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# Chapter 5

## MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

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Prof. Dr. Ali PINARBAŞI

Yildiz Technical University  
Mechanical Engineering Department  
Yildiz, ISTANBUL

# MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

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# Objectives

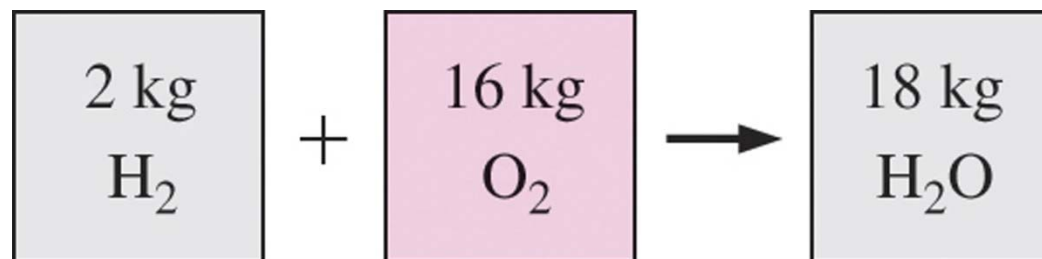
- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.
- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy.
- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers.
- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes.

# CONSERVATION OF MASS

**Conservation of mass:** Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

**Closed systems:** The mass of the system remain constant during a process.

**Control volumes:** Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.



Mass is conserved even during chemical reactions.

Mass  $m$  and energy  $E$  can be converted to each other:

$$E = mc^2$$

$c$  is the speed of light in a vacuum,  $c = 2.9979 \times 10^8$  m/s  
The mass change due to energy change is negligible.



# Conservation of Mass

The conservation of mass relation for a closed system undergoing a change is expressed as  $m_{\text{sys}} = \text{Const.}$  or  $dm_{\text{sys}}/dt = 0$ , which is the statement that the mass of the system remains constant during a process.

Mass balance for a control volume (CV) in rate form:

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt}$$

$\dot{m}_{\text{in}}$  and  $\dot{m}_{\text{out}}$

the total rates of mass flow into and out of the control volume

$dm_{\text{CV}}/dt$

the rate of change of mass within the control volume boundaries.

**Continuity equation:** In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the **continuity equation**.

# Conservation of momentum

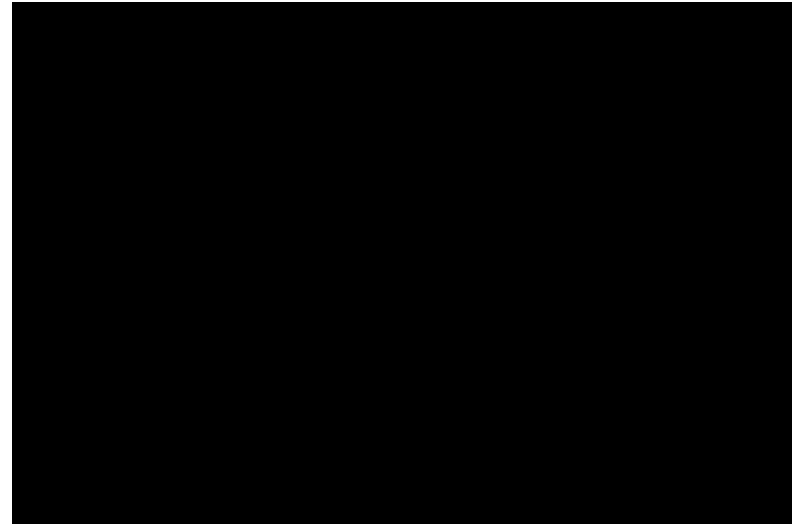
**Linear momentum:** The product of the mass and the velocity of a body is called the *linear momentum* or just the *momentum* of the body.

The momentum of a rigid body of mass  $m$  moving with a velocity  $V$  is  $mV$ .

**Newton's second law:** The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.



## Conservation of Momentum



**Conservation of momentum principle:** The momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such systems is conserved.

**Linear momentum equation:** In fluid mechanics, Newton's second law is usually referred to as the *linear momentum equation*.

# Moving or Deforming Control Volumes

The conservation of mass equation are also valid for moving or deforming control volumes provided that;

$$\frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA = 0$$

$$\frac{d}{dt} \int_{CV} \rho dV + \sum_{\text{out}} \int_A \rho V_n dA - \sum_{\text{in}} \int_A \rho V_n dA = 0$$

The **absolute velocity**  $V$  is replaced by the **relative velocity**  $V_r$ , which is the fluid velocity relative to the control surface. In the case of a nondeforming control volume, relative velocity is the fluid velocity observed by a person moving with the control volume and is expressed as;

$$\vec{V}_r = \vec{V} - \vec{V}_{CV}$$

Some practical problems (such as the injection of medication through the needle of a syringe by the forced motion of the plunger) involve deforming control volumes. The conservation of mass relations developed can still be used for such deforming control volumes provided that the velocity of the fluid crossing a deforming part of the control surface is expressed relative to the control surface (that is, the fluid velocity should be expressed relative to a reference frame attached to the deforming part of the control surface).

# MASS AND VOLUME FLOW RATES

**Mass flow rate:** The amount of mass flowing through a cross section per unit time.

The differential mass flow rate

$$\delta \dot{m} = \rho V_n dA_c$$

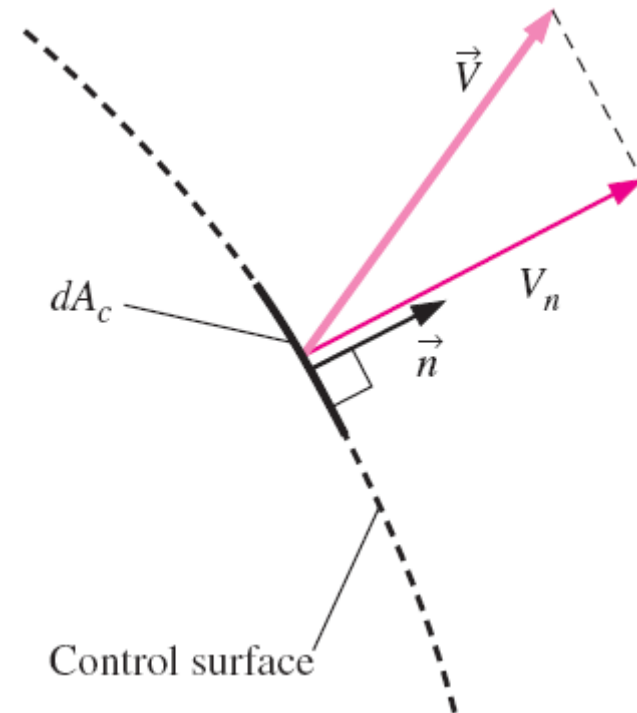
Point functions have *exact differentials*

$$\int_1^2 dA_c = A_{c2} - A_{c1} = \pi(r_2^2 - r_1^2)$$

Path functions have *inexact differentials*

$$\int_1^2 \delta \dot{m} = \dot{m}_{\text{total}}$$

$$\text{not } \dot{m}_2 - \dot{m}_1$$



The normal velocity  $V_n$  for a surface is the component of velocity perpendicular to the surface.

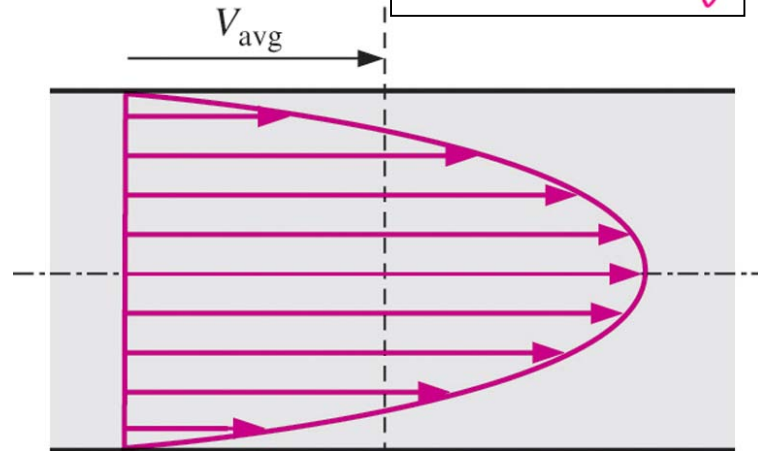
$$\delta \dot{m} = \rho V_n dA_c$$

$$\dot{m} = \int_{A_c} \delta \dot{m} = \int_{A_c} \rho V_n dA_c \quad (\text{kg/s})$$

### Mass flow rate

$$\dot{m} = \rho V_{\text{avg}} A_c \quad (\text{kg/s})$$

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$



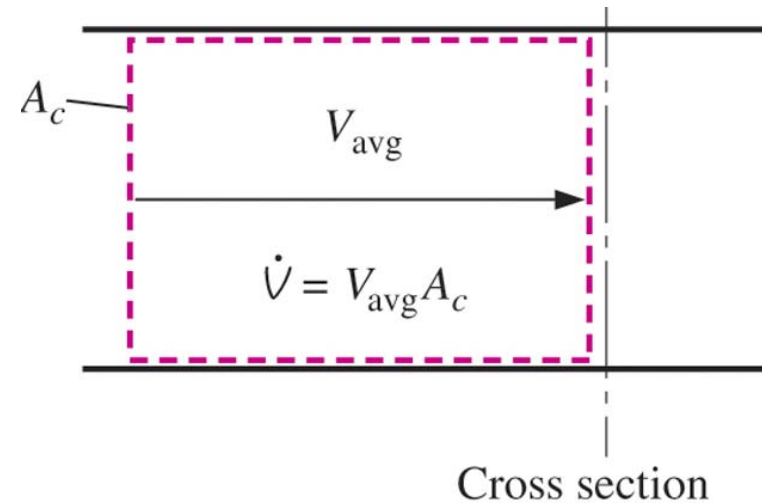
The average velocity  $V_{\text{avg}}$  is defined as the average speed through a cross section.

### Average velocity

$$V_{\text{avg}} = \frac{1}{A_c} \int_{A_c} V_n dA_c$$

### Volume flow rate

$$\dot{V} = \int_{A_c} V_n dA_c = V_{\text{avg}} A_c = V A_c \quad (\text{m}^3/\text{s})$$



The volume flow rate is the volume of fluid flowing through a cross section per unit time.

# CONSERVATION OF MASS PRINCIPLE

**The conservation of mass principle for a control volume:** The net mass transfer to or from a control volume during a time interval  $\Delta t$  is equal to the net change (increase or decrease) in the total mass within the control volume during  $\Delta t$ .

$$\left( \text{Total mass entering the CV during } \Delta t \right) - \left( \text{Total mass leaving the CV during } \Delta t \right) = \left( \text{Net change in mass within the CV during } \Delta t \right)$$



$$m_{in} - m_{out} = \Delta m_{CV} \quad (\text{kg})$$

$$\dot{m}_{in} - \dot{m}_{out} = dm_{CV}/dt \quad (\text{kg/s})$$

$\dot{m}_{in}$  and  $\dot{m}_{out}$

the total rates of mass flow into and out of the control volume

$dm_{CV}/dt$

the rate of change of mass within the control volume boundaries.

Conservation of mass principle for an ordinary bathtub.

Mass balance is applicable to any control volume undergoing any kind of process.

$$dm = \rho dV$$

Total mass within the CV:

$$m_{CV} = \int_{CV} \rho dV$$

Rate of change of mass within the CV:

$$\frac{dm_{CV}}{dt} = \frac{d}{dt} \int_{CV} \rho dV$$

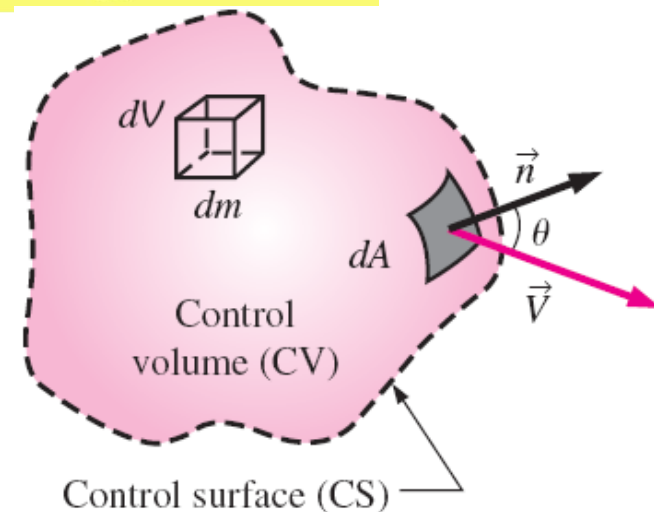
Normal component of velocity:

$$V_n = V \cos \theta = \vec{V} \cdot \vec{n}$$

Differential mass flow rate:  $\delta \dot{m} = \rho V_n dA = \rho (V \cos \theta) dA = \rho (\vec{V} \cdot \vec{n}) dA$

Net mass flow rate:  $\dot{m}_{net} = \int_{CS} \delta \dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho (\vec{V} \cdot \vec{n}) dA$

The differential control volume  $dV$  and the differential control surface  $dA$  used in the derivation of the conservation of mass relation.





# General conservation of mass

$$\frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho(\vec{V} \cdot \vec{n}) dA = 0$$

The time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

$$\frac{d}{dt} \int_{CV} \rho dV + \sum_{out} \int_A \rho V_n dA - \sum_{in} \int_A \rho V_n dA = 0$$

$$\frac{d}{dt} \int_{CV} \rho dV = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

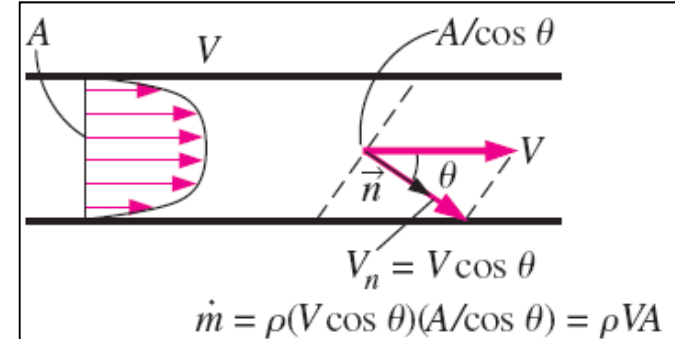
$$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m}$$

The conservation of mass equation is obtained by replacing  $B$  in the Reynolds transport theorem by mass  $m$ , and  $b$  by 1

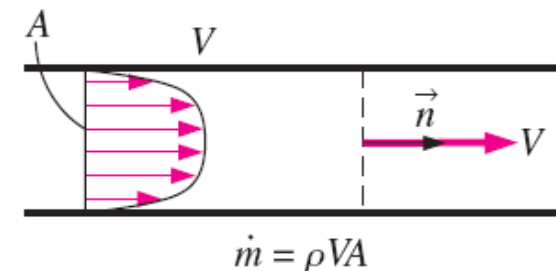
$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho b dV + \int_{CS} \rho b(\vec{V} \cdot \vec{n}) dA$$

$B = m$                        $b = 1$                        $b = 1$

$$\frac{dm_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho(\vec{V} \cdot \vec{n}) dA$$



(a) Control surface *at an angle* to flow



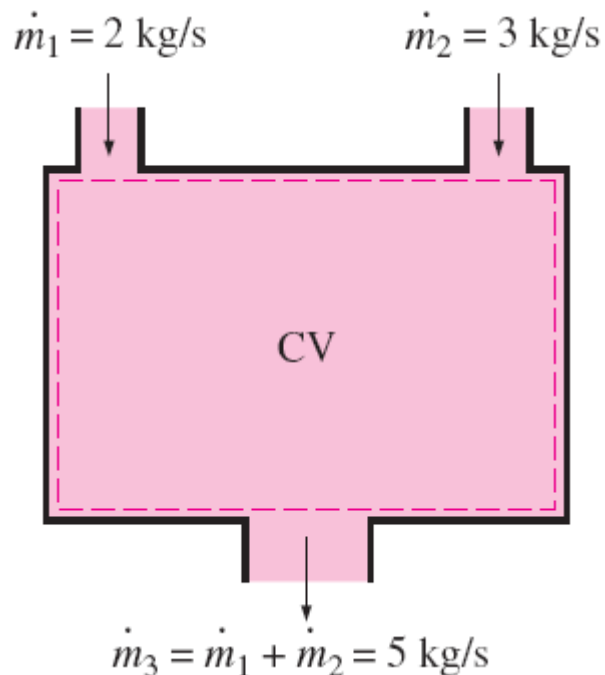
(b) Control surface *normal* to flow

A control surface should always be selected *normal to the flow* at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.

# MASS BALANCE FOR STEADY-FLOW PROCESSES

During a steady-flow process, the total amount of mass contained within a control volume does not change with time ( $m_{CV} = \text{constant}$ ).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate*.

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \quad (\text{kg/s})$$

Multiple inlets and exits

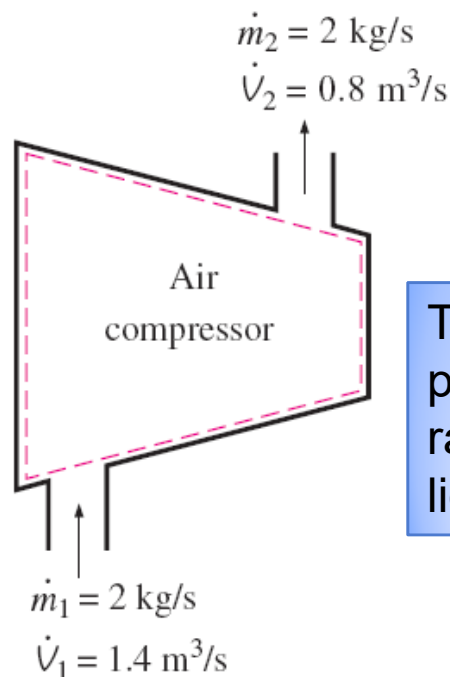
$$\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

## SPECIAL CASE: INCOMPRESSIBLE FLOW

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.



During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \quad (\text{m}^3/\text{s})$$

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible

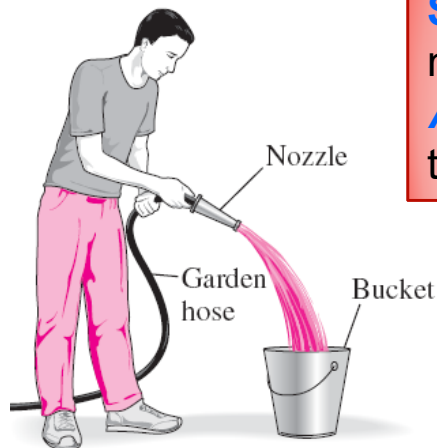
Steady, incompressible flow  
(single stream)

There is no such thing as a “**conservation of volume**” principle. However, for steady flow of liquids the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

The conservation of mass principle is based on experimental observations and requires every bit of mass to be accounted for during a process. If you can balance your checkbook (by keeping track of deposits and withdrawals, or by simply observing the “conservation of money” principle), you should have no difficulty applying the conservation of mass principle to engineering systems.

### EXAMPLE 5-1

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket with water, determine (a) the volume and mass flow rates of water through the hose, and (b) the average velocity of water at the nozzle exit.



**Solution** A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

**Assumptions** 1 Water is an incompressible substance. 2 Flow through the hose is steady. 3 There is no waste of water by splashing.

(a) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10 \text{ gal}}{50 \text{ s}} \left( \frac{3.7854 \text{ L}}{1 \text{ gal}} \right) = \mathbf{0.757 \text{ L/s}}$$
$$\dot{m} = \rho \dot{V} = (1 \text{ kg/L})(0.757 \text{ L/s}) = \mathbf{0.757 \text{ kg/s}}$$

(b) The cross-sectional area of the nozzle exit is

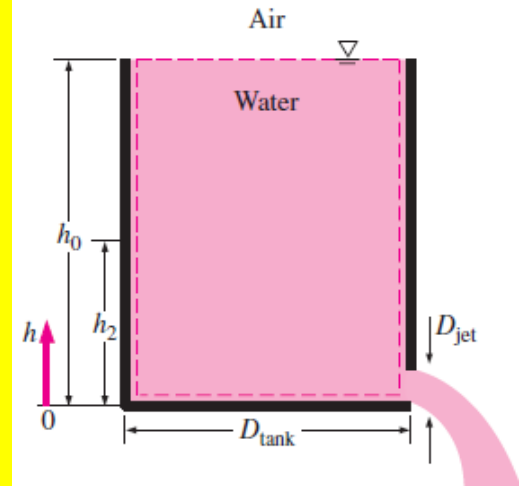
$$A_e = \pi r_e^2 = \pi (0.4 \text{ cm})^2 = 0.5027 \text{ cm}^2 = 0.5027 \times 10^{-4} \text{ m}^2$$

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757 \text{ L/s}}{0.5027 \times 10^{-4} \text{ m}^2} \left( \frac{1 \text{ m}^3}{1000 \text{ L}} \right) = \mathbf{15.1 \text{ m/s}}$$

**Discussion** It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.

## EXAMPLE 5-2

A 1.2 m-high, 0.9-m-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 1.3 m streams out. The average velocity of the jet is given by  $V = \sqrt{2gh}$  where  $h$  is the height of water in the tank measured from the center of the hole (a variable) and  $g$  is the gravitational acceleration. Determine how long it will take for the water level in the tank to drop to 0.6 m from the bottom.



$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt}$$

$$\dot{m}_{\text{out}} = (\rho VA)_{\text{out}} = \rho \sqrt{2gh} A_{\text{jet}}$$

$$m_{\text{CV}} = \rho V = \rho A_{\text{tank}} h$$

$$-\rho \sqrt{2gh} A_{\text{jet}} = \frac{d(\rho A_{\text{tank}} h)}{dt} \rightarrow -\rho \sqrt{2gh} (\pi D_{\text{jet}}^2 / 4) = \frac{\rho (\pi D_{\text{tank}}^2 / 4) dh}{dt}$$

$$dt = \frac{D_{\text{tank}}^2}{D_{\text{jet}}^2} \frac{dh}{\sqrt{2gh}}$$

$$\int_0^t dt = - \frac{D_{\text{tank}}^2}{D_{\text{jet}}^2 \sqrt{2g}} \int_{h_0}^{h_2} \frac{dh}{\sqrt{h}} \rightarrow t = \frac{\sqrt{h_0} - \sqrt{h_2}}{\sqrt{g/2}} \left( \frac{D_{\text{tank}}}{D_{\text{jet}}} \right)^2$$

$$t = \frac{\sqrt{1.2 \text{ m}} - \sqrt{0.6 \text{ m}}}{\sqrt{9.807/2 \text{ m/s}^2}} \left( \frac{0.9 \text{ m}}{0.013 \text{ m}} \right)^2 = 694 \text{ s} = \mathbf{11.6 \text{ dak}}$$

**Discussion** Using the same relation with  $h_2 = 0$  gives  $t = 43.1$  min for the discharge of the entire amount of water in the tank. Therefore, emptying the bottom half of the tank takes much longer than emptying the top half. This is due to the decrease in the average discharge velocity of water with decreasing  $h$ .

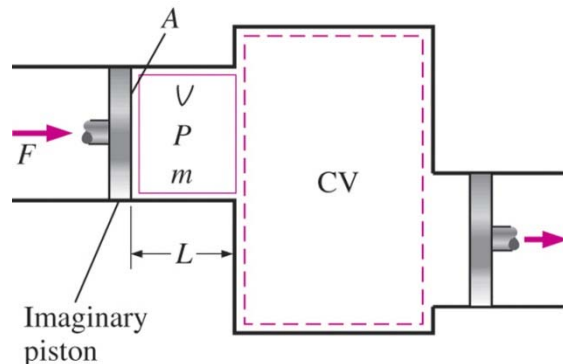
# FLOW WORK AND THE ENERGY OF A FLOWING FLUID

**Flow work, or flow energy:** The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

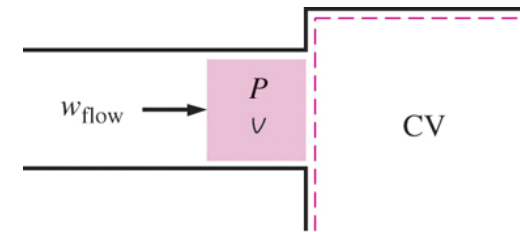
$$F = PA$$

$$W_{\text{flow}} = FL = PAL = P\mathcal{V} \quad (\text{kJ})$$

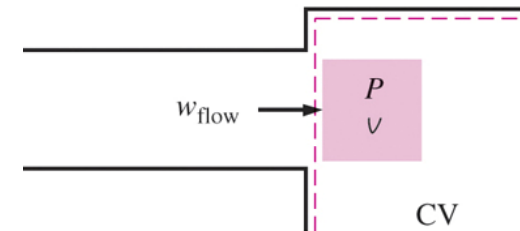
$$w_{\text{flow}} = P\mathcal{V} \quad (\text{kJ/kg})$$



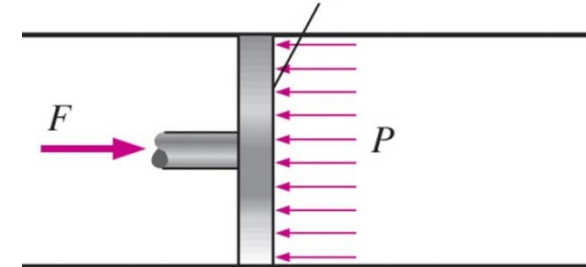
Schematic for flow work.



(a) Before entering



(b) After entering



In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

# TOTAL ENERGY OF A FLOWING FLUID

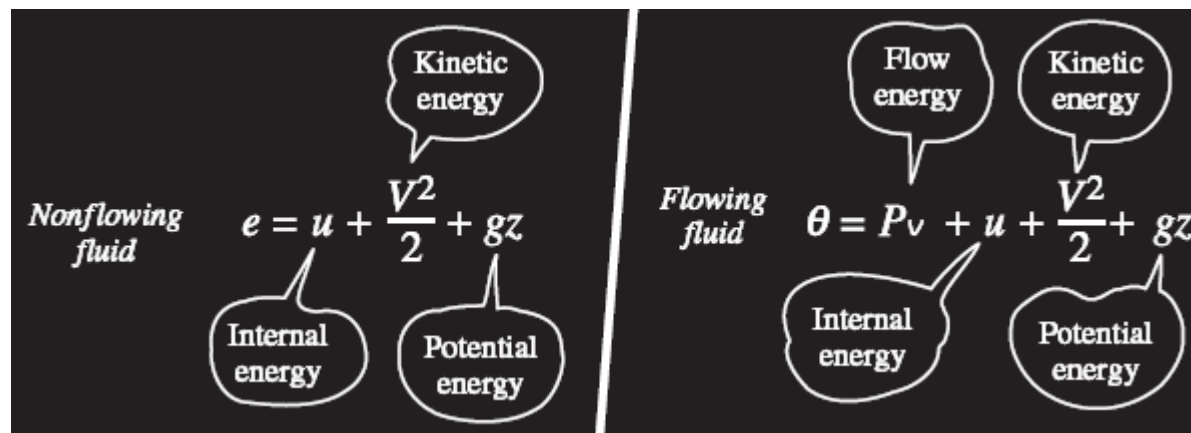
$$e = u + ke + pe = u + \frac{V^2}{2} + gz \quad (\text{kJ/kg})$$

$$\theta = Pv + e = Pv + (u + ke + pe)$$

$$h = u + Pv$$

$$\theta = h + ke + pe = h + \frac{V^2}{2} + gz \quad (\text{kJ/kg})$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.



The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.



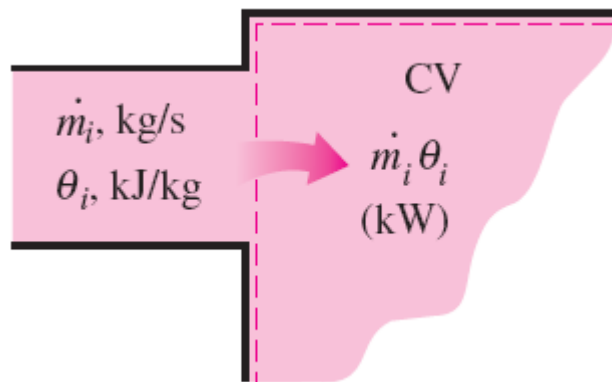
# ENERGY TRANSPORT BY MASS

Amount of energy transport:  $E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \quad (\text{kJ})$

Rate of energy transport:  $\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \quad (\text{kW})$

When the kinetic and potential energies of a fluid stream are negligible

$$E_{\text{mass}} = mh \quad \dot{E}_{\text{mass}} = \dot{m}h$$



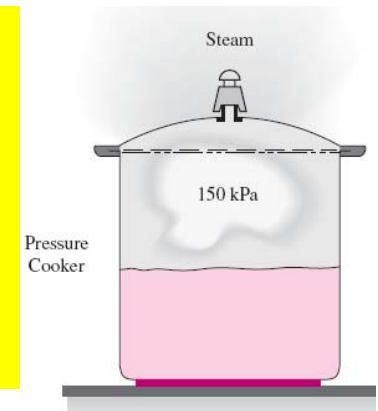
When the properties of the mass at each inlet or exit change with time as well as over the cross section

$$E_{\text{in, mass}} = \int_{m_i} \theta_i \delta m_i = \int_{m_i} \left( h_i + \frac{V_i^2}{2} + gz_i \right) \delta m_i$$

The product  $\dot{m}_i \theta_i$  is the energy transported into control volume by mass per unit time.

### EXAMPLE 5-3

Steam is leaving a 4-L pressure cooker whose operating pressure is 150 kPa. It is observed that the amount of liquid in the cooker has decreased by 0.6 L in 40 min after the steady operating conditions are established, and the cross-sectional area of the exit opening is 8 mm<sup>2</sup>. Determine (a) the mass flow rate of the steam and the exit velocity, (b) the total and flow energies of the steam per unit mass, and (c) the rate at which energy is leaving the cooker by steam.



**Solution** Steam is leaving a pressure cooker at a specified pressure. The velocity, flow rate, the total and flow energies, and the rate of energy transfer by mass are to be determined.

**Assumptions** 1 The flow is steady, and the initial start-up period is disregarded. 2 The kinetic and potential energies are negligible and thus they are not considered. 3 Saturation conditions exist within the cooker at all times so that steam leaves the cooker as a saturated vapor at the cooker pressure.

(a) the mass flow rate of the steam and the exit velocity,

$$m = \frac{\Delta V_{\text{liquid}}}{v_f} = \frac{0.6 \text{ L}}{0.001053 \text{ m}^3/\text{kg}} \left( \frac{1 \text{ m}^3}{1000 \text{ L}} \right) = 0.570 \text{ kg}$$
$$\dot{m} = \frac{m}{\Delta t} = \frac{0.570 \text{ kg}}{40 \text{ min}} = 0.0142 \text{ kg/min} = 2.37 \times 10^{-4} \text{ kg/s}$$
$$V = \frac{\dot{m}}{\rho_g A_c} = \frac{\dot{m} v_g}{A_c} = \frac{(2.37 \times 10^{-4} \text{ kg/s})(1.1594 \text{ m}^3/\text{kg})}{8 \times 10^{-6} \text{ m}^2} = 34.3 \text{ m/s}$$

(b) the total and flow energies of the steam per unit mass

$$e_{\text{flow}} = Pv = h - u = 2693.1 - 2519.2 = 173.9 \text{ kJ/kg}$$

$$\theta = h + \text{ke} + \text{pe} \cong h = 2693.1 \text{ kJ/kg}$$

The kinetic energy in this case is  $\text{ke} = V^2/2 = (34.3 \text{ m/s})^2/2 = 588 \text{ m}^2/\text{s}^2 = 0.588 \text{ kJ/kg}$ , which is small compared to enthalpy.

(c) The rate at which energy is leaving the cooker by mass is simply the product of the mass flow rate and the total energy of the exiting steam per unit mass,

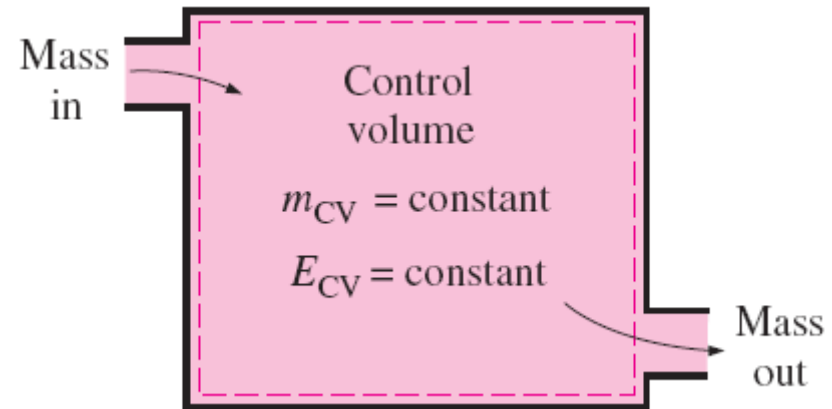
$$\dot{E}_{\text{mass}} = \dot{m}\theta = (2.37 \times 10^{-4} \text{ kg/s})(2693.1 \text{ kJ/kg}) = 0.638 \text{ kJ/s} = 0.638 \text{ kW}$$

**Discussion** The numerical value of the energy leaving the cooker with steam alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity is the difference between the enthalpies of the exiting vapor and the liquid inside (which is  $h_{fg}$ ) since it relates directly to the amount of energy supplied to the cooker.

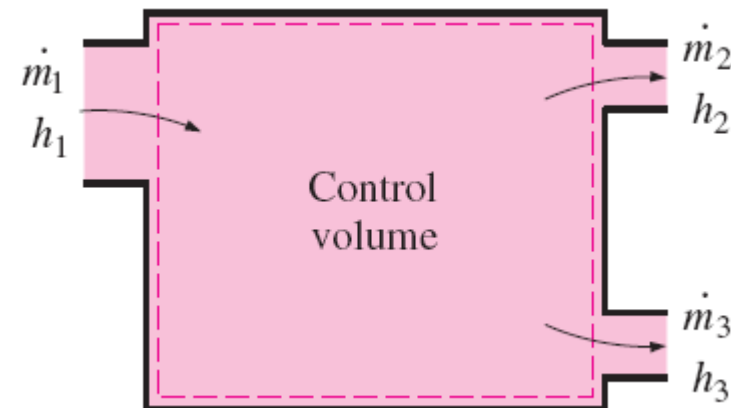
# ENERGY ANALYSIS OF STEADY-FLOW SYSTEMS



Many engineering systems such as power plants operate under steady conditions.



Under steady-flow conditions, the mass and energy contents of a control volume remain constant.



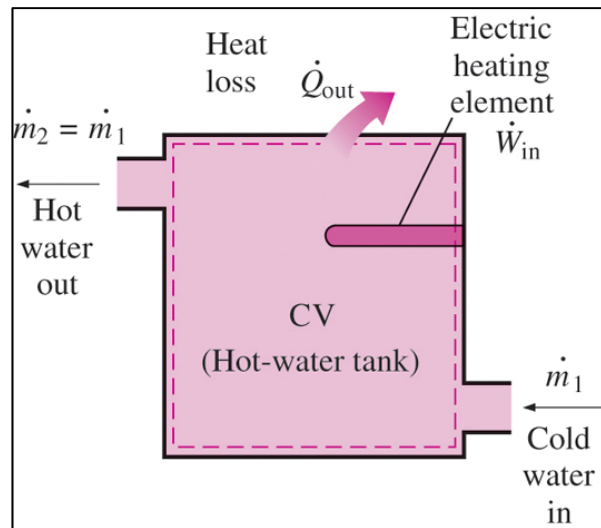
Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

# MASS AND ENERGY BALANCES FOR A STEADY-FLOW PROCESS

Mass balance for steady-flow systems:  $\dot{m}_{in} = \dot{m}_{out}$  (kg/s)

Multiple inlets and exits:  $\sum \dot{m}_i = \sum \dot{m}_e$  (kg/s)

One inlet and one exit:  $\dot{m}_1 = \dot{m}_2$  or  $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$



A water heater in steady operation.

## Energy balance

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} = 0 \quad \text{0 (steady)}$$

$$\underbrace{\dot{E}_{in}}_{\text{Rate of net energy transfer in by heat, work, and mass}} = \underbrace{\dot{E}_{out}}_{\text{Rate of net energy transfer out by heat, work, and mass}} \quad (\text{kW})$$

$$\dot{Q}_{in} + \dot{W}_{in} + \sum \dot{m}_i \theta_i = \dot{Q}_{out} + \dot{W}_{out} + \sum \dot{m}_e \theta_e$$

$$\dot{Q}_{in} + \dot{W}_{in} + \underbrace{\sum \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \underbrace{\sum \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$\dot{Q} - \dot{W} = \underbrace{\sum \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)}_{\text{for each exit}} - \underbrace{\sum \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right)}_{\text{for each inlet}}$$

Energy balance relations  
with sign conventions

(heat input and work  
output are positive)

$$\dot{Q} - \dot{W} = \dot{m} \left[ h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right]$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

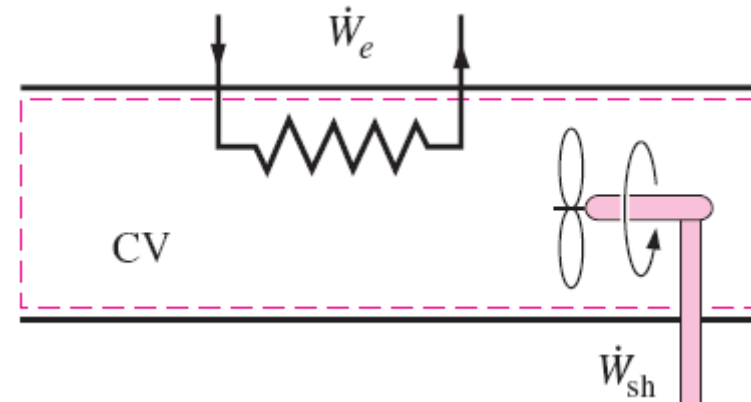
when kinetic and potential energy changes are negligible

$$q - w = h_2 - h_1$$

$$\frac{\text{J}}{\text{kg}} \equiv \frac{\text{N} \cdot \text{m}}{\text{kg}} \equiv \left( \text{kg} \frac{\text{m}}{\text{s}^2} \right) \frac{\text{m}}{\text{kg}} \equiv \frac{\text{m}^2}{\text{s}^2}$$

$$\left( \text{Also, } \frac{\text{Btu}}{\text{lbm}} \equiv 25,037 \frac{\text{ft}^2}{\text{s}^2} \right)$$

Some energy unit equivalents

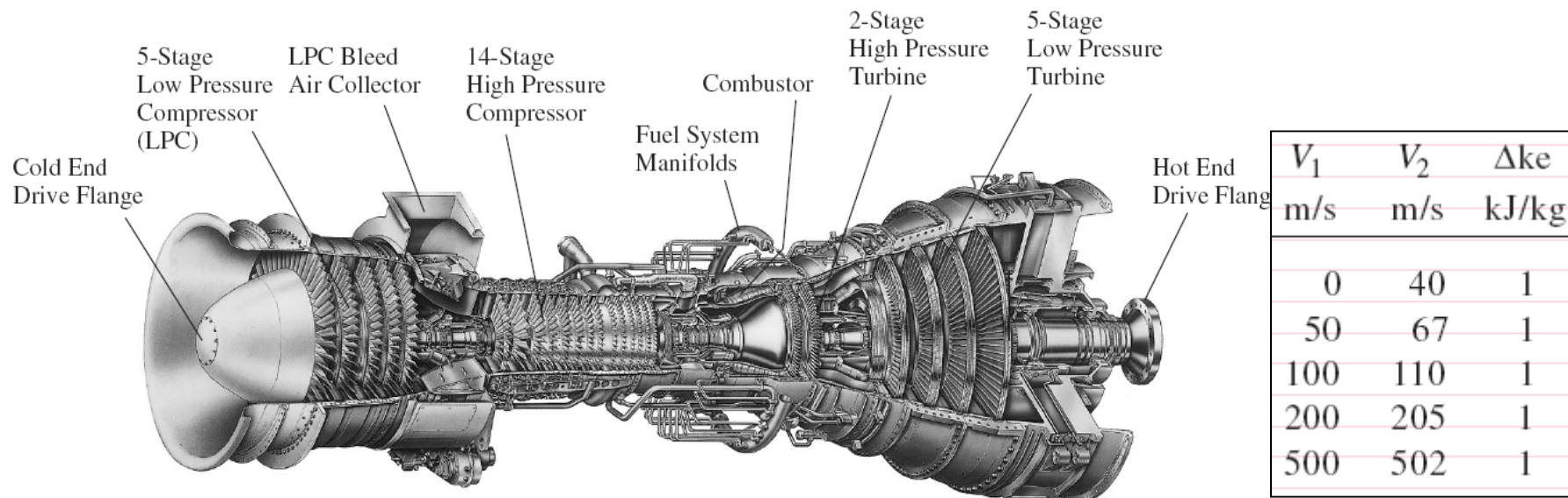


Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.



# SOME STEADY-FLOW ENGINEERING DEVICES

Many engineering devices operate essentially under the same conditions for long periods of time. The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance. Therefore, these devices can be conveniently analyzed as steady-flow devices.

