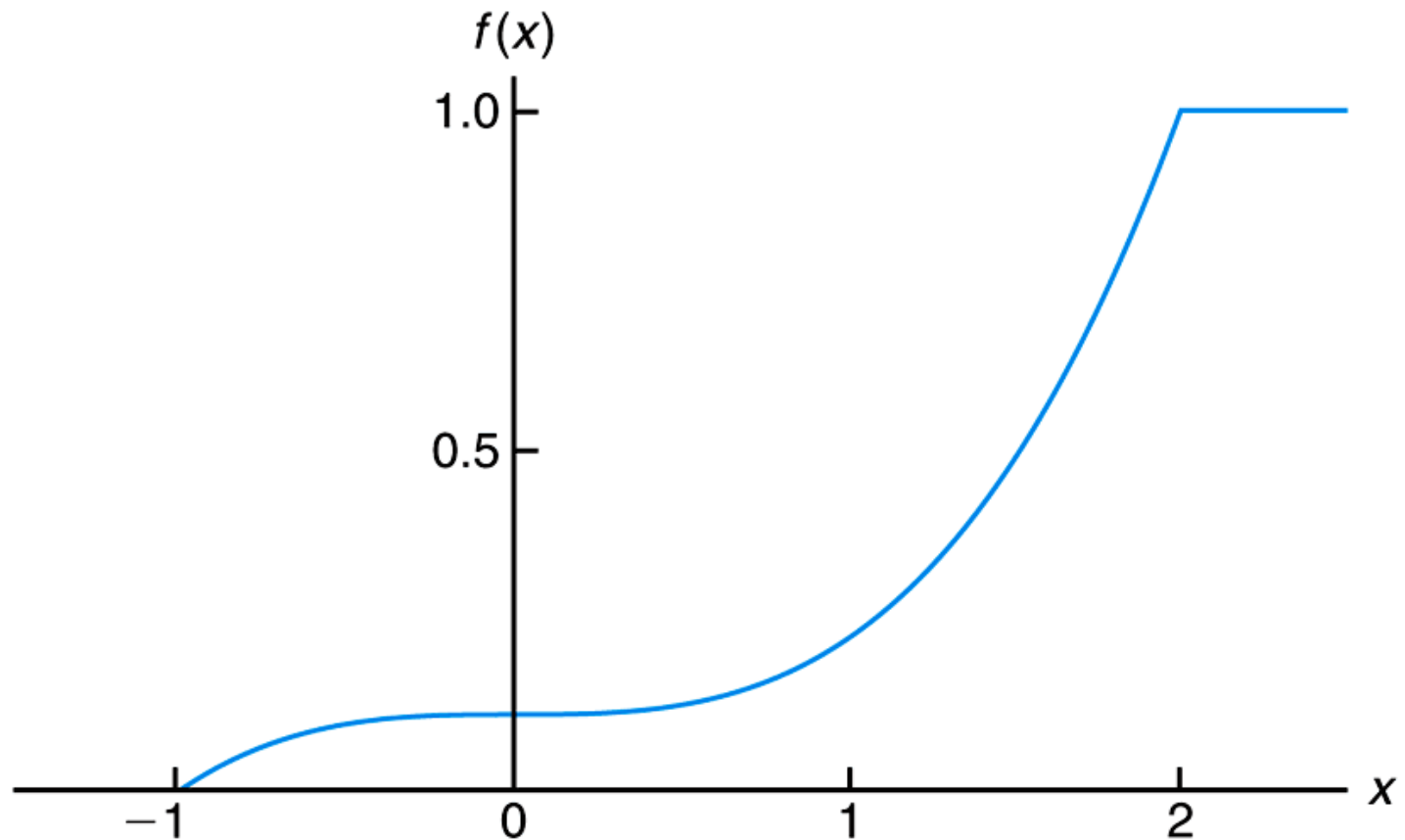
Definition 3.7



The **cumulative distribution function** $F(x)$ of a continuous random variable X with density function $f(x)$ is

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt, \quad \text{for } -\infty < x < \infty.$$

Figure 3.6 Continuous cumulative distribution function



Section 3.4

Joint Probability Distributions

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Definition 3.8



The function $f(x, y)$ is a **joint probability distribution** or **probability mass function** of the discrete random variables X and Y if

1. $f(x, y) \geq 0$ for all (x, y) ,
2. $\sum_x \sum_y f(x, y) = 1$,
3. $P(X = x, Y = y) = f(x, y)$.

For any region A in the xy plane, $P[(X, Y) \in A] = \sum_A f(x, y)$.

Table 3.1 Joint Probability Distribution for Example 3.14



$f(x, y)$		x			Row Totals
		0	1	2	
y	0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
	1	$\frac{3}{14}$	$\frac{3}{14}$	0	$\frac{3}{7}$
	2	$\frac{1}{28}$	0	0	$\frac{1}{28}$
Column Totals		$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$	1

Definition 3.9



The function $f(x, y)$ is a **joint density function** of the continuous random variables X and Y if

1. $f(x, y) \geq 0$, for all (x, y) ,
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$,
3. $P[(X, Y) \in A] = \int \int_A f(x, y) dx dy$, for any region A in the xy plane.

Definition 3.10



The **marginal distributions** of X alone and of Y alone are

$$g(x) = \sum_y f(x, y) \quad \text{and} \quad h(y) = \sum_x f(x, y)$$

for the discrete case, and

$$g(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad \text{and} \quad h(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

for the continuous case.

Definition 3.11



Let X and Y be two random variables, discrete or continuous. The **conditional distribution** of the random variable Y given that $X = x$ is

$$f(y|x) = \frac{f(x, y)}{g(x)}, \text{ provided } g(x) > 0.$$

Similarly, the conditional distribution of X given that $Y = y$ is

$$f(x|y) = \frac{f(x, y)}{h(y)}, \text{ provided } h(y) > 0.$$

Definition 3.12



Let X and Y be two random variables, discrete or continuous, with joint probability distribution $f(x, y)$ and marginal distributions $g(x)$ and $h(y)$, respectively. The random variables X and Y are said to be **statistically independent** if and only if

$$f(x, y) = g(x)h(y)$$

for all (x, y) within their range.

Definition 3.13



Let X_1, X_2, \dots, X_n be n random variables, discrete or continuous, with joint probability distribution $f(x_1, x_2, \dots, x_n)$ and marginal distribution $f_1(x_1), f_2(x_2), \dots, f_n(x_n)$, respectively. The random variables X_1, X_2, \dots, X_n are said to be mutually **statistically independent** if and only if

$$f(x_1, x_2, \dots, x_n) = f_1(x_1)f_2(x_2) \cdots f_n(x_n)$$

for all (x_1, x_2, \dots, x_n) within their range.

Section 3.5

Potential
Misconceptions
and Hazards;
Relationship to
Material in Other
Chapters

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