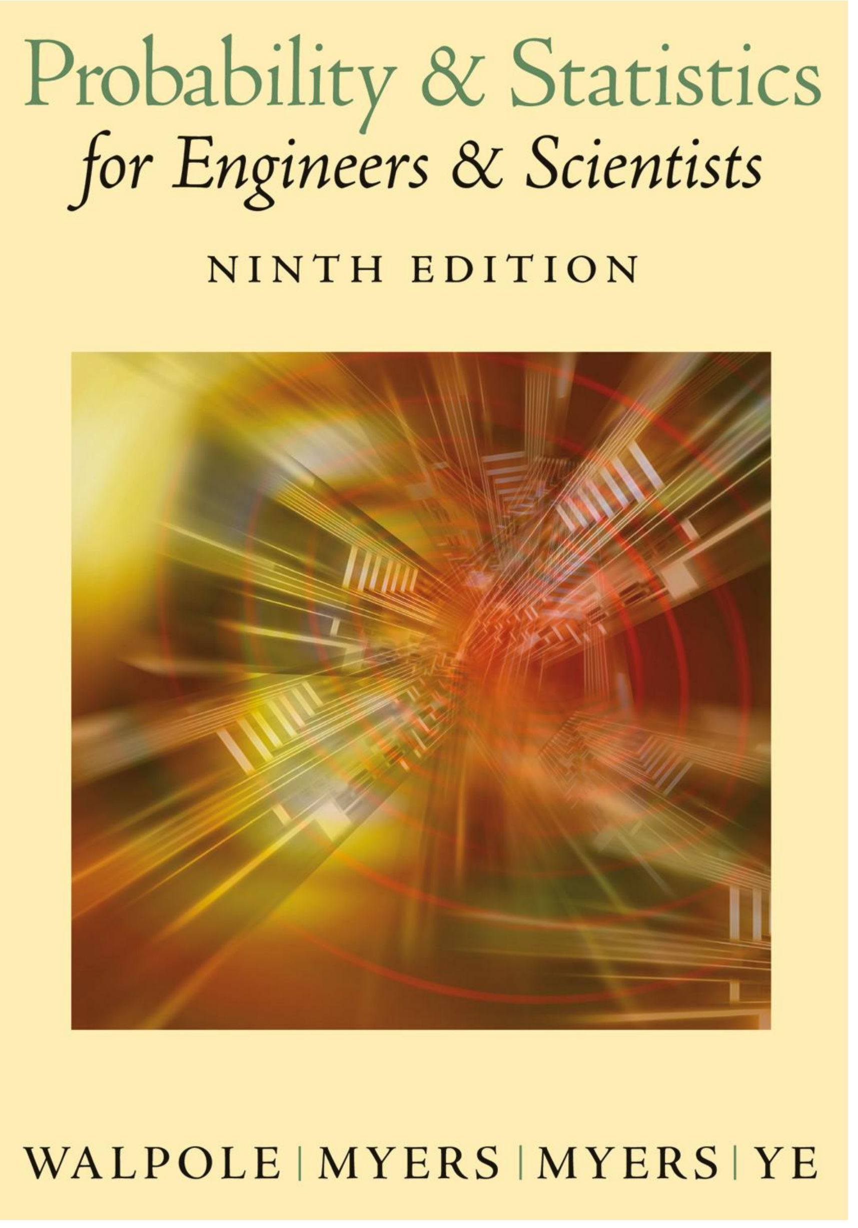
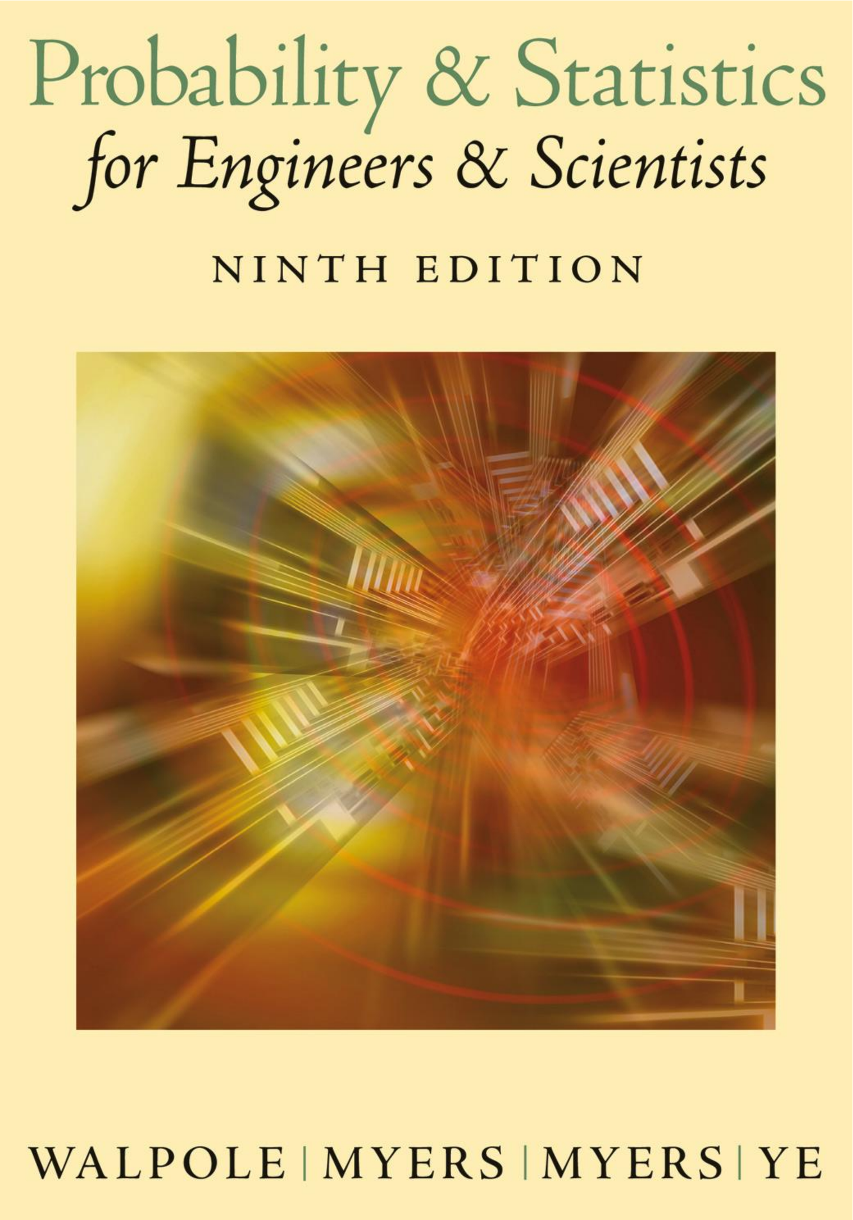
Chapter 9