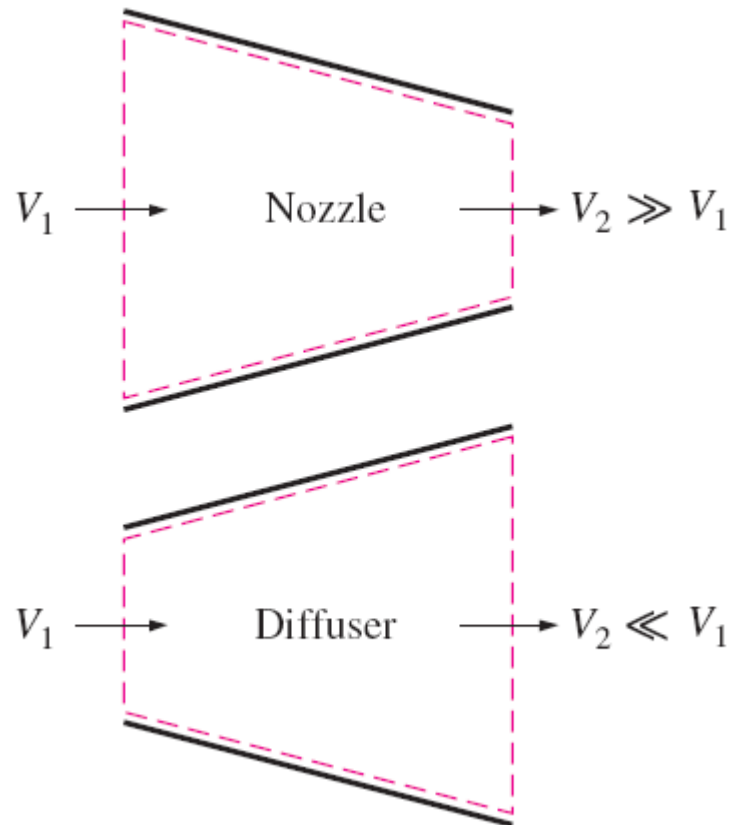


A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

At very high velocities, even small changes in velocities can cause significant changes in the kinetic energy of the fluid.



# Nozzles and Diffusers



Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

Energy balance for a nozzle or diffuser:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$
$$\dot{m} \left( h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left( h_2 + \frac{V_2^2}{2} \right)$$

(since  $\dot{Q} \cong 0$ ,  $\dot{W} = 0$ , and  $\Delta p_e \cong 0$ )

# Conservation of Energy

**The conservation of energy principle (the energy balance):** The net energy transfer to or from a system during a process be equal to the change in the energy content of the system.

Energy can be transferred to or from a closed system by heat or work.

Control volumes also involve energy transfer via mass flow.

$$\text{Conservation of energy:} \quad \dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{CV}}}{dt}$$

$\dot{E}_{\text{in}}$  and  $\dot{E}_{\text{out}}$

the total rates of energy transfer into and out of the control volume

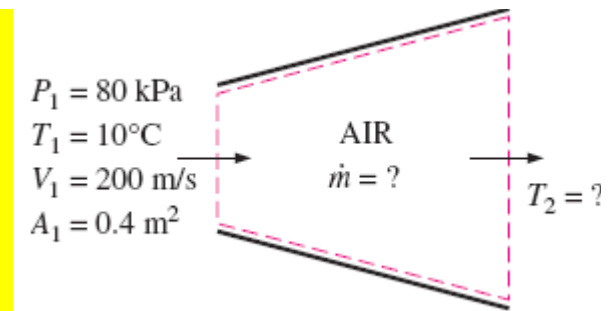
$dE_{\text{CV}}/dt$

the rate of change of energy within the control volume boundaries

In fluid mechanics, we usually limit our consideration to mechanical forms of energy only.

### EXAMPLE 5-4

Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m<sup>2</sup>. The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (a) the mass flow rate of the air and (b) the temperature of the air leaving the diffuser.



**Solution** Air enters the diffuser of a jet engine steadily at a specified velocity. The mass flow rate of air and the temperature at the diffuser exit are to be determined.

**Assumptions 1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{CV} = 0$  and  $\Delta E_{CV} = 0$ . **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The potential energy change is zero,  $\Delta pe = 0$ . **4** Heat transfer is negligible. **5** Kinetic energy at the diffuser exit is negligible. **6** There are no work interactions.

(a) To determine the mass flow rate, we need to find the specific volume of the air first. This is determined from the ideal-gas relation at the inlet conditions:

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(283 \text{ K})}{80 \text{ kPa}} = 1.015 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{1}{v_1} V_1 A_1 = \frac{1}{1.015 \text{ m}^3/\text{kg}} (200 \text{ m/s})(0.4 \text{ m}^2) = \mathbf{78.8 \text{ kg/s}}$$

(b) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\begin{aligned}\dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\ \dot{m} \left( h_1 + \frac{V_1^2}{2} \right) &= \dot{m} \left( h_2 + \frac{V_2^2}{2} \right) \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, \text{ and } \Delta pe \cong 0) \\ h_2 &= h_1 - \frac{V_2^2 - V_1^2}{2} \quad h_1 = h_{@ 283 \text{ K}} = 283.14 \text{ kJ/kg}\end{aligned}$$

The exit velocity of a diffuser is usually small compared with the inlet velocity ( $V_2 \ll V_1$ ); thus, the kinetic energy at the exit can be neglected. The enthalpy of air at the diffuser inlet is determined from the air table

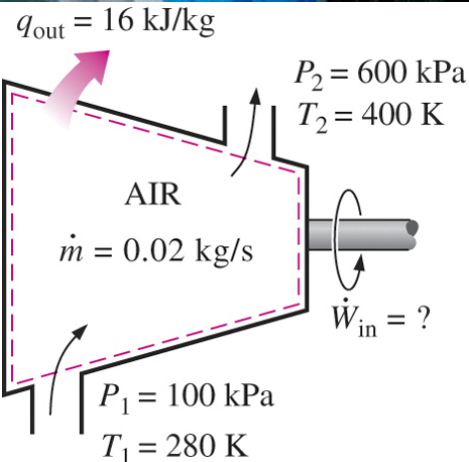
$$h_2 = 283.14 \text{ kJ/kg} - \frac{0 - (200 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 303.14 \text{ kJ/kg}$$

the temperature corresponding to this enthalpy value is

$$T_2 = 303 \text{ K}$$

**Discussion** This result shows that the temperature of the air increased by about 20°C as it was slowed down in the diffuser. The temperature rise of the air is mainly due to the conversion of kinetic energy to internal energy.

# Turbines and Compressors



Energy balance for the compressor in this figure:

**Turbine** drives the electric generator In steam, gas, or hydroelectric power plants.

As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

**Compressors**, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

A **fan** increases the pressure of a gas slightly and is mainly used to mobilize a gas.

A **compressor** is capable of compressing the gas to very high pressures.

**Pumps** work very much like compressors except that they handle liquids instead of gases.

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2$$

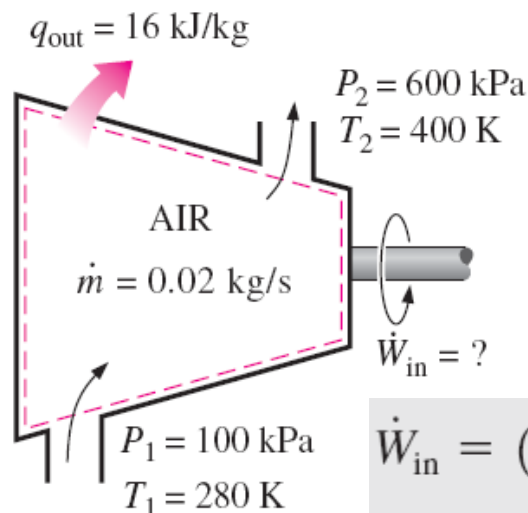
$$(\text{since } \Delta \text{ke} = \Delta \text{pe} \cong 0)$$

## EXAMPLE 5-6

Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

**Solution** Air is compressed steadily by a compressor to a specified temperature and pressure. The power input to the compressor is to be determined.

**Assumptions** **1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{CV}=0$  and  $\Delta E_{CV}=0$ . **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3**  $\Delta ke = \Delta pe = 0$ .



$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta ke = \Delta pe \cong 0)$$

$$\dot{W}_{\text{in}} = \dot{m}q_{\text{out}} + \dot{m}(h_2 - h_1)$$

$$h_1 = h_{@ 280 \text{ K}} = 280.13 \text{ kJ/kg}$$

$$h_2 = h_{@ 400 \text{ K}} = 400.98 \text{ kJ/kg}$$

$$\begin{aligned} \dot{W}_{\text{in}} &= (0.02 \text{ kg/s})(16 \text{ kJ/kg}) + (0.02 \text{ kg/s})(400.98 - 280.13) \text{ kJ/kg} \\ &= \mathbf{2.74 \text{ kW}} \end{aligned}$$

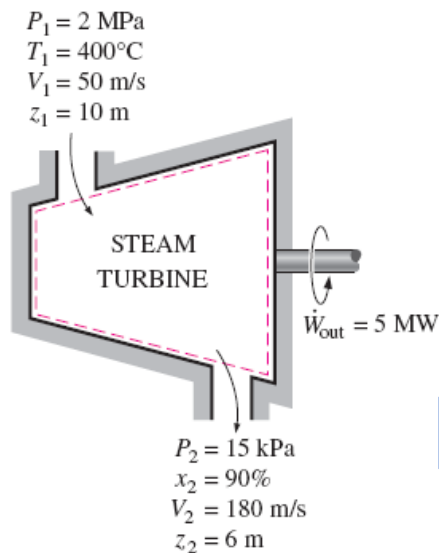
**Discussion** Note that the mechanical energy input to the compressor manifests itself as a rise in enthalpy of air and heat loss from the compressor.



## EXAMPLE 5-7

The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are;

- (a) Compare the magnitudes of  $\Delta h$ ,  $\Delta ke$ , and  $\Delta pe$ .
- (b) Determine the work done per unit mass of the steam flowing through the turbine.
- (c) Calculate the mass flow rate of the steam.



**Solution** The inlet and exit conditions of a steam turbine and its power output are given. The changes in kinetic energy, potential energy, and enthalpy of steam, as well as the work done per unit mass and the mass flow rate of steam are to be determined.

**Assumptions 1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{\text{CV}} = 0$  and  $\Delta E_{\text{CV}} = 0$ . **2** The system is adiabatic and thus there is no heat transfer.

- (a) At the inlet, steam is in a superheated vapor state, and its enthalpy is

$$\left. \begin{array}{l} P_1 = 2 \text{ MPa} \\ T_1 = 400^\circ\text{C} \end{array} \right\} h_1 = 3248.4 \text{ kJ/kg} \quad (\text{Table A-6})$$

At the turbine exit, we obviously have a saturated liquid–vapor mixture at 15-kPa pressure. The enthalpy at this state is

$$h_2 = h_f + x_2 h_{fg} = [225.94 + (0.9)(2372.3)] \text{ kJ/kg} = 2361.01 \text{ kJ/kg}$$



$$\Delta h = h_2 - h_1 = (2361.01 - 3248.4) \text{ kJ/kg} = -887.39 \text{ kJ/kg}$$

$$\Delta ke = \frac{V_2^2 - V_1^2}{2} = \frac{(180 \text{ m/s})^2 - (50 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 14.95 \text{ kJ/kg}$$

$$\Delta pe = g(z_2 - z_1) = (9.81 \text{ m/s}^2)[(6 - 10) \text{ m}] \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = -0.04 \text{ kJ/kg}$$

(b) The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{m} \left( h_1 + \frac{V_1^2}{2} + gz_1 \right) = \dot{W}_{\text{out}} + \dot{m} \left( h_2 + \frac{V_2^2}{2} + gz_2 \right) \quad (\text{since } \dot{Q} = 0)$$

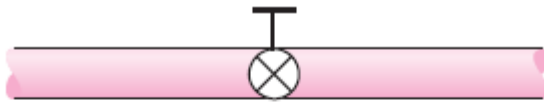
$$\begin{aligned} w_{\text{out}} &= - \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = -(\Delta h + \Delta ke + \Delta pe) \\ &= -[-887.39 + 14.95 - 0.04] \text{ kJ/kg} = 872.48 \text{ kJ/kg} \end{aligned}$$

(c) The required mass flow rate for a 5-MW power output is

$$\dot{m} = \frac{\dot{W}_{\text{out}}}{w_{\text{out}}} = \frac{5000 \text{ kJ/s}}{872.48 \text{ kJ/kg}} = 5.73 \text{ kg/s}$$

**Discussion** Two observations; First, the change in potential energy is insignificant in comparison to the changes in enthalpy and kinetic energy. Second, as a result of low pressure and thus high specific volume, the steam velocity at the turbine exit can be very high.

# Throttling valves



(a) An adjustable valve



(b) A porous plug



(c) A capillary tube

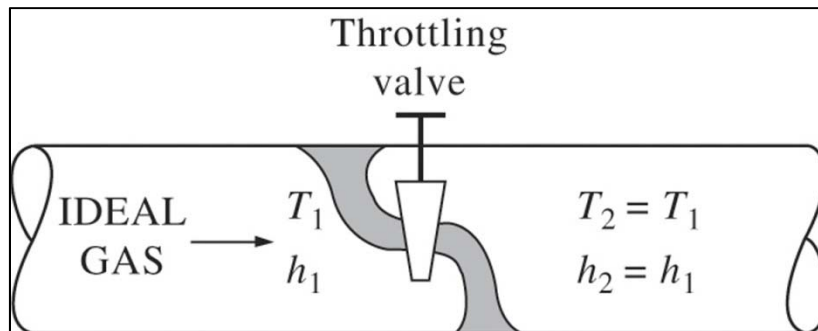
**Throttling valves** are *any kind of flow-restricting* devices that cause a significant pressure drop in the fluid.

***What is the difference between a turbine and a throttling valve?***

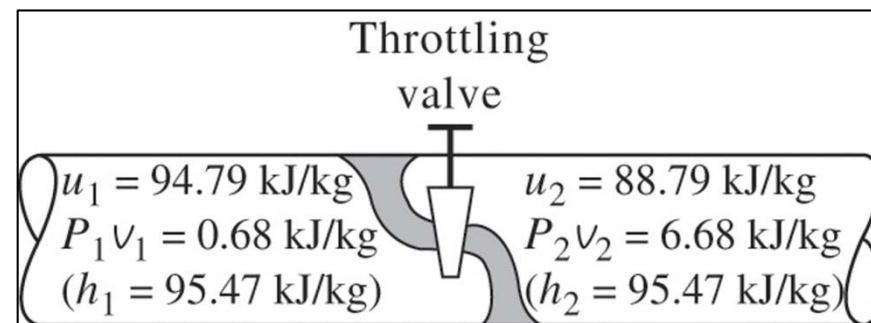
The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

$$h_2 \cong h_1 \quad (\text{kJ/kg}) \quad u_1 + P_1 v_1 = u_2 + P_2 v_2$$

Internal energy + Flow energy = Constant



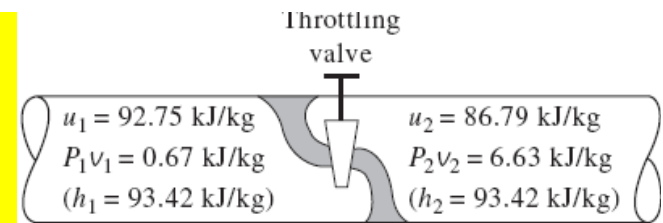
The temperature of an ideal gas does not change during a throttling ( $h = \text{cons.}$ ) process since  $h = h(T)$ .



During a throttling process, the enthalpy of a fluid remains constant. But internal and flow energies may be converted to each other.

### EXAMPLE 5-8

Refrigerant-134a enters the capillary tube of a refrigerator as saturated liquid at 0.8 MPa and is throttled to a pressure of 0.12 MPa. Determine the quality of the refrigerant at the final state and the temperature drop during this process.



**Solution** Refrigerant-134a that enters a capillary tube as saturated liquid is throttled to a specified pressure.

**Assumptions** Heat transfer and Kinetic energy change of the refrigerant is negligible.

$$\text{At inlet: } \left. \begin{array}{l} P_1 = 0.8 \text{ MPa} \\ \text{sat. liquid} \end{array} \right\} \begin{array}{l} T_1 = T_{\text{sat}} @ 0.8 \text{ MPa} = 31.31^\circ\text{C} \\ h_1 = h_f @ 0.8 \text{ MPa} = 95.47 \text{ kJ/kg} \end{array}$$

$$\text{At exit: } \begin{array}{l} P_2 = 0.12 \text{ MPa} \\ (h_2 = h_1) \end{array} \longrightarrow \begin{array}{l} h_f = 22.49 \text{ kJ/kg} \\ h_g = 236.97 \text{ kJ/kg} \end{array} \quad T_{\text{sat}} = -22.32^\circ\text{C}$$

The quality at this state is

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{95.47 - 22.49}{236.97 - 22.49} = \mathbf{0.340}$$

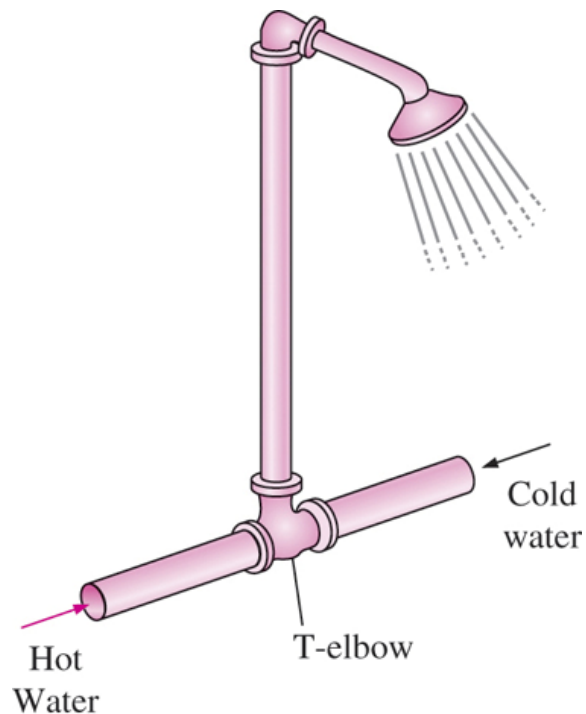
Since the exit state is a saturated mixture at 0.12 MPa, the exit temperature must be the saturation temperature at this pressure, which is  $-22.32^\circ\text{C}$ .

$$\Delta T = T_2 - T_1 = (-22.32 - 31.31)^\circ\text{C} = \mathbf{-53.63^\circ\text{C}}$$

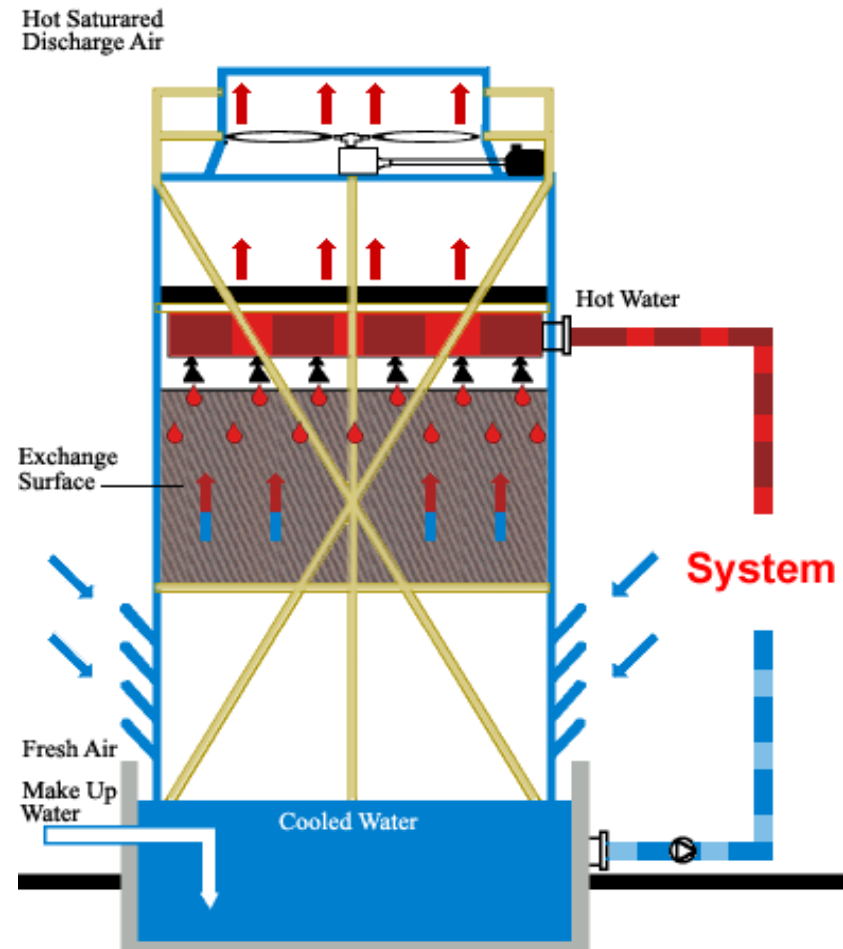
**Discussion** 34.0 % of the refrigerant vaporizes during this throttling process, and the energy needed to vaporize this refrigerant is absorbed from the refrigerant itself.

# Mixing chambers

In engineering applications, the section where the mixing process takes place is commonly referred to as a **mixing chamber**.



The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.



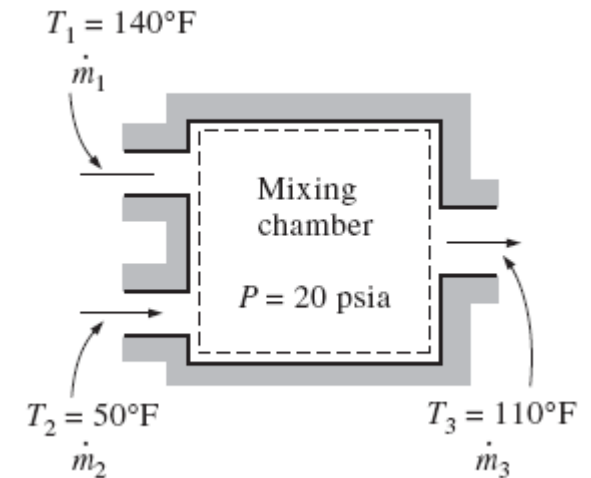
Energy balance for the adiabatic mixing chamber in the figure is:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

(since  $\dot{Q} \cong 0$ ,  $\dot{W} = 0$ ,  $ke \cong pe \cong 0$ )

### EXAMPLE 5-8

Consider an ordinary shower where hot water at 140°F is mixed with cold water at 50°F. If it is desired that a steady stream of warm water at 110°F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia.



$$\text{Mass balance:} \quad \dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \Delta \dot{m}_{\text{system}} \xrightarrow{0 \text{ (steady)}} = 0$$

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, \text{ke} \cong \text{pe} \cong 0)$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$y h_1 + h_2 = (y + 1) h_3$$

where  $y = \dot{m}_1 / \dot{m}_2$  is the desired mass flow rate ratio.

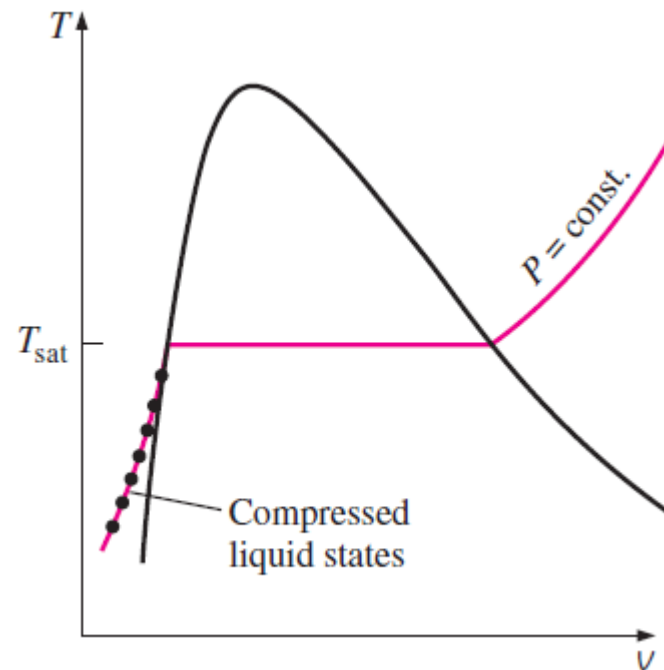
$$h_1 \cong h_f @ 140^\circ\text{F} = 107.96 \text{ Btu/lbm}$$

$$h_2 \cong h_f @ 50^\circ\text{F} = 18.07 \text{ Btu/lbm}$$

$$h_3 \cong h_f @ 110^\circ\text{F} = 78.02 \text{ Btu/lbm}$$

$$y = \frac{h_3 - h_2}{h_1 - h_3} = \frac{78.02 - 18.07}{107.99 - 78.02} = 2.0$$

**Discussion** Note that the mass flow rate of the hot water must be twice the mass flow rate of the cold water for the mixture to leave at  $110^\circ\text{F}$ .

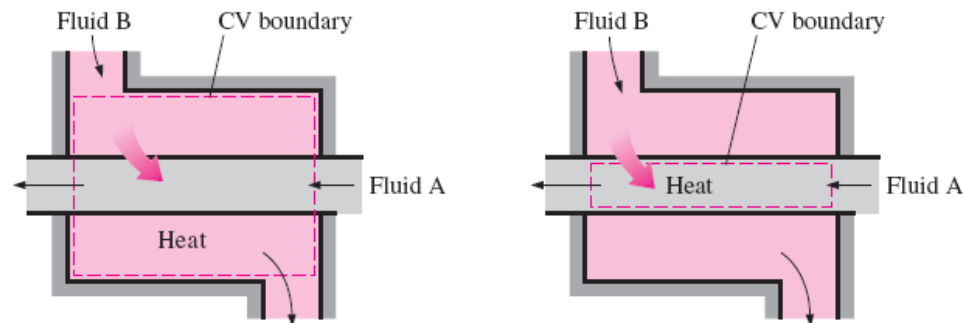


A substance exists as a compressed liquid at temperatures below the saturation temperatures at the given pressure.



# Heat exchangers

**Heat exchangers** are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.

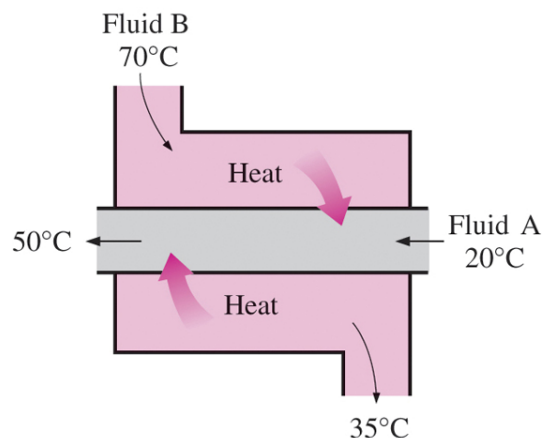


(a) System: Entire heat exchanger ( $Q_{CV} = 0$ )

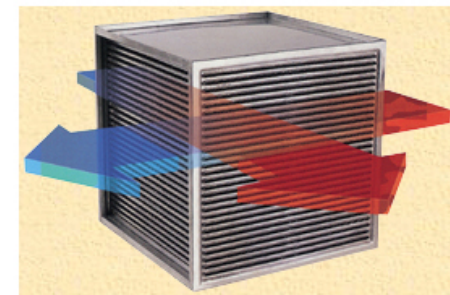
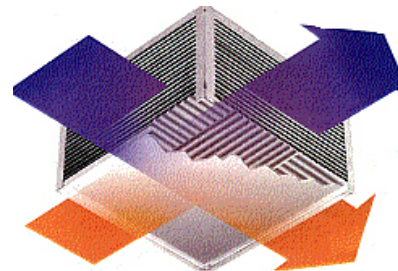
(b) System: Fluid A ( $Q_{CV} \neq 0$ )

The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.

Mass and energy balances for the adiabatic heat exchanger in the figure is:



A heat exchanger can be as simple as two concentric pipes.



$$\dot{E}_{in} = \dot{E}_{out}$$

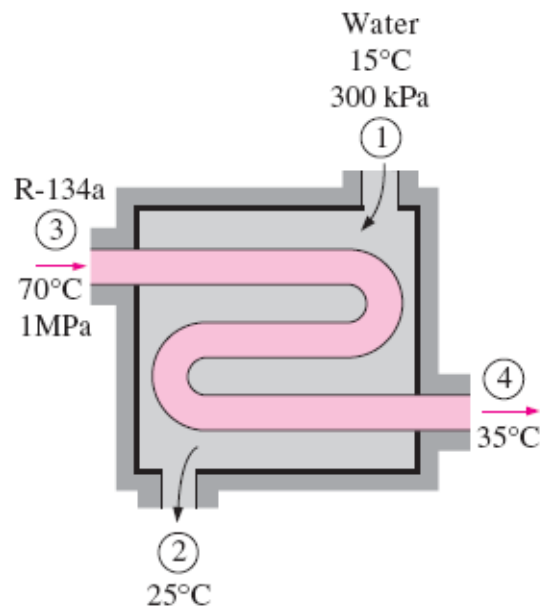
$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$

## EXAMPLE 5-10

Refrigerant-134a is to be cooled by water in a condenser. The refrigerant enters the condenser with a mass flow rate of 6 kg/min at 1 MPa and 70°C and leaves at 35°C. The cooling water enters at 300 kPa and 15°C and leaves at 25°C. Neglecting any pressure drops, determine (a) the mass flow rate of the cooling water required and (b) the heat transfer rate from the refrigerant to water.



**Solution** Refrigerant-134a is cooled by water in a condenser. The mass flow rate of the cooling water and the rate of heat transfer from the refrigerant to the water are to be determined.

**Assumptions** 1 This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{CV}=0$  and  $\Delta E_{CV}=0$ . 2 The kinetic and potential energies are negligible,  $\Delta ke \cong \Delta pe \cong 0$ . 3 Heat losses from the system are negligible and thus  $\dot{Q} \cong 0$ . 4 There is no work interaction.

(a) Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows

Mass balance:

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4 \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, ke \cong pe \cong 0)$$

$$\dot{m}_w(h_1 - h_2) = \dot{m}_R(h_4 - h_3)$$

$$h_1 \cong h_{f@15^\circ\text{C}} = 62.982 \text{ kJ/kg}$$

$$h_2 \cong h_{f@25^\circ\text{C}} = 104.83 \text{ kJ/kg}$$

$$\left. \begin{array}{l} P_3 = 1 \text{ MPa} \\ T_3 = 70^\circ\text{C} \end{array} \right\} h_3 = 303.85 \text{ kJ/kg}$$

$$\left. \begin{array}{l} P_4 = 1 \text{ MPa} \\ T_4 = 35^\circ\text{C} \end{array} \right\} h_4 \cong h_{f@35^\circ\text{C}} = 100.87 \text{ kJ/kg}$$

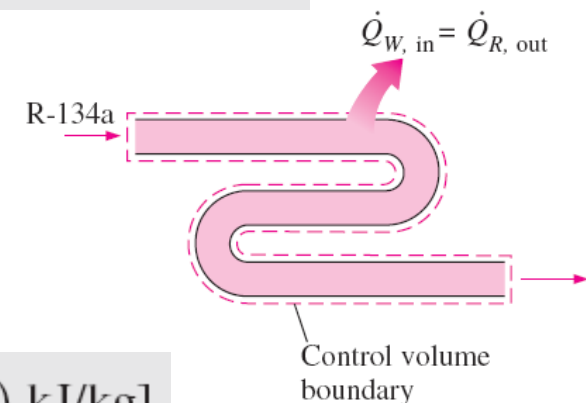
$$\dot{m}_w(62.982 - 104.83) \text{ kJ/kg} = (6 \text{ kg/min})[(100.87 - 303.85) \text{ kJ/kg}]$$

$$\dot{m}_w = \mathbf{29.10 \text{ kg/min}}$$

(b) the heat transfer rate from the refrigerant to water.

$$\begin{aligned} \dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\ \dot{Q}_{w,\text{in}} + \dot{m}_w h_1 &= \dot{m}_w h_2 \end{aligned}$$

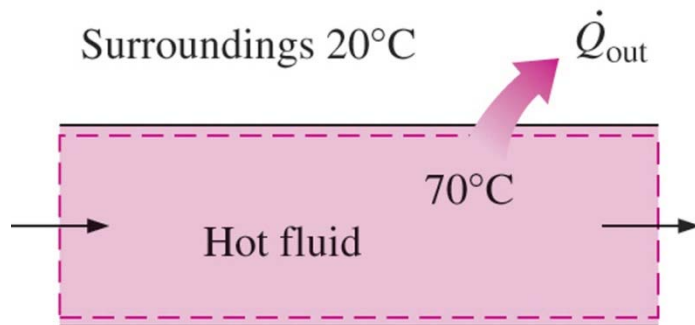
$$\begin{aligned} \dot{Q}_{w,\text{in}} + \dot{m}_w(h_2 - h_1) &= (29.10 \text{ kg/min})[(104.83 - 62.982) \text{ kJ/kg}] \\ &= \mathbf{1218 \text{ kJ/min}} \end{aligned}$$



**Discussion** Had we chosen the volume occupied by the refrigerant as the control volume, we would have obtained the same result for  $\dot{Q}_{R,\text{out}}$  since the heat gained by the water is equal to the heat lost by the refrigerant.

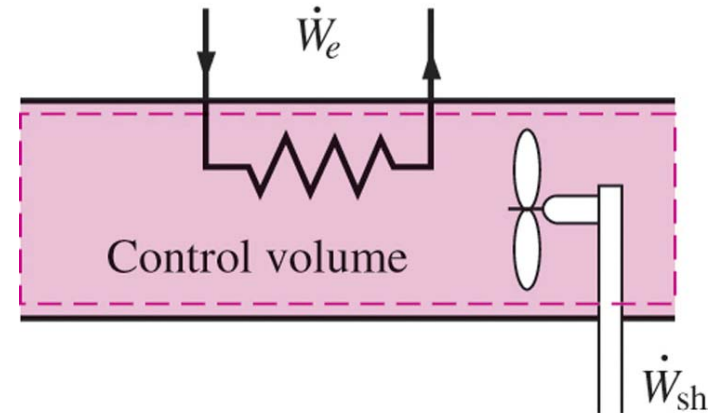
# Pipe and duct flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions.



Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

$$\begin{aligned}\dot{E}_{in} &= \dot{E}_{out} \\ \dot{W}_{e,in} + \dot{m}h_1 &= \dot{Q}_{out} + \dot{m}h_2 \\ \dot{W}_{e,in} - \dot{Q}_{out} &= \dot{m}c_p(T_2 - T_1)\end{aligned}$$

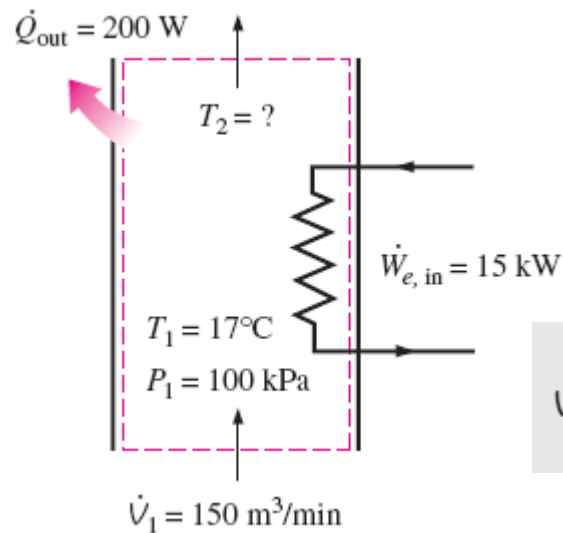


Pipe or duct flow may involve more than one form of work at the same time.



## EXAMPLE 5-11

The electric heating systems used in many houses consist of a simple duct with resistance wires. Air is heated as it flows over resistance wires. Consider a 15-kW electric heating system. Air enters the heating section at 100 kPa and 17°C with a volume flow rate of 150 m<sup>3</sup>/min. If heat is lost from the air in the duct to the surroundings at a rate of 200 W, determine the exit temperature of air.



$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{W}_{e, \text{in}} + \dot{m}h_1 = \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \Delta \text{ke} \cong \Delta \text{pe} \cong 0)$$

$$\dot{W}_{e, \text{in}} - \dot{Q}_{\text{out}} = \dot{m}c_p(T_2 - T_1)$$

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(290 \text{ K})}{100 \text{ kPa}} = 0.832 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{150 \text{ m}^3/\text{min}}{0.832 \text{ m}^3/\text{kg}} \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 3.0 \text{ kg/s}$$

$$(15 \text{ kJ/s}) - (0.2 \text{ kJ/s}) = (3 \text{ kg/s})(1.005 \text{ kJ/kg} \cdot ^\circ\text{C})(T_2 - 17)^\circ\text{C}$$

$$T_2 = \mathbf{21.9^\circ\text{C}}$$

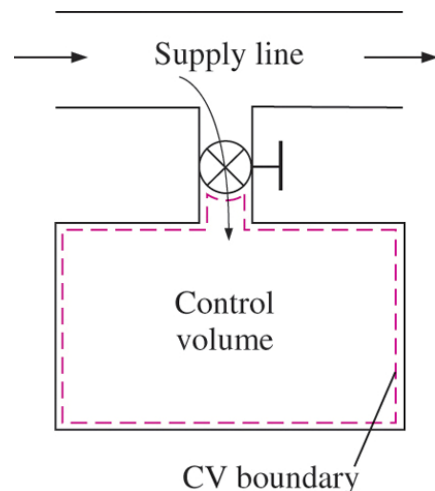
**Discussion** Note that heat loss from the duct reduces the exit temperature of air.



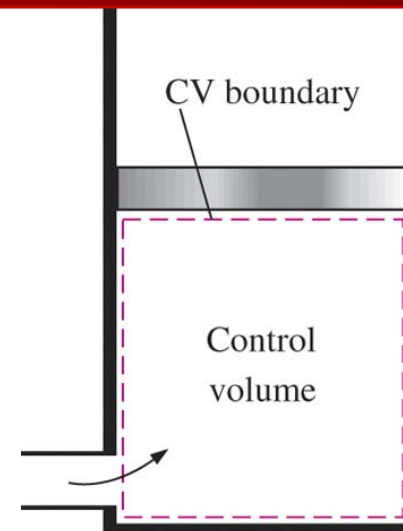
# ENERGY ANALYSIS OF UNSTEADY-FLOW PROCESSES

Many processes of interest, however, involve *changes* within the control volume with time. Such processes are called *unsteady-flow*, or *transient-flow*, processes. Most unsteady-flow processes can be represented reasonably well by the *uniform-flow process*.

**Uniform-flow process:** The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.



Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.



The shape and size of a control volume may change during an unsteady-flow process.



# Mass balance

Mass balance:  $m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{system}}$

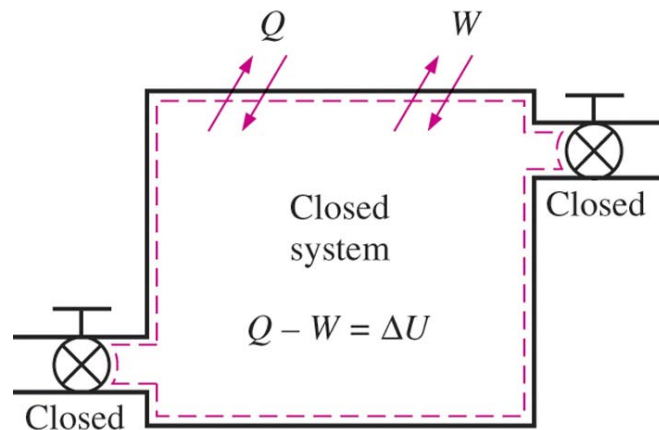
$$\Delta m_{\text{system}} = m_{\text{final}} - m_{\text{initial}}$$

$$\sum m_i - \sum m_e = (m_2 - m_1)_{\text{system}}$$

## Energy balance

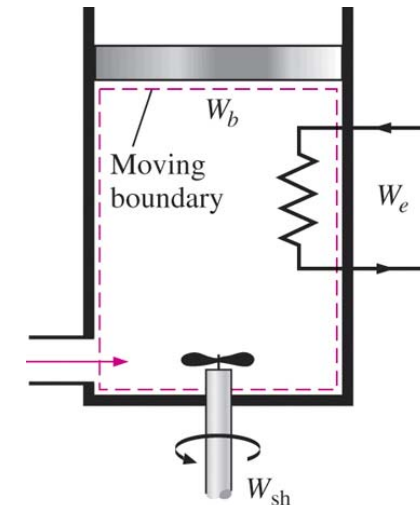
$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ})$$

$$\left( Q_{\text{in}} + W_{\text{in}} + \sum m_i \theta_i \right) - \left( Q_{\text{out}} + W_{\text{out}} + \sum m_e \theta_e \right) = (m_2 e_2 - m_1 e_1)_{\text{system}}$$



$$\theta = h + \text{ke} + \text{pe}$$

$$e = u + \text{ke} + \text{pe}$$

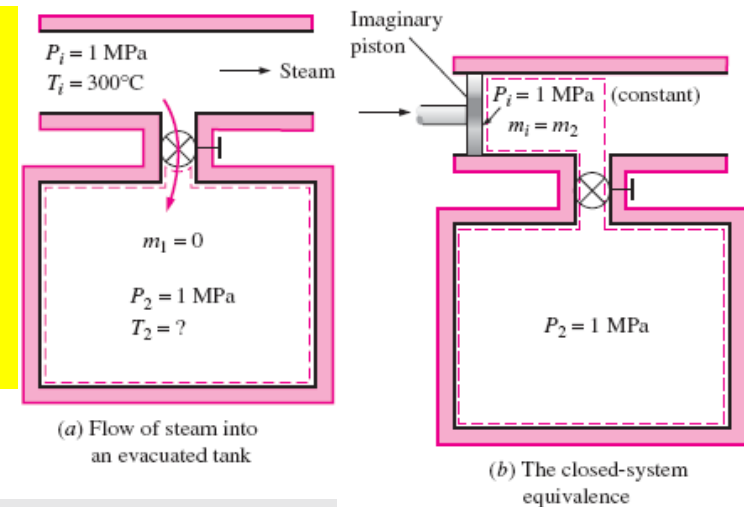


The energy equation of a uniform-flow system reduces to that of a closed system when all the inlets and exits are closed.