One- and Two-Sample Estimation Problems



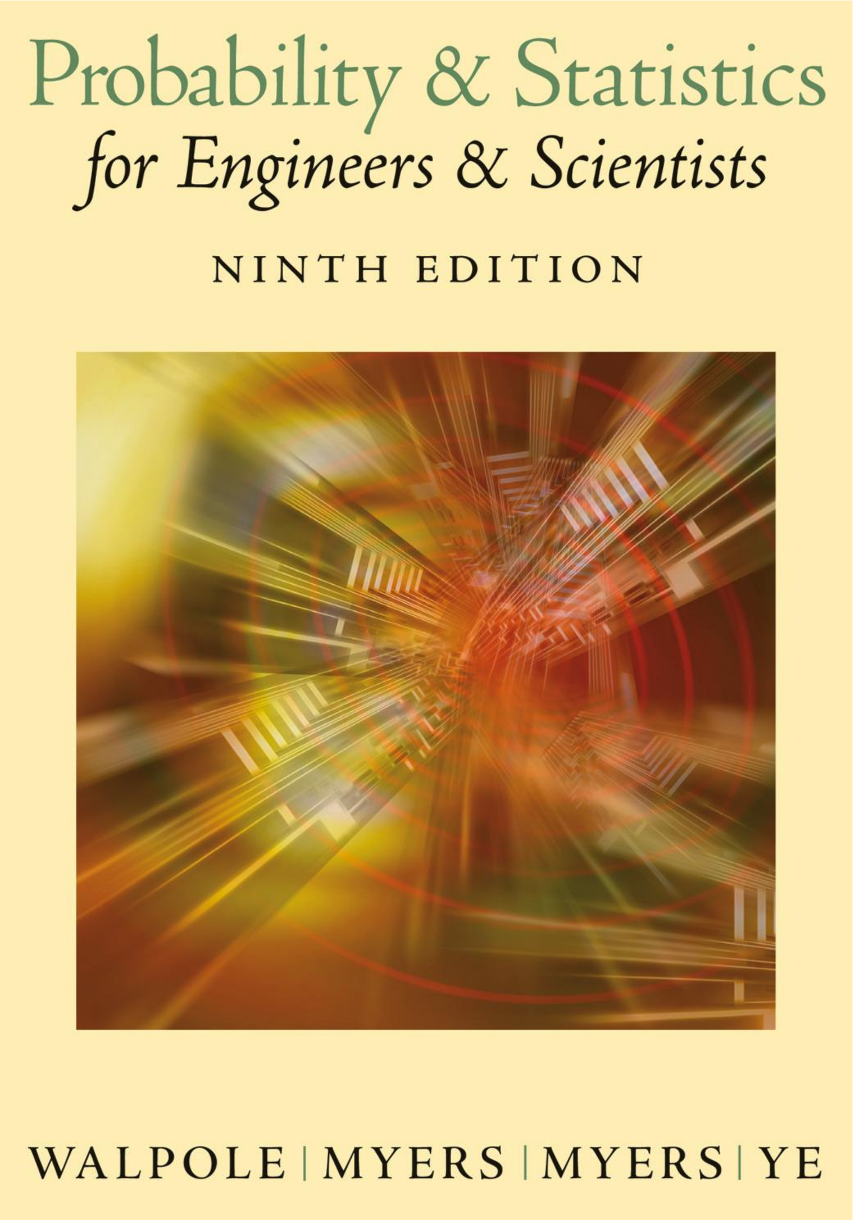
Section 9.1

Introduction



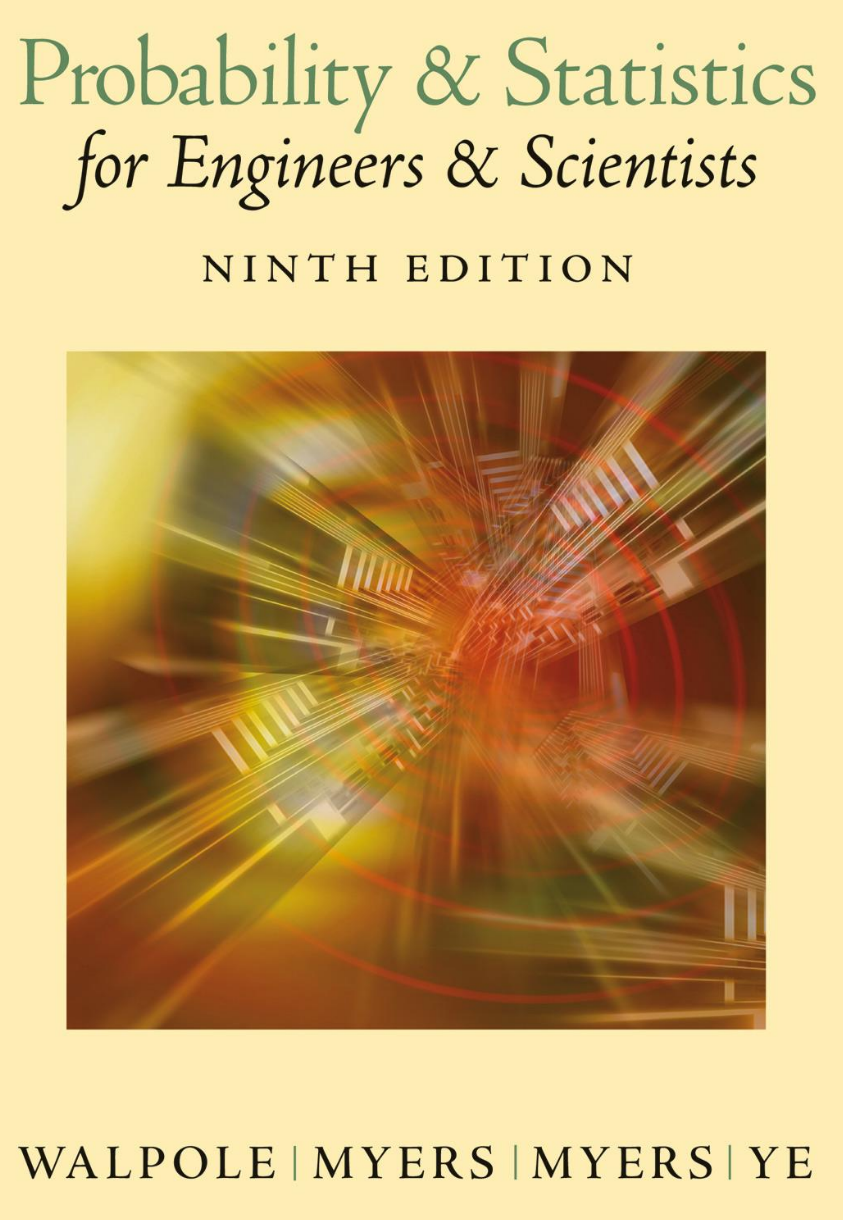
Section 9.2

Statistical Inference



Section 9.3

Classical Methods of Estimation



Definition 9.1

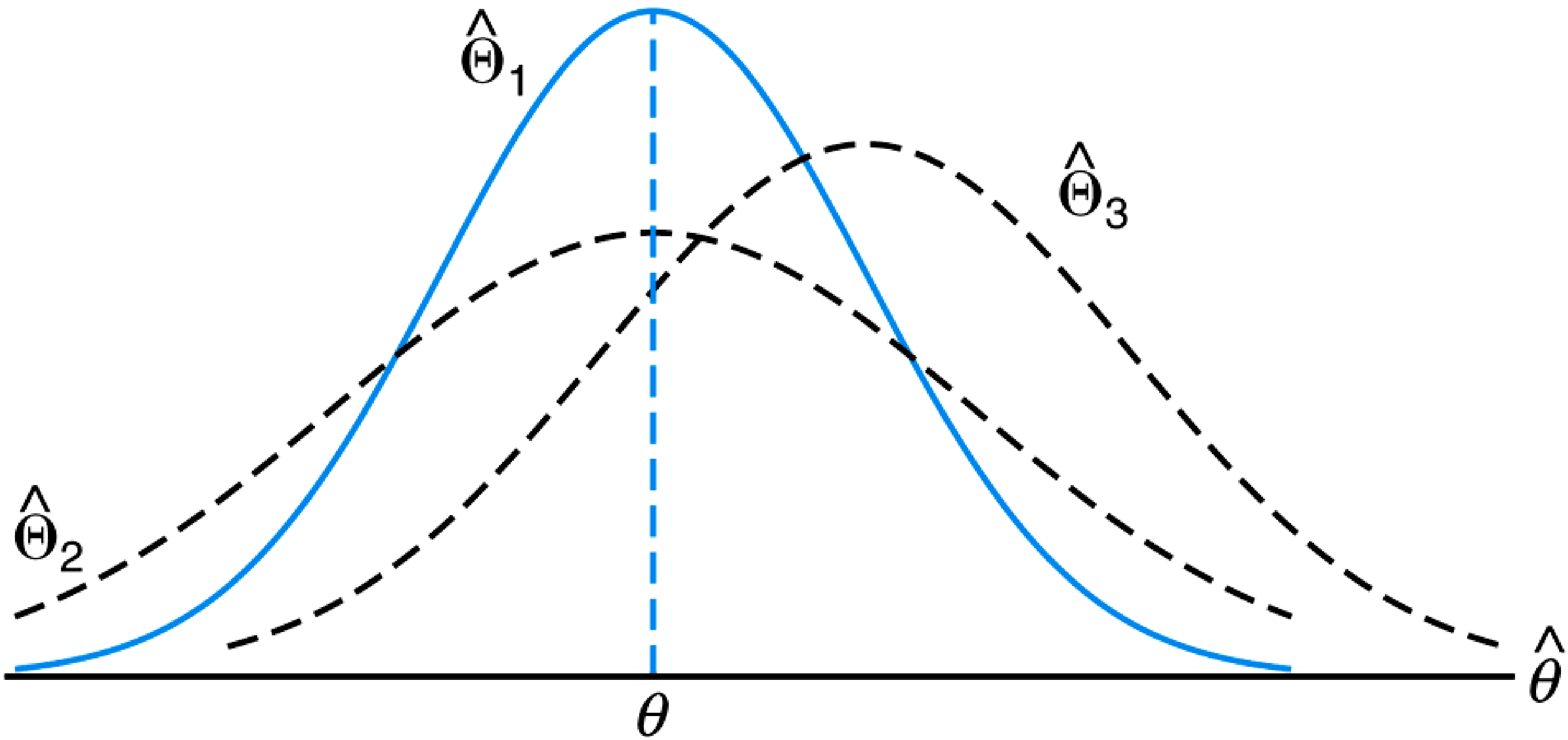
A statistic $\hat{\Theta}$ is said to be an **unbiased estimator** of the parameter θ if

$$\mu_{\hat{\Theta}} = E(\hat{\Theta}) = \theta.$$

Definition 9.2

If we consider all possible unbiased estimators of some parameter θ, the one with the smallest variance is called the **most efficient estimator** of θ.

Figure 9.1 Sampling distributions of different estimators of θ



Section 9.4

Single Sample: Estimating the Mean

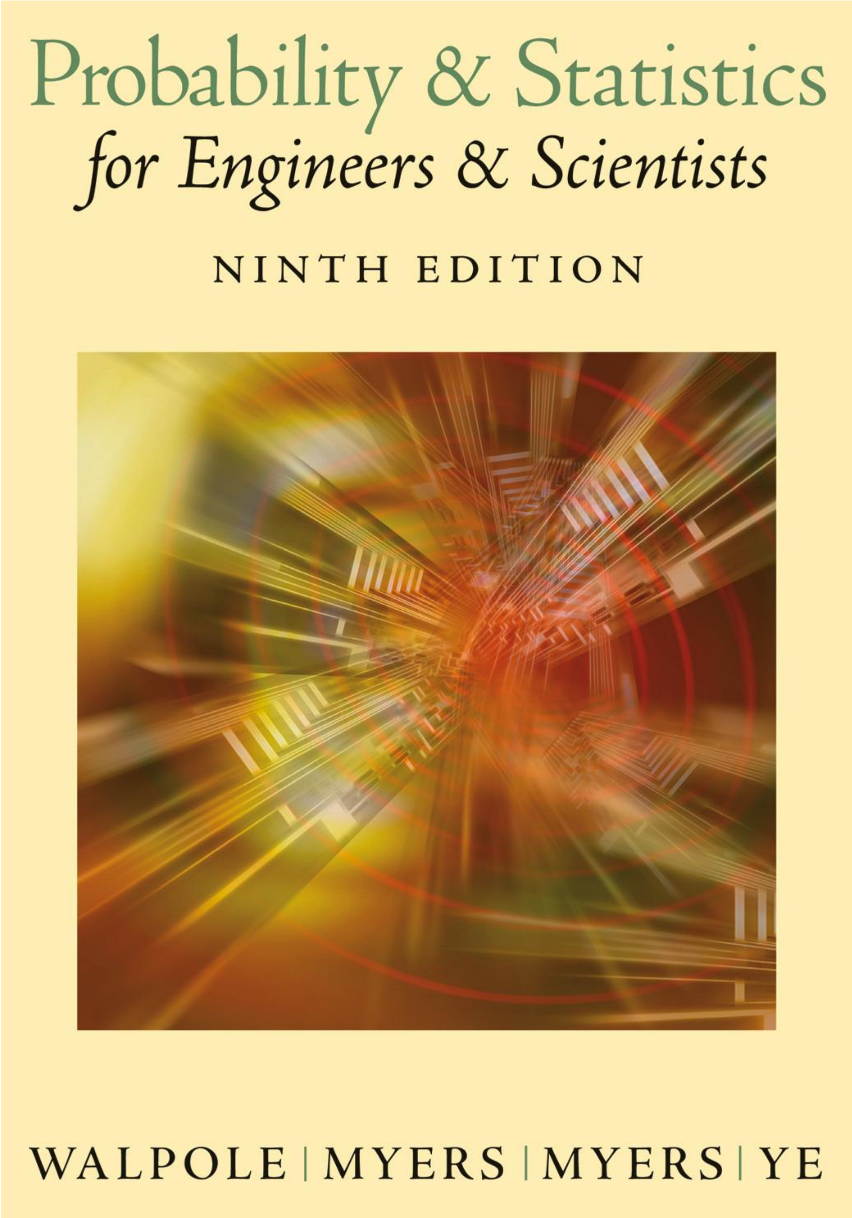
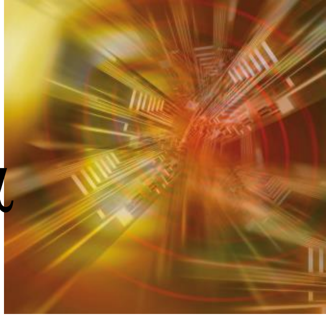