A uniform-flow system may involve electrical, shaft, and boundary work all at once.

## EXAMPLE 5-12

A rigid, insulated tank that is initially evacuated is connected through a valve to a supply line that carries steam at 1 MPa and 300°C. Now the valve is opened, and steam is allowed to flow slowly into the tank until the pressure reaches 1 MPa, at which point the valve is closed. Determine the final temperature of the steam in the tank.



$$\text{Mass balance: } m_i - m_e = \Delta m_{\text{system}} \rightarrow m_i = m_2 - \overset{0}{m_1} = m_2$$

$$m_i h_i = m_2 u_2 \quad (\text{since } W = Q = 0, \text{ ke} \cong \text{pe} \cong 0, m_1 = 0)$$

$$u_2 = h_i$$

$$\left. \begin{array}{l} P_i = 1 \text{ MPa} \\ T_i = 300^\circ\text{C} \end{array} \right\} h_i = 3051.6 \text{ kJ/kg}$$

$$\left. \begin{array}{l} P_2 = 1 \text{ MPa} \\ u_2 = 3051.6 \text{ kJ/kg} \end{array} \right\} T_2 = 456.1^\circ\text{C}$$

**Discussion** Note that the temperature of the steam in the tank has increased by  $156.1^\circ\text{C}$ . This result may be surprising at first, and you may be wondering where the energy to raise the temperature of the steam came from. The answer lies in the enthalpy term  $h = u + Pv$ . Part of the energy represented by enthalpy is the flow energy  $Pv$ , and this flow energy is converted to sensible internal energy once the flow ceases to exist in the control volume, and it shows up as an increase in temperature.

# SUMMARY

- Conservation of mass
  - Mass and volume flow rates
  - Mass balance for a steady-flow process
  - Mass balance for incompressible flow
- Flow work and the energy of a flowing fluid
  - Energy transport by mass
- Energy analysis of steady-flow systems
- Some steady-flow engineering devices
  - Nozzles and Diffusers
  - Turbines and Compressors
  - Throttling valves
  - Mixing chambers and Heat exchangers
  - Pipe and Duct flow
- Energy analysis of unsteady-flow processes

Thermodynamics: An Engineering Approach, 5<sup>th</sup> Edition  
Yunus A. Cengel, Michael A. Boles  
McGraw-Hill, 2008

# Chapter 6

## The SECOND LAW OF THERMODYNAMICS

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Prof. Dr. Ali PINARBAŞI

Yildiz Technical University  
Mechanical Engineering Department  
Yildiz, ISTANBUL

# THE SECOND LAW OF THERMODYNAMICS

6-1 Introduction to the Second Law

6-2 Thermal Energy Reservoirs

6-3 Heat Engines

Thermal Efficiency

Can We Save  $Q_{out}$ ?

The Second Law of Thermodynamics: Kelvin-Planck Statement

6-4 Refrigerators and Heat Pumps

Coefficient of Performance

Heat Pumps

The Second Law of Thermodynamics: Clausius Statement

Equivalence of the Two Statements

6-5 Perpetual-Motion Machines

6-6 Reversible and Irreversible Processes

Irreversibilities

Internally and Externally Reversible Processes

6-7 The Carnot Cycle

The Reversed Carnot Cycle

6-8 The Carnot Principles

6-9 The Thermodynamic Temperature Scale

6-10 The Carnot Heat Engine

The Quality of Energy

Quantity versus Quality in Daily Life

6-11 The Carnot Refrigerator and Heat Pump

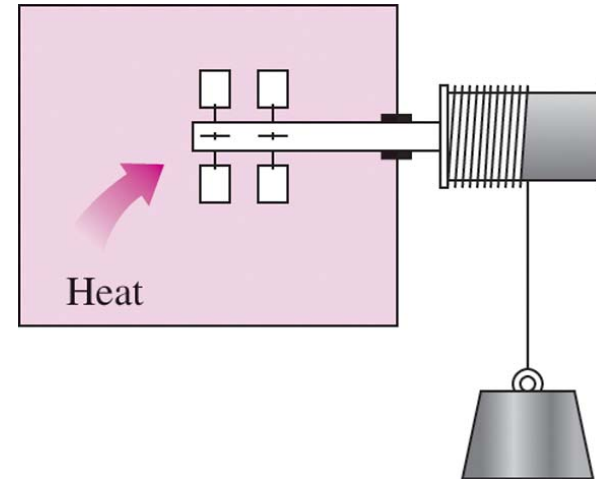
# Objectives

- Introduce the second law of thermodynamics.
- Identify valid processes as those that satisfy both the first and second laws of thermodynamics.
- Discuss thermal energy reservoirs, reversible and irreversible processes, heat engines, refrigerators, and heat pumps.
- Describe the Kelvin–Planck and Clausius statements of the second law of thermodynamics.
- Discuss the concepts of perpetual-motion machines.
- Apply the second law of thermodynamics to cycles and cyclic devices.
- Apply the second law to develop the absolute thermodynamic temperature scale.
- Describe the Carnot cycle.
- Examine the Carnot principles, idealized Carnot heat engines, refrigerators, and heat pumps.
- Determine the expressions for the thermal efficiencies and coefficients of performance for reversible heat engines, heat pumps, and refrigerators.

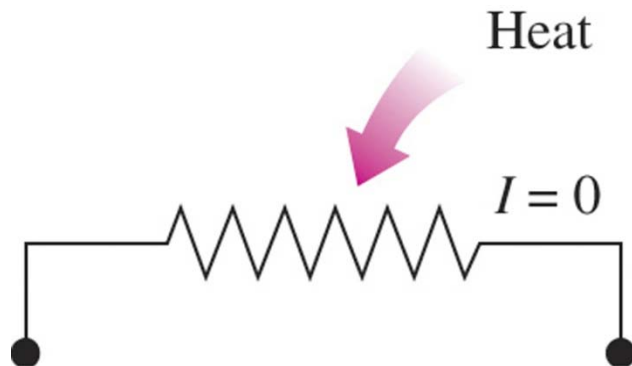


# INTRODUCTION TO THE SECOND LAW

A cup of hot coffee does not get hotter in a cooler room.



Transferring heat to a paddle wheel will not cause it to rotate.

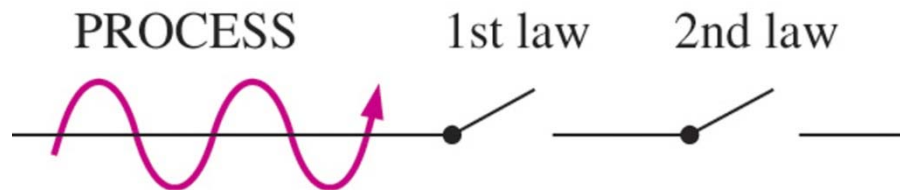


Transferring heat to a wire will not generate electricity.

**These processes cannot occur even though they are not in violation of the first law.**

ONE WAY

Processes occur in a certain direction, and not in the reverse direction.

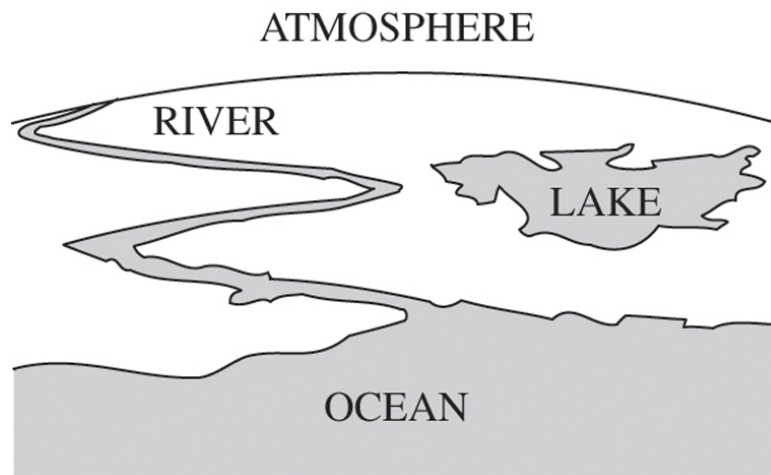


A process must satisfy both the first and second laws of thermodynamics to proceed.

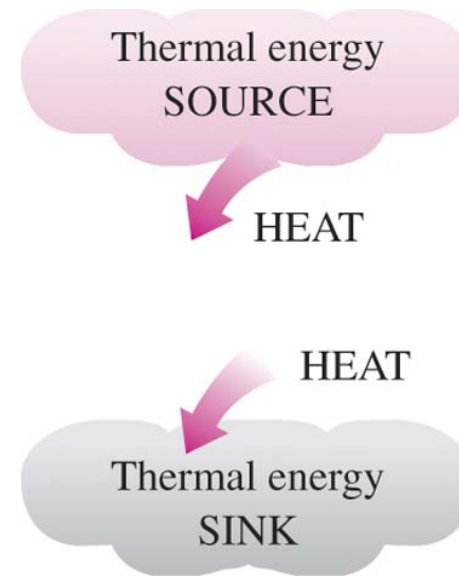
## MAJOR USES OF THE SECOND LAW

1. The second law may be used to identify the **direction** of processes.
2. The second law also asserts that energy has **quality** as well as quantity. The first law is concerned with the quantity of energy and the transformations of energy from one form to another with no regard to its quality. The second law provides the necessary means to determine the quality as well as the degree of degradation of energy during a process.
3. The second law of thermodynamics is also used in determining the **theoretical limits** for the performance of commonly used engineering systems, such as heat engines and refrigerators, as well as predicting the *degree of completion* of chemical reactions.

# THERMAL ENERGY RESERVOIRS



Bodies with relatively large thermal masses can be modeled as thermal energy reservoirs.



A source supplies energy in the form of heat, and a sink absorbs it.

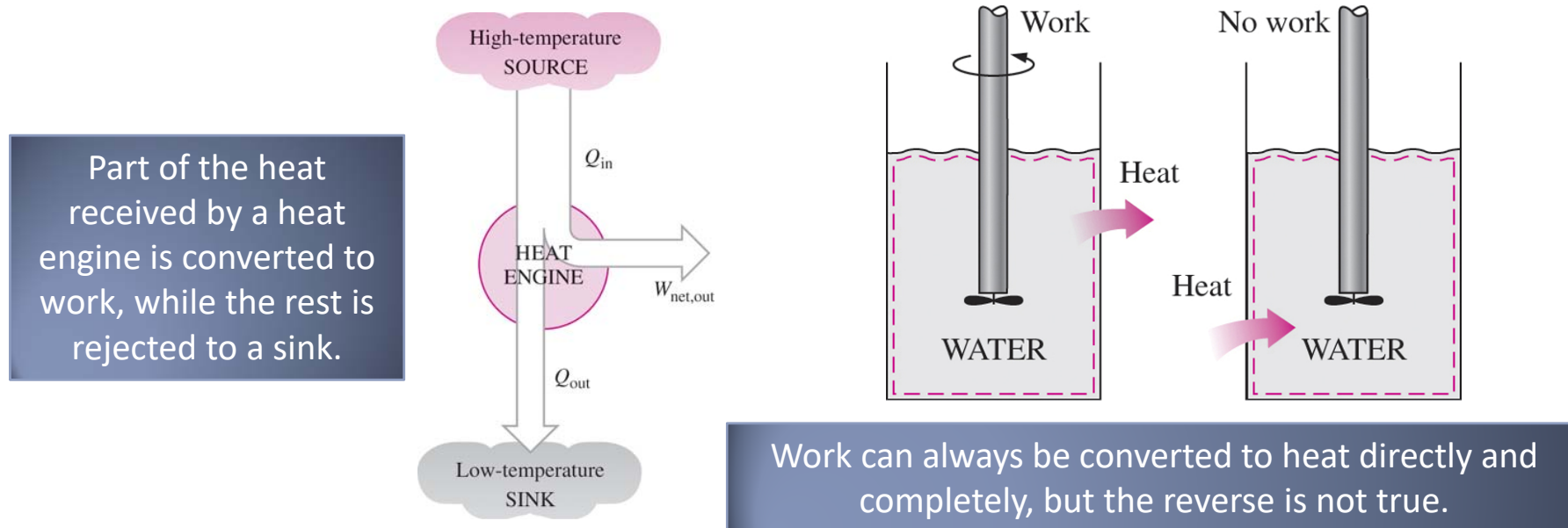
- A hypothetical body with a relatively large *thermal energy capacity* (mass  $\times$  specific heat) that can supply or absorb finite amounts of heat without undergoing any change in temperature is called a **thermal energy reservoir, or just a reservoir**.
- In practice, large bodies of water such as oceans, lakes, and rivers as well as the atmospheric air can be modeled accurately as thermal energy reservoirs because of their large thermal energy storage capabilities or thermal masses.

# HEAT ENGINES

The devices that convert heat to work.

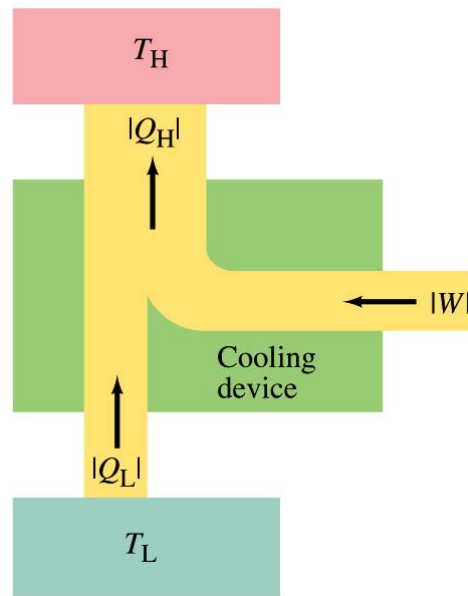
1. They receive heat from a high-temperature source (solar energy, oil furnace, nuclear reactor, etc.).
2. They convert part of this heat to work (usually in the form of a rotating shaft.)
3. They reject the remaining waste heat to a low-temperature sink (the atmosphere, rivers, etc.).
4. They operate on a cycle.

Heat engines and other cyclic devices usually involve a fluid to and from which heat is transferred while undergoing a cycle. This fluid is called the **working fluid**.

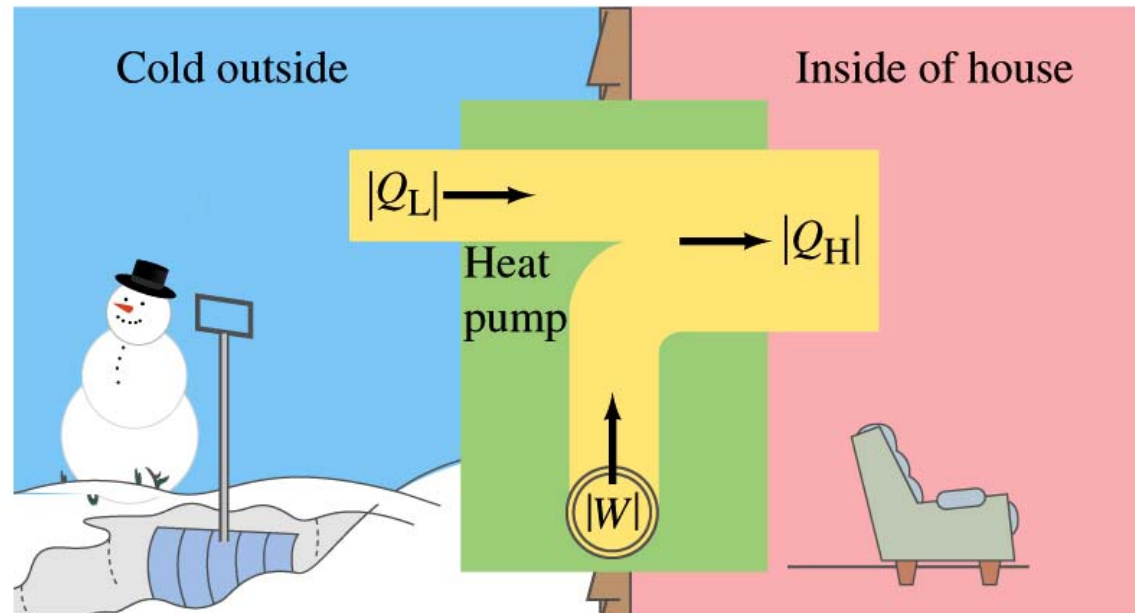
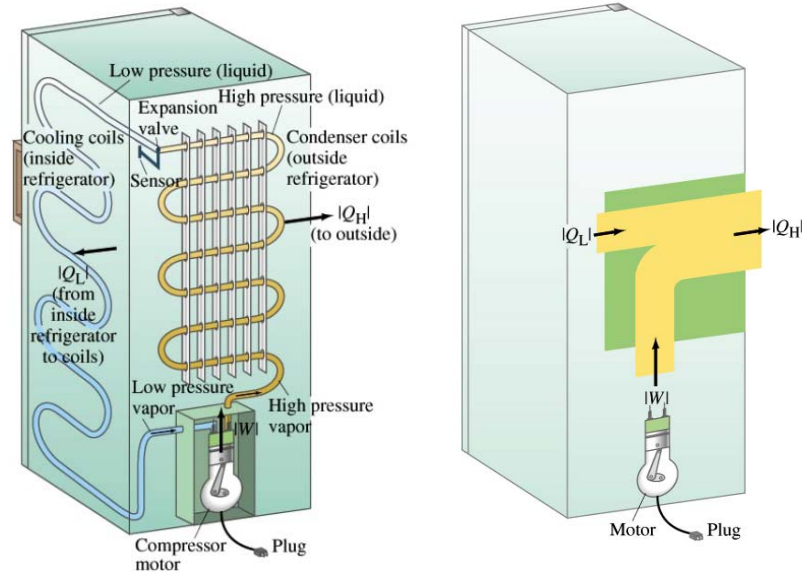


# Heat Pump

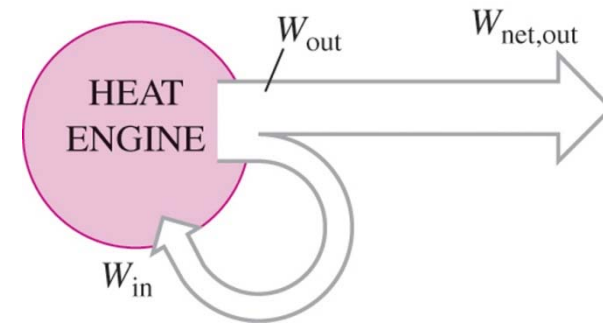
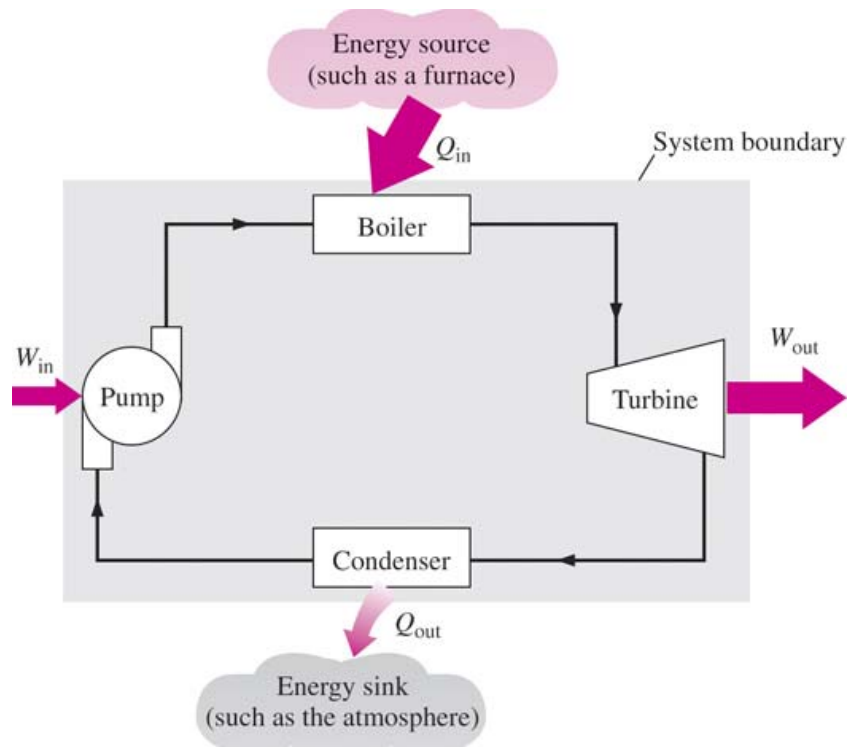
You cannot get cool the home opening the refrigerator door.



We just pay for the work not the heat absorbing.



# A steam power plant



A portion of the work output of a heat engine is consumed internally to maintain continuous operation.

$$W_{net,out} = W_{out} - W_{in} \quad (\text{kJ})$$

$$W_{net,out} = Q_{in} - Q_{out} \quad (\text{kJ})$$

$Q_{in}$  = amount of heat supplied to steam in boiler from a high-temperature source (furnace)

$Q_{out}$  = amount of heat rejected from steam in condenser to a low-temperature sink (the atmosphere, a river, etc.)

$W_{out}$  = amount of work delivered by steam as it expands in turbine

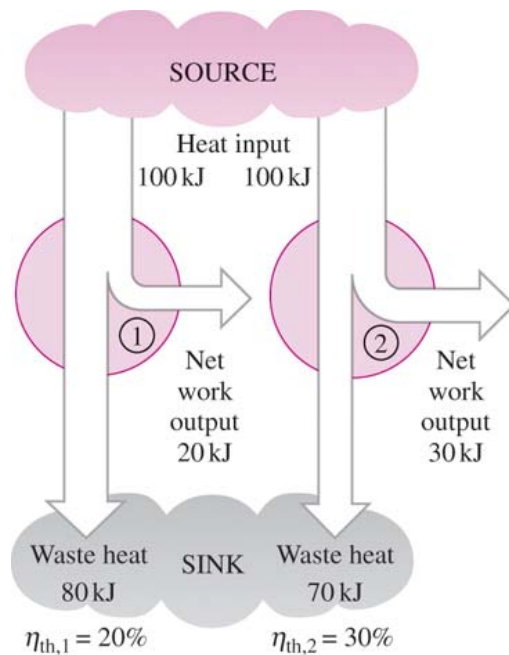
$W_{in}$  = amount of work required to compress water to boiler pressure



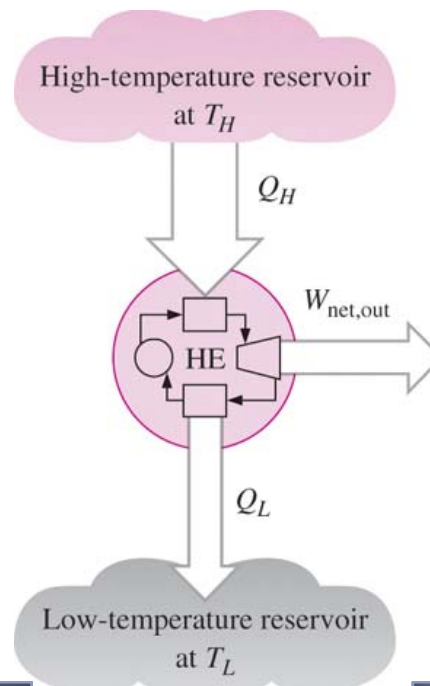
# Thermal efficiency

The fraction of the heat input that is converted to net work output is a measure of the performance of a heat engine and is called the **thermal efficiency**  $\eta_{th}$

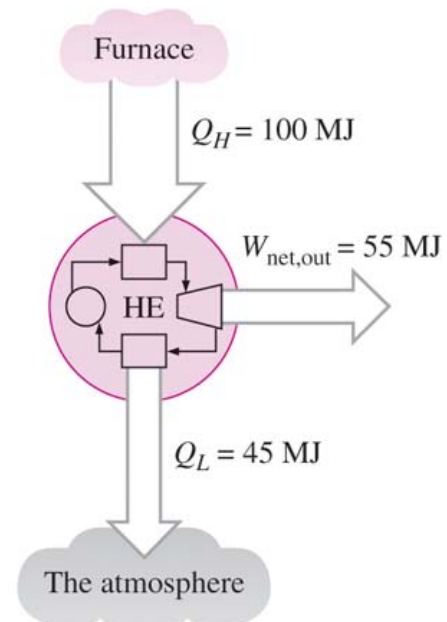
$$\text{Thermal efficiency} = \frac{\text{Net work output}}{\text{Total heat input}}$$



Some heat engines perform better than others (convert more of the heat they receive to work).



Schematic of a heat engine.



Even the most efficient heat engines reject almost one-half of the energy they receive as waste heat.

$$\text{Thermal efficiency} = \frac{\text{Net work output}}{\text{Total heat input}}$$

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_{\text{in}}}$$

$$\eta_{\text{th}} = 1 - \frac{Q_{\text{out}}}{Q_{\text{in}}}$$

$$W_{\text{net, out}} = Q_H - Q_L$$

$$W_{\text{net,out}} = Q_H - Q_L$$

$$\eta_{\text{th}} = \frac{W_{\text{net,out}}}{Q_H}$$

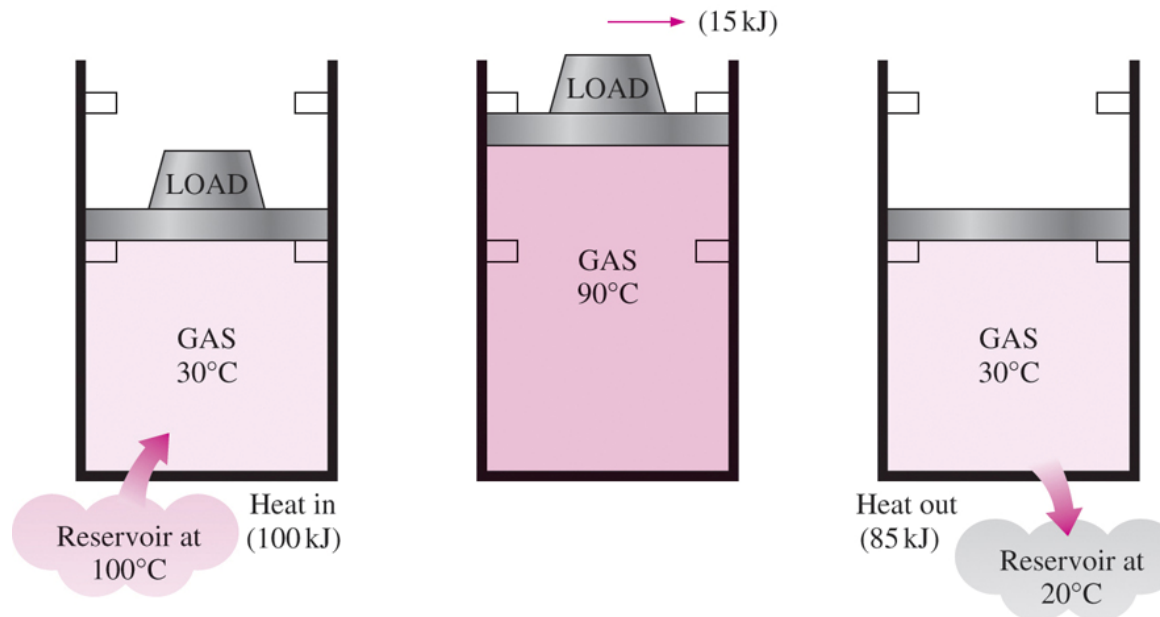
$$\eta_{\text{th}} = 1 - \frac{Q_L}{Q_H}$$

The thermal efficiencies of work-producing devices are relatively low.

Ordinary spark-ignition automobile engines have a thermal efficiency of **about 25 percent**. That is, an automobile engine converts about 25 percent of the chemical energy of **the gasoline** to mechanical work.

This number is as high as **40 percent** for **diesel engines** and large gas-turbine plants and as high as **60 percent** for large **combined gas-steam** power plants.

## Can we save $Q_{\text{out}}$ ?



A heat-engine cycle cannot be completed without rejecting some heat to a low-temperature sink.

Every heat engine must *waste* some energy by transferring it to a low-temperature reservoir in order to complete the cycle, even under idealized conditions.

In a steam power plant, the condenser is the device where large quantities of waste heat is rejected to rivers, lakes, or the **atmosphere**.

Can we not just take the condenser out of the plant and save all that waste energy?

The answer is, unfortunately, a firm **no** for the simple reason that without a heat rejection process in a condenser, the cycle cannot be completed.

### Example 6-1

Heat is transferred to a heat engine from a furnace at a rate of 80 MW. If the rate of waste heat rejection to a nearby river is 50 MW, determine the net power output and the thermal efficiency for this heat engine.

**Solution** The rates of heat transfer to and from a heat engine are given. The net power output and the thermal efficiency are to be determined.  
**Assumptions** Heat losses through the pipes and other components are negligible.

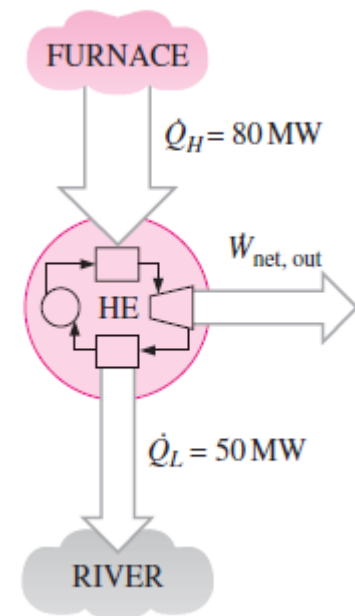
$$\dot{Q}_H = 80 \text{ MW} \quad \text{and} \quad \dot{Q}_L = 50 \text{ MW}$$

The net power output of this heat engine is

$$\dot{W}_{\text{net, out}} = \dot{Q}_H - \dot{Q}_L = (80 - 50) \text{ MW} = \mathbf{30 \text{ MW}}$$

Then the thermal efficiency is

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net, out}}}{\dot{Q}_H} = \frac{30 \text{ MW}}{80 \text{ MW}} = \mathbf{0.375 \text{ (or 37.5\%)}}$$



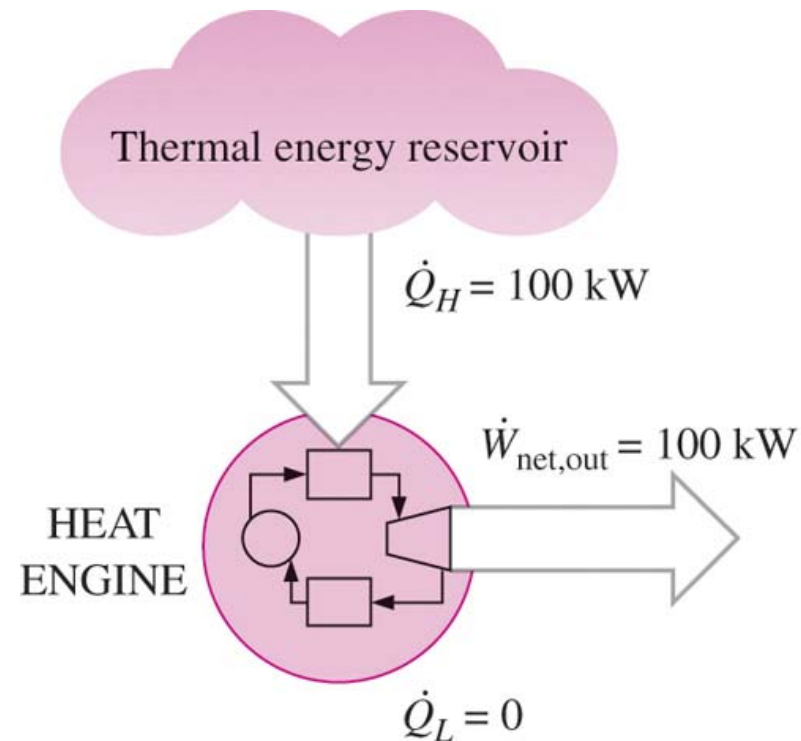
**Discussion** Note that the heat engine converts 37.5 percent of the heat it receives to work.

# The Second Law of Thermodynamics: Kelvin-Planck Statement

It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce a net amount of work.

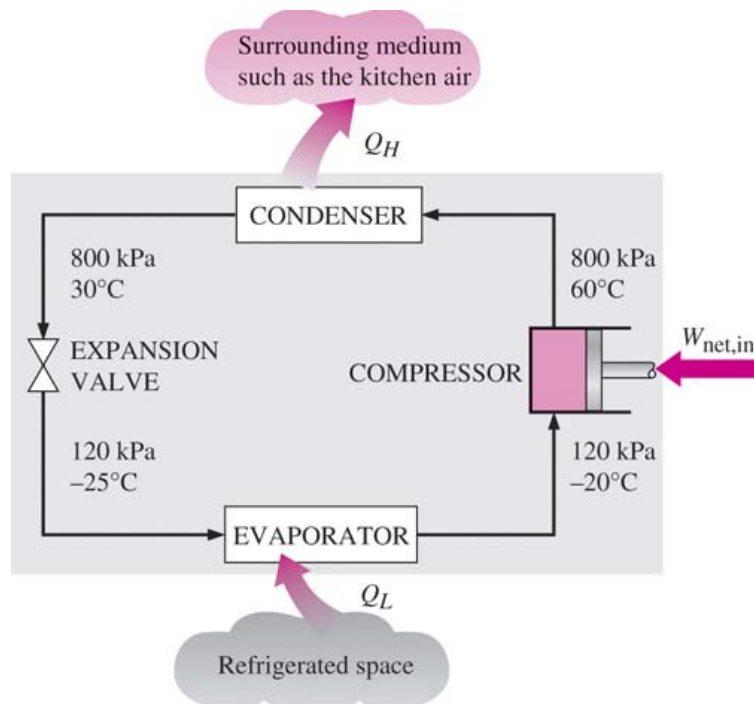
*No heat engine can have a thermal efficiency of 100 percent, or as for a power plant to operate, the working fluid must exchange heat with the environment as well as the furnace.*

The impossibility of having a 100% efficient heat engine is not due to friction or other dissipative effects. It is a limitation that applies to both the idealized and the actual heat engines.



A heat engine that violates the Kelvin–Planck statement of the second law.

# REFRIGERATORS AND HEAT PUMPS

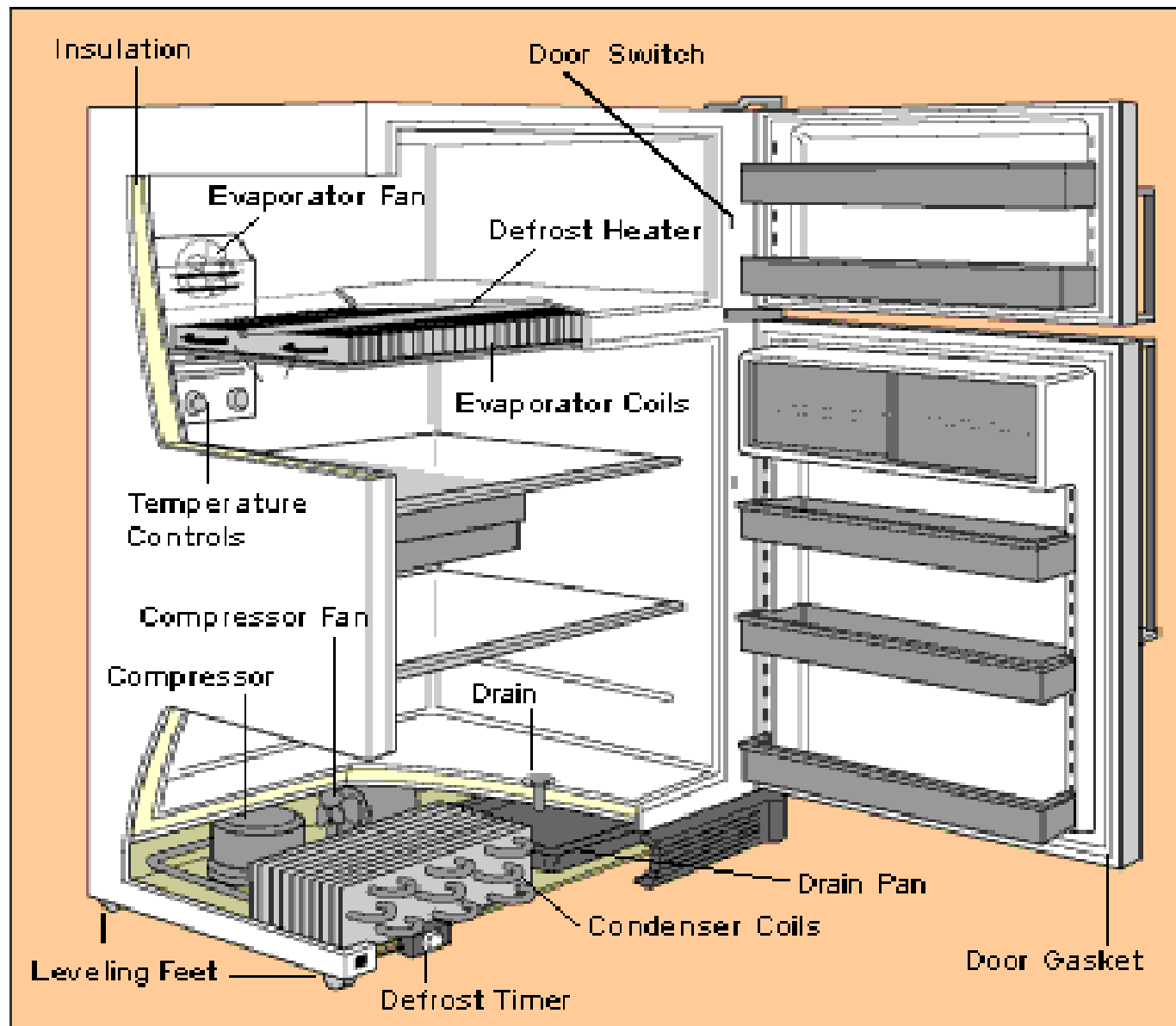


Basic components of a refrigeration system and typical operating conditions.

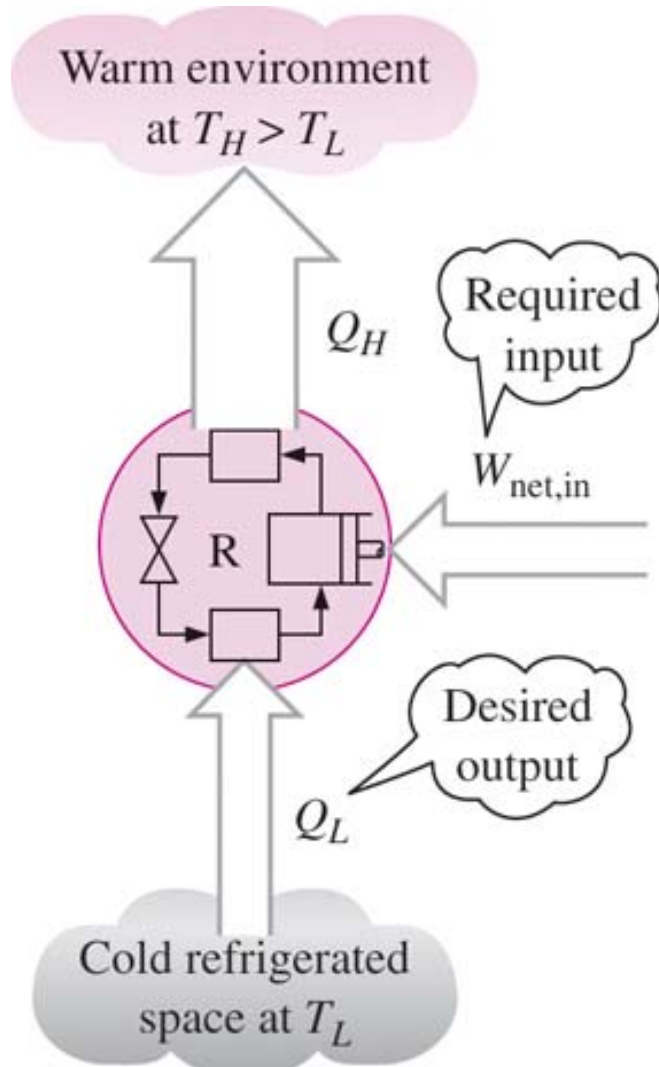
- The transfer of heat from a low-temperature medium to a high-temperature one requires special devices called **refrigerators**.
- Refrigerators, like heat engines, are cyclic devices.
- The working fluid used in the refrigeration cycle is called a **refrigerant**.
- The most frequently used refrigeration cycle is the **vapor-compression refrigeration cycle**.

In a household refrigerator, the freezer compartment where heat is absorbed by the refrigerant serves as the evaporator, and the coils usually behind the refrigerator where heat is dissipated to the kitchen air serve as the condenser.





# Coefficient of Performance



The objective of a refrigerator is to remove  $Q_L$  from the cooled space.

The *efficiency* of a refrigerator is expressed in terms of the **coefficient of performance (COP)**.

The objective of a refrigerator is to remove heat ( $Q_L$ ) from the refrigerated space.

$$\text{COP}_R = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{W_{\text{net,in}}}$$

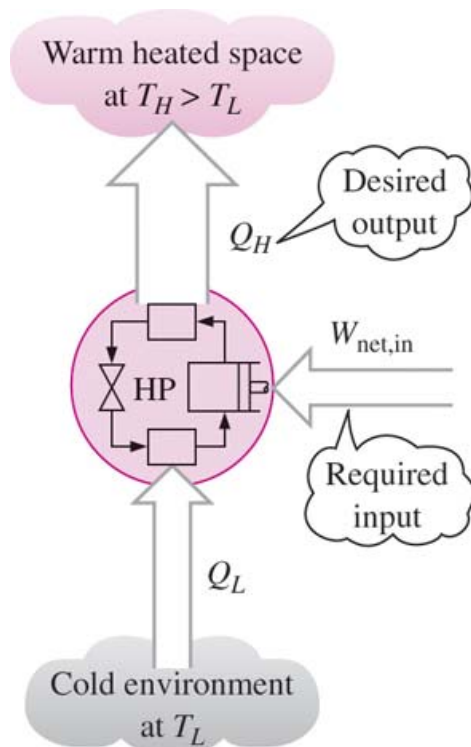
$$W_{\text{net,in}} = Q_H - Q_L \quad (\text{kJ})$$

$$\text{COP}_R = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1}$$

Can the value of  $\text{COP}_R$  be greater than unity?