Figure 9.2 $P(-z_{\alpha/2} < Z < z_{\alpha/2}) = 1-\alpha$



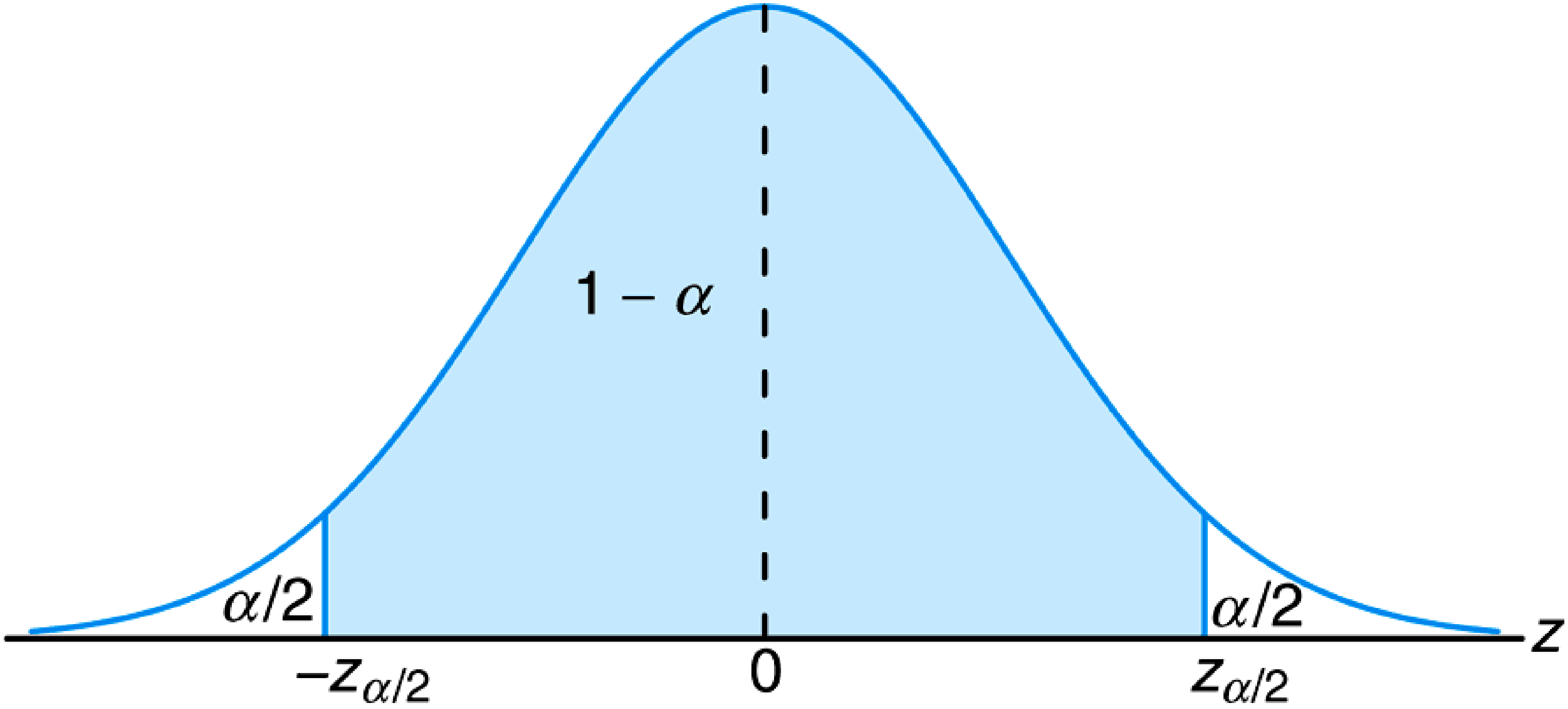


Figure 9.3 Interval estimates of μ for different samples

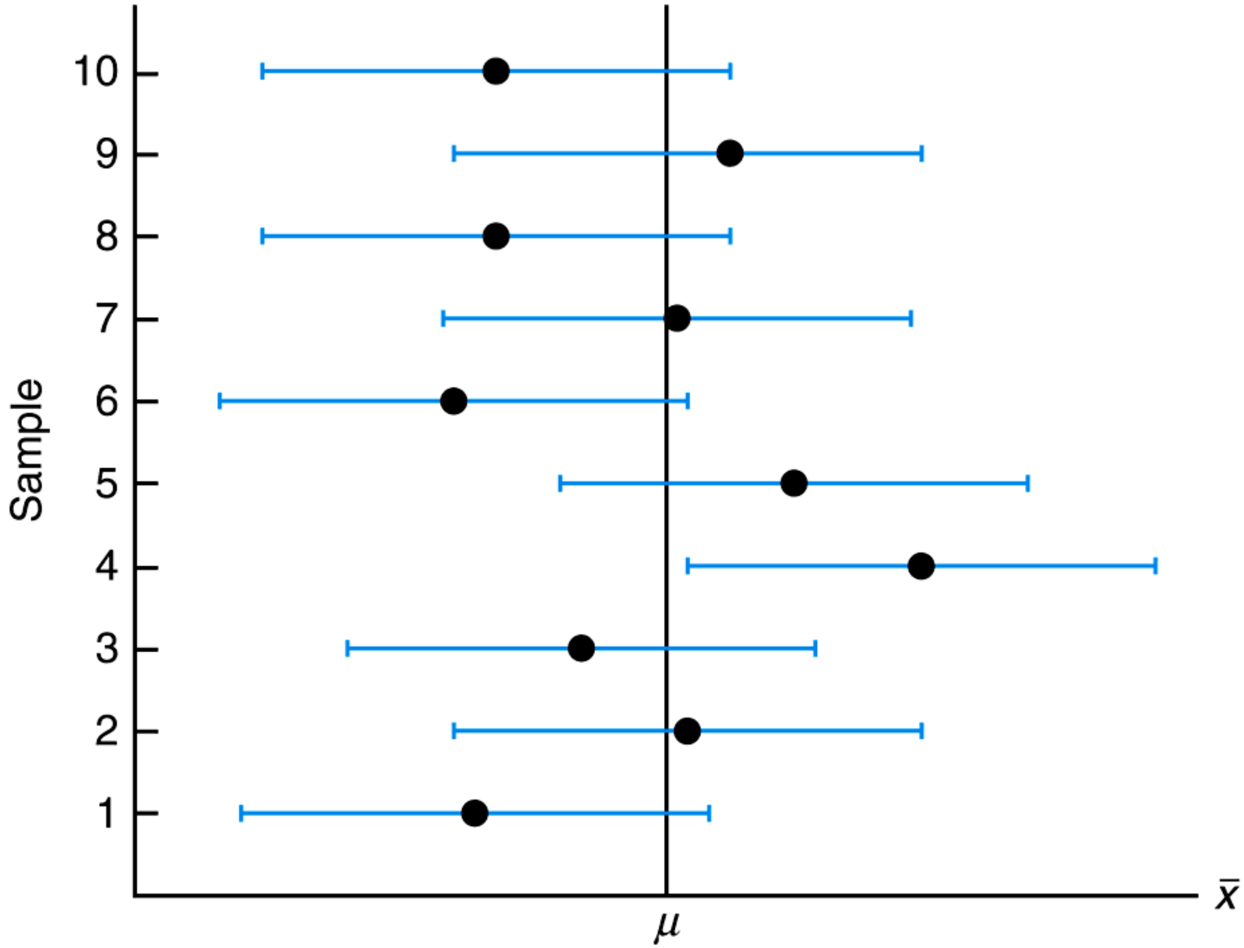
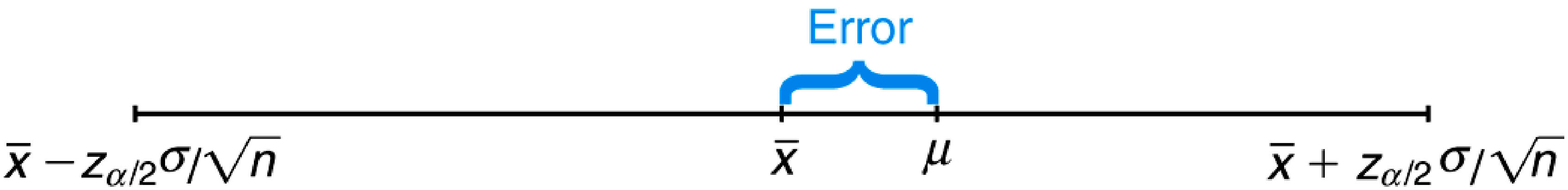


Figure 9.4 Error in estimating μ by x



Theorem 9.1

If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$.

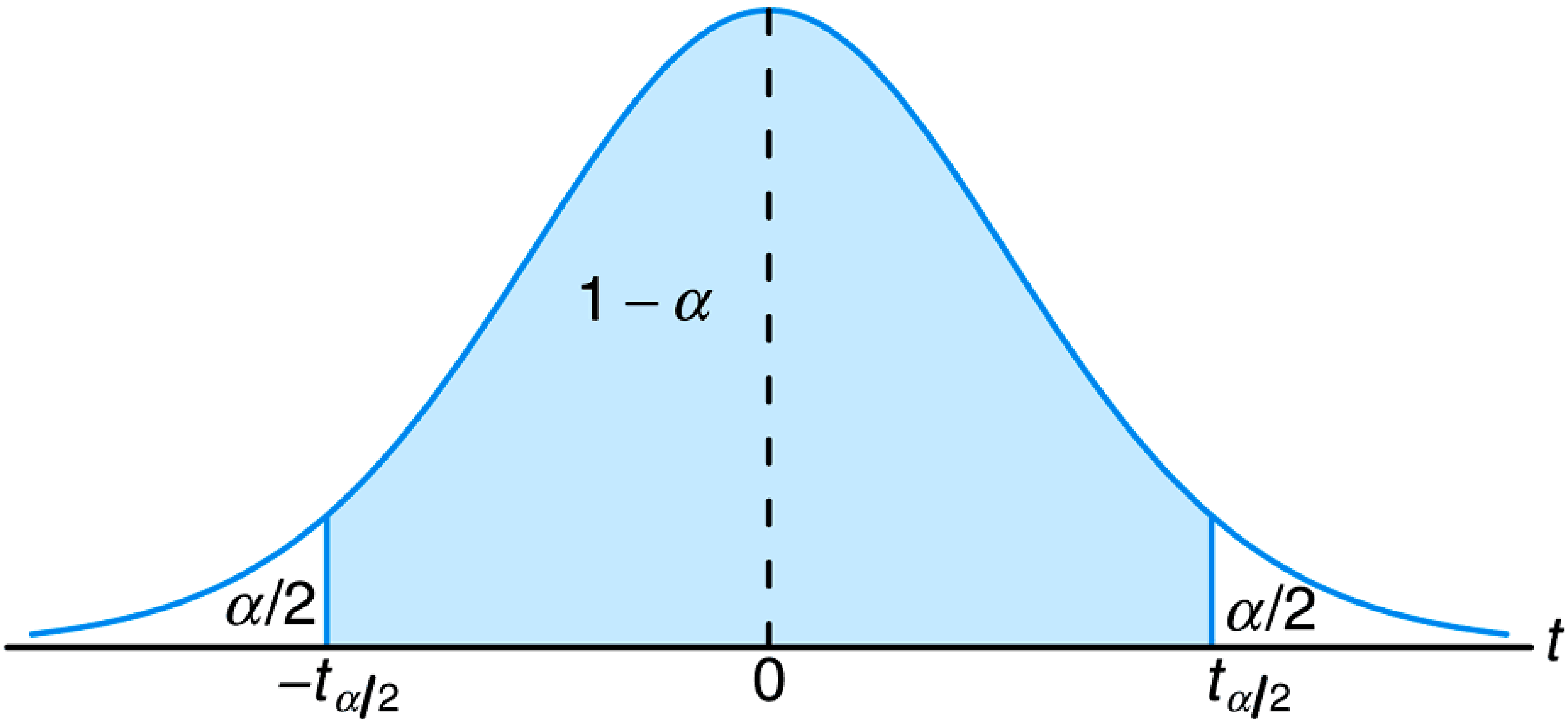
Theorem 9.2

If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error will not exceed a specified amount e when the sample size is