# Heat Pumps

Another device that transfers heat from a low-temperature medium to a high-temperature one is the **heat pump**. Refrigerators and heat pumps operate on the same cycle but differ in their objectives



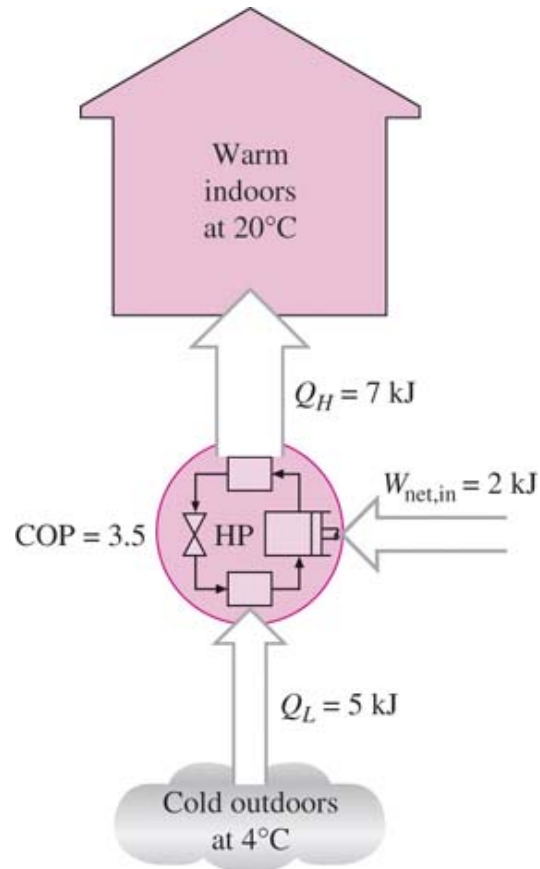
The objective of a heat pump is to supply heat  $Q_H$  into the warmer space.

The objective of a refrigerator is to maintain the refrigerated space at a low temperature by removing heat from it. Discharging this heat to a higher-temperature medium is merely a necessary part of the operation, not the purpose.

The objective of a heat pump, however, is to maintain a heated space at a high temperature. This is accomplished by absorbing heat from a low-temperature source, such as well water or cold outside air in winter, and supplying this heat to the high-temperature medium such as a house

$$\text{COP}_{\text{HP}} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_H}{W_{\text{net,in}}}$$

$$\text{COP}_{\text{HP}} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H}$$



for fixed values of  $Q_L$  and  $Q_H$

$$\text{COP}_{\text{HP}} = \text{COP}_{\text{R}} + 1$$

Can the value of  $\text{COP}_{\text{HP}}$  be lower than unity?

What does  $\text{COP}_{\text{HP}}=1$  represent?

The work supplied to a heat pump is used to extract energy from the cold outdoors and carry it into the warm indoors.



When installed backward, an air conditioner functions as a heat pump.

$$\text{EER} = 3.412 \text{ COP}_R$$

- Most heat pumps in operation today have a seasonally averaged COP of 2 to 3.
- Most existing heat pumps use the cold outside air as the heat source in winter (*air-source* HP).
- In cold climates their efficiency drops considerably when temperatures are below the freezing point.
- In such cases, *geothermal* (*ground-source*) HP that use the ground as the heat source can be used.
- Such heat pumps are more expensive to install, but they are also more efficient.
- **Air conditioners** are basically refrigerators whose refrigerated space is a room or a building instead of the food compartment.
- The COP of a refrigerator decreases with decreasing refrigeration temperature.
- Therefore, it is not economical to refrigerate to a lower temperature than needed.

**Energy efficiency rating (EER):** The amount of heat removed from the cooled space in Btu's for 1 Wh (watthour) of electricity consumed.

### Example 6-3

The food compartment of a refrigerator is maintained at 4°C by removing heat from it at a rate of 360 kJ/min. If the required power input to the refrigerator is 2 kW, determine  
(a) the coefficient of performance of the refrigerator and  
(b) the rate of heat rejection to the room that houses the refrigerator.

**Solution** The power consumption of a refrigerator is given. The COP and the rate of heat rejection are to be determined.

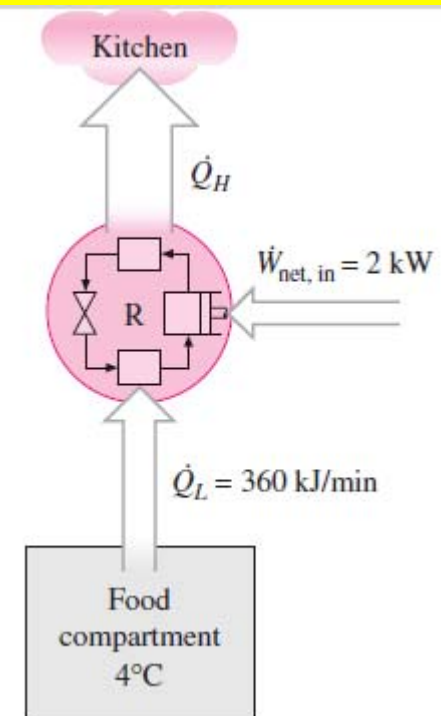
**Assumptions** Steady operating conditions exist.

(a) The coefficient of performance of the refrigerator is

$$\text{COP}_R = \frac{\dot{Q}_L}{\dot{W}_{\text{net, in}}} = \frac{360 \text{ kJ/min}}{2 \text{ kW}} \left( \frac{1 \text{ kW}}{60 \text{ kJ/min}} \right) = 3$$

(b) The rate at which heat is rejected to the room that

$$\dot{Q}_H = \dot{Q}_L + \dot{W}_{\text{net, in}} = 360 \text{ kJ/min} + (2 \text{ kW}) \left( \frac{60 \text{ kJ/min}}{1 \text{ kW}} \right) = 480 \text{ kJ/min}$$



**Discussion** Both the energy removed from the refrigerated space as heat and the energy supplied to the refrigerator as electrical work eventually show up in the room air and become part of the internal energy of the air. This demonstrates that energy can change from one form to another, can move from one place to another, but is never destroyed during a process.



### Example 6-4

A heat pump is used to meet the heating requirements of a house and maintain it at 20°C. On a day when the outdoor air temperature drops to -2°C, the house is estimated to lose heat at a rate of 80,000 kJ/h. If the heat pump under these conditions has a COP of 2.5, determine

- (a) the power consumed by the heat pump and
- (b) the rate at which heat is absorbed from the cold outdoor air.

**Solution** The COP of a heat pump is given.

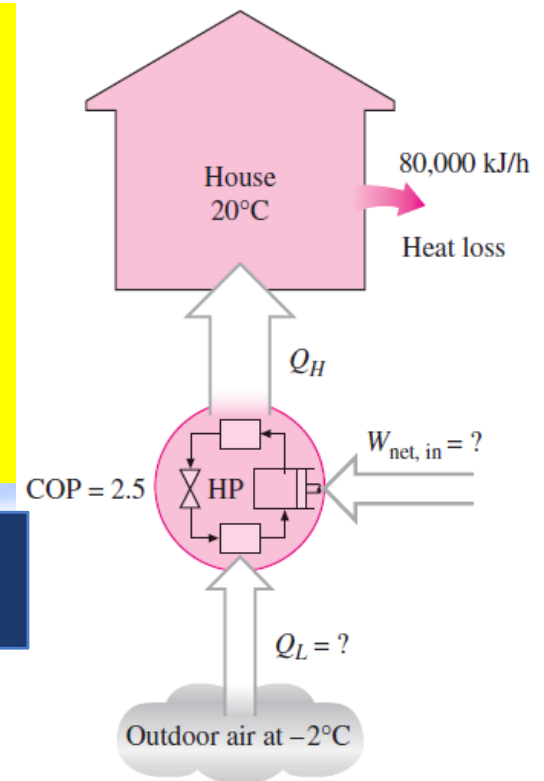
**Assumptions** Steady operating conditions exist.

- (a) The power consumed by this heat pump

$$\dot{W}_{\text{net, in}} = \frac{\dot{Q}_H}{\text{COP}_{\text{HP}}} = \frac{80,000 \text{ kJ/h}}{2.5} = \mathbf{32,000 \text{ kJ/h}} \text{ (or 8.9 kW)}$$

- (b) The house is losing heat at a rate of 80,000 kJ/h. If the house is to be maintained at a constant temperature of 20°C, the heat pump must deliver heat to the house at the same rate, that is, at a rate of 80,000 kJ/h. Then the rate of heat transfer from the outdoor becomes

$$\dot{Q}_L = \dot{Q}_H - \dot{W}_{\text{net, in}} = (80,000 - 32,000) \text{ kJ/h} = \mathbf{48,000 \text{ kJ/h}}$$



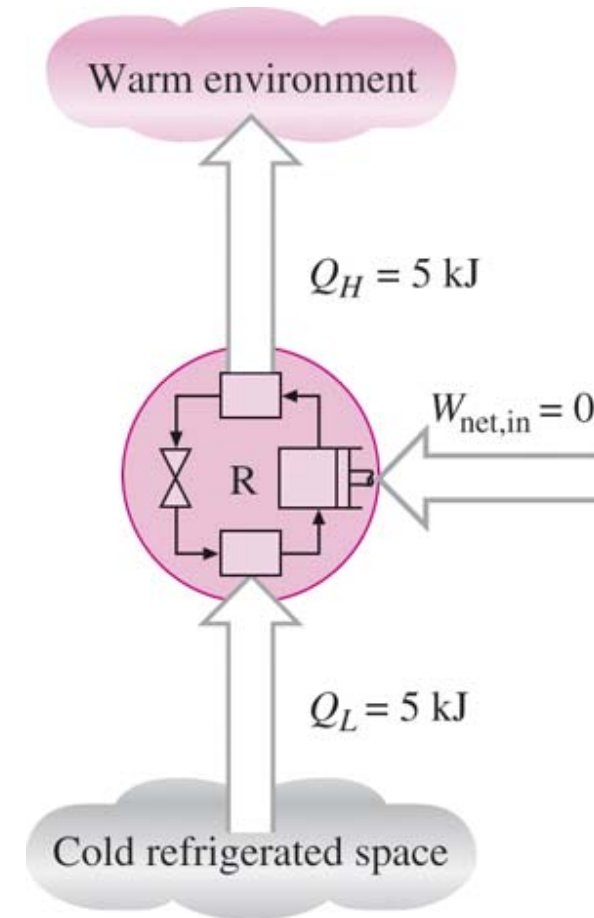
# The Second Law of Thermodynamics: Clausius Statement

It is impossible to construct a device that operates in a cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body.

*It states that a refrigerator cannot operate unless its compressor is driven by an external power source, such as an electric motor.*

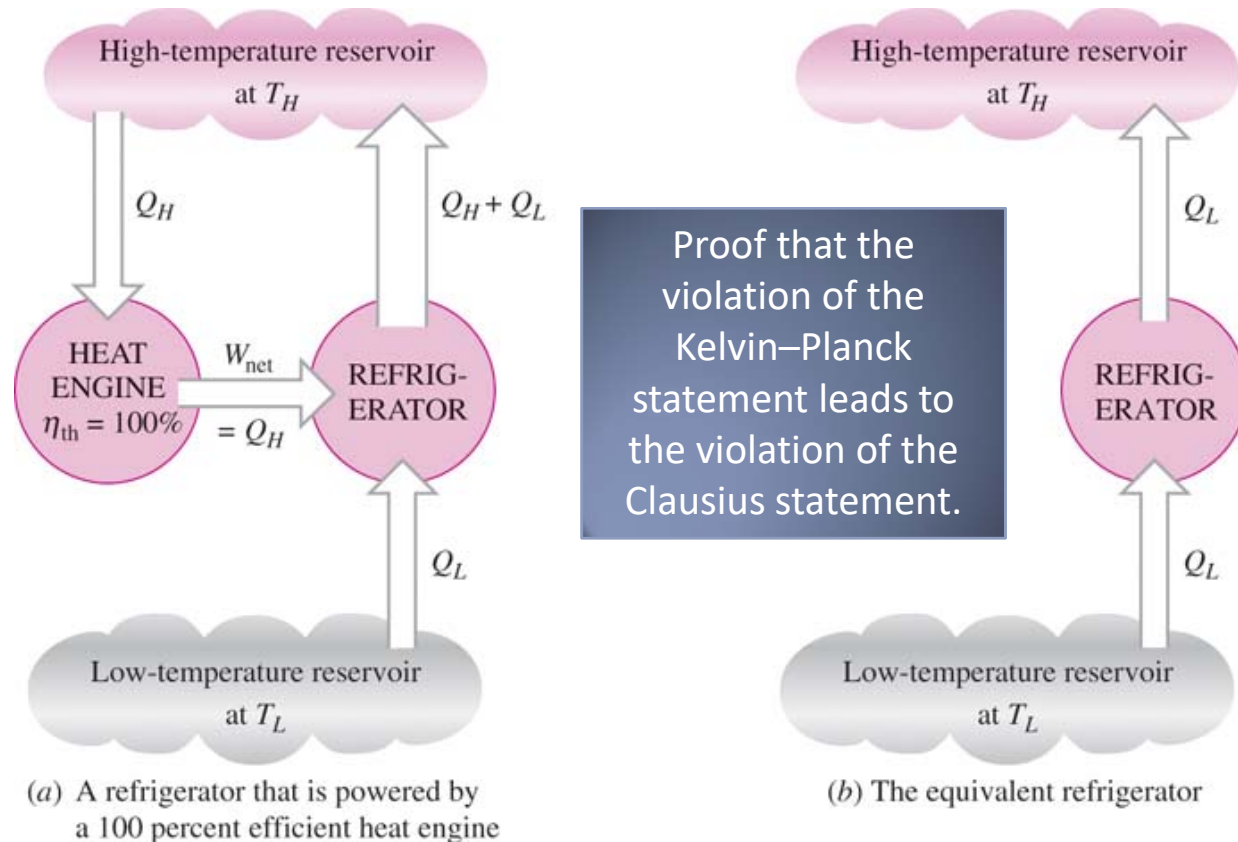
This way, the net effect on the surroundings involves the consumption of some energy in the form of work, in addition to the transfer of heat from a colder body to a warmer one.

To date, no experiment has been conducted that contradicts the second law, and this should be taken as sufficient proof of its validity.



A refrigerator that violates the Clausius statement of the second law.

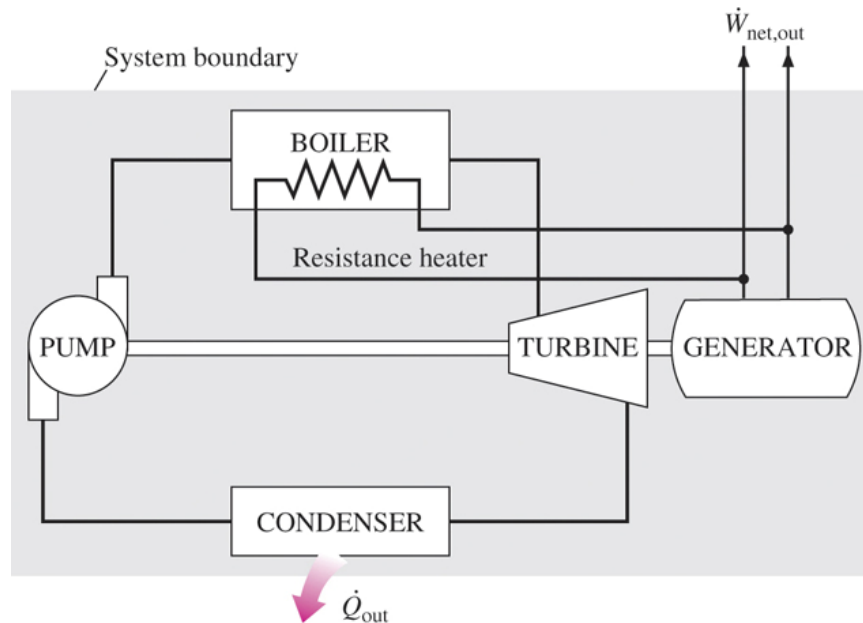
# Equivalence of the two Statements



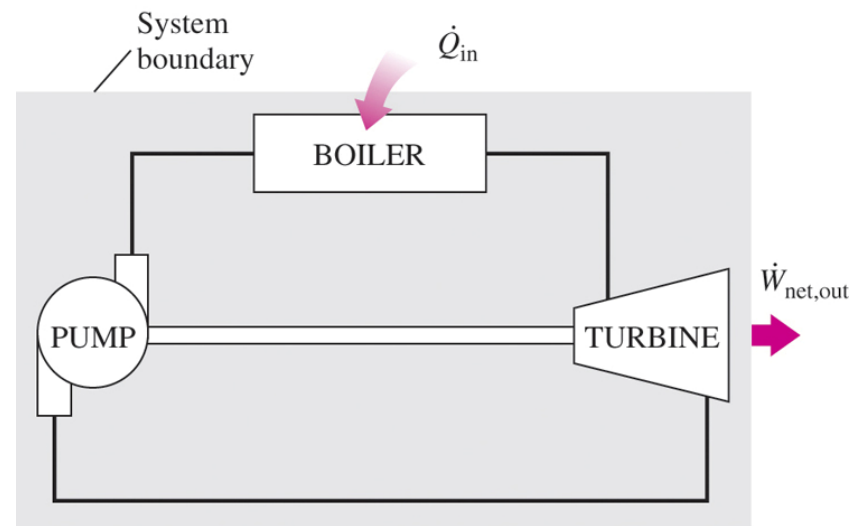
The Kelvin–Planck and the Clausius statements are equivalent in their consequences, and either statement can be used as the expression of the second law of thermodynamics.

Any device that violates the Kelvin–Planck statement also violates the Clausius statement, and vice versa.

# PERPETUAL-MOTION MACHINES



A perpetual-motion machine that violates the first law (PMM1).



A perpetual-motion machine that violates the second law of thermodynamics (PMM2).

**Perpetual-motion machine:** Any device that violates the first or the second law.

A device that violates the first law (by *creating* energy) is called a **PMM1**.

A device that violates the second law is called a **PMM2**.

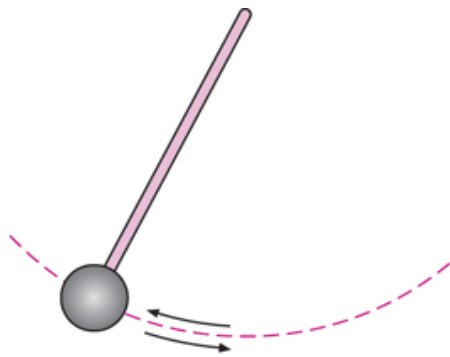
Despite numerous attempts, no perpetual-motion machine is known to have worked.

***If something sounds too good to be true, it probably is.***

# REVERSIBLE AND IRREVERSIBLE PROCESSES

**Reversible process:** A process that can be reversed without leaving any trace on the surroundings.

**Irreversible process:** A process that is not reversible.

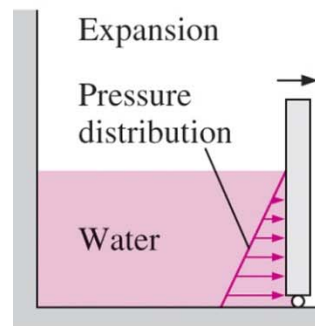


(a) Frictionless pendulum

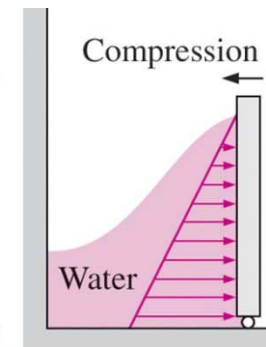
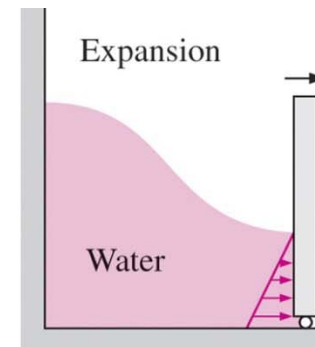
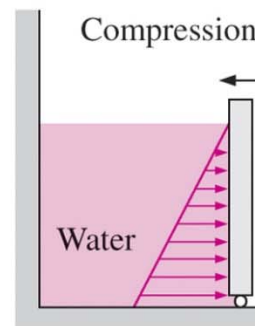
- All the processes occurring in nature are irreversible.
- **Why are we interested in reversible processes?**
- (1) they are easy to analyze and (2) they serve as idealized models (theoretical limits) to which actual processes can be compared.
- Some processes are more irreversible than others.
- We try to approximate reversible processes. **Why?**



(b) Quasi-equilibrium expansion and compression of a gas



(a) Slow (reversible) process



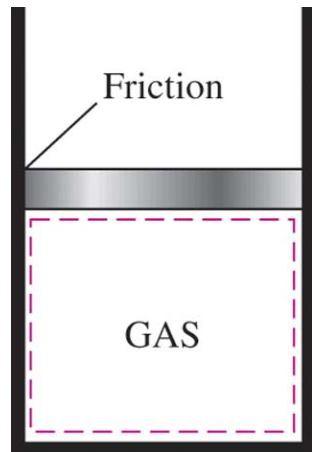
(b) Fast (irreversible) process

Two familiar reversible processes.

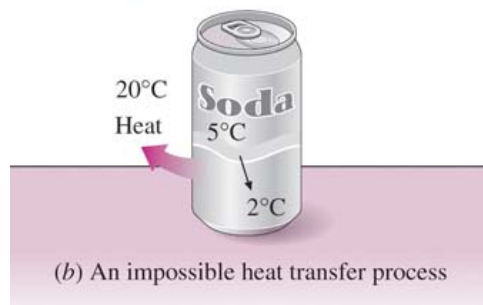
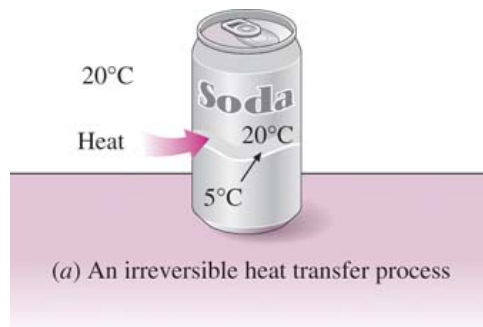
Reversible processes deliver the most and consume the least work.

# Irreversibilities

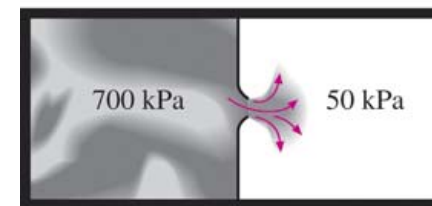
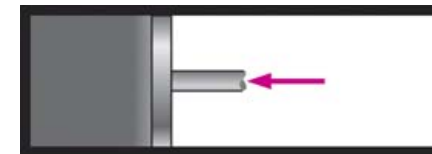
- The factors that cause a process to be irreversible are called **irreversibilities**.
- They include **friction**, **unrestrained expansion**, **mixing of two fluids**, **heat transfer across a finite temperature difference**, **electric resistance**, **inelastic deformation of solids**, and **chemical reactions**.
- The presence of any of these effects renders a process irreversible.



Friction renders a process irreversible.



(a) Heat transfer through a temperature difference is irreversible, and (b) the reverse process is impossible.

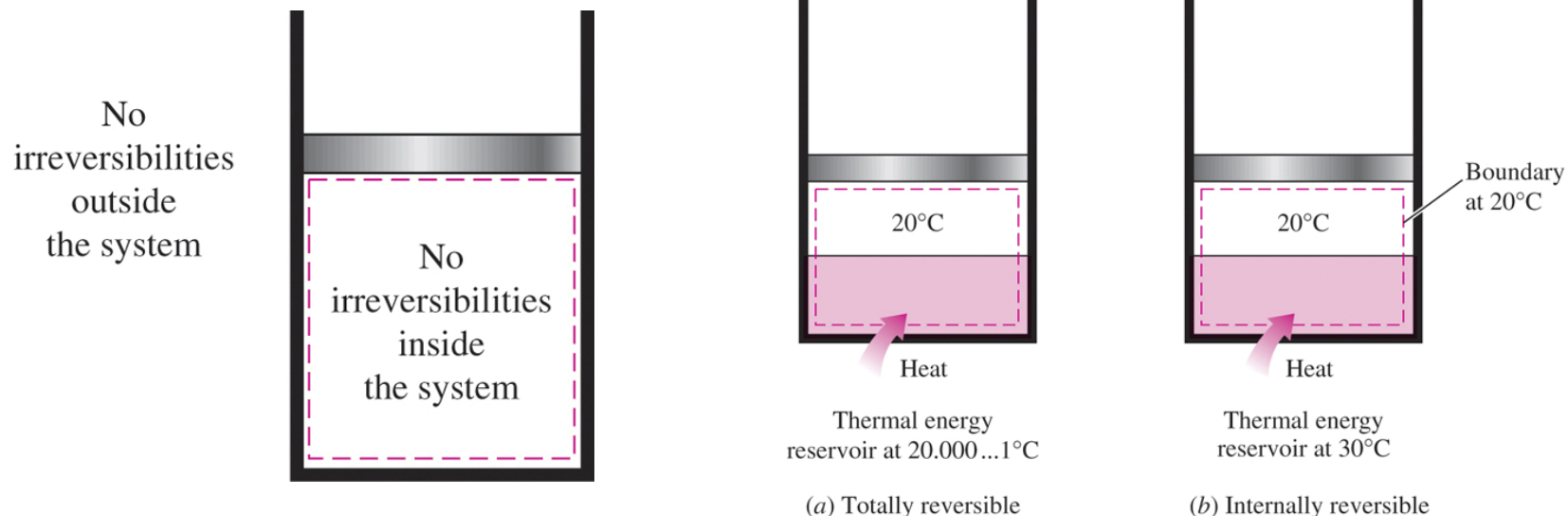


Irreversible compression and expansion processes.



# Internally and Externally Reversible Processes

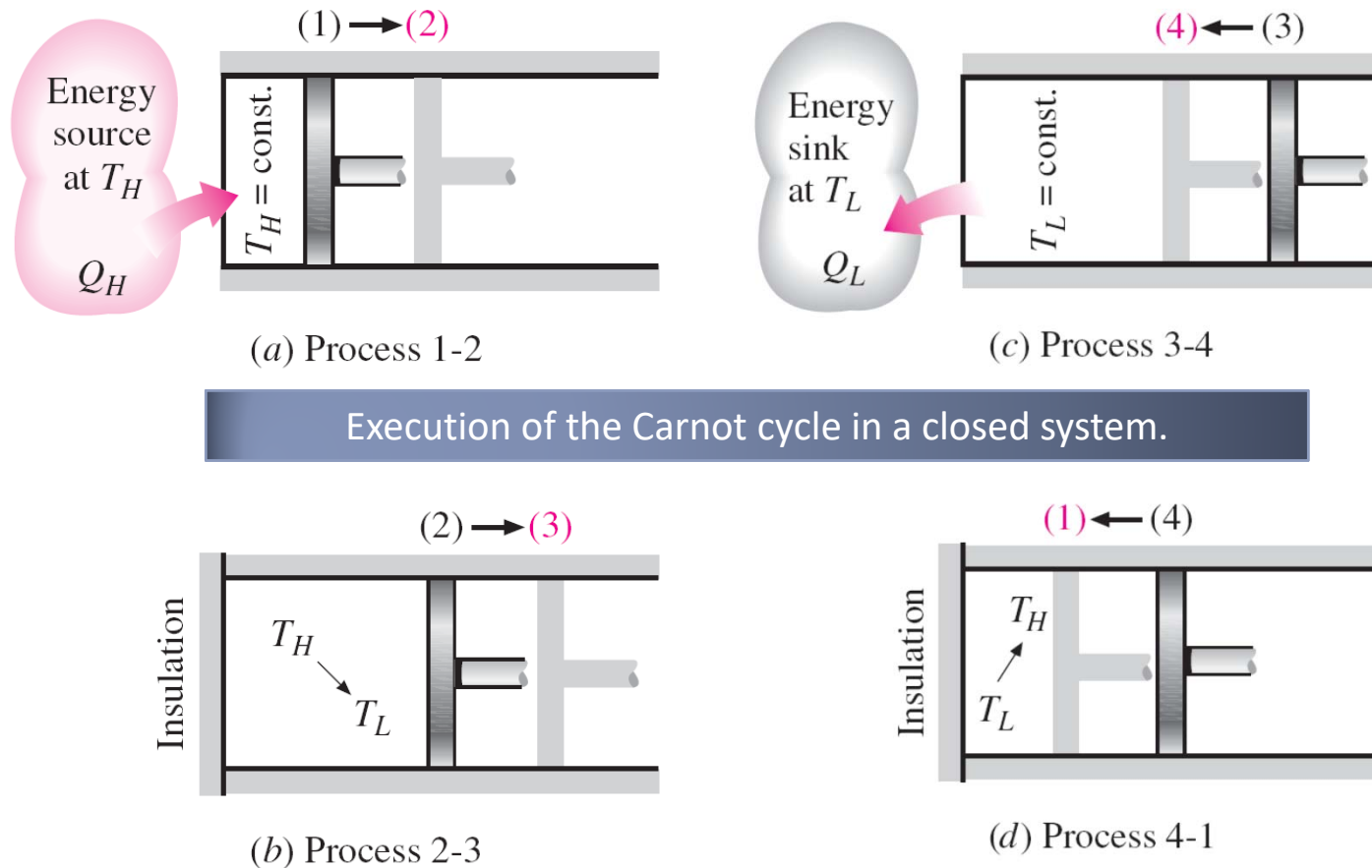
- **Internally reversible process:** If no irreversibilities occur within the boundaries of the system during the process.
- **Externally reversible:** If no irreversibilities occur outside the system boundaries.
- **Totally reversible process:** It involves no irreversibilities within the system or its surroundings.
- A totally reversible process involves no heat transfer through a finite temperature difference, no nonquasi-equilibrium changes, and no friction or other dissipative effects.



A reversible process involves no internal and external irreversibilities

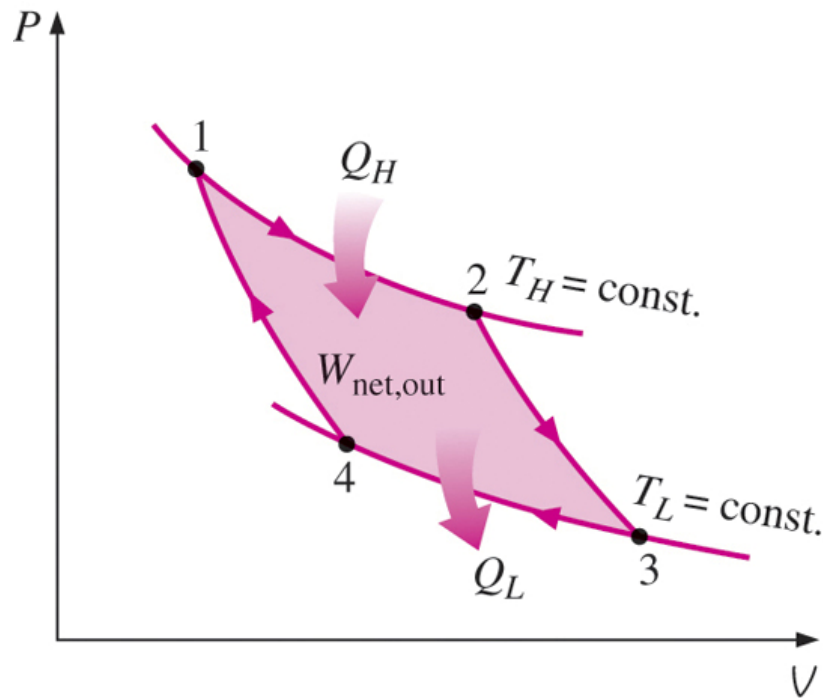
Totally and internally reversible heat transfer processes.

# THE CARNOT CYCLE

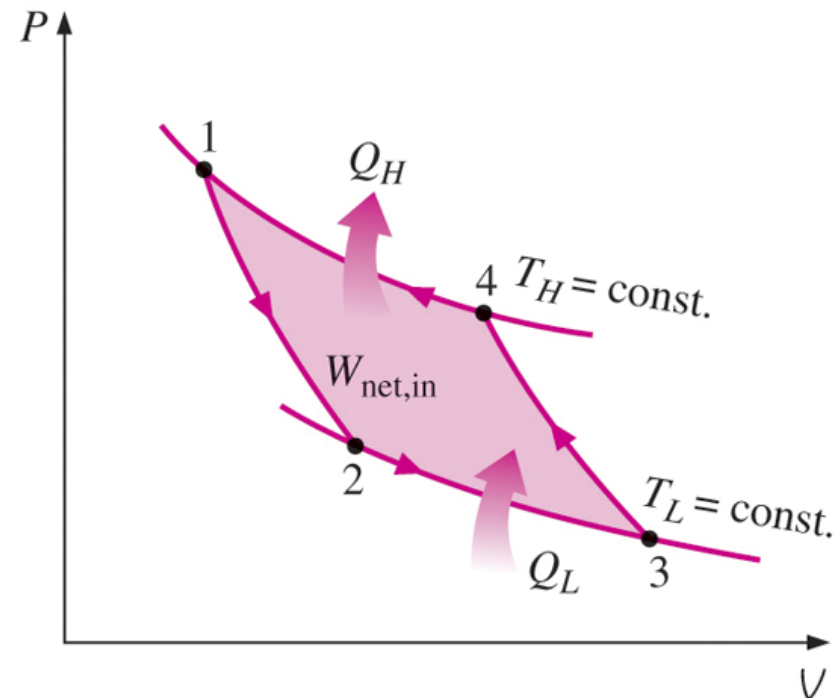


Reversible Isothermal Expansion (process 1-2,  $T_H = \text{constant}$ )  
 Reversible Adiabatic Expansion (process 2-3, temperature drops from  $T_H$  to  $T_L$ )  
 Reversible Isothermal Compression (process 3-4,  $T_L = \text{constant}$ )  
 Reversible Adiabatic Compression (process 4-1, temperature rises from  $T_L$  to  $T_H$ )

# The Reversed Carnot Cycle

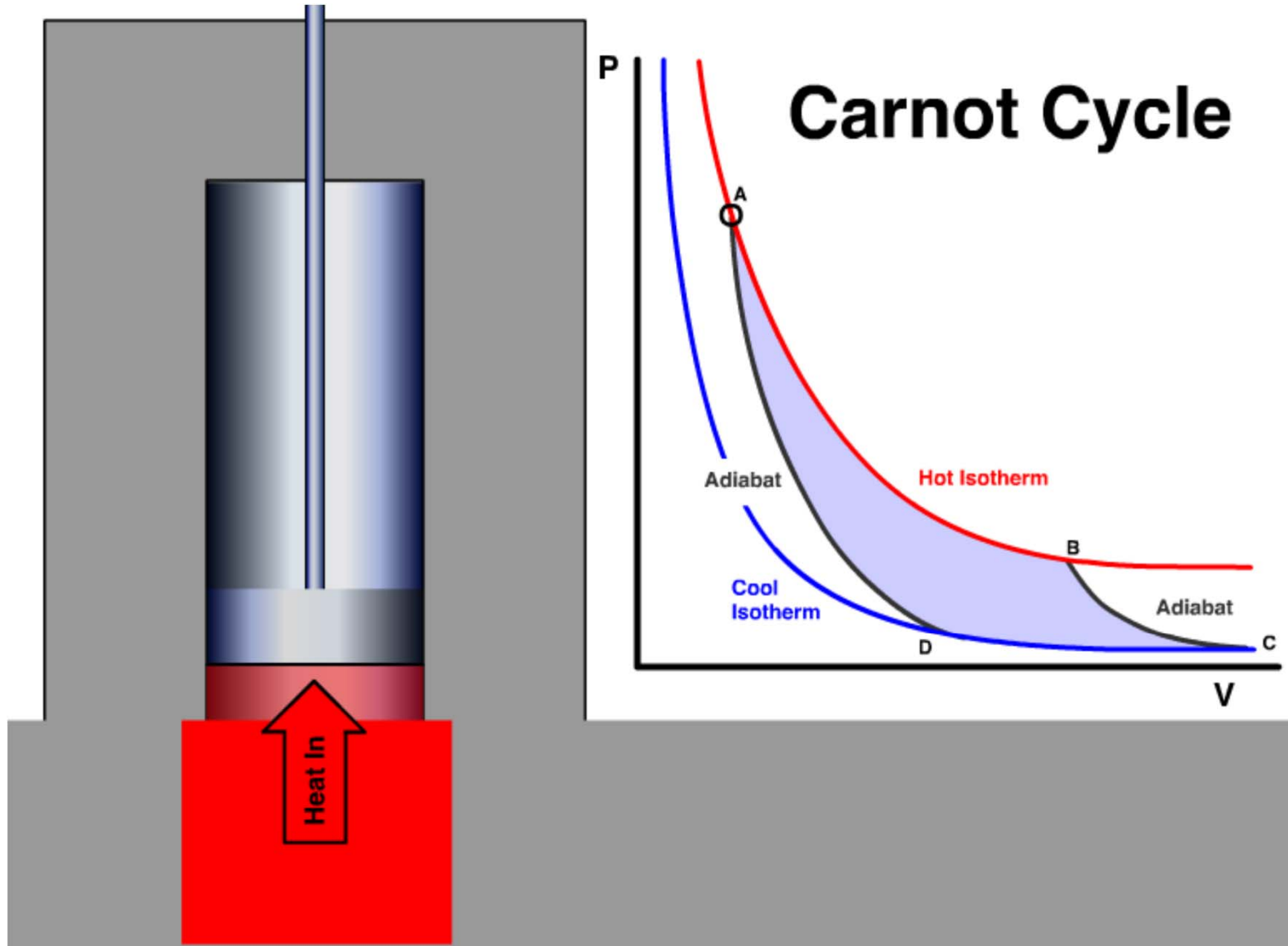


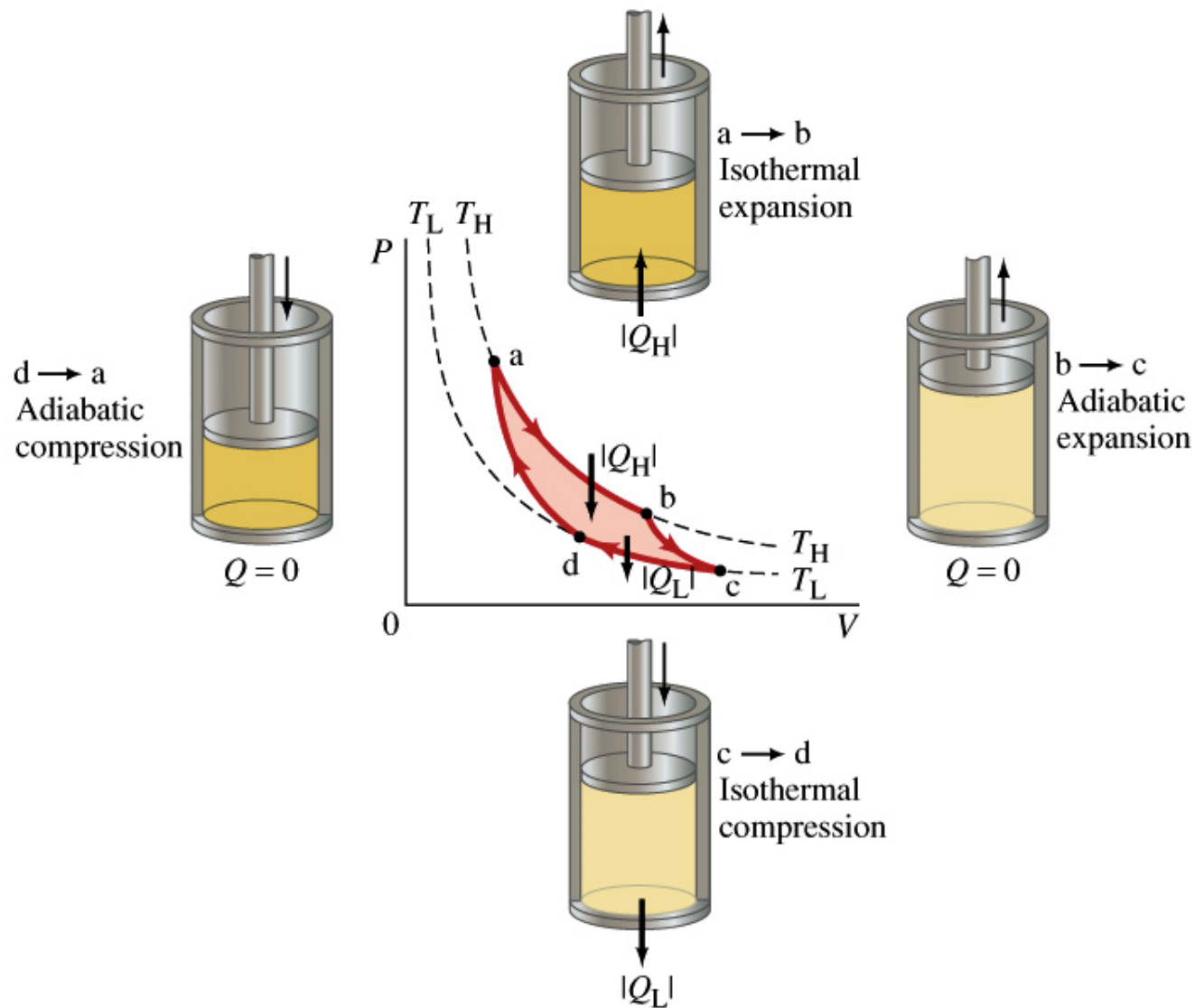
P-V diagram of the Carnot cycle.