$$n = \left(\frac{z_{\alpha/2}\sigma}{e}\right)^2.$$

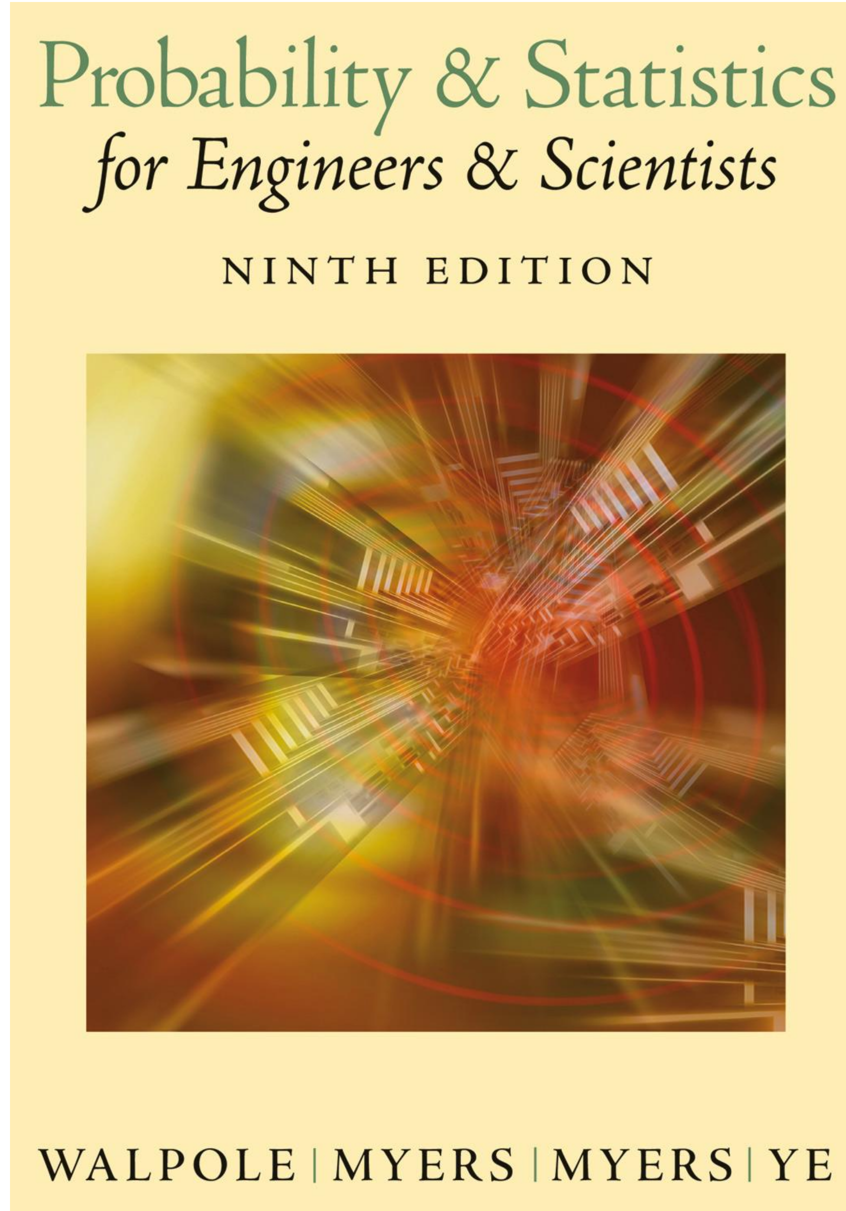
Figure 9.5

$$P(-t_{\alpha/2} < T < t_{\alpha/2}) = 1-\alpha$$



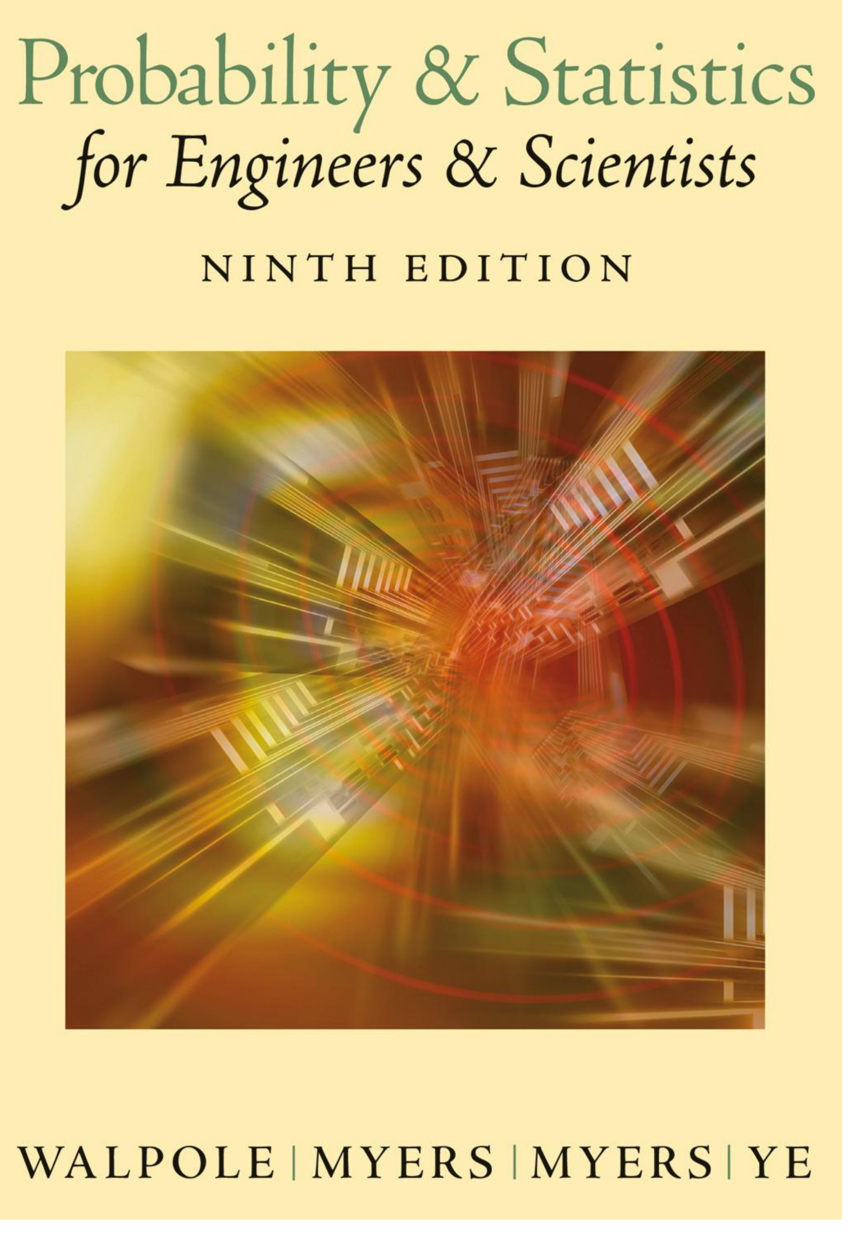
Section 9.5

Standard Error of a Point Estimate



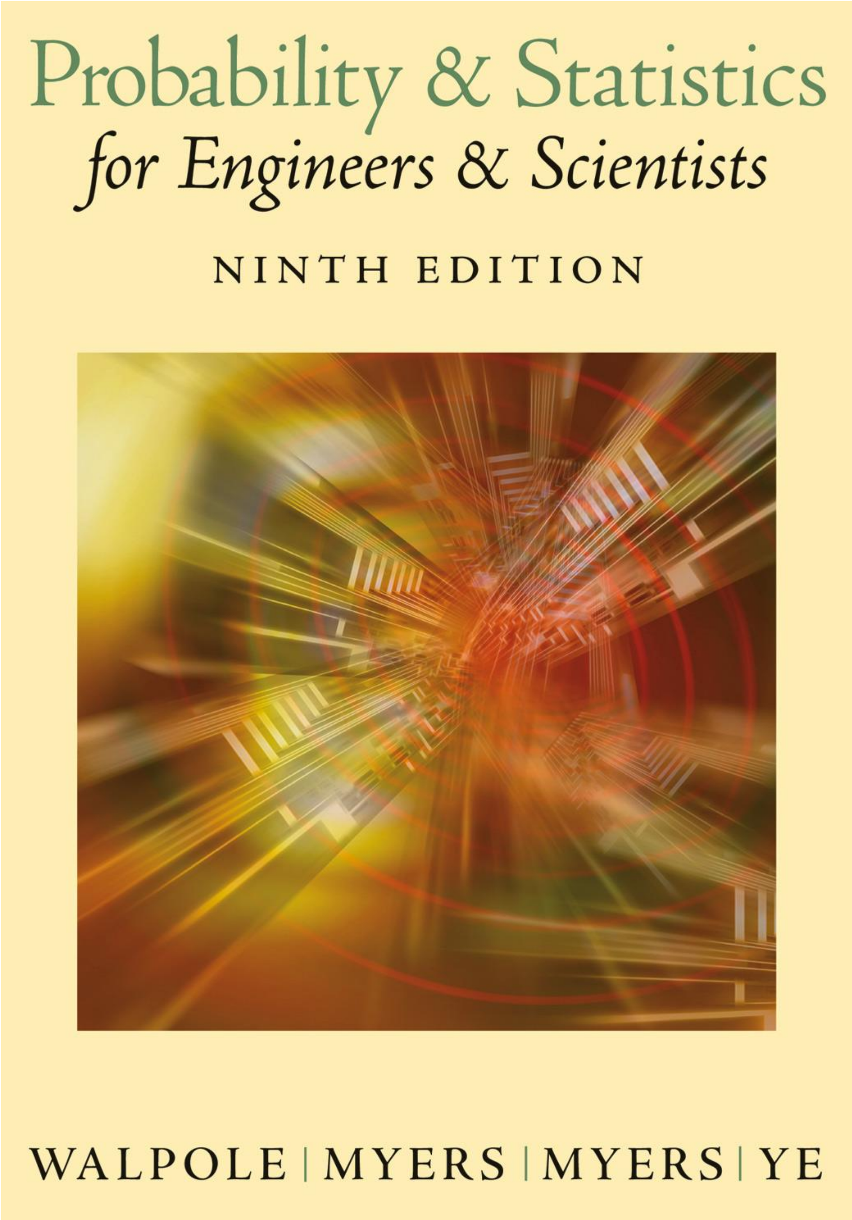
Section 9.6

Prediction Intervals



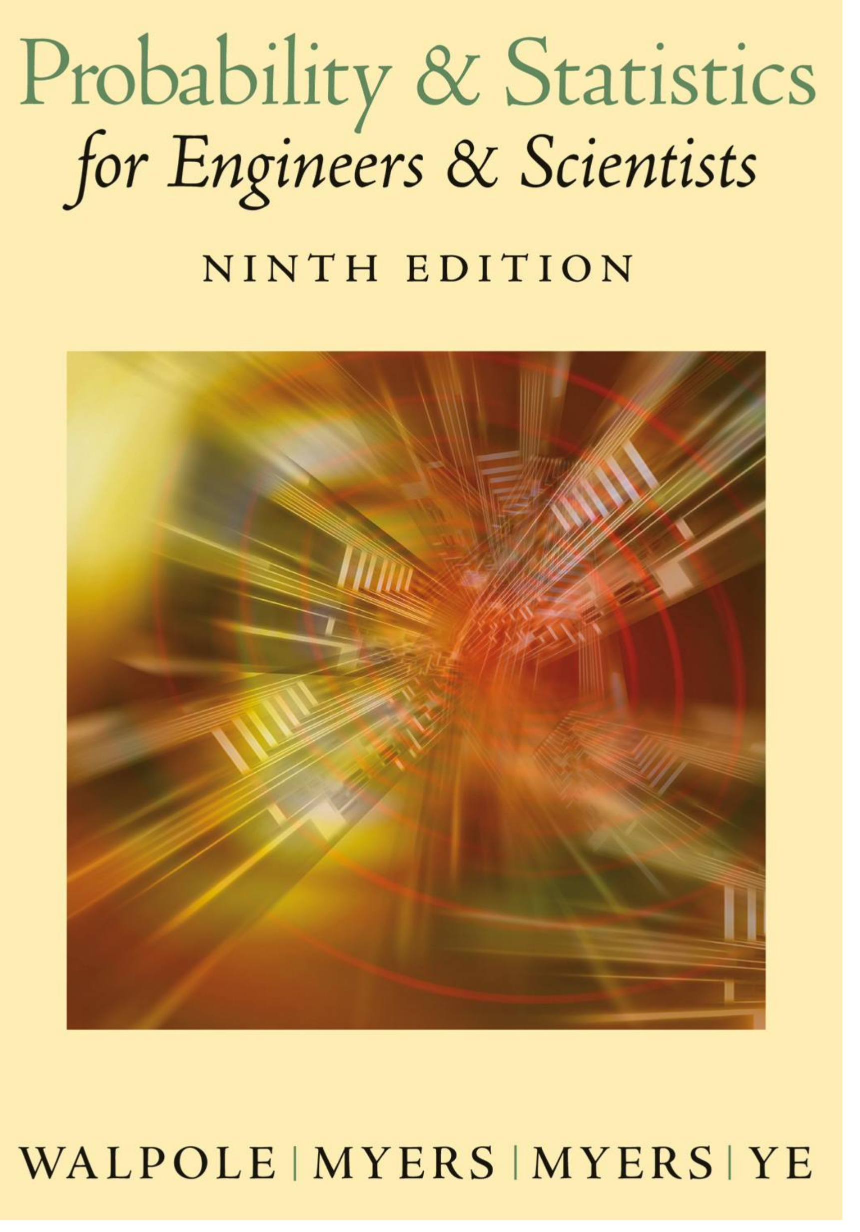
Section 9.7

Tolerance Limits



Section 9.8

Two Samples: Estimating the Difference between Two Means



Section 9.9

Paired Observations

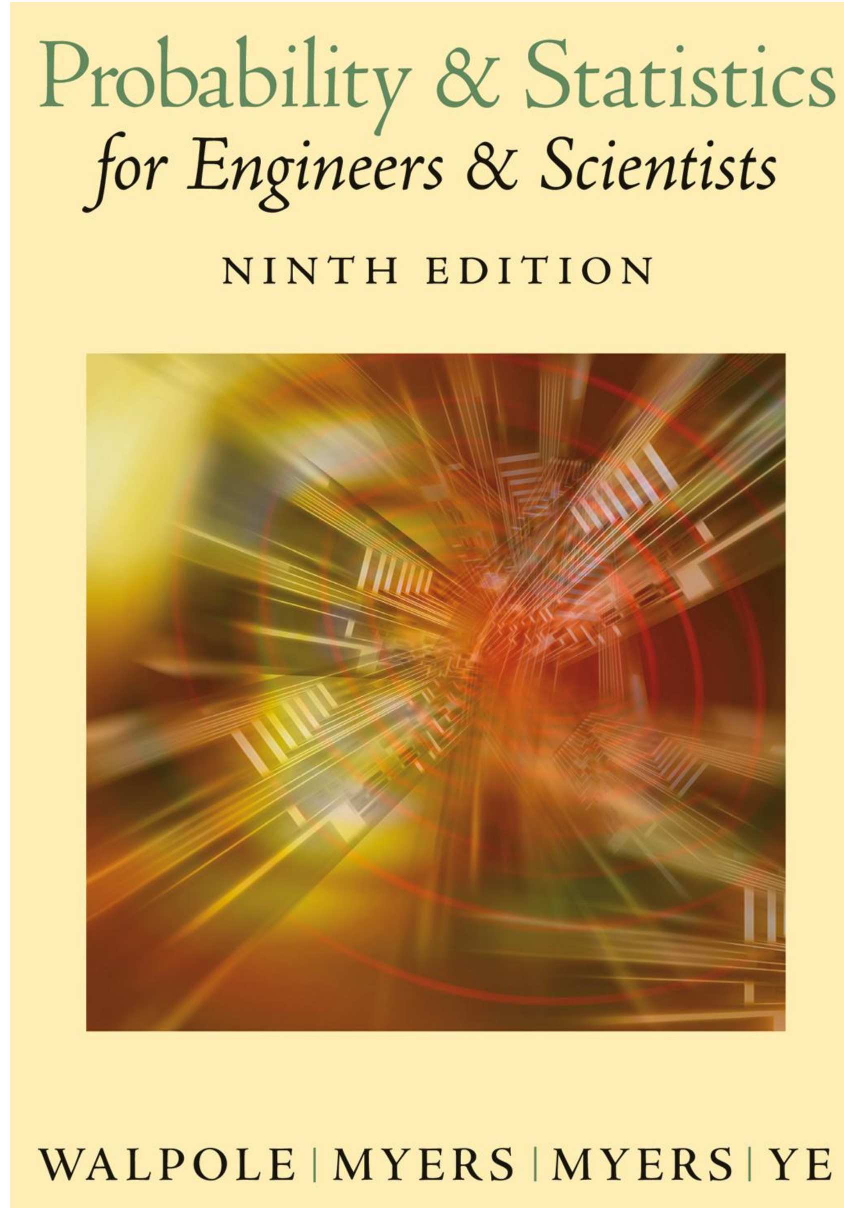


Table 9.1 Data for Example 9.13

Veteran	TCDD Levels in Plasma	TCDD Levels in Fat Tissue	d_i	Veteran	TCDD Levels in Plasma	TCDD Levels in Fat Tissue	d_i
1	2.5	4.9	−2.4	11	6.9	7.0	−0.1
2	3.1	5.9	−2.8	12	3.3	2.9	0.4
3	2.1	4.4	−2.3	13	4.6	4.6	0.0
4	3.5	6.9	−3.4	14	1.6	1.4	0.2
5	3.1	7.0	−3.9	15	7.2	7.7	−0.5
6	1.8	4.2	−2.4	16	1.8	1.1	0.7
7	6.0	10.0	−4.0	17	20.0	11.0	9.0
8	3.0	5.5	−2.5	18	2.0	2.5	−0.5
9	36.0	41.0	−5.0	19	2.5	2.3	0.2
10	4.7	4.4	0.3	20	4.1	2.5	1.6