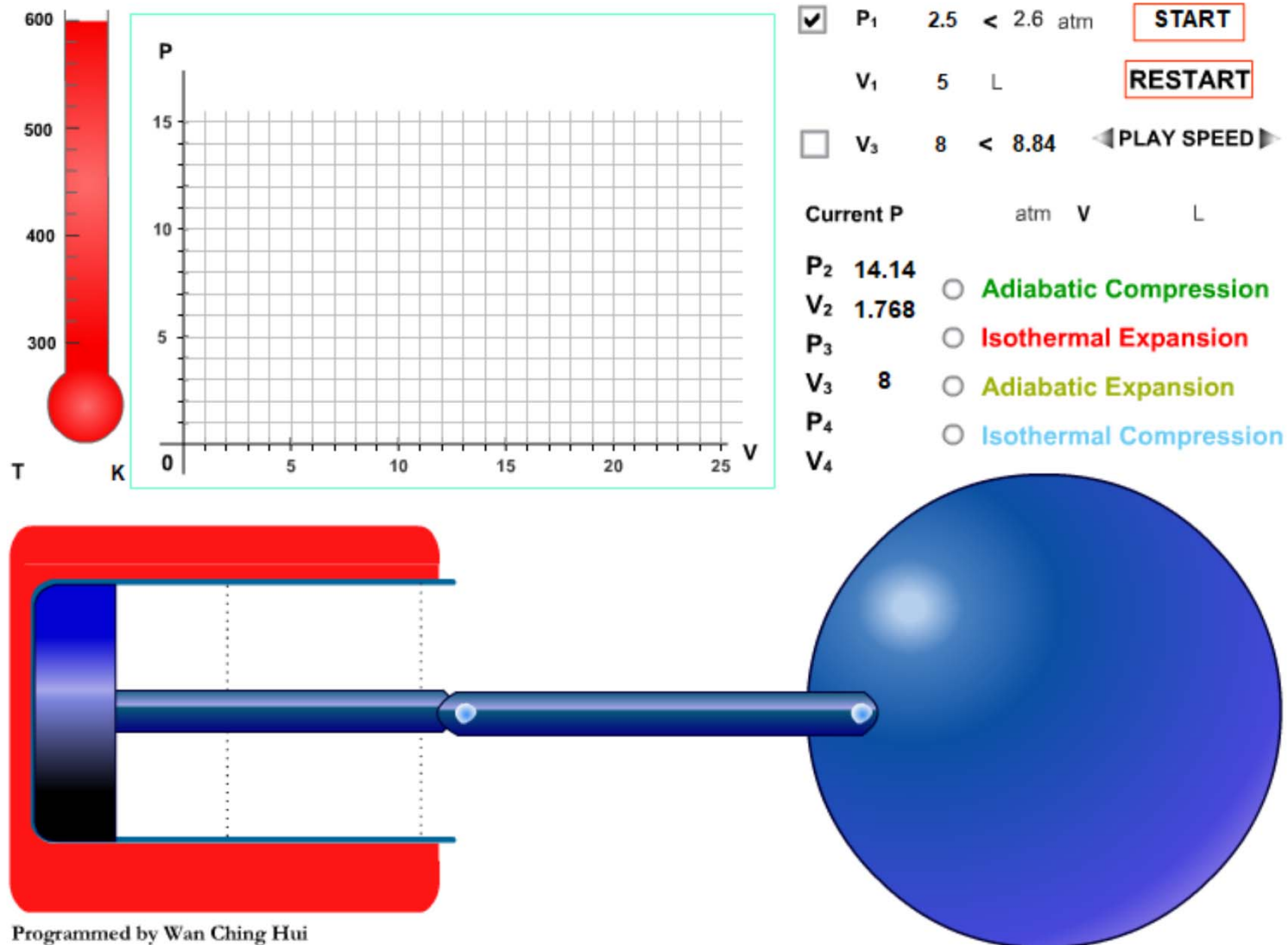


P-V diagram of the reversed Carnot cycle.

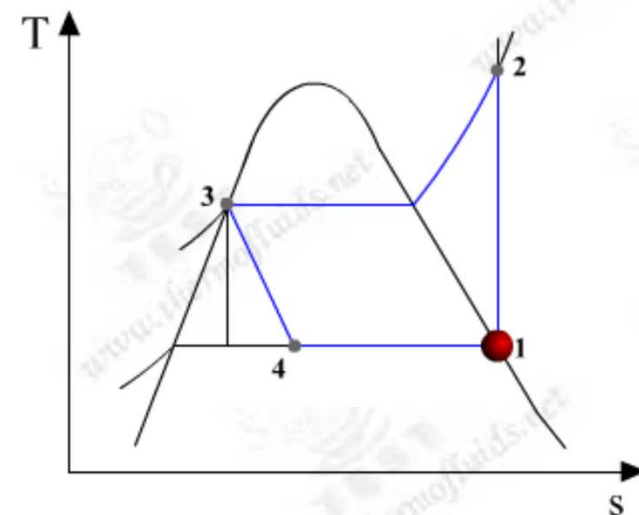
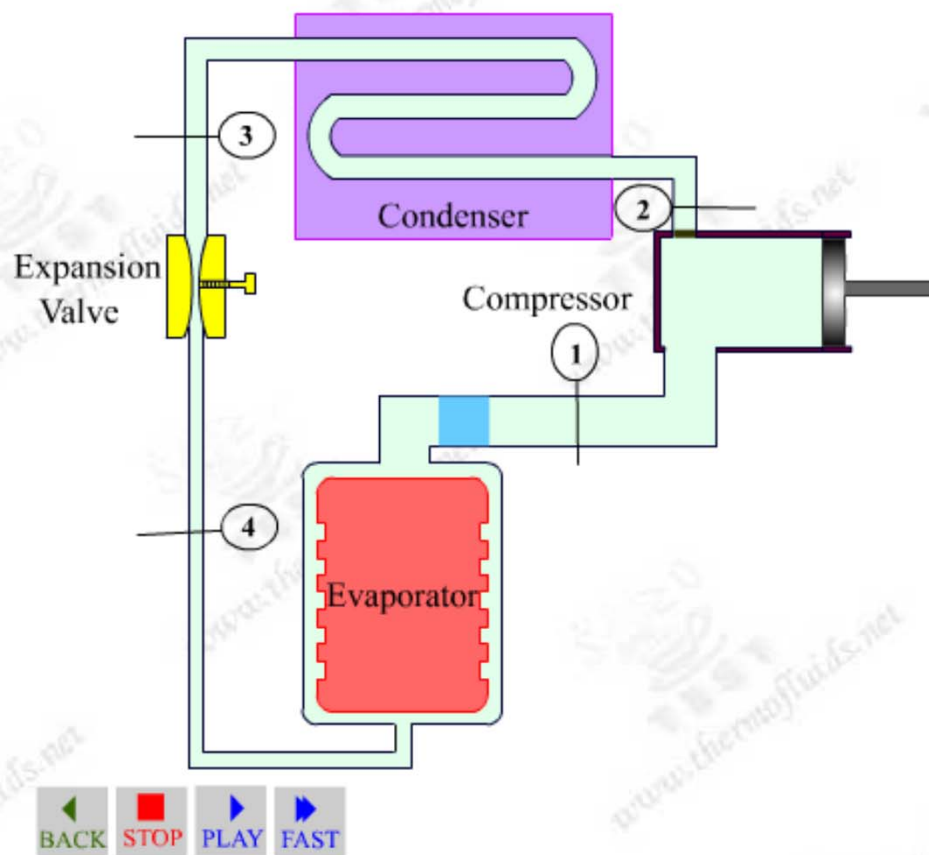
The Carnot heat-engine cycle is a totally reversible cycle. Therefore, all the processes that comprise it can be *reversed*, in which case it becomes the **Carnot refrigeration cycle**.

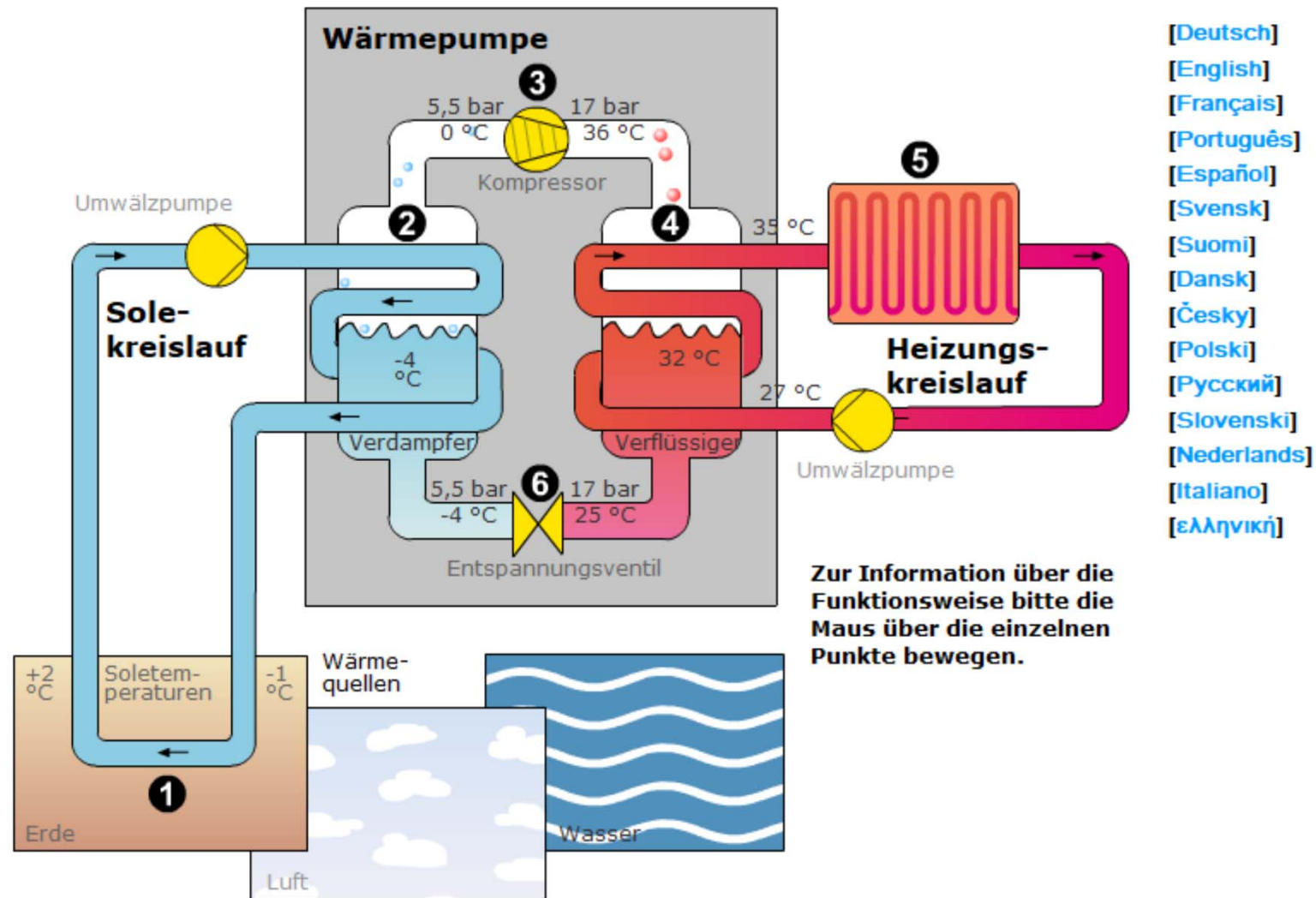












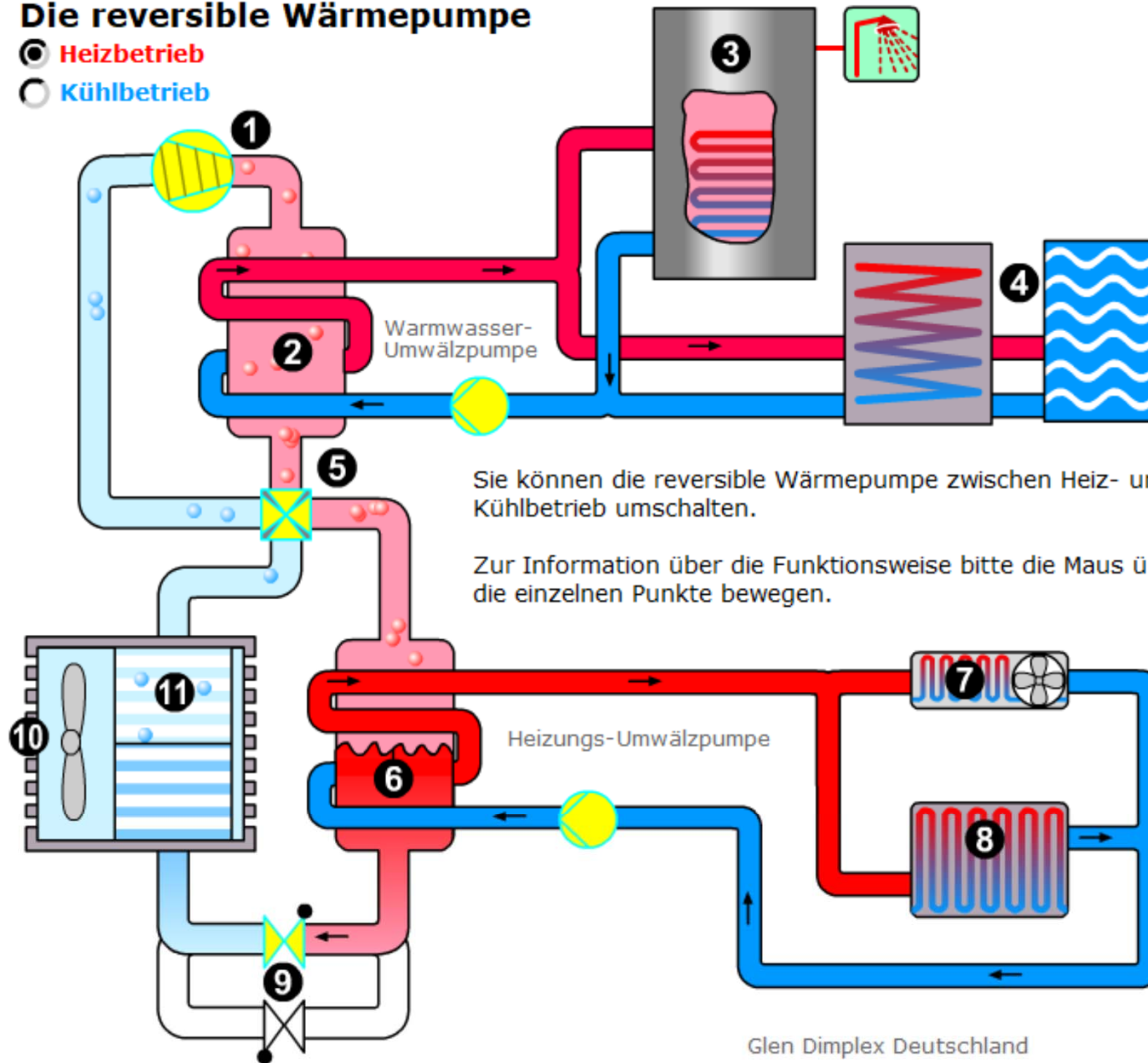
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Glen Dimplex Deutschland

## Die reversible Wärmepumpe

● Heizbetrieb

○ Kühlbetrieb



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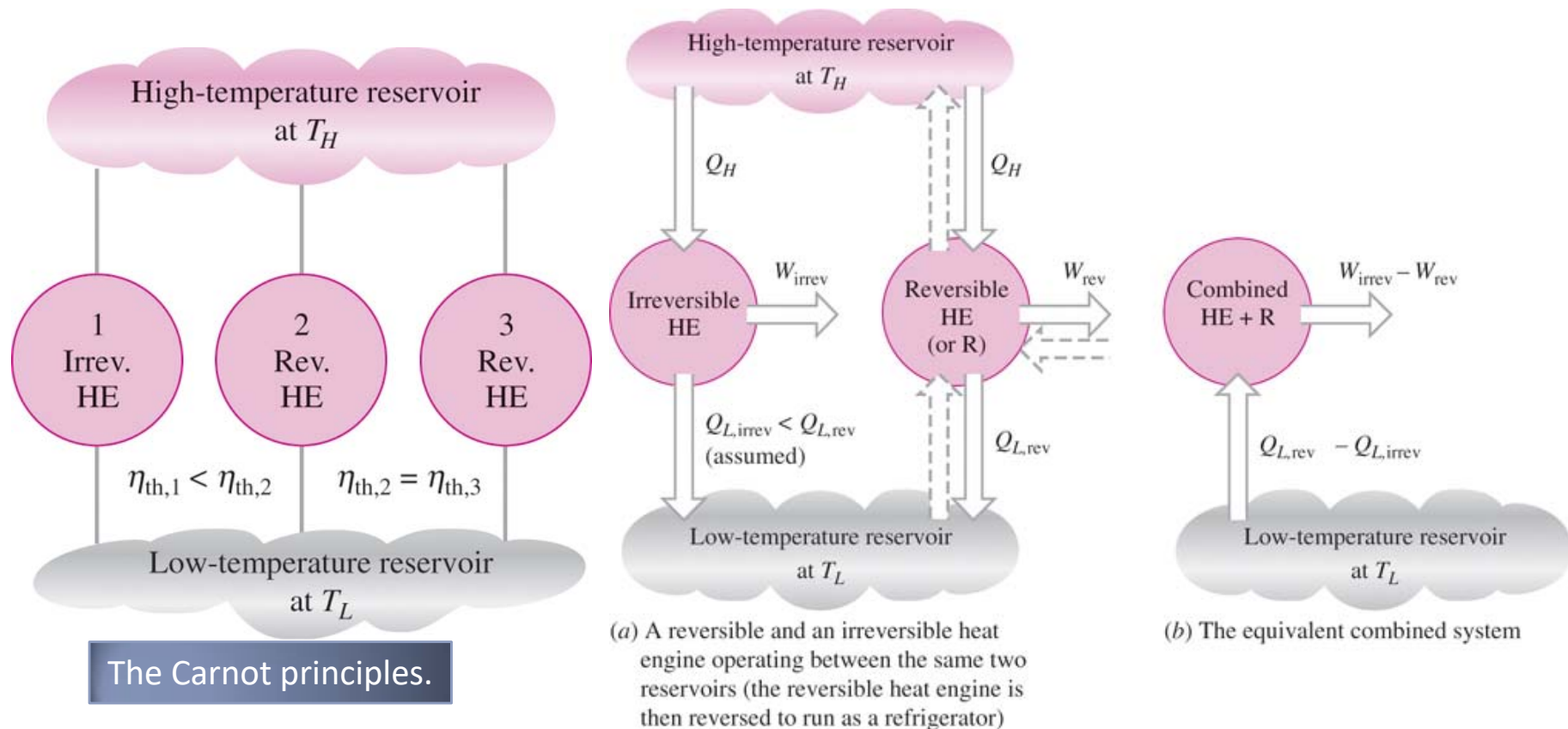
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# THE CARNOT PRINCIPLES



The Carnot principles.

(a) A reversible and an irreversible heat engine operating between the same two reservoirs (the reversible heat engine is then reversed to run as a refrigerator)

(b) The equivalent combined system

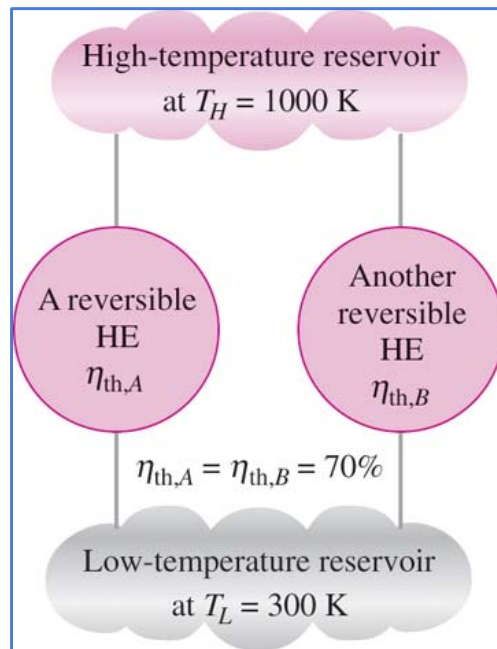
Proof of the first Carnot principle.

1. The efficiency of an irreversible heat engine is always less than the efficiency of a reversible one operating between the same two reservoirs.
2. The efficiencies of all reversible heat engines operating between the same two reservoirs are the same.

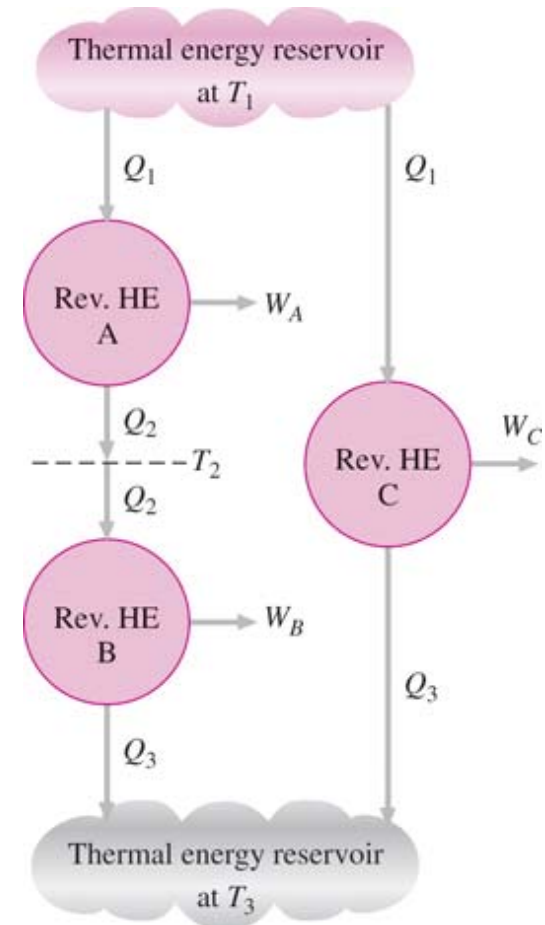
# THE THERMODYNAMIC TEMPERATURE SCALE

A temperature scale that is independent of the properties of the substances that are used to measure temperature is called a **thermodynamic temperature scale**.

Such a temperature scale offers great conveniences in thermodynamic calculations.



All reversible heat engines operating between the same two reservoirs have the same efficiency.



to develop the thermodynamic temperature scale.

$$\eta_{\text{th, rev}} = g(T_H, T_L)$$

$$\frac{Q_H}{Q_L} = f(T_H, T_L)$$

Since the heat input to engine C is the same as the heat input to the combined engines A and B, both systems must reject the same amount of heat.

Applying to all three engines separately,

$$\frac{Q_1}{Q_2} = f(T_1, T_2), \quad \frac{Q_2}{Q_3} = f(T_2, T_3), \quad \text{and} \quad \frac{Q_1}{Q_3} = f(T_1, T_3)$$

$$\frac{Q_1}{Q_3} = \frac{Q_1}{Q_2} \frac{Q_2}{Q_3}$$

$$f(T_1, T_3) = f(T_1, T_2) \cdot f(T_2, T_3)$$

$$f(T_1, T_2) = \frac{\phi(T_1)}{\phi(T_2)} \quad \text{and} \quad f(T_2, T_3) = \frac{\phi(T_2)}{\phi(T_3)} \quad \frac{Q_1}{Q_3} = f(T_1, T_3) = \frac{\phi(T_1)}{\phi(T_3)}$$

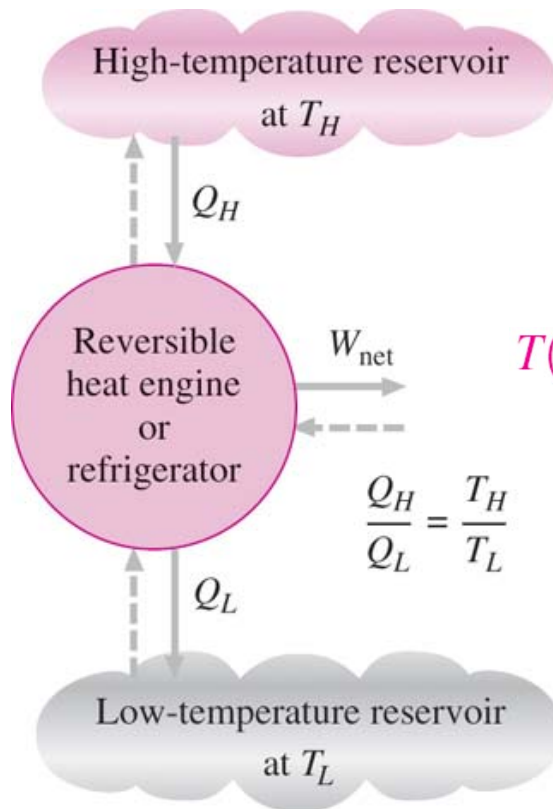
For a reversible heat engine operating between two reservoirs at  $T_H$  and  $T_L$

$$\frac{Q_H}{Q_L} = \frac{\phi(T_H)}{\phi(T_L)}$$

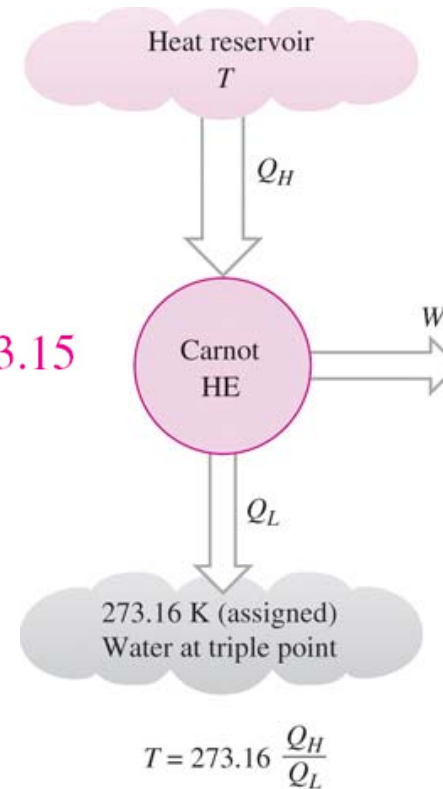
$$\left( \frac{Q_H}{Q_L} \right)_{\text{rev}} = \frac{T_H}{T_L}$$

This temperature scale is called the **Kelvin scale**, and the temperatures on this scale are called **absolute temperatures**.





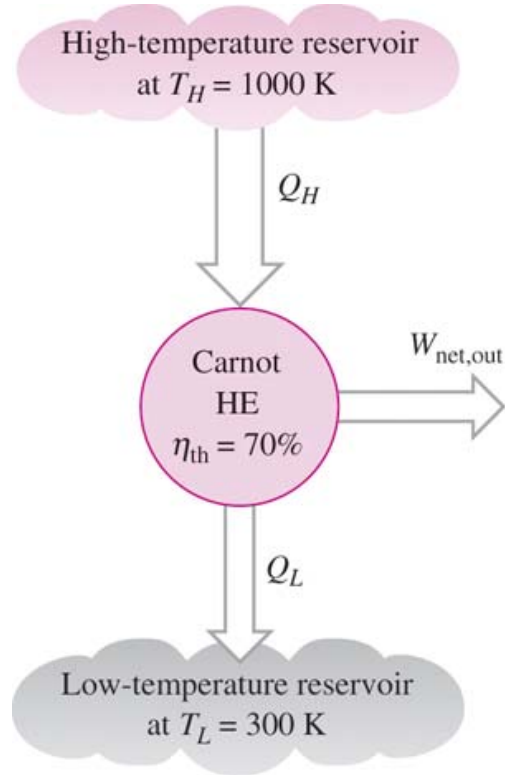
$$T(^{\circ}\text{C}) = T(\text{K}) - 273.15$$



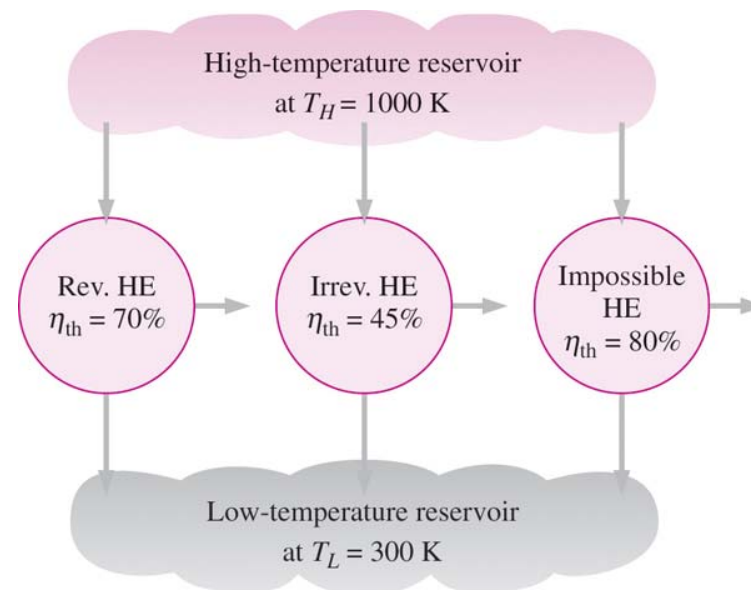
For reversible cycles, the heat transfer ratio  $Q_H/Q_L$  can be replaced by the absolute temperature ratio  $T_H/T_L$ .

A conceptual experimental setup to determine thermodynamic temperatures on the Kelvin scale by measuring heat transfers  $Q_H$  and  $Q_L$ .

# THE CARNOT HEAT ENGINE



The Carnot heat engine is the most efficient of all heat engines operating between the same high- and low-temperature reservoirs.



No heat engine can have a higher efficiency than a reversible heat engine operating between the same high- and low-temperature reservoirs.

$$\eta_{th} \begin{cases} < \eta_{th, rev} & \text{irreversible heat engine} \\ = \eta_{th, rev} & \text{reversible heat engine} \\ > \eta_{th, rev} & \text{impossible heat engine} \end{cases}$$

Any heat engine  $\eta_{th} = 1 - \frac{Q_L}{Q_H}$

Carnot heat engine  $\eta_{th, rev} = 1 - \frac{T_L}{T_H}$

### Example 6-5

A Carnot heat engine receives 500 kJ of heat per cycle from a high-temperature source at 652°C and rejects heat to a low-temperature sink at 30°C. Determine (a) the thermal efficiency of this Carnot engine and (b) the amount of heat rejected to the sink per cycle.

**Solution** The heat supplied to a Carnot heat engine is given. The thermal efficiency and the heat rejected are to be determined.

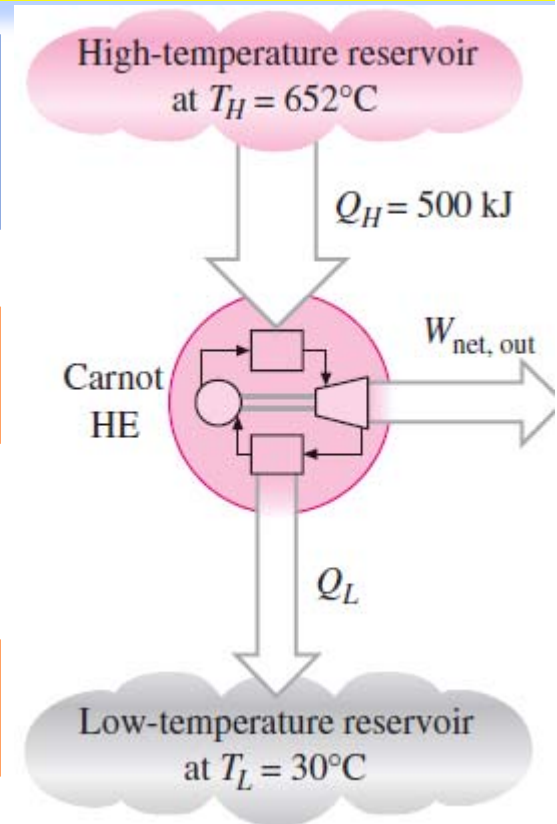
(a) The Carnot heat engine is a reversible heat engine, and so its efficiency can be determined

$$\eta_{\text{th}, C} = \eta_{\text{th}, \text{rev}} = 1 - \frac{T_L}{T_H} = 1 - \frac{(30 + 273) \text{ K}}{(652 + 273) \text{ K}} = \mathbf{0.672}$$

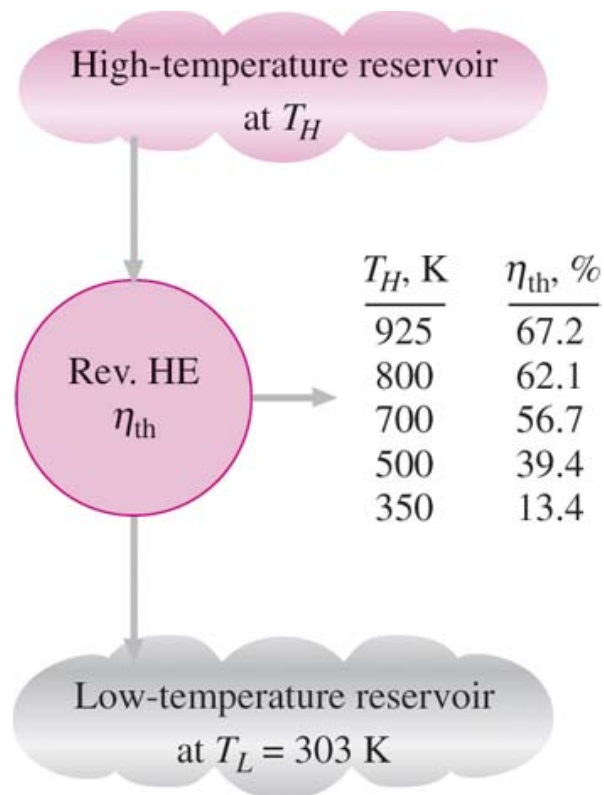
b) The amount of heat rejected  $Q_L$  by this reversible heat engine is easily determined

$$Q_{L, \text{rev}} = \frac{T_L}{T_H} Q_{H, \text{rev}} = \frac{(30 + 273) \text{ K}}{(652 + 273) \text{ K}} (500 \text{ kJ}) = \mathbf{164 \text{ kJ}}$$

**Discussion** Note that this Carnot heat engine rejects to a low-temperature sink 164 kJ of the 500 kJ of heat it receives during each cycle



# The Quality of Energy



The fraction of heat that can be converted to work as a function of source temperature.

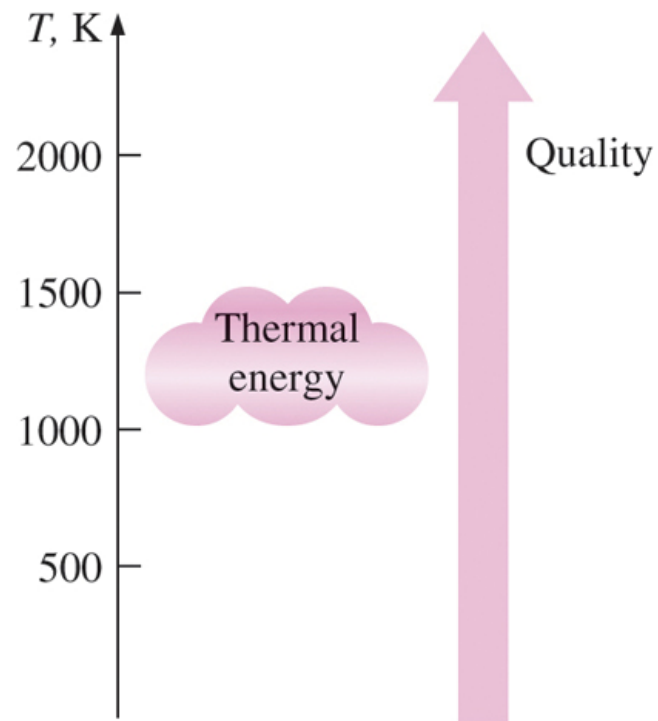
$$\eta_{th,rev} = 1 - \frac{T_L}{T_H}$$

The Carnot heat engine in Example 6–5 receives heat from a source at 925 K and converts 67.2 % of it to work while rejecting the rest (32.8 %) to a sink at 303 K.

Now let us examine how the thermal efficiency varies with the source temperature when the sink temperature is held constant.

Can we use °C unit for temperature here?

How do you increase the thermal efficiency of a Carnot heat engine? How about for actual heat engines?

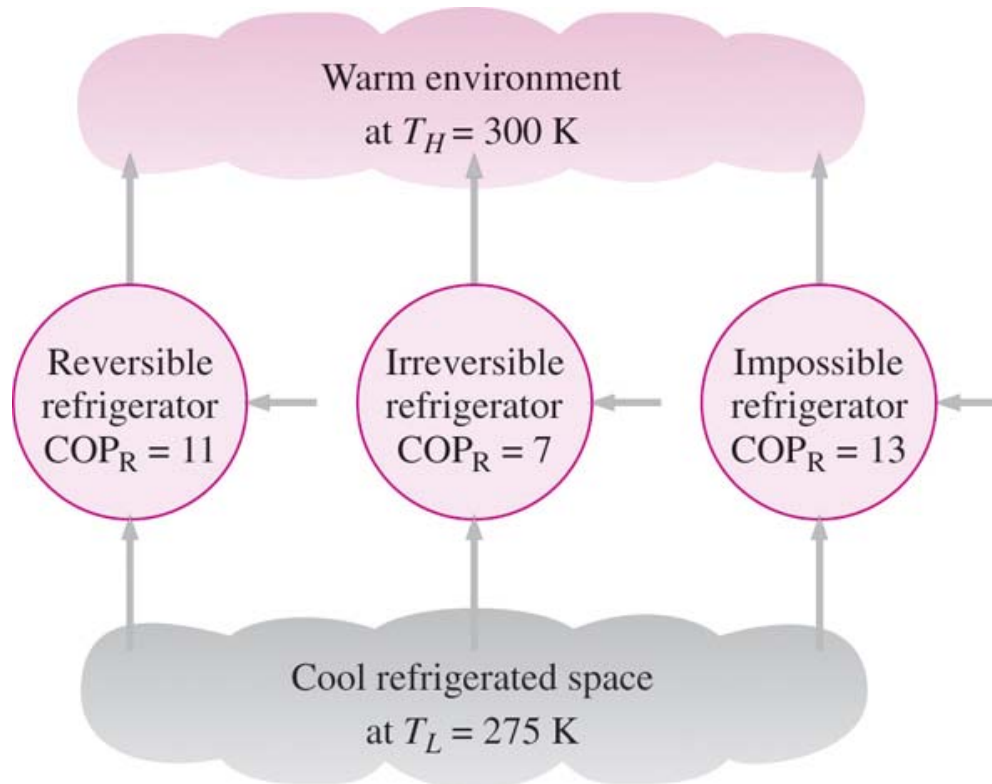


The higher the temperature of the thermal energy, the higher its quality.

*more of the high-temperature thermal energy can be converted to work. Therefore, the higher the temperature, the higher the quality of the energy*

# THE CARNOT REFRIGERATOR AND HEAT PUMP

A refrigerator or a heat pump that operates on the reversed Carnot cycle is called a **Carnot refrigerator**, or a **Carnot heat pump**.



No refrigerator can have a higher COP than a reversible refrigerator operating between the same temperature limits.

Any refrigerator or heat pump

$$\text{COP}_R = \frac{1}{Q_H/Q_L - 1}$$

$$\text{COP}_{\text{HP}} = \frac{1}{1 - Q_L/Q_H}$$

Carnot refrigerator or heat pump

$$\text{COP}_{\text{HP,rev}} = \frac{1}{1 - T_L/T_H}$$

$$\text{COP}_{R,\text{rev}} = \frac{1}{T_H/T_L - 1}$$

How do you increase the COP of a Carnot refrigerator or heat pump? How about for actual ones?



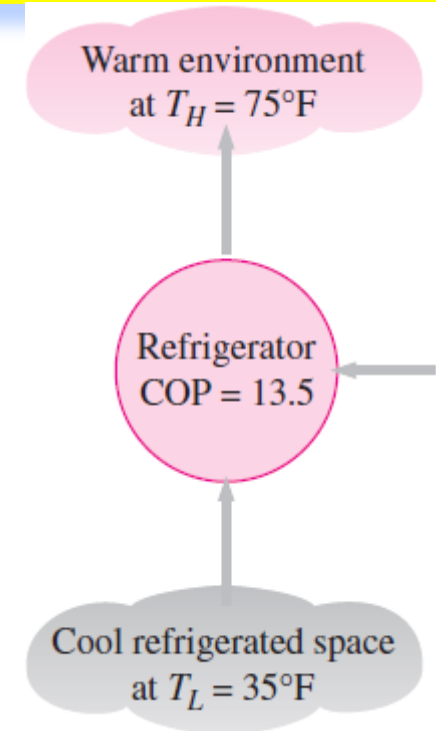
### Example 6-6

An inventor claims to have developed a refrigerator that maintains the refrigerated space at 2°C while operating in a room where the temperature is 25°C and that has a COP of 13.5. Is this claim reasonable?

**Solution** An extraordinary claim made for the performance of a refrigerator is to be evaluated.

**Assumptions** Steady operating conditions exist.

$$\begin{aligned}\text{COP}_{\text{SM, maks}} &= \text{COP}_{\text{SM, tr}} = \frac{1}{T_H/T_L - 1} \\ &= \frac{1}{(25 + 273)/(2 + 273) - 1} = 12.0\end{aligned}$$



**Discussion** This is the highest COP a refrigerator can have when removing heat from a cool medium at 2°C to a warmer medium at 25°C. Since the COP claimed by the inventor is above this maximum value, the claim is *false*.

### Example 6-7

A heat pump is to be used to heat a house during the winter. The house is to be maintained at 21°C at all times. The house is estimated to be losing heat at a rate of 135,000 kJ/h when the outside temperature drops to 5°C. Determine the minimum power required to drive this heat pump.

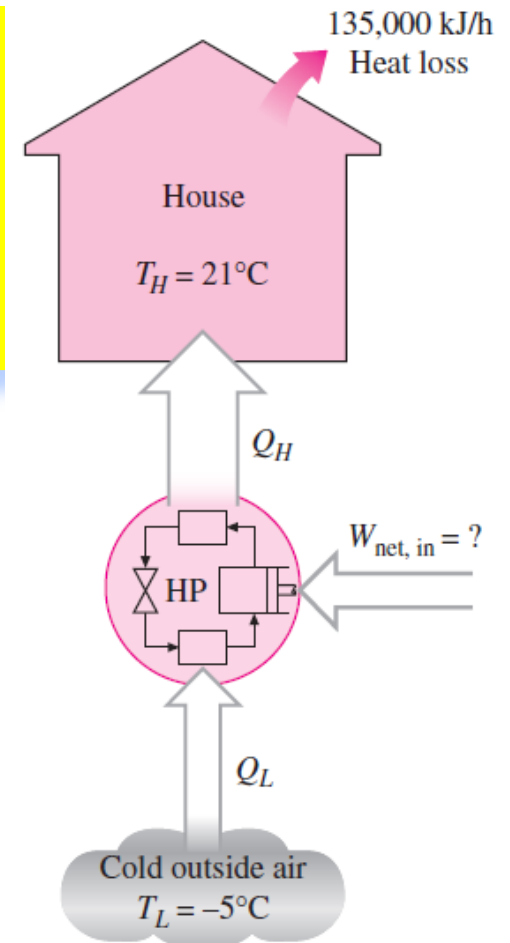
**Solution** A heat pump maintains a house at a fixed temperature. The required minimum power input to the heat pump is to be determined.

**Assumptions** Steady operating conditions exist.

$$\text{COP}_{\text{HP, rev}} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (-5 + 273 \text{ K})/(12 + 273 \text{ K})} = 11.3$$

$$\dot{W}_{\text{net, in}} = \frac{Q_H}{\text{COP}_{\text{HP}}} = \frac{37.5 \text{ kW}}{11.3} = 3.32 \text{ kW}$$

**Discussion** This heat pump can meet the heating requirements of this house by consuming electric power at a rate of 3.32 kW only. If this house were to be heated by electric resistance heaters instead, the power consumption would jump up 11.3 times to 37.5 kW. Notice that the heat pump does not create energy. It merely transports it from one medium (the cold outdoors) to another (the warm indoors).



# SUMMARY

- Introduction to the second law
- Thermal energy reservoirs
- Heat engines
  - Thermal efficiency
  - The 2<sup>nd</sup> law: Kelvin-Planck statement
- Refrigerators and heat pumps
  - Coefficient of performance (COP)
  - The 2<sup>nd</sup> law: Clausius statement
- Perpetual motion machines
- Reversible and irreversible processes
  - Irreversibilities, Internally and externally reversible processes
- The Carnot cycle
  - The reversed Carnot cycle
- The Carnot principles
- The thermodynamic temperature scale
- The Carnot heat engine
  - The quality of energy
- The Carnot refrigerator and heat pump

Thermodynamics: An Engineering Approach, 5<sup>th</sup> Edition  
Yunus A. Cengel, Michael A. Boles  
McGraw-Hill, 2008

# Chapter 7

## ENTROPY

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Prof. Dr. Ali PINARBAŞI

Yildiz Technical University  
Mechanical Engineering Department  
Yildiz, ISTANBUL

## 7-1 Entropy

A Special Case: Internally Reversible Isothermal Heat Transfer Processes

## 7-2 The Increase of Entropy Principle

Some Remarks about Entropy

## 7-3 Entropy Change of Pure Substances

## 7-4 Isentropic Processes

## 7-5 Property Diagrams Involving Entropy

## 7-6 What Is Entropy?

Entropy and Entropy Generation in Daily Life

## 7-7 The T ds Relations

## 7-8 Entropy Change of Liquids and Solids

## 7-9 The Entropy Change of Ideal Gases

Constant Specific Heats (Approximate Analysis)

Variable Specific Heats (Exact Analysis)

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

Variable Specific Heats (Exact Analysis)

Relative Pressure and Relative Specific Volume

## 7-10 Reversible Steady-Flow Work

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

## 7-11 Minimizing the Compressor Work

Multistage Compression with Intercooling

## 7-12 Isentropic Efficiencies of Steady-Flow Devices

Isentropic Efficiency of Turbines

Isentropic Efficiencies of Compressors and Pumps

Isentropic Efficiency of Nozzles

## 7-13 Entropy Balance

Entropy Change of a System,  $\Delta S_{\text{system}}$

Mechanisms of Entropy Transfer,  $S_{\text{in}}$  and  $S_{\text{out}}$

1 Heat Transfer

2 Mass Flow

Entropy Generation,  $S_{\text{gen}}$

Closed Systems

Control Volumes

Entropy Generation Associated with a Heat Transfer Process



# Objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

# ENTROPY

2nd law of thermodynamics leads to expressions that involve inequalities.

An irreversible (i.e., actual) heat engine, for example, is less efficient than a reversible one operating between the same two thermal energy reservoirs.

Likewise, an irreversible refrigerator or a heat pump has a lower coefficient of performance (COP) than a reversible one operating between the same temperature limits.

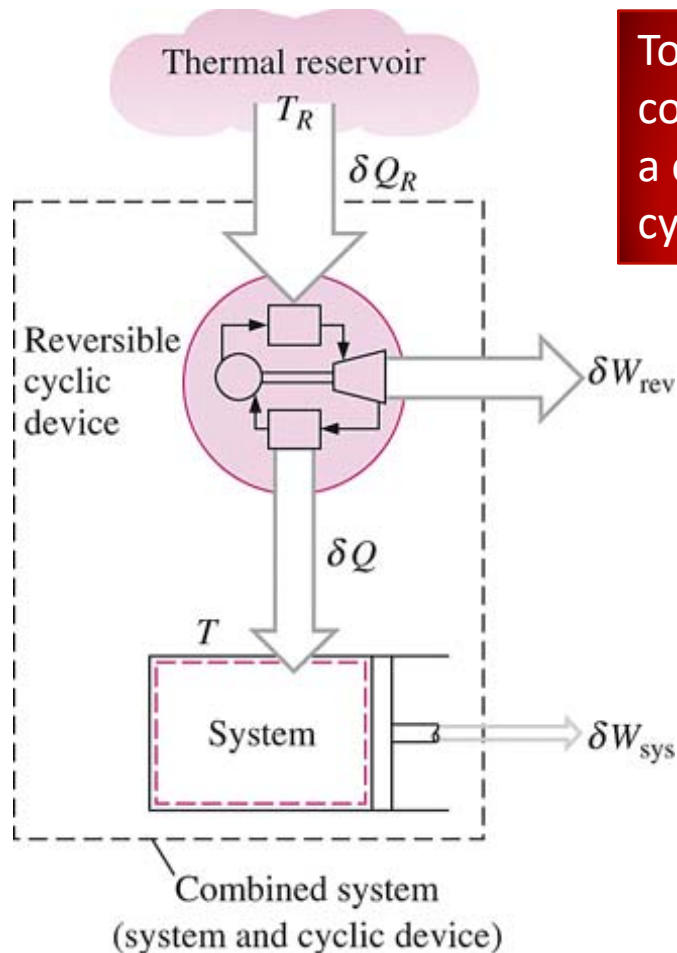
Another important inequality that has major consequences in thermodynamics is the **Clausius inequality**.

It was first stated by the German physicist R. J. E. Clausius (1822–1888), one of the founders of thermodynamics, and is expressed as

$$\oint \frac{\delta Q}{T} \leq 0$$

*That is, the cyclic integral of  $dQ/T$  is always less than or equal to zero.*

# ENTROPY



To demonstrate the validity of the Clausius inequality, consider a system connected to a thermal energy reservoir at a constant absolute temperature of  $T_R$  through a *reversible* cyclic device.

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\delta W_C = \delta Q_R - dE_C$$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T}$$

Eliminating  $dQ_R$  from the two relations above yields

$$\delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

Then the preceding relation becomes

$$W_C = T_R \oint \frac{\delta Q}{T}$$

Clausius inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\oint \left( \frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

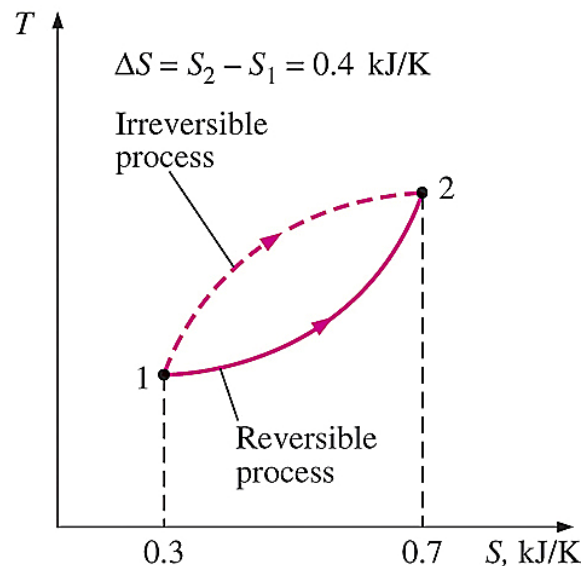
The system considered in the development of the Clausius inequality.

## Formal definition of entropy

$$dS = \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.



Notice that we have actually defined the **change** in entropy instead of entropy itself, just as we defined the change in energy instead of the energy itself when we developed the first-law relation.

The entropy change between two specified states is the same whether the process is reversible or irreversible.

## A Special Case: Internally Reversible Isothermal Heat Transfer Processes

$$\Delta S = \int_1^2 \left( \frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left( \frac{\delta Q}{T_0} \right)_{\text{int rev}} = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}}$$

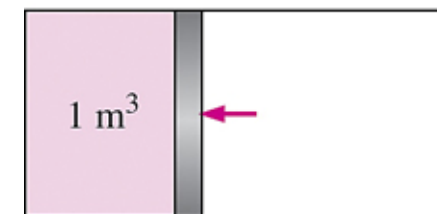
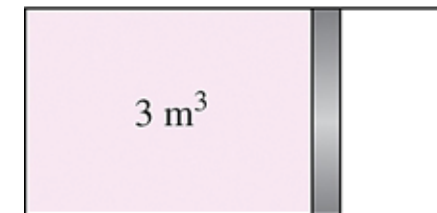
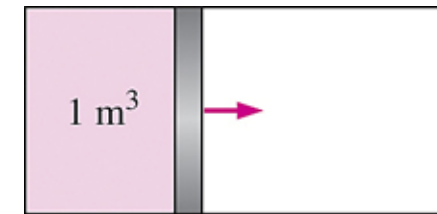
A quantity whose cyclic integral is zero (i.e., a property like volume)

$$\oint \left( \frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

Entropy is an extensive property of a system.

$$\Delta S = \frac{Q}{T_0}$$

This equation is particularly useful for determining the entropy changes of thermal energy reservoirs.

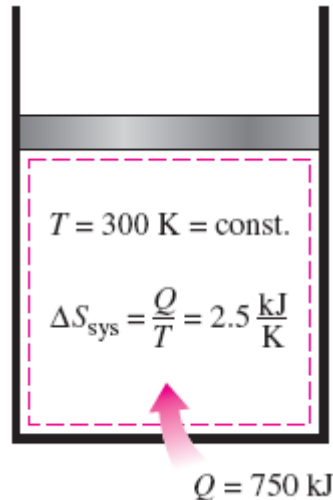


$$\oint dV = \Delta V_{\text{cycle}} = 0$$

The net change in volume (a property) during a cycle is always zero.

## EXAMPLE 7-1

A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.



**Solution** Heat is transferred to a liquid–vapor mixture of water in a piston–cylinder device at constant pressure. The entropy change of water is to be determined.

**Assumptions** No irreversibilities occur within the system boundaries during the process.

The system undergoes an internally reversible, isothermal process, and thus its entropy change can be determined directly from

$$\Delta S_{\text{sys, isothermal}} = \frac{Q}{T_{\text{sys}}} = \frac{750 \text{ kJ}}{300 \text{ K}} = 2.5 \text{ kJ/K}$$

**Discussion** Note that the entropy change of the system is positive, as expected, since heat transfer is *to* the system.

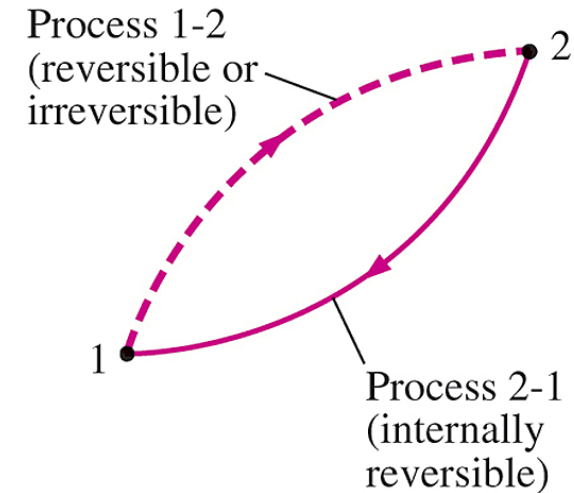


# THE INCREASE OF ENTROPY PRINCIPLE

Consider a cycle that is made up of two processes: process 1-2, which is arbitrary (reversible or irreversible), and process 2-1, which is internally reversible,

$$\oint \frac{\delta Q}{T} \leq 0 \quad \int_1^2 \frac{\delta Q}{T} + \int_2^1 \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0 \quad S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$



A cycle composed of a reversible and an irreversible process.

The equality holds for an internally reversible process and the inequality for an irreversible process.

$$dS \geq \frac{\delta Q}{T}$$

Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation  $S_{\text{gen}}$  is always a *positive* quantity or zero.  
Can the entropy of a system during a process decrease?

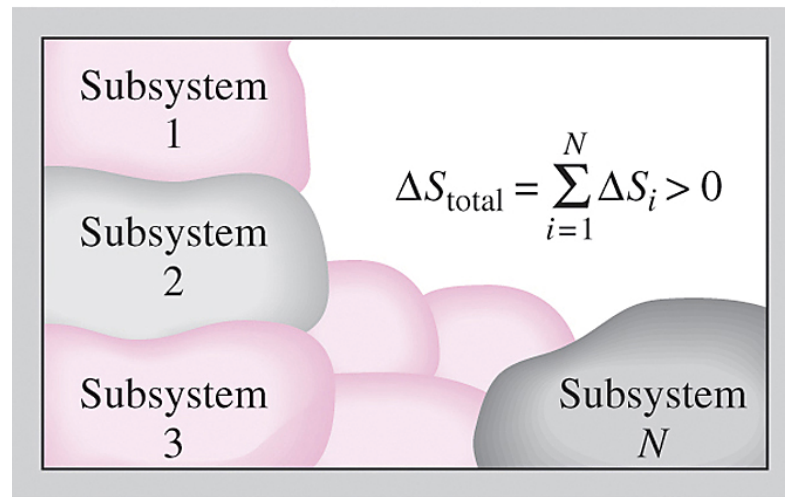
$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

*the entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant. In other words, it never decreases. This is known as **the increase of entropy principle**.*

$$\Delta S_{\text{isolated}} \geq 0$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

(Isolated)

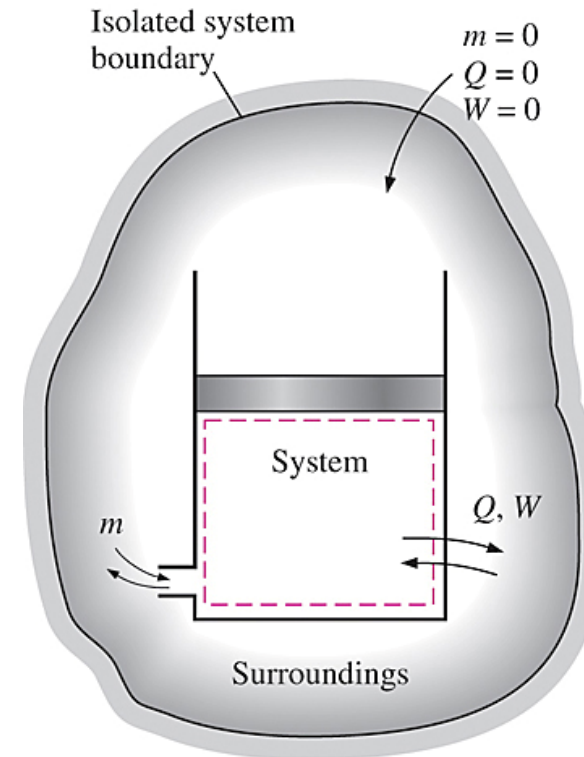


The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.

$$\Delta S_{\text{isolated}} \geq 0$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

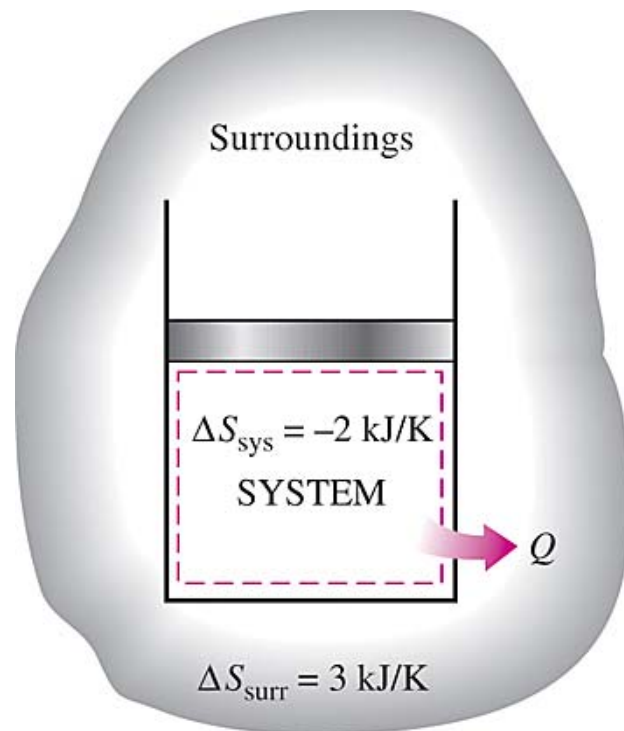
$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$



A system and its surroundings form an isolated system.

**The increase of entropy principle**

# Some Remarks about Entropy



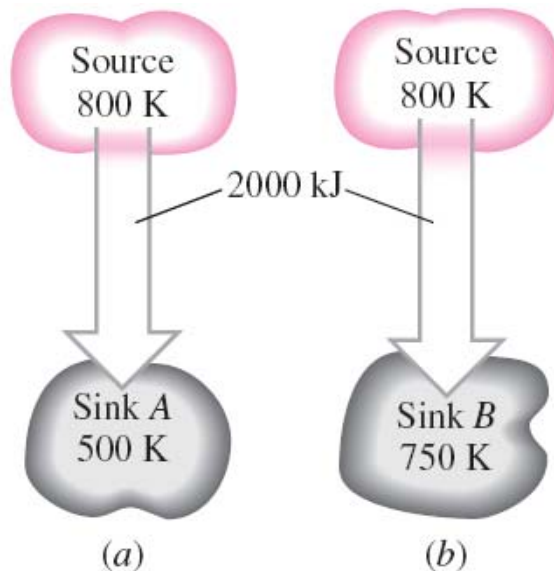
$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} = 1 \text{ kJ/K}$$

The entropy change of a system can be negative, but the entropy generation cannot.

1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is,  $S_{\text{gen}} \geq 0$ . A process that violates this principle is impossible.
2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

## EXAMPLE 7-2

A heat source at 800 K loses 2000 kJ of heat to a sink at (a) 500 K and (b) 750 K. Determine which heat transfer process is more irreversible.



**Solution** Heat is transferred from a heat source to two heat sinks at different temperatures. The heat transfer process that is more irreversible is to be determined.

**Analysis** Both cases involve heat transfer through a finite temperature difference, and therefore both are irreversible. The magnitude of the irreversibility associated with each process can be determined by calculating the total entropy change for each case. The total entropy change for a heat transfer process involving two reservoirs (a source and a sink) is the sum of the entropy changes of each reservoir since the two reservoirs form an adiabatic system.

(a) For the heat transfer process to a sink at 500 K:

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000 \text{ kJ}}{800 \text{ K}} = -2.5 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000 \text{ kJ}}{500 \text{ K}} = +4.0 \text{ kJ/K}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0) \text{ kJ/K} = \mathbf{+1.5 \text{ kJ/K}}$$

Therefore, 1.5 kJ/K of entropy is generated during this process. Noting that both reservoirs have undergone internally reversible processes, the entire entropy generation took place in the partition.

(b) Repeating the calculations in part (a) for a sink temperature of 750 K, we obtain

$$\Delta S_{\text{source}} = -2.5 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = +2.7 \text{ kJ/K}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = (-2.5 + 2.7) \text{ kJ/K} = \mathbf{+0.2 \text{ kJ/K}}$$

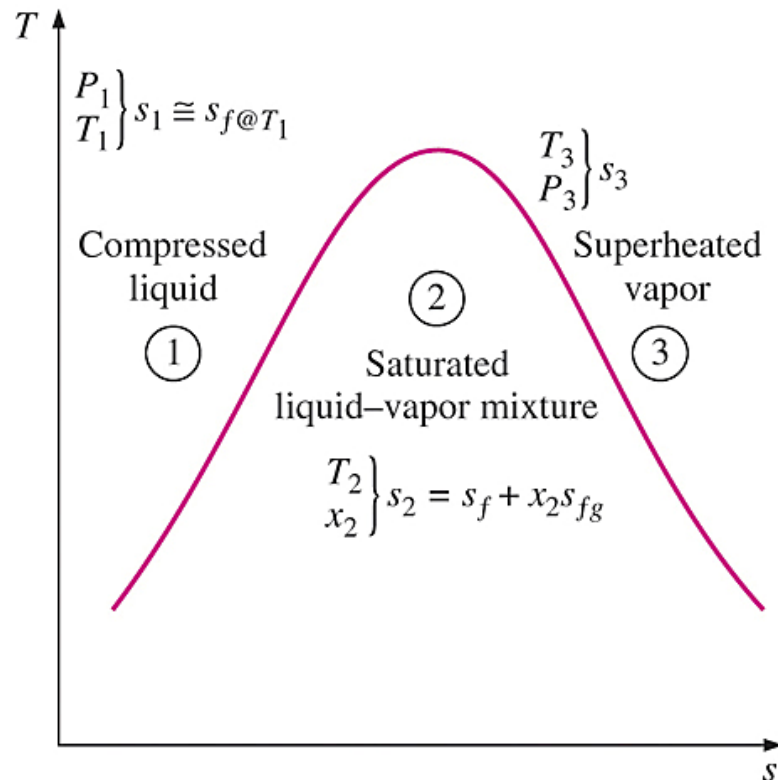
The total entropy change for the process in part (b) is smaller, and therefore it is less irreversible. This is expected since the process in (b) involves a smaller temperature difference and thus a smaller irreversibility.

**Discussion** The irreversibilities associated with both processes could be eliminated by operating a Carnot heat engine between the source and the sink. For this case it can be shown that  $\Delta S_{\text{total}} = 0$ .

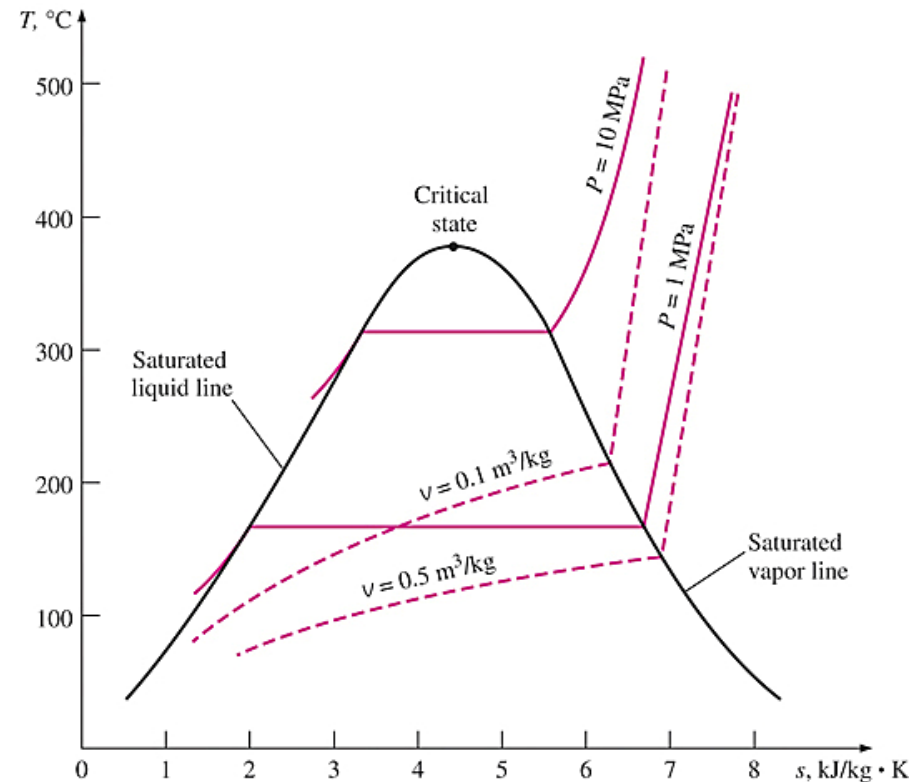


# ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.



The entropy of a pure substance is determined from the tables (like other properties).



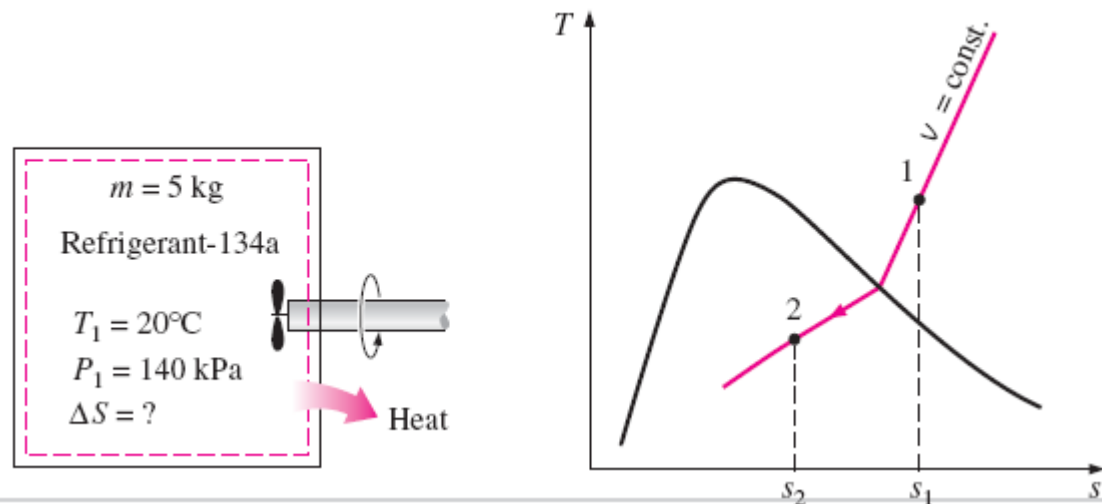
Schematic of the  $T$ - $s$  diagram for water.

Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \quad (\text{kJ/K})$$

### EXAMPLE 7-3

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.



**Solution** The refrigerant in a rigid tank is cooled while being stirred until the pressure drops to a specified value. The entropy change of the refrigerant is to be determined.

**Assumptions** The volume of the tank is constant and thus  $v_2 = v_1$ .

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 140 \text{ kPa} \\ T_1 = 20^\circ\text{C} \end{array} \right\} \quad \begin{array}{l} s_1 = 1.0624 \text{ kJ/kg} \cdot \text{K} \\ v_1 = 0.16544 \text{ m}^3/\text{kg} \end{array}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 100 \text{ kPa} \\ (v_2 = v_1) \end{array} \right\} \quad \begin{array}{l} v_f = 0.0007259 \text{ m}^3/\text{kg} \\ v_g = 0.19254 \text{ m}^3/\text{kg} \end{array}$$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.16544 - 0.0007259}{0.19254 - 0.0007259} = 0.859$$

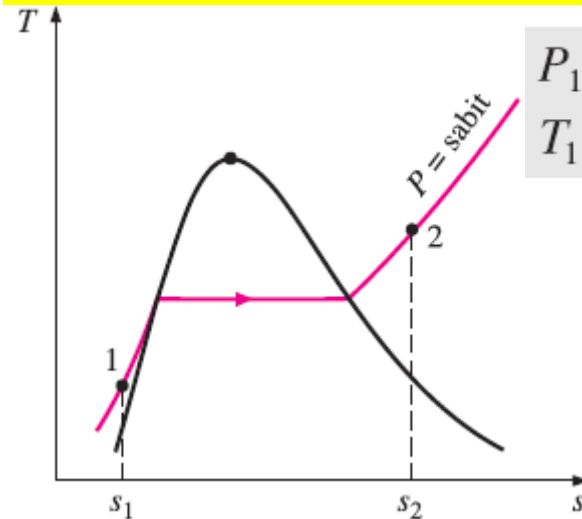
$$s_2 = s_f + x_2 s_{fg} = 0.07188 + (0.859)(0.87995) = 0.8278 \text{ kJ/kg} \cdot \text{K}$$

$$\begin{aligned}\Delta S &= m(s_2 - s_1) = (5 \text{ kg})(0.8278 - 1.0624) \text{ kJ/kg} \cdot \text{K} \\ &= -1.173 \text{ kJ/K}\end{aligned}$$

**Discussion** The negative sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the *entropy generation*  $S_{\text{gen}}$  that cannot be negative.

### EXAMPLE 7-4

A piston–cylinder device initially contains 1.5 kg of liquid water at 150 kPa and 20°C. The water is now heated at constant pressure by the addition of 4000 kJ of heat. Determine the entropy change of the water during this process.



$$\left. \begin{array}{l} P_1 = 150 \text{ kPa} \\ T_1 = 20^\circ\text{C} \end{array} \right\} \begin{array}{l} s_1 \cong s_{f@20^\circ\text{C}} = 0.2965 \text{ kJ/kg} \cdot \text{K} \\ h_1 \cong h_{f@20^\circ\text{C}} = 83.915 \text{ kJ/kg} \end{array}$$

$$\Delta U + W_b = \Delta H$$

$$Q_g - W_s = \Delta U$$

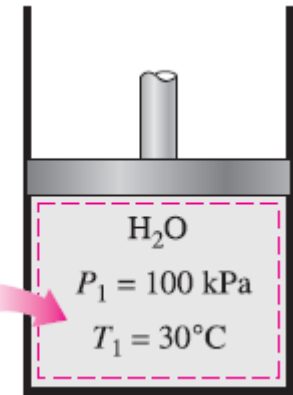
$$Q_g = \Delta H = m(h_2 - h_1)$$

$$4000 \text{ kJ} = (1.5 \text{ kg})(h_2 - 83.915 \text{ kJ/kg})$$

$$h_2 = 2750.6 \text{ kJ/kg}$$

$$\left. \begin{array}{l} P_2 = 150 \text{ kPa} \\ h_2 = 2750.6 \text{ kJ/kg} \end{array} \right\} \begin{array}{l} s_2 = 7.3674 \text{ kJ/kg} \cdot \text{K} \\ \text{(Tablo A-6, doğrusal oranlama)} \end{array}$$

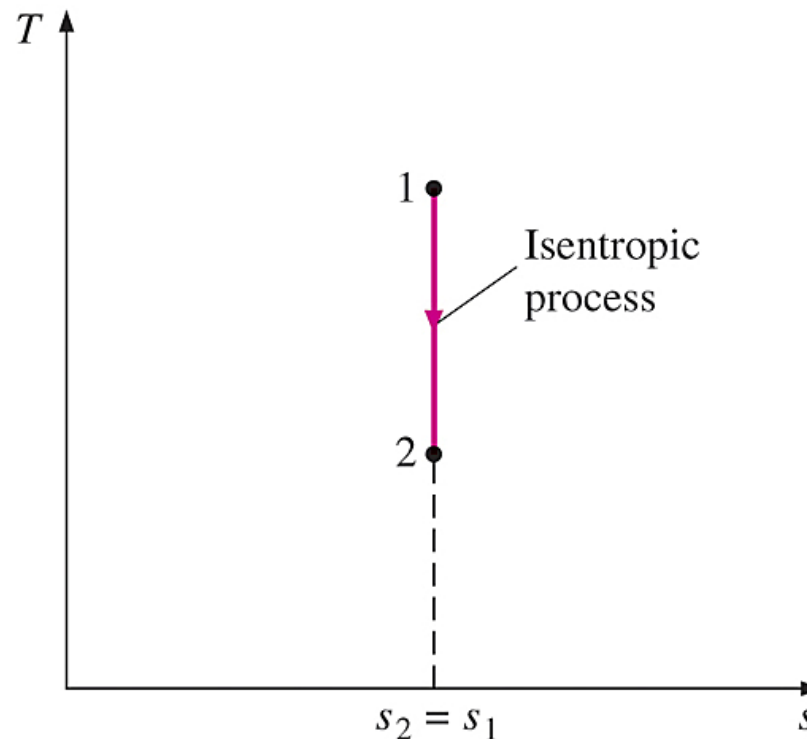
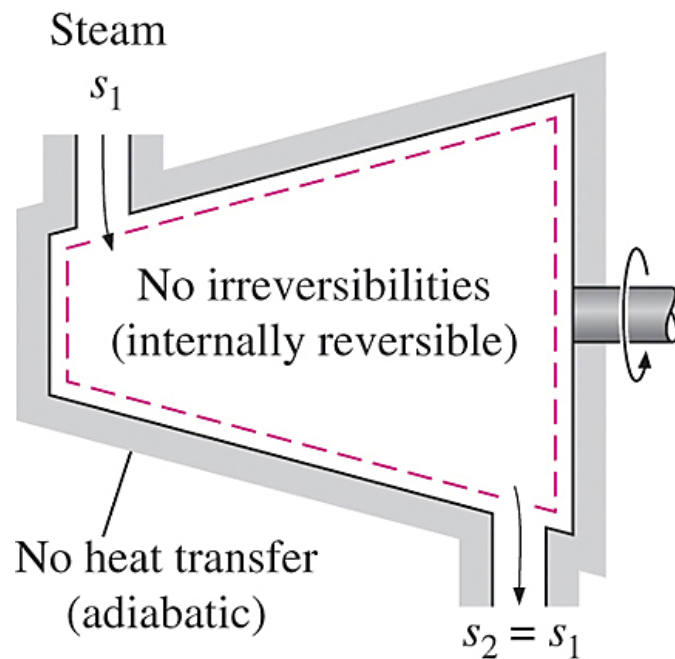
$$\begin{aligned} \Delta S &= m(s_2 - s_1) = (1.5 \text{ kg})(7.3674 - 0.2965) \text{ kJ/kg} \cdot \text{K} \\ &= \mathbf{10.61 \text{ kJ/kg}} \end{aligned}$$



# ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **isentropic process**.

$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1 \quad (\text{kJ/kg} \cdot \text{K})$$



During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

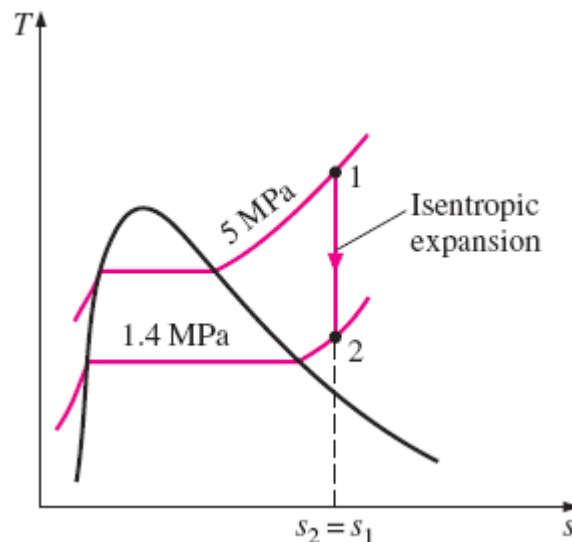
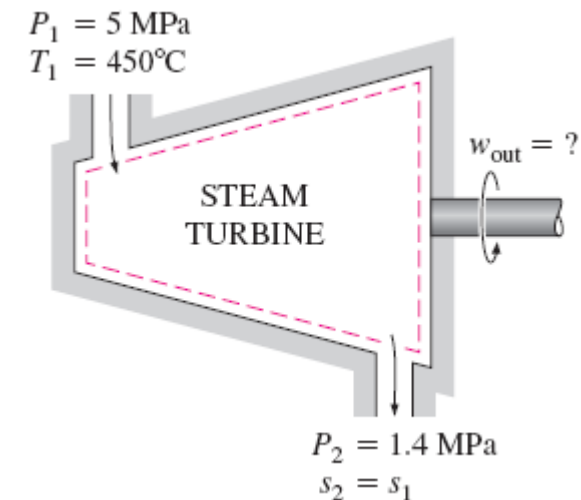
The isentropic process appears as a *vertical* line segment on a  $T-s$  diagram.

## EXAMPLE 7-5

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.

**Solution** Steam is expanded in an adiabatic turbine to a specified pressure in a reversible manner. The work output of the turbine is to be determined.

**Assumptions** 1 This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{CV}=0$ ,  $\Delta E_{CV}=0$ , and  $\Delta S_{CV}=0$ . 2 The process is reversible. 3 Kinetic and potential energies are negligible. 4 The turbine is adiabatic and thus there is no heat transfer.



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

$$\begin{aligned} \dot{E}_{in} &= \dot{E}_{out} \\ \dot{m}h_1 &= \dot{W}_{out} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0, \text{ ke} \approx \text{pe} \approx 0) \\ \dot{W}_{out} &= \dot{m}(h_1 - h_2) \end{aligned}$$



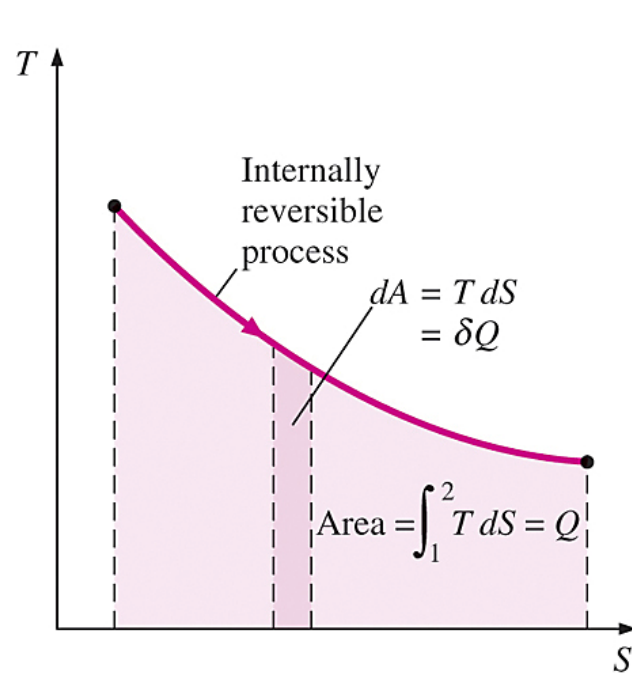
The inlet state is completely specified since two properties are given. But only one property (pressure) is given at the final state, and we need one more property to fix it. The second property comes from the observation that the process is reversible and adiabatic, and thus isentropic. Therefore,  $s_2 = s_1$ , and

$$\begin{array}{ll} \text{State 1:} & \left. \begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 450^\circ\text{C} \end{array} \right\} \begin{array}{l} h_1 = 3317.2 \text{ kJ/kg} \\ s_1 = 6.8210 \text{ kJ/kg} \cdot \text{K} \end{array} \\ \\ \text{State 2:} & \left. \begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} h_2 = 2967.4 \text{ kJ/kg} \end{array}$$

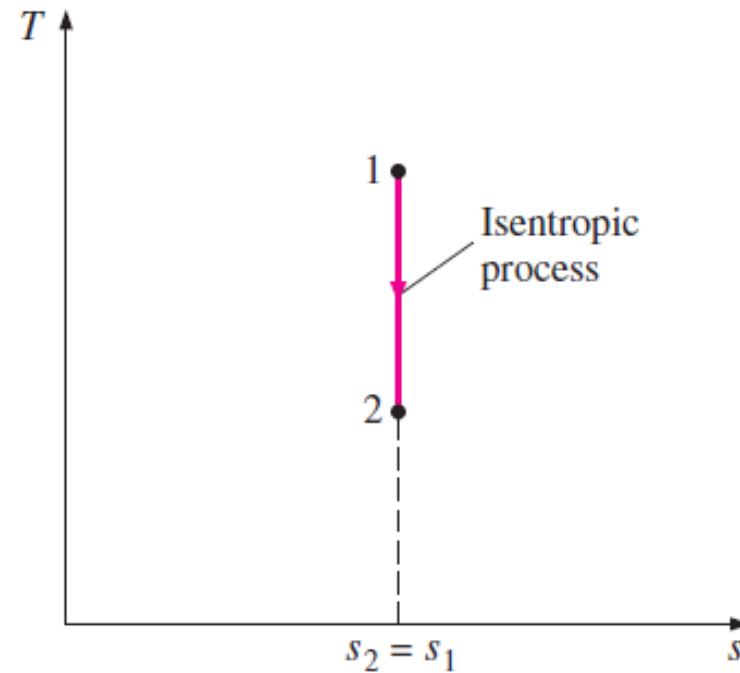
Then the work output of the turbine per unit mass of the steam becomes

$$w_{\text{out}} = h_1 - h_2 = 3317.2 - 2967.4 = \mathbf{349.8 \text{ kJ/kg}}$$

# PROPERTY DIAGRAMS INVOLVING ENTROPY



On a  $T$ - $S$  diagram, the area under the process curve represents the heat transfer for internally reversible processes.