Source: Schecter, A. et al. "Partitioning of 2,3,7,8-chlorinated dibenzo-*p*-dioxins and dibenzofurans between adipose tissue and plasma lipid of 20 Massachusetts Vietnam veterans," *Chemosphere*, Vol. 20, Nos. 7–9, 1990, pp. 954–955 (Tables I and II).

Section 9.10

Single Sample: Estimating a Proportion

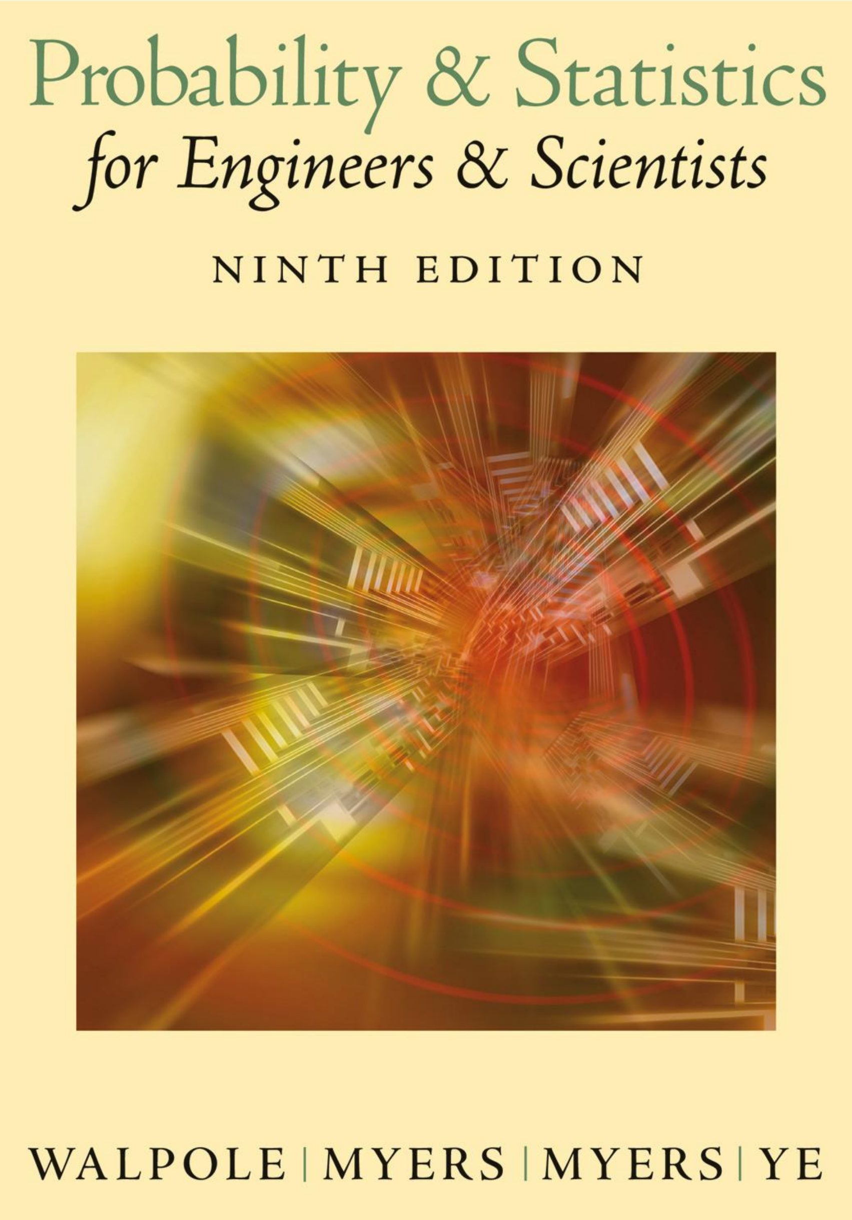
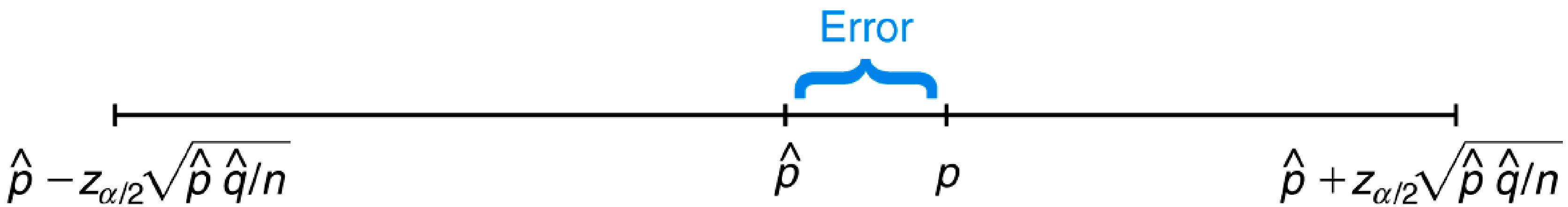


Figure 9.6 Error in estimating p by $\hat{p}$



Theorem 9.3

If $\hat{p}$ is used as an estimate of p, we can be $100(1-\alpha)\%$ confident that the error will not exceed $z_{\alpha/2}\sqrt{\hat{p}\hat{q}/n}$.

Theorem 9.4

If $\hat{p}$ is used as an estimate of p, we can be $100(1-\alpha)\%$ confident that the error will be less than a specified amount e when the sample size is approximately

$$n = \frac{z_{\alpha/2}^2 \hat{p}\hat{q}}{e^2}.$$

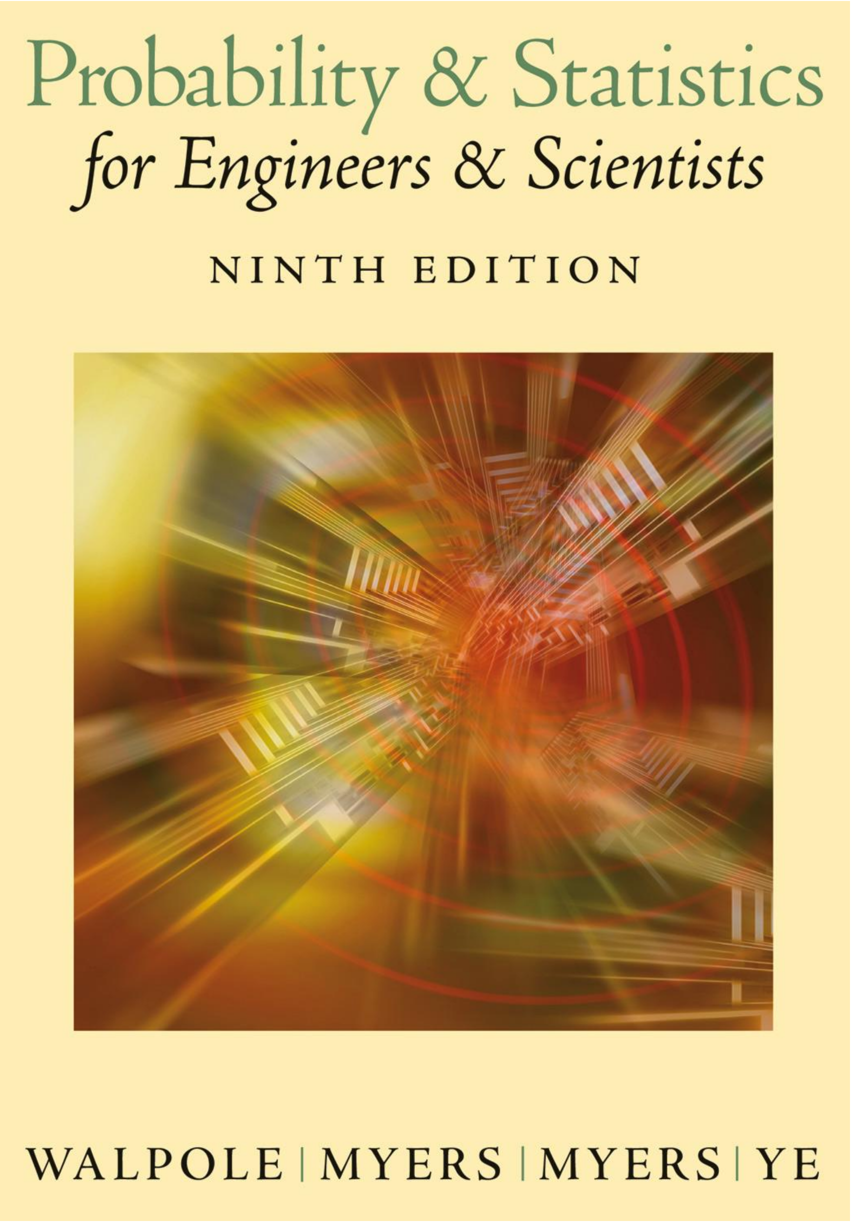
Theorem 9.5

If $\hat{p}$ is used as an estimate of p, we can be **at least** $100(1-\alpha)\%$ confident that the error will not exceed a specified amount e when the sample size is

$$n = \frac{z_{\alpha/2}^2}{4e^2}.$$

Section 9.11

Two Samples: Estimating the Difference between Two Proportions



Section 9.12

Single Sample: Estimating the Variance

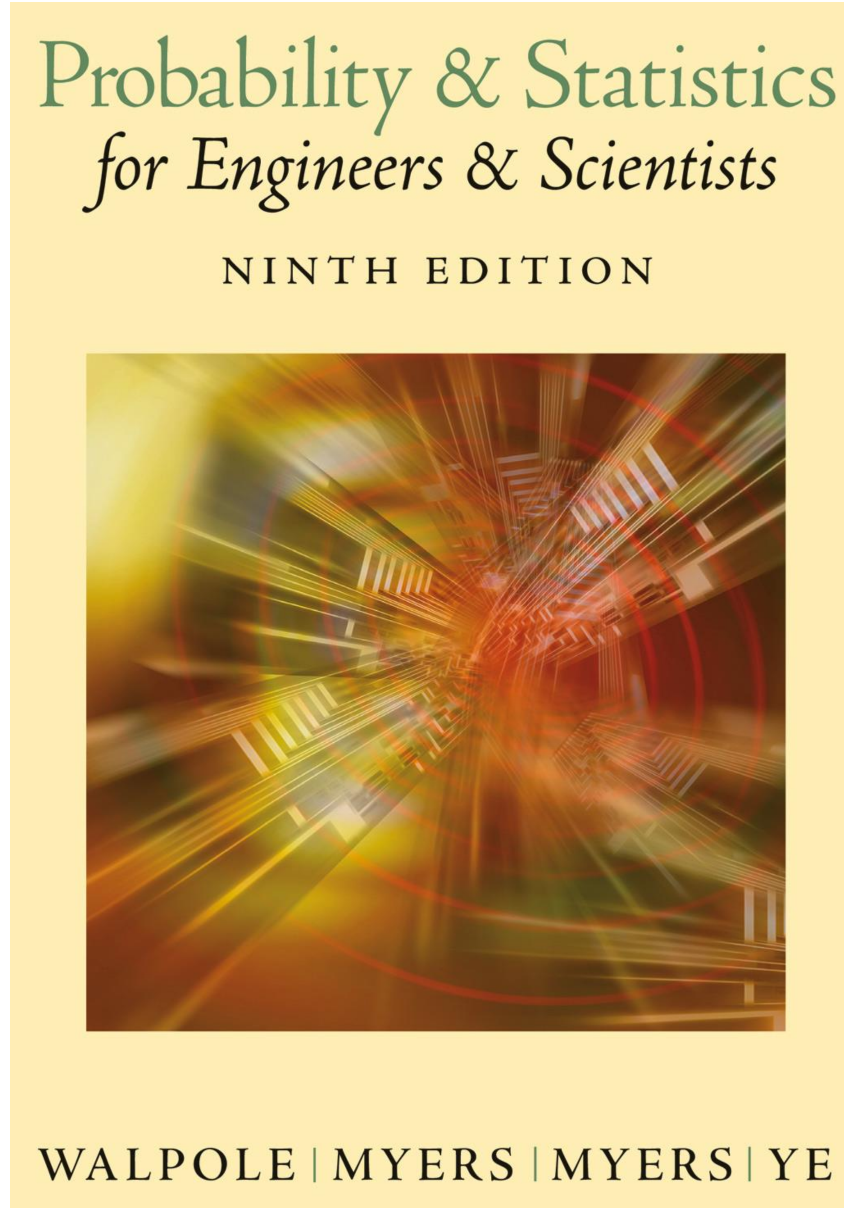
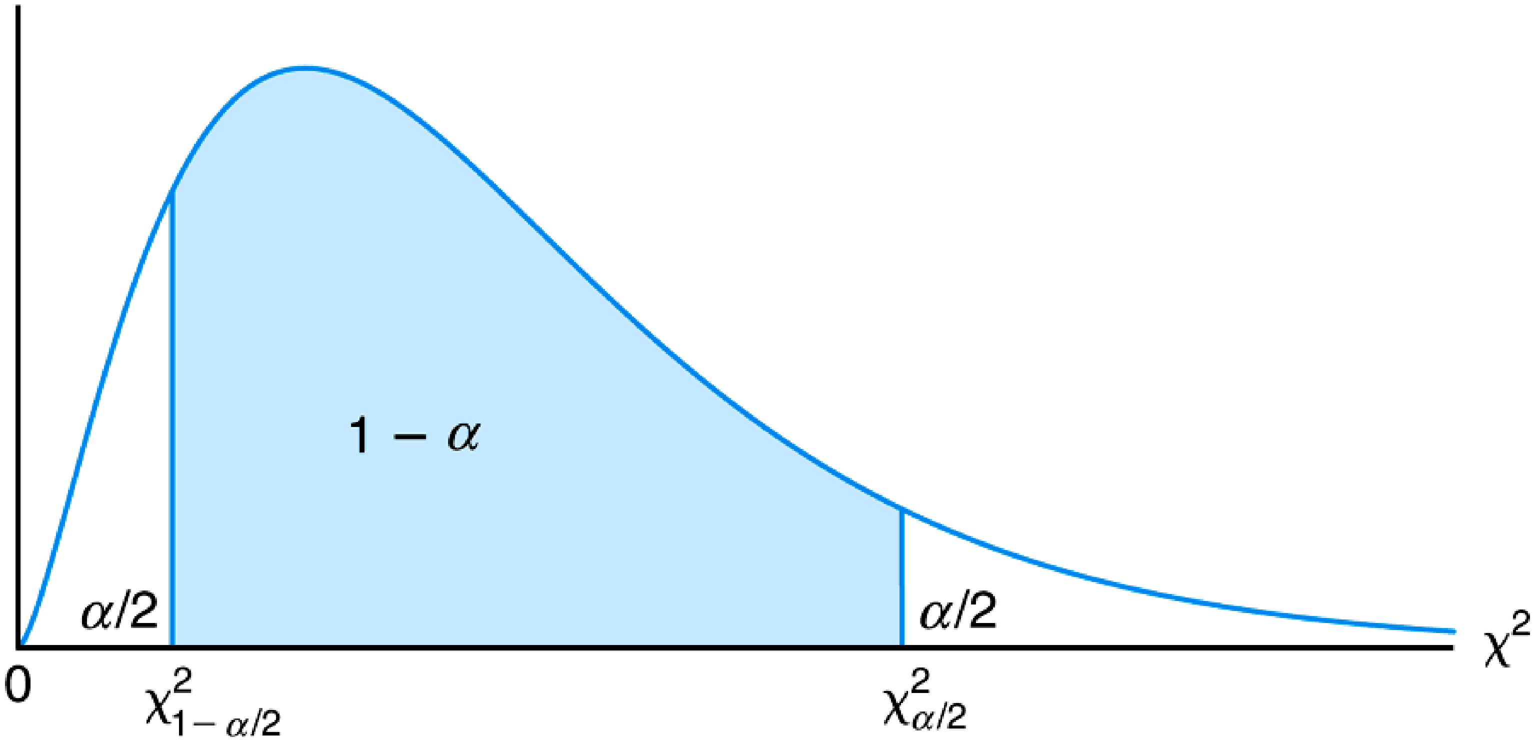


Figure 9.7

$$P(\chi^2_{1-\alpha/2} < X^2 < \chi^2_{\alpha/2}) = 1 - \alpha$$



Section 9.13

Two Samples: Estimating the Ratio of Two Variances

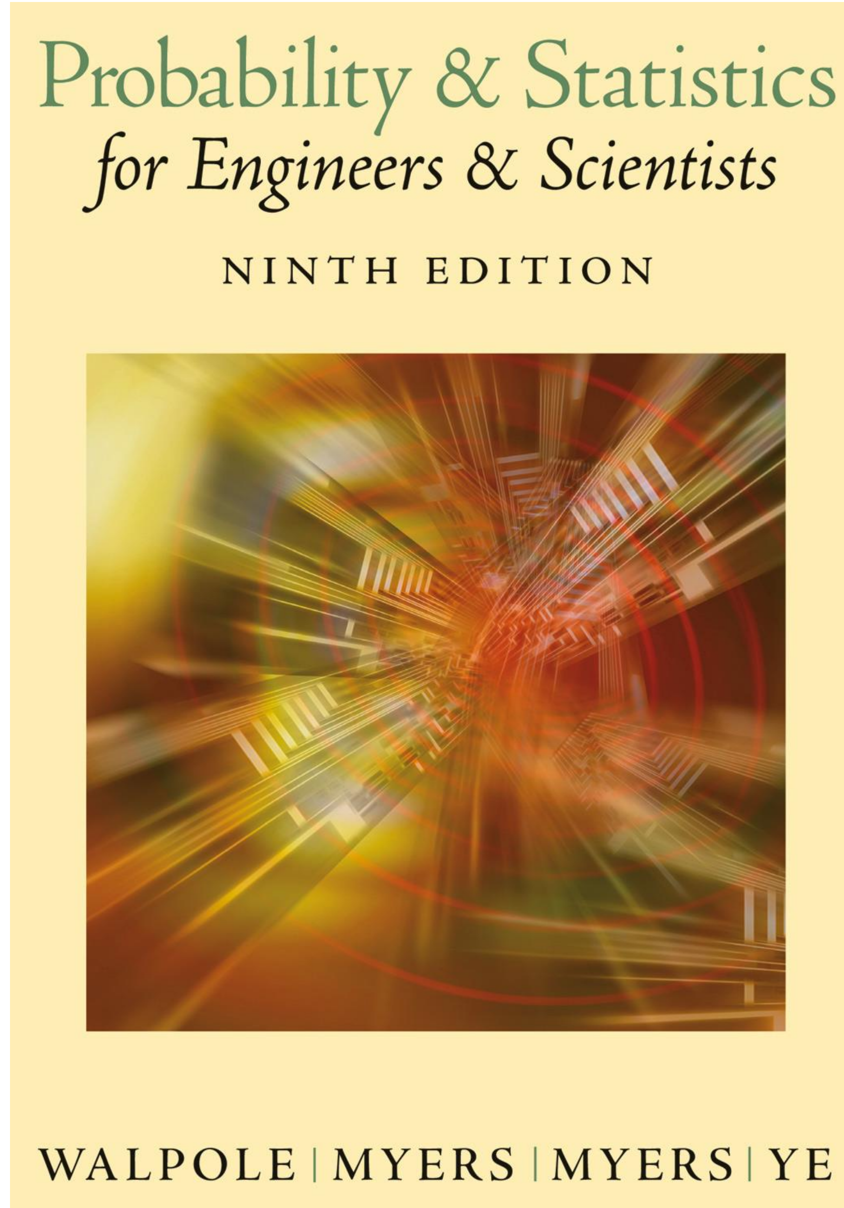
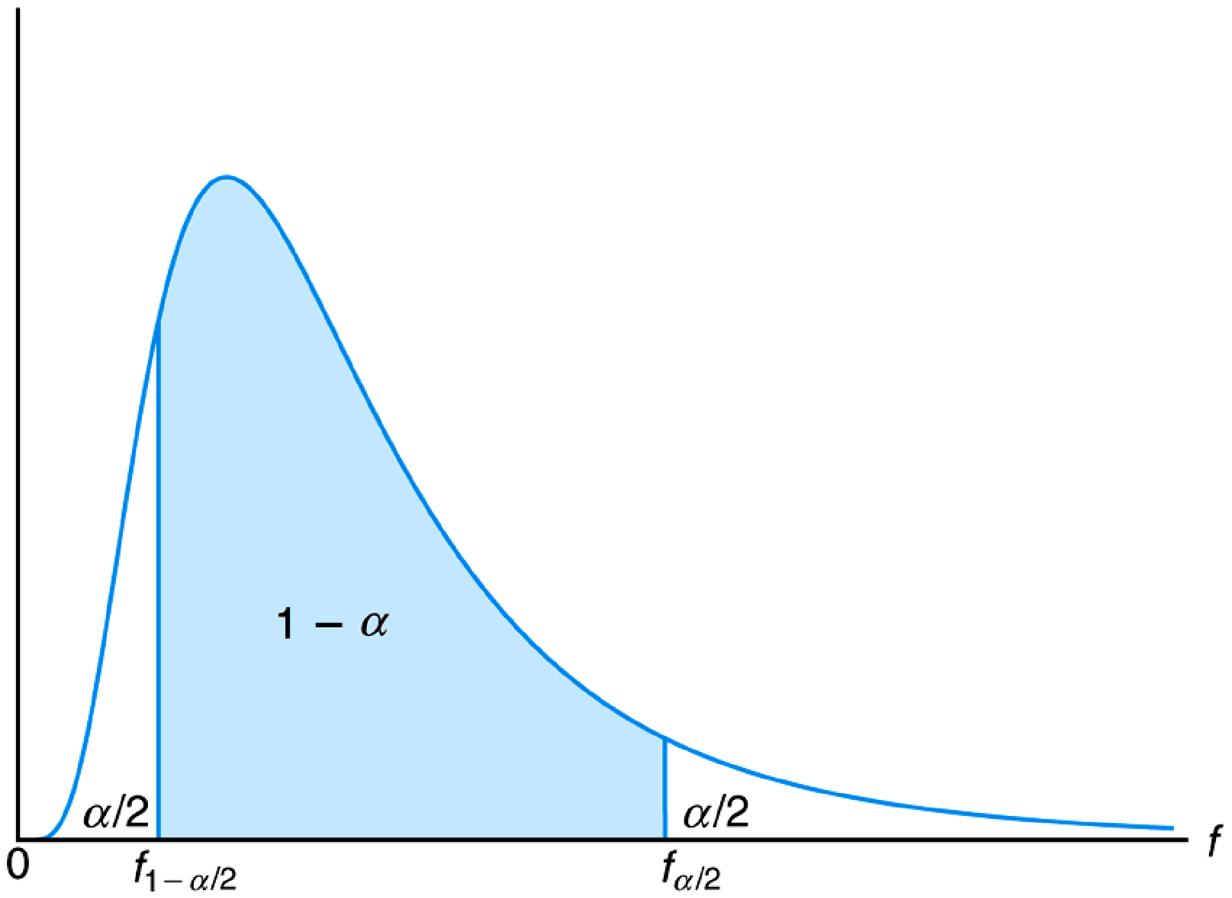


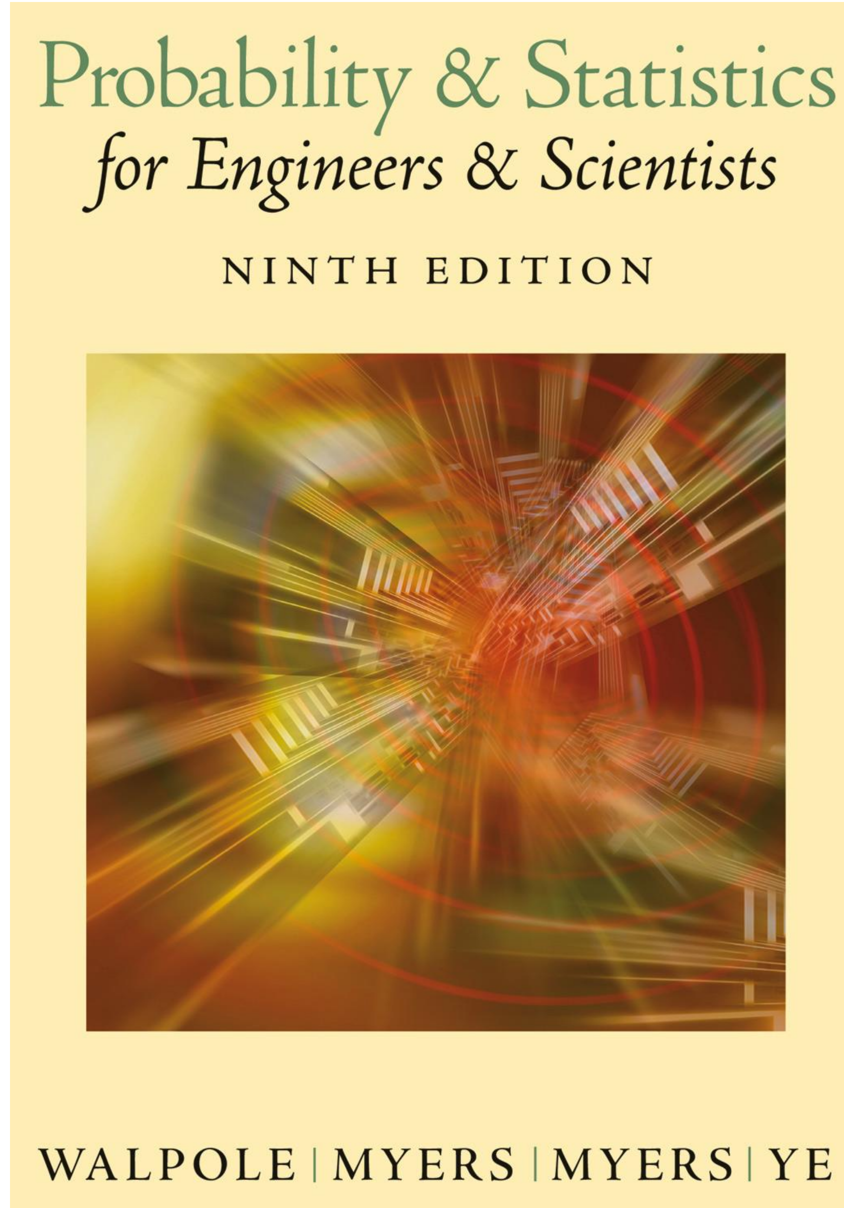
Figure 9.8

$$P[f_{1-\alpha/2}(v_1, v_2) < F < f_{\alpha/2}(v_1, v_2)] = 1 - \alpha$$



Section 9.14

Maximum Likelihood Estimation (Optional)



Definition 9.3

Given independent observations $x_1, x_2, \ldots, x_n$ from a probability density function (continuous case) or probability mass function (discrete case) $f(\mathbf{x}; \theta)$, the maximum likelihood estimator $\hat{\theta}$ is that which maximizes the likelihood function

$$L(x_1, x_2, \ldots, x_n; \theta) = f(\mathbf{x}; \theta) = f(x_1, \theta) f(x_2, \theta) \cdots f(x_n, \theta).$$

Section 9.15

Potential Misconceptions and Hazards; Relationship to Material in Other Chapters