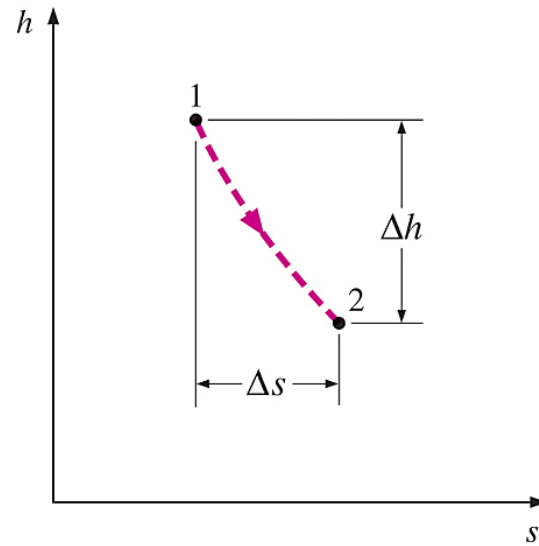
The isentropic process appears as a *vertical* line segment on a  $T$ - $s$  diagram.

$$\delta Q_{\text{int rev}} = T dS \quad (\text{kJ})$$

$$\delta q_{\text{int rev}} = T ds \quad (\text{kJ/kg})$$

$$Q_{\text{int rev}} = \int_1^2 T dS \quad (\text{kJ})$$

$$q_{\text{int rev}} = \int_1^2 T ds \quad (\text{kJ/kg})$$



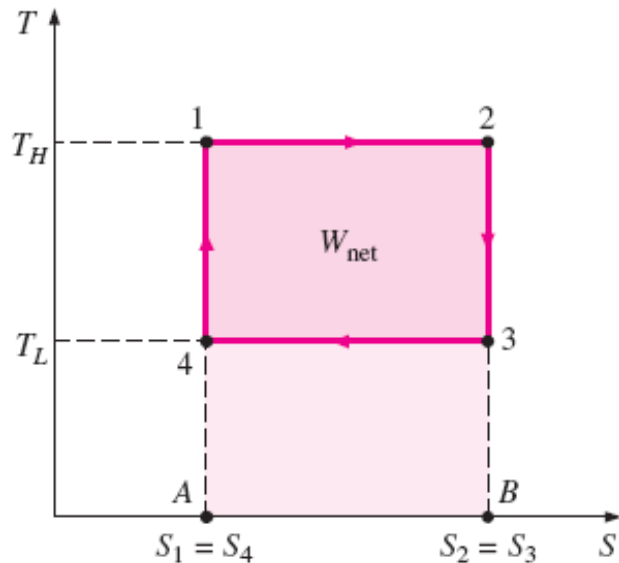
For adiabatic steady-flow devices, the vertical distance  $\Delta h$  on an  $h$ - $s$  diagram is a measure of work, and the horizontal distance  $\Delta s$  is a measure of irreversibilities.

$$Q_{\text{int rev}} = T_0 \Delta S \quad (\text{kJ})$$

$$q_{\text{int rev}} = T_0 \Delta s \quad (\text{kJ/kg})$$

## EXAMPLE 7-6

Show the Carnot cycle on a  $T$ - $S$  diagram and indicate the areas that represent the heat supplied  $Q_H$ , heat rejected  $Q_L$ , and the net work output  $W_{\text{net, out}}$  on this diagram.



**Solution** The Carnot cycle is to be shown on a  $T$ - $S$  diagram, and the areas that represent  $Q_H$ ,  $Q_L$ , and  $W_{\text{net, out}}$  are to be indicated.

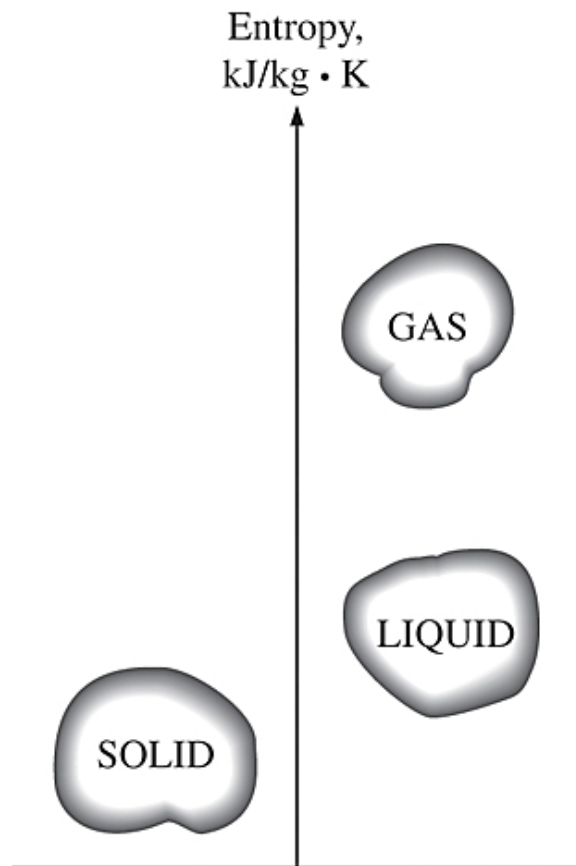
**Analysis** Recall that the Carnot cycle is made up of two reversible isothermal ( $T=\text{constant}$ ) processes and two isentropic ( $s=\text{constant}$ ) processes. These four processes form a rectangle on a  $T$ - $S$  diagram,

On a  $T$ - $S$  diagram, the area under the process curve represents the heat transfer for that process. Thus the area  $A12B$  represents  $Q_H$ , the area  $A43B$  represents  $Q_L$ , and the difference between these two (the area in color) represents the net work since

$$W_{\text{net, out}} = Q_H - Q_L$$

Therefore, the area enclosed by the path of a cycle (area 1234) on a  $T$ - $S$  diagram represents the net work. Recall that the area enclosed by the path of a cycle also represents the net work on a  $P$ - $V$  diagram.

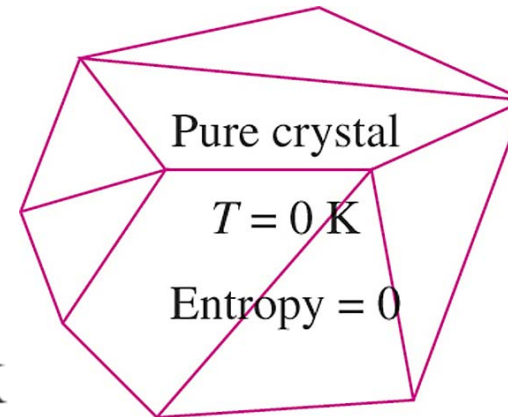
# WHAT IS ENTROPY?



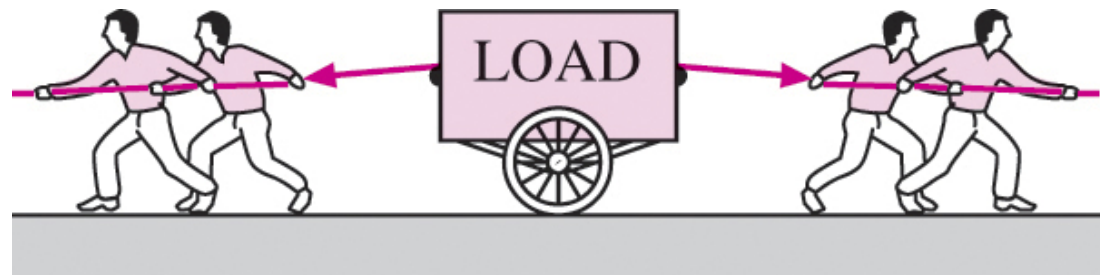
Boltzmann relation

$$S = k \ln p$$

$$k = 1.3806 \times 10^{-23} \text{ J/K}$$

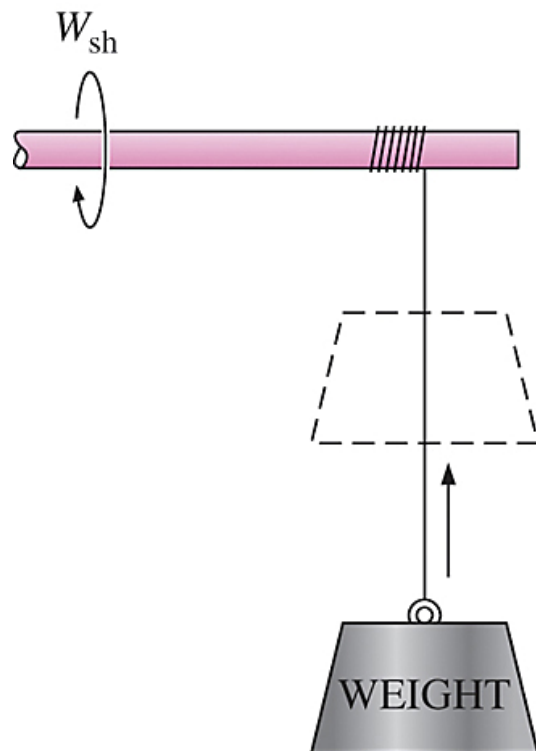


A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero **(the third law of thermodynamics)**.

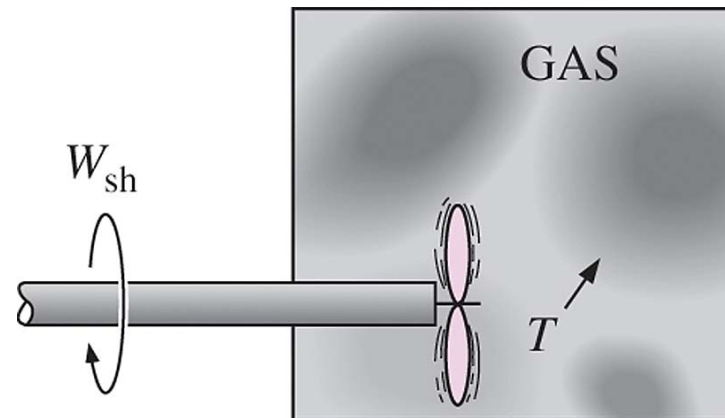


The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

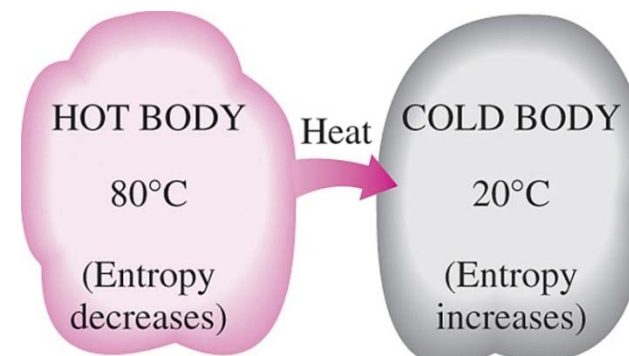
Disorganized energy does not create much useful effect, no matter how large it is.



In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.



The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

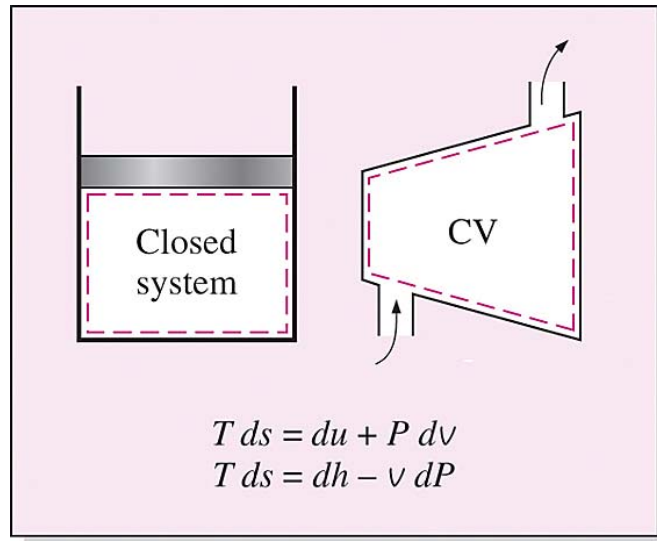


During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)



# THE $T ds$ RELATIONS

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev, out}} = dU$$



The  $T ds$  relations are valid for both reversible and irreversible processes and for both closed and open systems.

**the second  $T ds$  equation**

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

$$\delta Q_{\text{int rev}} = T dS$$

$$\delta W_{\text{int rev, out}} = P dV$$

$$T dS = dU + P dV \quad (\text{kJ})$$

$$T ds = du + P dv \quad (\text{kJ/kg})$$

**the first  $T ds$ , or Gibbs equation**

$$h = u + Pv$$

$$\left. \begin{array}{l} dh = du + P dv + v dP \\ T ds = du + P dv \end{array} \right\} T ds = dh - v dP$$

Differential changes in entropy in terms of other properties

# ENTROPY CHANGE OF LIQUIDS AND SOLIDS

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

$$ds = \frac{du}{T} + \frac{P}{T} dv$$

$$dv \cong 0 \text{ for liquids and solids}$$

$$ds = \frac{du}{T} = \frac{c}{T} dT$$

$$\text{since } c_p = c_v = c \text{ and } du = c dT$$

$$\text{Liquids, solids: } s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{av} \ln \frac{T_2}{T_1} \quad (\text{kJ/kg} \cdot \text{K})$$

For an isentropic process of an incompressible substance

$$s_2 - s_1 = c_{av} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

## EXAMPLE 7-7

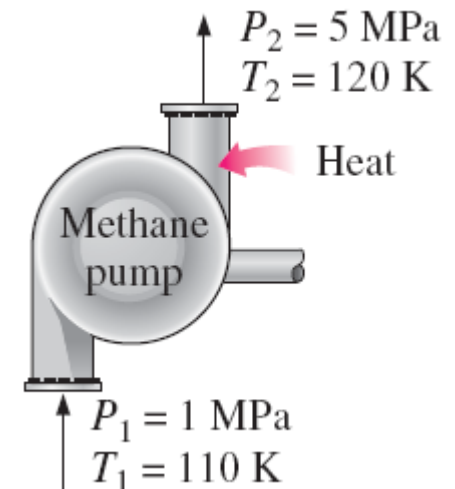
Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or 82°C), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 7–1. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (a) using actual data for methane and (b) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?

**Solution** Liquid methane undergoes a process between two specified states. The entropy change of methane is to be determined by using compressed liquid data and by assuming methane to be incompressible, and the results are to be compared.

(a) We consider a unit mass of liquid methane. The entropies of the methane at the initial and final states are

State 1:	$\left. \begin{array}{l} P_1 = 1 \text{ MPa} \\ T_1 = 110 \text{ K} \end{array} \right\}$	$\begin{array}{l} s_1 = 4.875 \text{ kJ/kg} \cdot \text{K} \\ c_{p1} = 3.471 \text{ kJ/kg} \cdot \text{K} \end{array}$
State 2:	$\left. \begin{array}{l} P_2 = 5 \text{ MPa} \\ T_2 = 120 \text{ K} \end{array} \right\}$	$\begin{array}{l} s_2 = 5.145 \text{ kJ/kg} \cdot \text{K} \\ c_{p2} = 3.486 \text{ kJ/kg} \cdot \text{K} \end{array}$

$$\Delta s = s_2 - s_1 = 5.145 - 4.875 = \mathbf{0.270 \text{ kJ/kg} \cdot \text{K}}$$



(b) Approximating liquid methane as an incompressible substance, its entropy change is determined to be

$$\Delta s = c_{av} \ln \frac{T_2}{T_1} = (3.4785 \text{ kJ/kg} \cdot \text{K}) \ln \frac{120 \text{ K}}{110 \text{ K}} = \mathbf{0.303 \text{ kJ/kg} \cdot \text{K}}$$

$$c_{p, av} = \frac{c_{p1} + c_{p2}}{2} = \frac{3.471 + 3.486}{2} = 3.4785 \text{ kJ/kg} \cdot \text{K}$$

Therefore, the error involved in approximating liquid methane as an incompressible substance is

$$\text{Error} = \frac{|\Delta s_{\text{actual}} - \Delta s_{\text{ideal}}|}{\Delta s_{\text{actual}}} = \frac{|0.270 - 0.303|}{0.270} = \mathbf{0.122 \text{ (or 12.2\%)}}$$

**Discussion** This result is not surprising since the density of liquid methane changes during this process from 425.8 to 415.2 kg/m<sup>3</sup> (about 3 %), which makes us question the validity of the incompressible substance assumption. Still, this assumption enables us to obtain reasonably accurate results with less effort, which proves to be very convenient in the absence of compressed liquid data.

## EXAMPLE 7-8

A cryogenic manufacturing facility handles liquid methane at 115 K and 5 MPa at a rate of 0.280 m<sup>3</sup>/s. A process requires dropping the pressure of liquid methane to 1 MPa, which is done by throttling the liquid methane by passing it through a flow resistance such as a valve. A recently hired engineer proposes to replace the throttling valve by a turbine in order to produce power while dropping the pressure to 1 MPa. Using data from Table 7–1, determine the maximum amount of power that can be produced by such a turbine. Also, determine how much this turbine will save the facility from electricity usage costs per year if the turbine operates continuously (8760 h/yr) and the facility pays \$0.075/kWh for electricity.

Properties of liquid methane

Temp., $T$ , K	Pressure, $P$ , MPa	Density, $\rho$ , kg/m <sup>3</sup>	Enthalpy, $h$ , kJ/kg	Entropy, $s$ , kJ/kg · K	Specific heat, $c_p$ , kJ/kg · K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486



**Solution** Liquid methane is expanded in a turbine to a specified pressure at a specified rate. The maximum power that this turbine can produce and the amount of money it can save per year are to be determined.

**Assumptions** **1** This is a steady-flow process since there is no change with time at any point and thus  $m_{CV} = 0$ ,  $E_{CV} = 0$ , and  $SCV = 0$ . **2** The turbine is adiabatic and thus there is no heat transfer. **3** The process is reversible. **4** Kinetic and potential energies are negligible.

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 115 \text{ K} \end{array} \right\} \quad \begin{array}{l} h_1 = 232.2 \text{ kJ/kg} \\ s_1 = 4.9945 \text{ kJ/kg} \cdot \text{K} \\ \rho_1 = 422.15 \text{ kg/s} \end{array}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 1 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} \quad h_2 = 222.8 \text{ kJ/kg}$$

$$\dot{m} = \rho_1 \dot{V}_1 = (422.15 \text{ kg/m}^3)(0.280 \text{ m}^3/\text{s}) = 118.2 \text{ kg/s}$$

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} = 0$$

$$\begin{aligned} \dot{E}_{\text{in}} &= \dot{E}_{\text{out}} \\ \dot{m}h_1 &= \dot{W}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0, \text{ ke} \cong \text{pe} \cong 0) \\ \dot{W}_{\text{out}} &= \dot{m}(h_1 - h_2) \\ &= (118.2 \text{ kg/s})(232.3 - 222.8) \text{ kJ/kg} \\ &= \mathbf{1123 \text{ kW}} \end{aligned}$$

$$\begin{aligned}\text{Annual power production} &= \dot{W}_{\text{out}} \times \Delta t = (1123 \text{ kW})(8760 \text{ h/yr}) \\ &= 0.9837 \times 10^7 \text{ kWh/yr}\end{aligned}$$

At \$0.075/kWh, the amount of money this turbine will save the facility is

$$\begin{aligned}\text{Annual power savings} &= (\text{Annual power production})(\text{Unit cost of power}) \\ &= (0.9837 \times 10^7 \text{ kWh/yr})(\$0.075/\text{kWh}) \\ &= \text{\textcolor{red}{\$737,800/yr}}\end{aligned}$$

**Discussion** This example shows the importance of the property entropy since it enabled us to quantify the work potential that is being wasted. In practice, the turbine will not be isentropic, and thus the power produced will be less. The analysis above gave us the upper limit. An actual turbine-generator assembly can utilize about 80 % of the potential and produce more than 900 kW of power while saving the facility more than \$600,000 a year. It can also be shown that the temperature of methane will drop to 113.9 K (a drop of 1.1 K) during the isentropic expansion process in the turbine instead of remaining constant at 115 K as would be the case if methane were assumed to be an incompressible substance. The temperature of methane would rise to 116.6 K (a rise of 1.6 K) during the throttling process.



# THE ENTROPY CHANGE OF IDEAL GASES

From the first  $T ds$  relation

$$ds = \frac{du}{T} + \frac{P dv}{T} \quad \begin{aligned} du &= c_v dT \\ P &= RT/v \end{aligned}$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

From the second  $T ds$  relation

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

$$dh = c_p dT \quad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$\begin{aligned} Pv &= RT \\ du &= C_v dT \\ dh &= C_p dT \end{aligned}$$

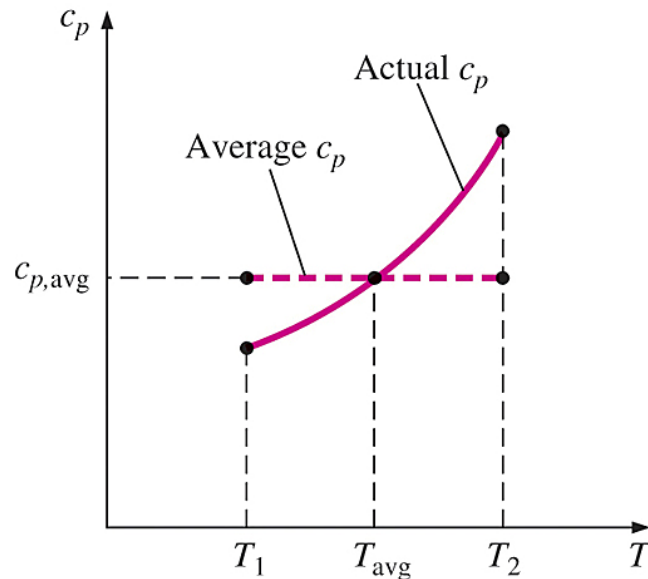
# Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad (\text{kJ/kg} \cdot \text{K})$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$s_2 - s_1 = c_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kg} \cdot \text{K})$$



Entropy change of an ideal gas on a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,av} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \quad (\text{kJ/mol} \cdot \text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,av} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/mol} \cdot \text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

# Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function  $s^\circ$  as

$$s^\circ = \int_0^T c_p(T) \frac{dT}{T}$$

$$\int_1^2 c_p(T) \frac{dT}{T} = s_2^\circ - s_1^\circ$$

On a unit-mass basis

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kg} \cdot \text{K})$$

On a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

$T, \text{K}$	$s^\circ(T), \text{kJ/kg} \cdot \text{K}$
.	.
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.

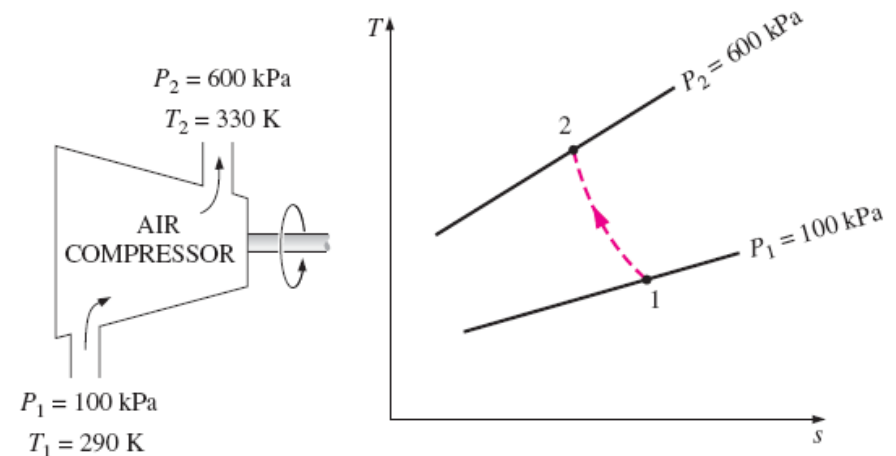
The entropy of an ideal gas depends on both  $T$  and  $P$ . The function  $s$  represents only the temperature-dependent part of entropy.

## EXAMPLE 7-9

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using (a) property values from the air table and (b) average specific heats.

**Solution** Air is compressed between two specified states. The entropy change of air is to be determined by using tabulated property values and also by using average specific heats.

**Assumptions** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. Therefore, entropy change relations developed under the ideal-gas assumption are applicable.



(a) The properties of air are given in the air table (Table A–21). Reading  $s$  values at given temperatures and substituting, we find

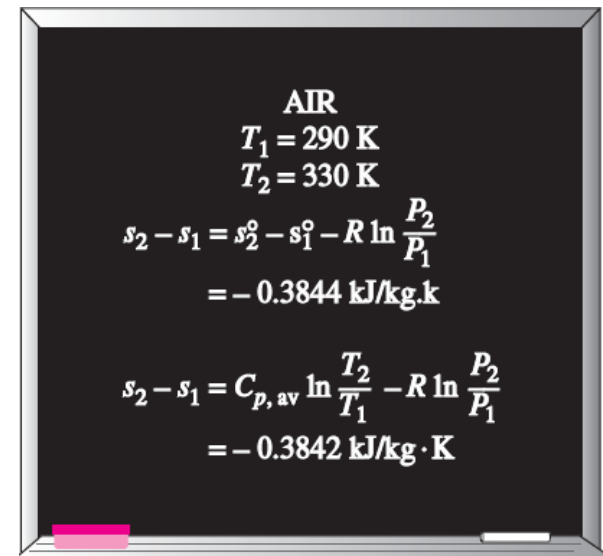
$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$

$$\begin{aligned} &= [(1.79783 - 1.66802) \text{ kJ/kg} \cdot \text{K}] - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \\ &= \mathbf{-0.3844 \text{ kJ/kg} \cdot \text{K}} \end{aligned}$$

(b) The entropy change of air during this process can also be determined approximately from Eq. 7–34 by using a  $c_p$  value at the average temperature of 37°C (Table A–2b) and treating it as a constant:

$$\begin{aligned}s_2 - s_1 &= c_{p, \text{av}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\&= (1.006 \text{ kJ/kg} \cdot \text{K}) \ln \frac{330 \text{ K}}{290 \text{ K}} - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \\&= \mathbf{-0.3842 \text{ kJ/kg} \cdot \text{K}}\end{aligned}$$

**Discussion** The two results above are almost identical since the change in temperature during this process is relatively small. When the temperature change is large, however, they may differ significantly. For those cases, Eq. 7–39 should be used instead of Eq. 7–34 since it accounts for the variation of specific heats with temperature.



AIR  
 $T_1 = 290 \text{ K}$   
 $T_2 = 330 \text{ K}$

$$\begin{aligned}s_2 - s_1 &= s_2^o - s_1^o - R \ln \frac{P_2}{P_1} \\&= -0.3844 \text{ kJ/kg.k}\end{aligned}$$
$$\begin{aligned}s_2 - s_1 &= c_{p, \text{av}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\&= -0.3842 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

# Isentropic Processes of Ideal Gases

## Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad (\text{kJ/kg} \cdot \text{K})$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_1}{v_2} \quad \ln \frac{T_2}{T_1} = \ln \left( \frac{v_1}{v_2} \right)^{R/c_v}$$

$$R = c_p - c_v, \quad k = c_p/c_v \quad R/c_v = k - 1$$

$$\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{v_1}{v_2} \right)^{k-1}$$

$$\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k}$$

$$\left( \frac{P_2}{P_1} \right)_{s=\text{const.}} = \left( \frac{v_1}{v_2} \right)^k$$

$$\begin{aligned} TV^{k-1} &= \text{constant} \\ TP^{(1-k)/k} &= \text{constant} \quad (\text{ideal gas}) \\ PV^k &= \text{constant} \end{aligned}$$

$$\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = \left( \frac{v_1}{v_2} \right)^{k-1}$$

VALID FOR  
\*ideal gas  
\*isentropic process  
\*constant specific heats

The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

# Isentropic Processes of Ideal Gases

## Variable Specific Heats (Exact Analysis)

$$0 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \longrightarrow s_2^\circ - s_1^\circ + R \ln \frac{P_2}{P_1}$$

## Relative Pressure and Relative Specific Volume

$$\frac{P_2}{P_1} = \exp \frac{s_2^\circ - s_1^\circ}{R}$$

$\exp(s^\circ/R)$  is the relative pressure  $P_r$

$$\frac{P_2}{P_1} = \frac{\exp(s_2^\circ/R)}{\exp(s_1^\circ/R)}$$

$$\left( \frac{P_2}{P_1} \right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \longrightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{T_2}{T_1} \frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

$$\left( \frac{v_2}{v_1} \right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$

$T/P_r$  is the relative specific volume  $v_r$



Process: isentropic

Given:  $P_1$ ,  $T_1$ , and  $P_2$

Find:  $T_2$

$T$	$P_r$
.	.
.	.
.	.
$T_2$	← read $P_{r2} = \frac{P_2}{P_1} P_{r1}$
.	.
.	.
.	.
$T_1$	→ read $P_{r1}$
.	.
.	.
.	.

The use of  $P_r$  data for calculating the final temperature during an isentropic process.

Process: isentropic

Given:  $v_1$ ,  $T_1$ , and  $v_2$

Find:  $T_2$

$T$	$v_r$
.	.
.	.
.	.
$T_2$	← read $v_{r2} = \frac{v_2}{v_1} v_{r1}$
.	.
.	.
.	.
$T_1$	→ read $v_{r1}$
.	.
.	.
.	.

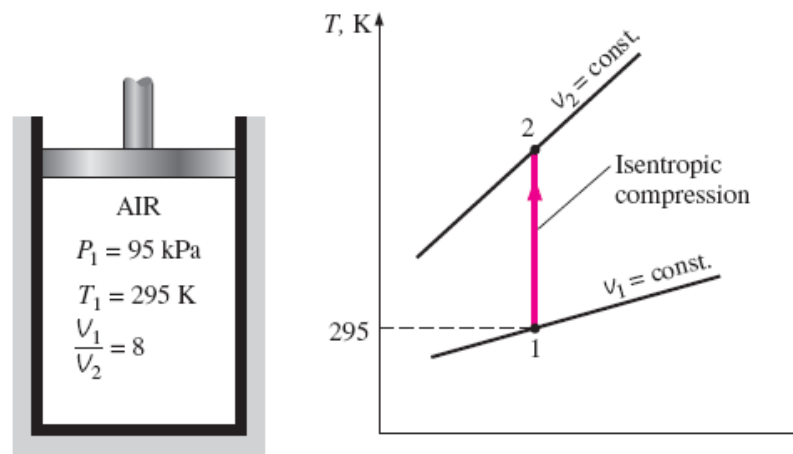
The use of  $v_r$  data for calculating the final temperature during an isentropic process

## EXAMPLE 7-10

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio  $V_1/V_2$  of this piston–cylinder device is 8, determine the final temperature of the air.

**Solution** Air is compressed in a car engine isentropically. For a given compression ratio, the final air temperature is to be determined.

**Assumptions** At specified conditions, air can be treated as an ideal gas. Therefore, the isentropic relations developed earlier for ideal gases are applicable.



For closed systems:  $\frac{V_2}{V_1} = \frac{v_2}{v_1}$       At  $T_1 = 295 \text{ K}$ :  $v_{r1} = 647.9$

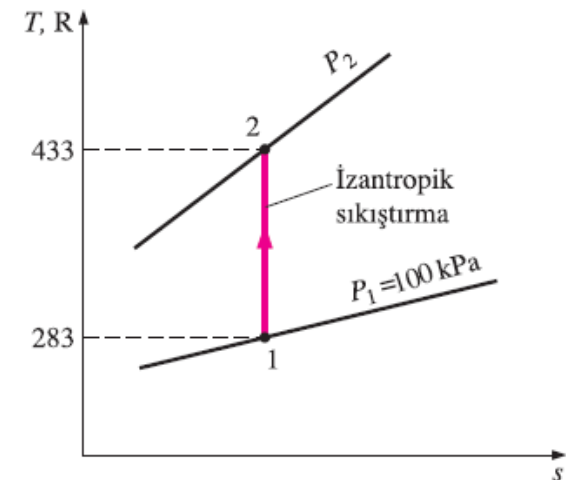
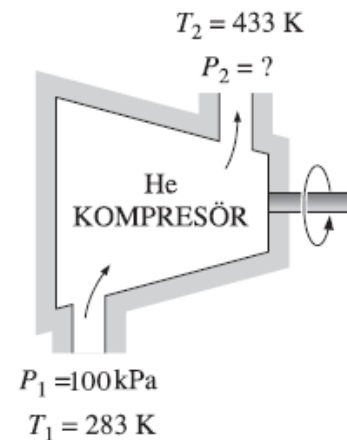
$$v_{r2} = v_{r1} \left( \frac{v_2}{v_1} \right) = (647.9) \left( \frac{1}{8} \right) = 80.99 \rightarrow T_2 = \mathbf{662.7 \text{ K}}$$

Therefore, the temperature of air will increase by 367.7°C during this process.

### EXAMPLE 7-11

Helium gas is compressed in an adiabatic compressor from an initial state of 100 kPa and 10°C to a final temperature of 160°C in a reversible manner. Determine the exit pressure of helium.

**Assumptions** At specified conditions helium can be treated as an ideal gas since it is at a high temperature relative to its critical-point value of 450F. Therefore, the isentropic relations developed earlier for ideal gases are applicable.



$$P_2 = P_1 \left( \frac{T_2}{T_1} \right)^{k/(k-1)} = (100 \text{ kPa}) \left( \frac{780 \text{ R}}{510 \text{ R}} \right)^{1.667/0.667} = \mathbf{289 \text{ kPa}}$$

# REVERSIBLE STEADY-FLOW WORK

The work done during a process depends on the path followed as well as on the properties at the end states. Recall that reversible (quasi-equilibrium) moving boundary work associated with closed systems is expressed in terms of the fluid properties as

$$\left. \begin{array}{l} \delta q_{\text{rev}} = T ds \\ T ds = dh - v dP \end{array} \right\} \delta q_{\text{rev}} = dh - v dP$$

$$-\delta w_{\text{rev}} = v dP + dke + dpe$$

$$w_{\text{rev}} = - \int_1^2 v dP - \Delta ke - \Delta pe \quad (\text{kJ/kg})$$

When kinetic and potential energies are negligible

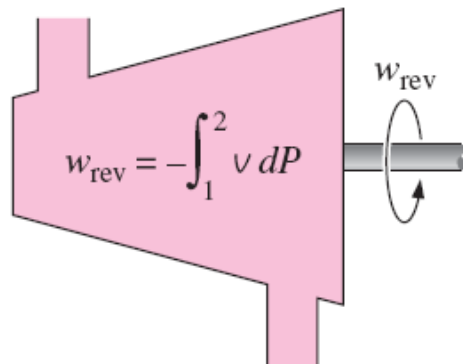
$$w_{\text{rev}} = - \int_1^2 v dP \quad (\text{kJ/kg})$$

$$w_{\text{rev, in}} = \int_1^2 v dP + \Delta ke + \Delta pe$$

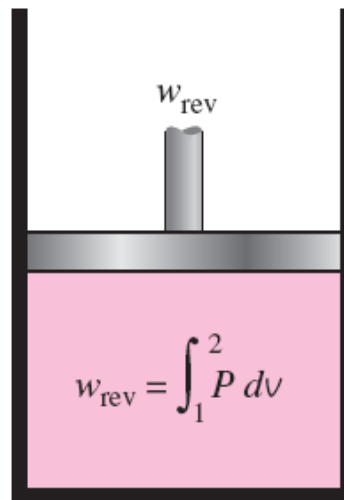
$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta ke - \Delta pe \quad (\text{kJ/kg})$$

For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (**Bernoulli equation**):

$$\nu(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

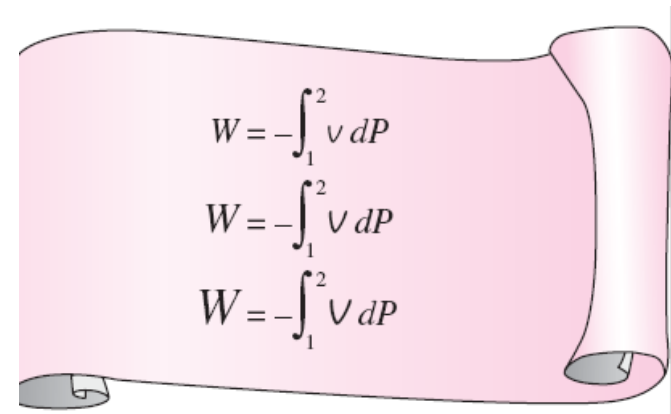


(a) Steady-flow system



(b) Closed system

Reversible work relations for steady-flow and closed systems.



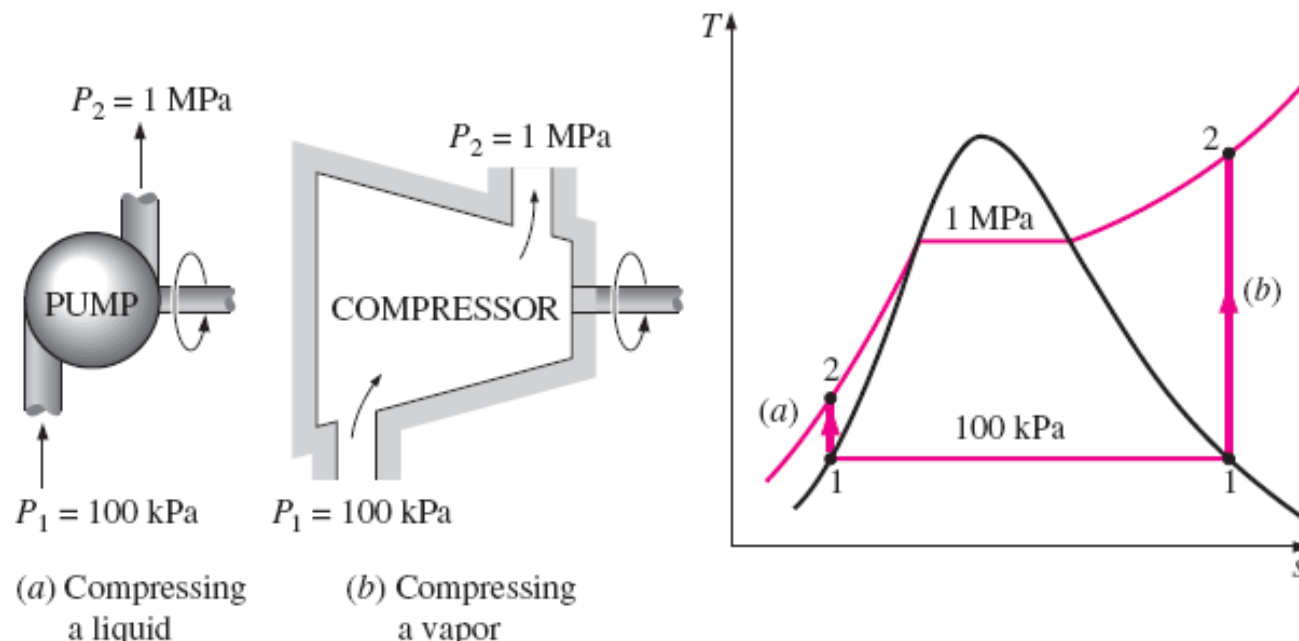
The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.

## EXAMPLE 7-12

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (a) saturated liquid and (b) saturated vapor at the inlet state.

**Solution** Steam is to be compressed from a given pressure to a specified pressure isentropically. The work input is to be determined for the cases of steam being a saturated liquid and saturated vapor at the inlet.

**Assumptions** 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible. 3 The process is given to be isentropic.



(a) In this case, steam is a saturated liquid initially, and its specific volume is

$$v_1 = v_{f@100 \text{ kPa}} = 0.001043 \text{ m}^3/\text{kg}$$

$$\begin{aligned} w_{\text{rev, in}} &= \int_1^2 v \, dP \cong v_1(P_2 - P_1) \\ &= (0.001043 \text{ m}^3/\text{kg})[(1000 - 100) \text{ kPa}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\ &= \mathbf{0.94 \text{ kJ/kg}} \end{aligned}$$

(b) This time, steam is a saturated vapor initially and remains a vapor during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how  $v$  varies with  $P$  to perform the integration in Eq. 7-53. This relation, in general, is not readily available. But for an isentropic process, it is easily obtained from the

$$\left. \begin{aligned} T \, ds &= dh - v \, dP \quad (\text{Eq. 7-24}) \\ ds &= 0 \quad (\text{isentropic process}) \end{aligned} \right\} v \, dP = dh$$

$$w_{\text{rev, in}} = \int_1^2 v \, dP = \int_1^2 dh = h_2 - h_1$$



$$\begin{array}{ll} \text{State 1:} & \left. \begin{array}{l} P_1 = 100 \text{ kPa} \\ \text{(sat. vapor)} \end{array} \right\} \begin{array}{l} h_1 = 2675.0 \text{ kJ/kg} \\ s_1 = 7.3589 \text{ kJ/kg} \cdot \text{K} \end{array} \\ \\ \text{State 2:} & \left. \begin{array}{l} P_2 = 1 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} h_2 = 3194.5 \text{ kJ/kg} \end{array}$$

$$w_{\text{rev, in}} = (3194.5 - 2675.0) \text{ kJ/kg} = \mathbf{519.5 \text{ kJ/kg}}$$

**Discussion** Note that compressing steam in the vapor form would require over 500 times more work than compressing it in the liquid form between the same pressure limits.

# Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + dke + dpe$$

Actual

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe$$

Reversible

$$\delta q_{\text{act}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta w_{\text{rev}}$$

$$\delta w_{\text{rev}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta q_{\text{act}}$$

$$\delta q_{\text{rev}} = T ds \quad ds \geq \frac{\delta q_{\text{act}}}{T}$$

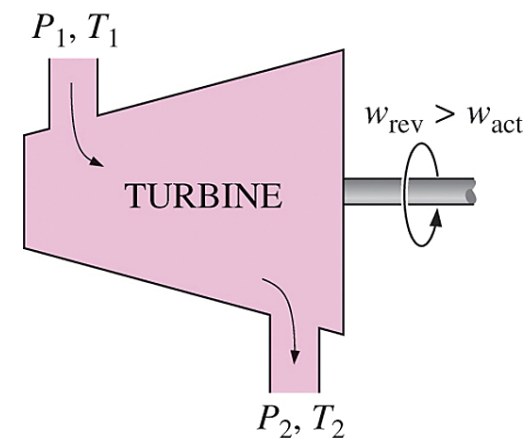
$$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + dke + dpe$$

$$\frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} = ds - \frac{\delta q_{\text{act}}}{T} \geq 0$$

$$\delta w_{\text{rev}} \geq \delta w_{\text{act}}$$

$$w_{\text{rev}} \geq w_{\text{act}}$$

A reversible turbine delivers more work than an irreversible one if both operate between the same end states.



Work-producing devices such as turbines deliver more work, and work-consuming devices such as pumps and compressors require less work when they operate reversibly.

# MINIMIZING THE COMPRESSOR WORK

When kinetic and potential energies are negligible

$$w_{\text{rev, in}} = \int_1^2 v \, dP$$

Isentropic ( $Pv^k = \text{constant}$ ):

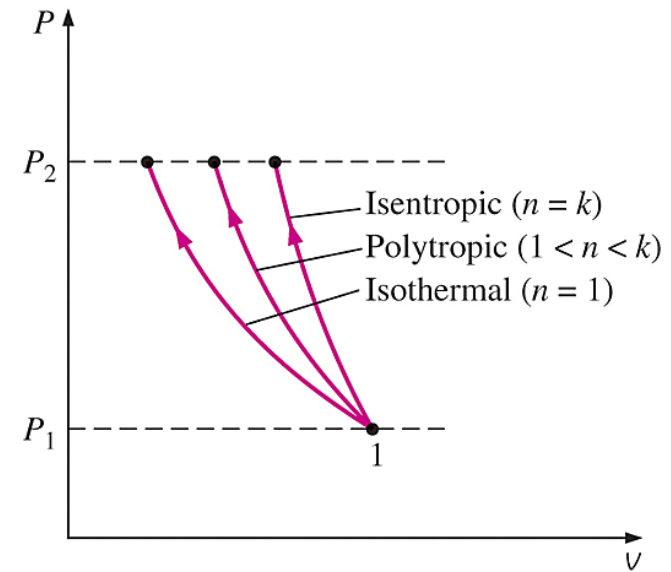
$$w_{\text{comp, in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

Polytropic ( $Pv^n = \text{constant}$ ):

$$w_{\text{comp, in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

Isothermal ( $Pv = \text{constant}$ ):

$$w_{\text{comp, in}} = RT \ln \frac{P_2}{P_1}$$

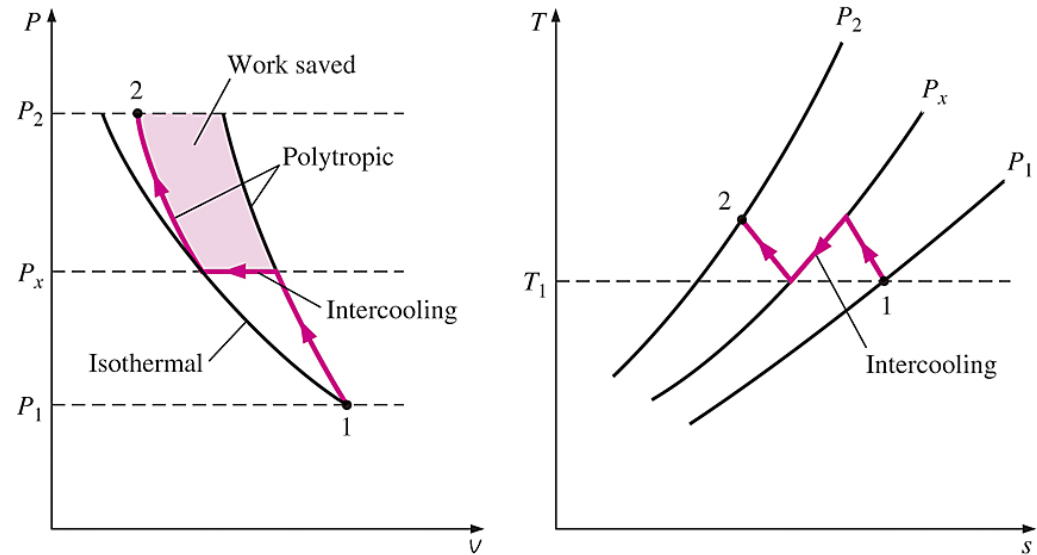


$P$ - $v$  diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

The adiabatic compression ( $Pv^k = \text{constant}$ ) requires the maximum work and the isothermal compression ( $T = \text{constant}$ ) requires the minimum. **Why?**

# Multistage Compression with Intercooling

The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an *intercooler*.



P-v and T-s diagrams for a two-stage steady-flow compression process.

$$W_{\text{comp, in}} = W_{\text{comp I, in}} + W_{\text{comp II, in}}$$

$$= \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]$$

To minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same.

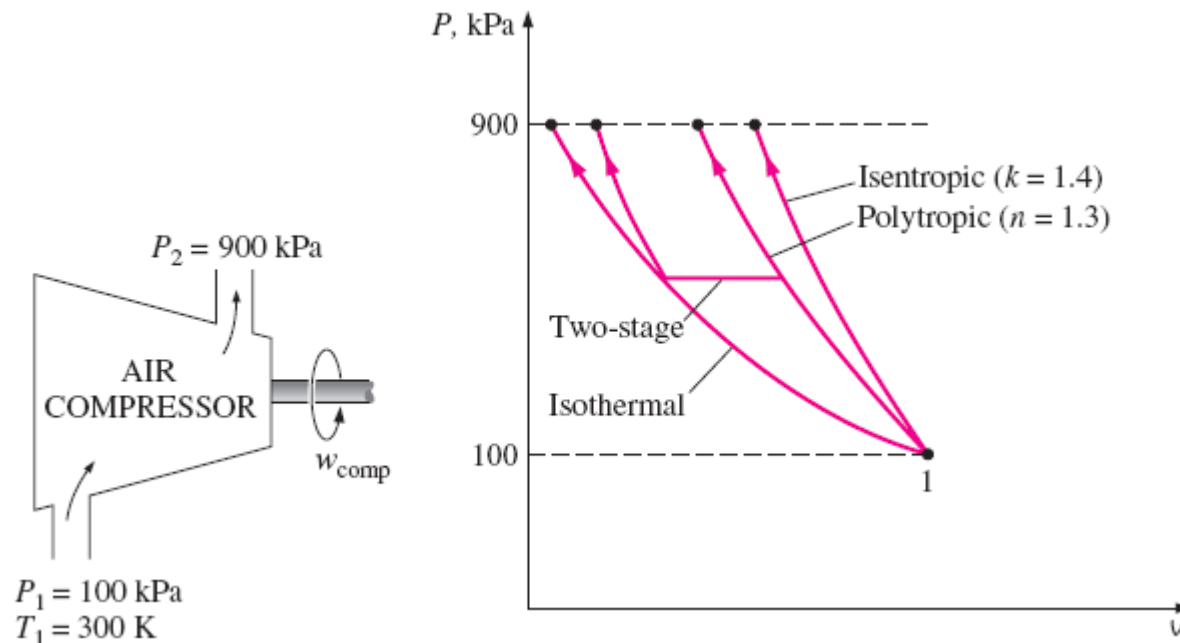
$$P_x = (P_1 P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

## EXAMPLE 7-13

Air is compressed steadily by a reversible compressor from an inlet state of 100 kPa and 300 K to an exit pressure of 900 kPa. Determine the compressor work per unit mass for (a) isentropic compression with  $k=1.4$ , (b) polytropic compression with  $n=1.3$ , (c) isothermal compression, and (d) ideal two stage compression with intercooling with a polytropic exponent of 1.3.

**Solution** Air is compressed reversibly from a specified state to a specified pressure. The compressor work is to be determined for the cases of isentropic, polytropic, isothermal, and two-stage compression.

**Assumptions** 1 Steady operating conditions exist. 2 At specified conditions, air can be treated as an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. 3 Kinetic and potential energy changes are negligible.



(a) Isentropic compression with  $k = 1.4$ :

$$\begin{aligned}w_{\text{comp, in}} &= \frac{kRT_1}{k - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right] \\&= \frac{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.4 - 1} \left[ \left( \frac{900 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.4-1)/1.4} - 1 \right] \\&= \mathbf{263.2 \text{ kJ/kg}}\end{aligned}$$

(b) Polytropic compression with  $n = 1.3$ :

$$\begin{aligned}w_{\text{comp, in}} &= \frac{nRT_1}{n - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] \\&= \frac{(1.3)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.3 - 1} \left[ \left( \frac{900 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.3-1)/1.3} - 1 \right] \\&= \mathbf{246.4 \text{ kJ/kg}}\end{aligned}$$

(c) Isothermal compression:

$$\begin{aligned}w_{\text{comp, in}} &= RT \ln \frac{P_2}{P_1} = (0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K}) \ln \frac{900 \text{ kPa}}{100 \text{ kPa}} \\&= \mathbf{189.2 \text{ kJ/kg}}\end{aligned}$$

(d) Ideal two-stage compression with intercooling ( $n=1.3$ ): In this case, the pressure ratio across each stage is the same, and its value is

$$P_x = (P_1 P_2)^{1/2} = [(100 \text{ kPa})(900 \text{ kPa})]^{1/2} = 300 \text{ kPa}$$

The compressor work across each stage is also the same. Thus the total compressor work is twice the compression work for a single stage:

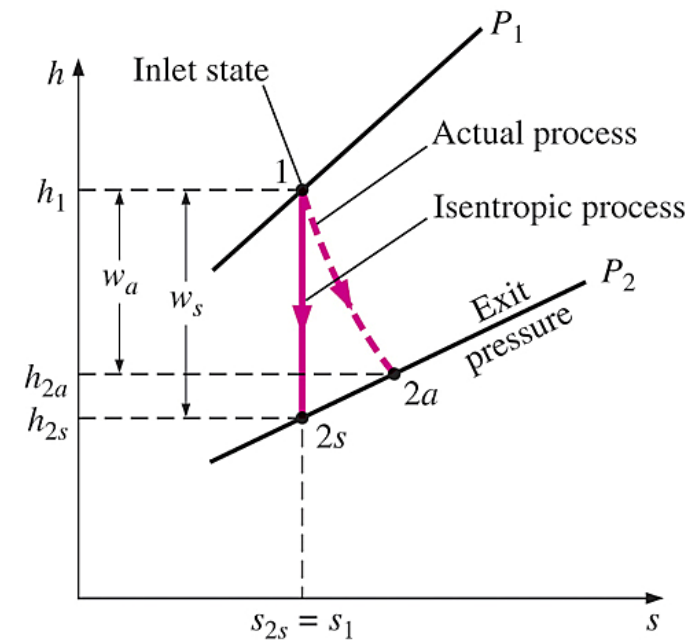
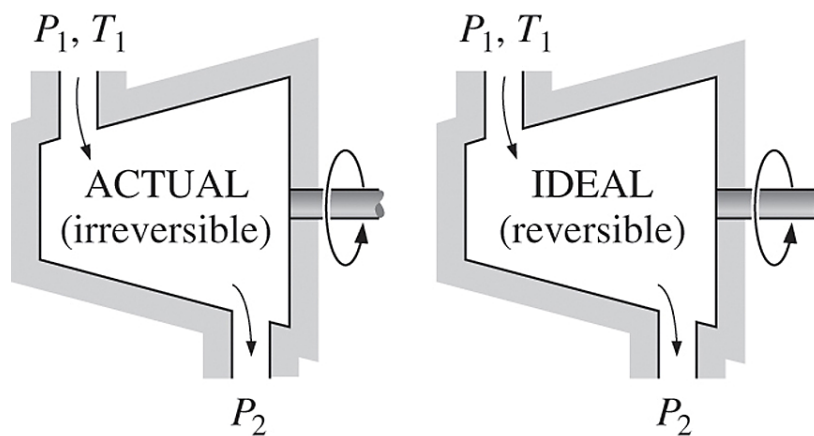
$$\begin{aligned} w_{\text{comp, in}} &= 2w_{\text{comp I, in}} = 2 \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] \\ &= \frac{2(1.3)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.3-1} \left[ \left( \frac{300 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.3-1)/1.3} - 1 \right] \\ &= \mathbf{215.3 \text{ kJ/kg}} \end{aligned}$$

**Discussion** Of all four cases considered, the isothermal compression requires the minimum work and the isentropic compression the maximum. The compressor work is decreased when two stages of polytropic compression are utilized instead of just one. As the number of compressor stages is increased, the compressor work approaches the value obtained for the isothermal case.



# ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.



The  $h-s$  diagram for the actual and isentropic processes of an adiabatic turbine.

## Isentropic Efficiency of Turbines

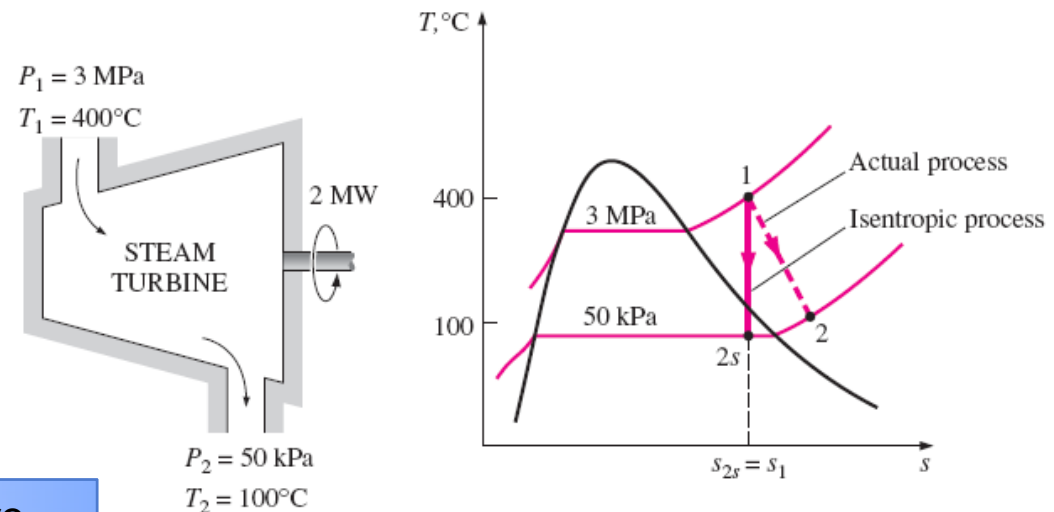
$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \quad \eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

## EXAMPLE 7-14

Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (a) the isentropic efficiency of the turbine and (b) the mass flow rate of the steam flowing through the turbine.

**Solution** Steam flows steadily between specified inlet and exit states. For a specified power output, the isentropic efficiency and the mass flow rate are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The changes in kinetic and potential energies are negligible. 3 The turbine is adiabatic.



(a) The enthalpies at various states are

State 1:	$P_1 = 3 \text{ MPa}$	$\left. \begin{array}{l} h_1 = 3231.7 \text{ kJ/kg} \\ s_1 = 6.9235 \text{ kJ/kg} \cdot \text{K} \end{array} \right\}$
	$T_1 = 400^\circ\text{C}$	
State 2a:	$P_{2a} = 50 \text{ kPa}$	$\left. \begin{array}{l} h_{2a} = 2682.4 \text{ kJ/kg} \end{array} \right\}$
	$T_{2a} = 100^\circ\text{C}$	

The exit enthalpy of the steam for the isentropic process  $h_{2s}$  is determined from the requirement that the entropy of the steam remain constant ( $s_{2s} = s_1$ ):

$$\text{State } 2s: \quad \begin{array}{l} P_{2s} = 50 \text{ kPa} \\ (s_{2s} = s_1) \end{array} \rightarrow \begin{array}{l} s_f = 1.0912 \text{ kJ/kg} \cdot \text{K} \\ s_g = 7.5931 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

$$h_{2s} = h_f + x_{2s}h_{fg} = 340.54 + 0.897(2304.7) = 2407.9 \text{ kJ/kg}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = \mathbf{0.667, \text{ or } 66.7\%}$$

(b) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{a, \text{out}} + \dot{m}h_{2a}$$

$$\dot{W}_{a, \text{out}} = \dot{m}(h_1 - h_{2a})$$

$$2 \text{ MW} \left( \frac{1000 \text{ kJ/s}}{1 \text{ MW}} \right) = \dot{m}(3231.7 - 2682.4) \text{ kJ/kg}$$
$$\dot{m} = \mathbf{3.64 \text{ kg/s}}$$

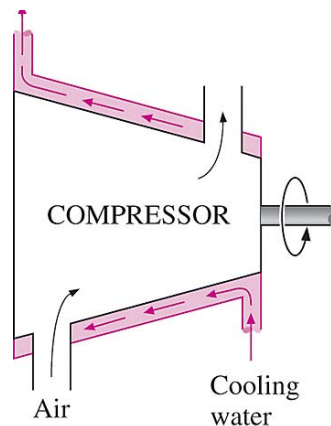
# Isentropic Efficiencies of Compressors and Pumps

$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

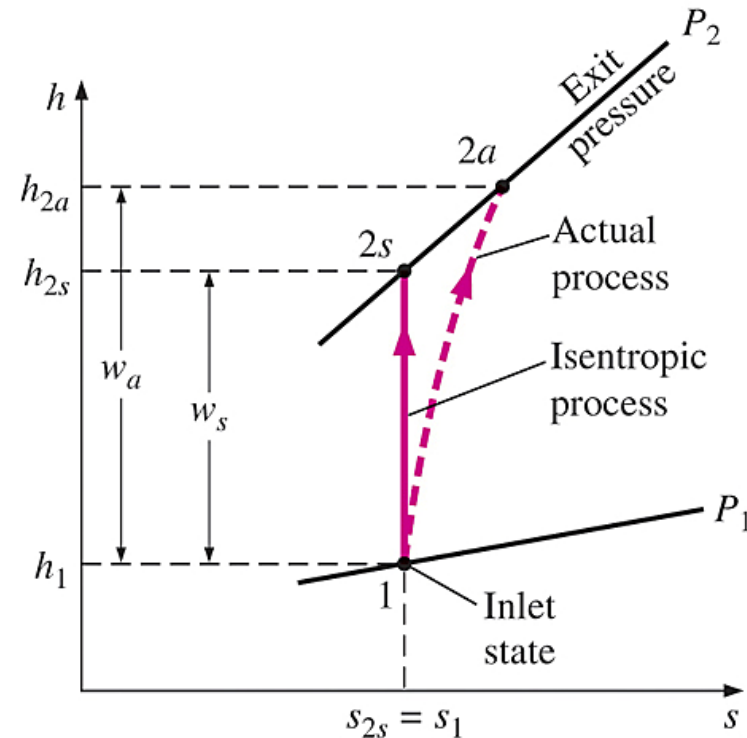
$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \text{When kinetic and potential energies are negligible}$$

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1} \quad \text{For a pump}$$

$$\eta_c = \frac{w_t}{w_a} \quad \text{Isothermal efficiency}$$



Compressors are sometimes intentionally cooled to minimize the work input.



The  $h$ - $s$  diagram of the actual and isentropic processes of an adiabatic compressor.

Can you use isentropic efficiency for a non-adiabatic compressor?

Can you use isothermal efficiency for an adiabatic compressor?

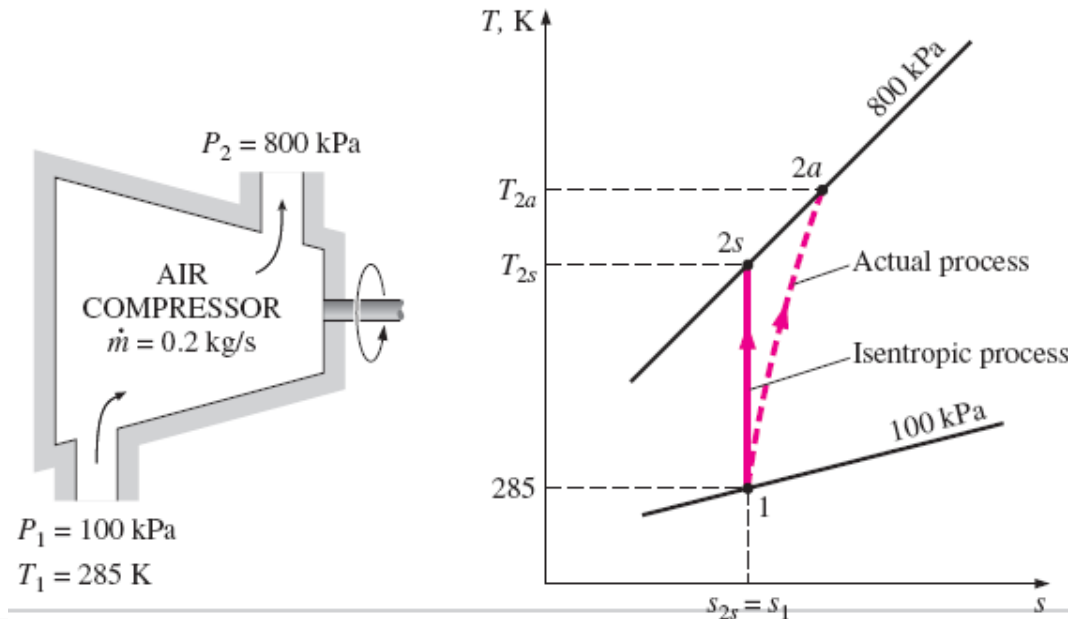
## EXAMPLE 7-15

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (a) the exit temperature of air and (b) the required power input to the compressor.

**Solution** Air is compressed to a specified pressure at a specified rate. For a given isentropic efficiency, the exit temperature and the power input are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The changes in kinetic and potential energies are negligible. 4 The compressor is adiabatic.

(a) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be determined with minimal effort in this case is  $h_{2a}$  since the isentropic efficiency of the compressor is given. At the compressor inlet,



$$T_1 = 285 \text{ K} \rightarrow h_1 = 285.14 \text{ kJ/kg}$$

$$(P_{r1} = 1.1584)$$

$$P_{r2} = P_{r1} \left( \frac{P_2}{P_1} \right) = 1.1584 \left( \frac{800 \text{ kPa}}{100 \text{ kPa}} \right) = 9.2672$$

$$P_{r2} = 9.2672 \rightarrow h_{2s} = 517.05 \text{ kJ/kg}$$

$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \rightarrow 0.80 = \frac{(517.05 - 285.14) \text{ kJ/kg}}{(h_{2a} - 285.14) \text{ kJ/kg}}$$

$$h_{2a} = 575.03 \text{ kJ/kg} \rightarrow T_{2a} = \mathbf{569.5 \text{ K}}$$

(b) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 + \dot{W}_{a,\text{in}} = \dot{m}h_{2a}$$

$$\dot{W}_{a,\text{in}} = \dot{m}(h_{2a} - h_1)$$

$$= (0.2 \text{ kg/s})[(575.03 - 285.14) \text{ kJ/kg}] = \mathbf{58.0 \text{ kW}}$$

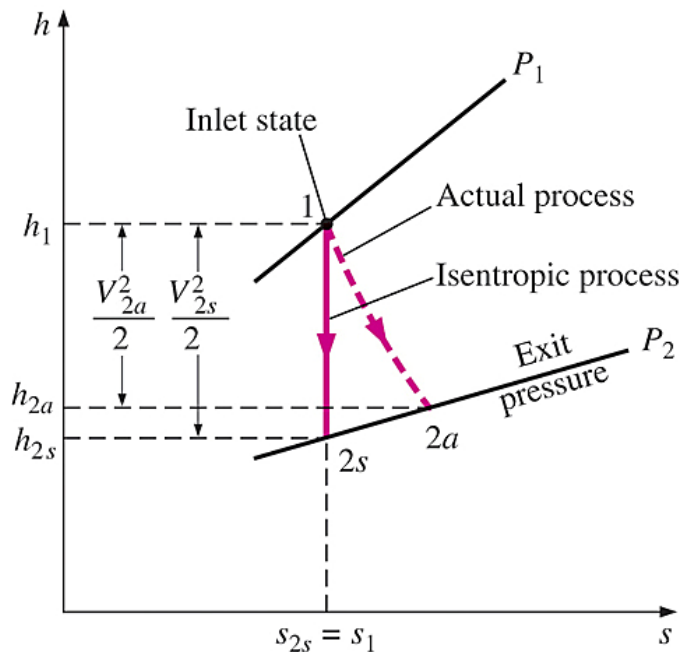
**Discussion** Notice that in determining the power input to the compressor, we used  $h_{2a}$  instead of  $h_{2s}$  since  $h_{2a}$  is the actual enthalpy of the air as it exits the compressor. The quantity  $h_{2s}$  is a hypothetical enthalpy value that the air would have if the process were isentropic.

# Isentropic Efficiency of Nozzles

Nozzles are essentially adiabatic devices and are used to accelerate a fluid. Therefore, the isentropic process serves as a suitable model for nozzles.

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

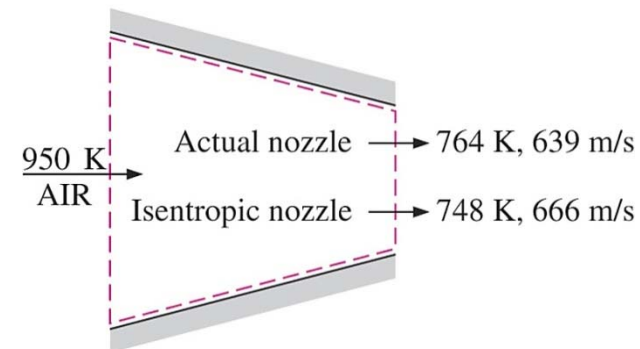


The  $h$ - $s$  diagram of the actual and isentropic processes of an adiabatic nozzle.

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

Then,

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.

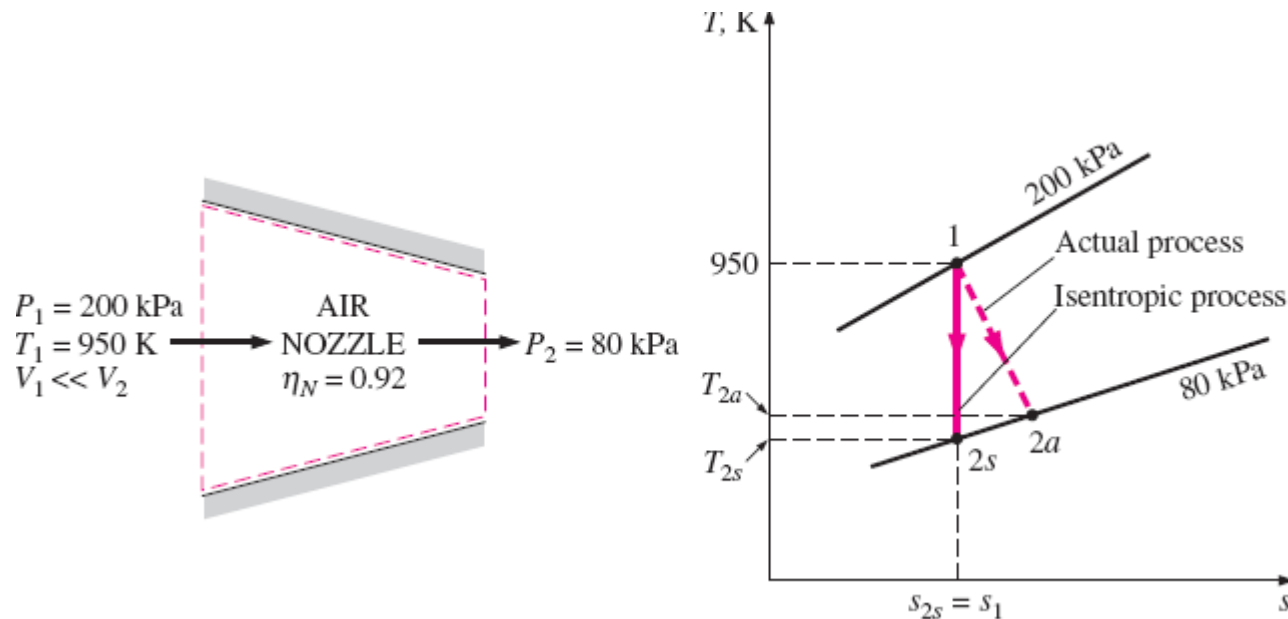


## EXAMPLE 7-16

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 80 kPa. If the isentropic efficiency of the nozzle is 92 percent, determine (a) the maximum possible exit velocity, (b) the exit temperature, and (c) the actual velocity of the air. Assume constant specific heats for air.

**Solution** The acceleration of air in a nozzle is considered. For specified exit pressure and isentropic efficiency, the maximum and actual exit velocities and the exit temperature are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The inlet kinetic energy is negligible. 4 The nozzle is adiabatic.



(a) the maximum possible exit velocity

$$\frac{T_{2s}}{T_1} = \left( \frac{P_{2s}}{P_1} \right)^{(k-1)/k}$$

$$T_{2s} = T_1 \left( \frac{P_{2s}}{P_1} \right)^{(k-1)/k} = (950 \text{ K}) \left( \frac{80 \text{ kPa}}{200 \text{ kPa}} \right)^{0.354/1.354} = 748 \text{ K}$$

$$e_{\text{in}} = e_{\text{out}}$$
$$h_1 + \frac{V_1^2}{2} = h_{2s} + \frac{V_{2s}^2}{2}$$

$$V_{2s} = \sqrt{2(h_1 - h_{2s})} = \sqrt{2c_{p, \text{av}}(T_1 - T_{2s})}$$
$$= \sqrt{2(1.099 \text{ kJ/kg} \cdot \text{K})[(950 - 748) \text{ K}] \left( \frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)}$$
$$= \mathbf{666 \text{ m/s}}$$

(b) The actual exit temperature of the air will be higher than the isentropic exit temperature evaluated above, and it is determined from

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{c_{p, \text{av}}(T_1 - T_{2a})}{c_{p, \text{av}}(T_1 - T_{2s})}$$

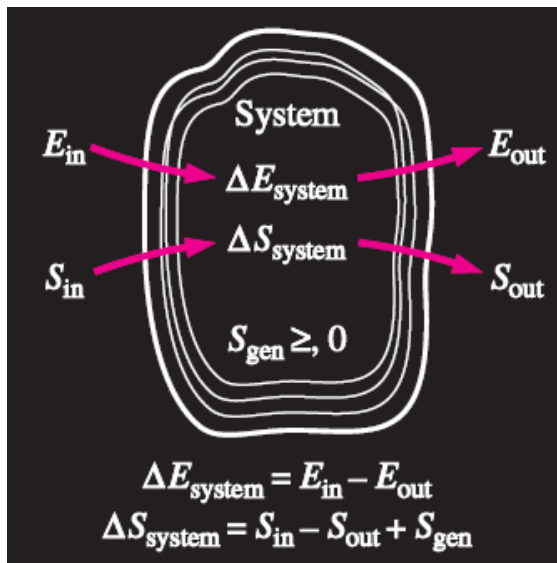
$$0.92 = \frac{950 - T_{2a}}{950 - 748} \rightarrow T_{2a} = \mathbf{764 \text{ K}}$$

(c) The actual exit velocity of air can be determined from the definition of isentropic efficiency of a nozzle,

$$\eta_N = \frac{V_{2a}^2}{V_{2s}^2} \rightarrow V_{2a} = \sqrt{\eta_N V_{2s}^2} = \mathbf{639 \text{ m/s}}$$

# ENTROPY BALANCE

The property **entropy** is a measure of molecular disorder or randomness of a system, and the second law of thermodynamics states that entropy can be created but it cannot be destroyed.



*The entropy change of a system during a process is equal to the net entropy transfer through the system boundary and the entropy generated within the system.*

$$\left( \begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{entering} \end{array} \right) - \left( \begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{leaving} \end{array} \right) + \left( \begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{generated} \end{array} \right) = \left( \begin{array}{c} \text{Change in the} \\ \text{total entropy} \\ \text{of the system} \end{array} \right)$$

Energy and entropy balances for a system.

$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}}$$

## Entropy Change of a System, $\Delta S_{\text{system}}$

Entropy change = Entropy at final state – Entropy at initial state

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

When the properties of the system are not uniform

the entropy change of steady-flow devices such as nozzles, compressors, turbines, pumps, and heat exchangers is zero during steady operation.

When the properties of the system are not uniform, the entropy of the system can be determined by integration from

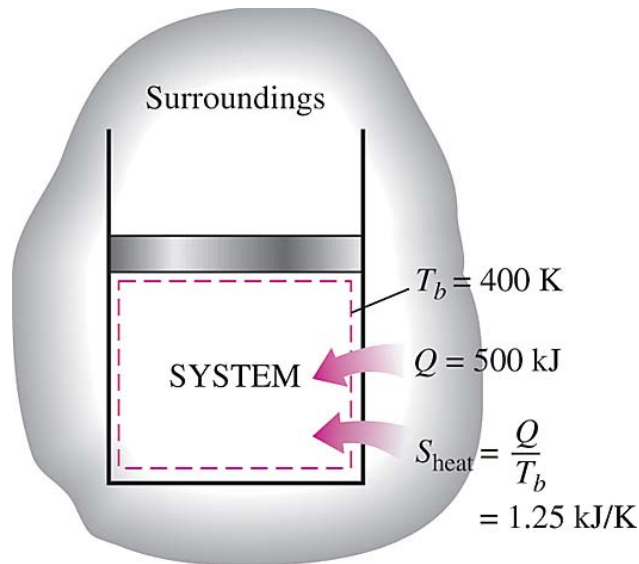
$$S_{\text{system}} = \int s \, \delta m = \int_V s \rho \, dV$$

# Mechanisms of Entropy Transfer, $S_{in}$ and $S_{out}$

## 1 Heat Transfer

Entropy transfer by heat transfer:

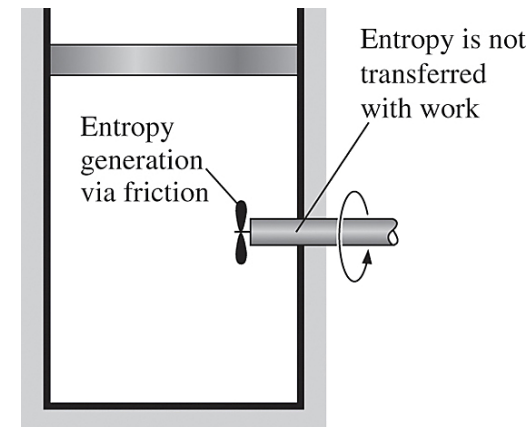
$$S_{\text{heat}} = \frac{Q}{T} \quad (T = \text{constant}) \quad S_{\text{heat}} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$



Heat transfer is always accompanied by entropy transfer in the amount of  $Q/T$ , where  $T$  is the boundary temperature.

Entropy transfer by work:

$$S_{\text{work}} = 0$$



No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.

# Mechanisms of Entropy Transfer, $S_{in}$ and $S_{out}$

## 2 Mass Flow

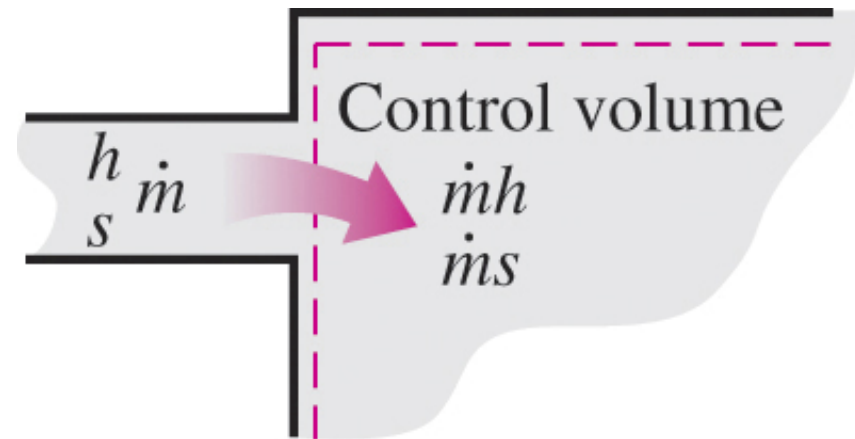
Entropy transfer by mass:

$$S_{\text{mass}} = ms$$

When the properties of the mass change during the process

$$\dot{S}_{\text{mass}} = \int_{A_c} s \rho V_n dA_c$$

$$S_{\text{mass}} = \int s \delta m = \int_{\Delta t} \dot{S}_{\text{mass}} dt$$



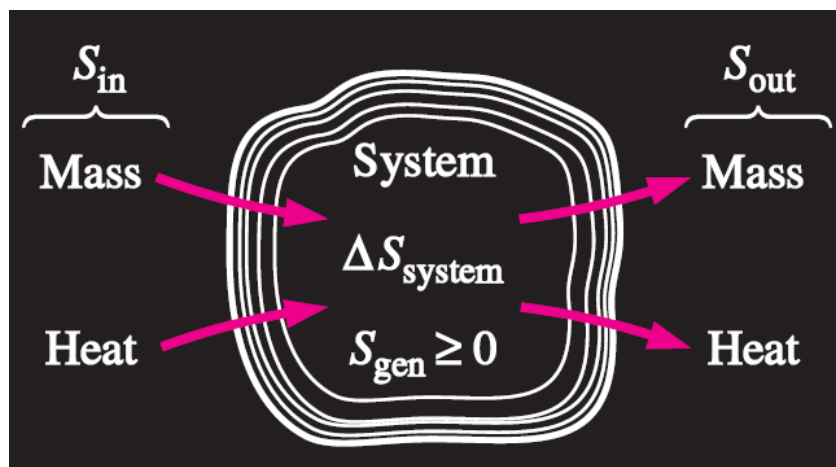
Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.

# Entropy Generation, $S_{\text{gen}}$

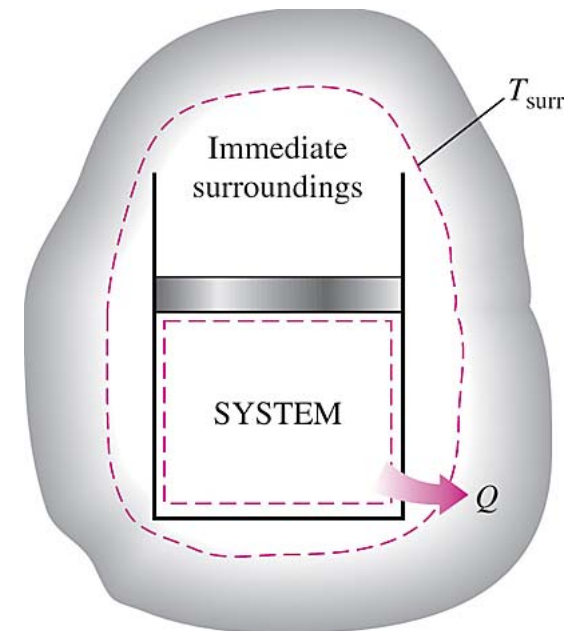
$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}} \quad (\text{kJ/K})$$

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \quad (\text{kW/K})$$

$$(s_{\text{in}} - s_{\text{out}}) + s_{\text{gen}} = \Delta s_{\text{system}} \quad (\text{kJ/kg} \cdot \text{K})$$



Mechanisms of entropy transfer for a general system.



Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.



# Closed Systems

$$\text{Closed system:} \quad \sum \frac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \quad (\text{kJ/K})$$

The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

$$\text{Adiabatic closed system:} \quad S_{\text{gen}} = \Delta S_{\text{adiabatic system}}$$

$$\text{System + Surroundings:} \quad S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$$

$$\Delta S_{\text{system}} = m(s_2 - s_1) \quad \Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$$

# Control Volumes

$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = (S_2 - S_1)_{\text{CV}}$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = \Delta \dot{S}_{\text{CV}}$$

*Steady-flow:*

$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

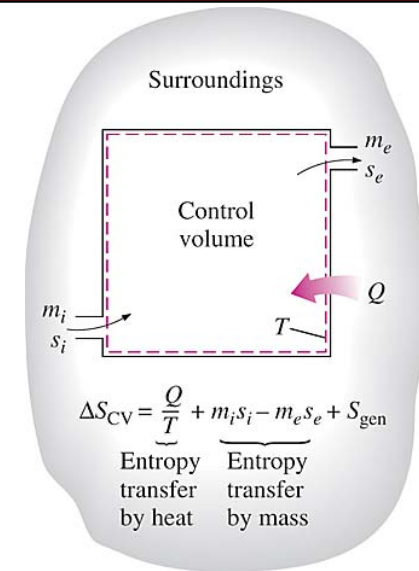
*Steady-flow, single-stream:*

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$$

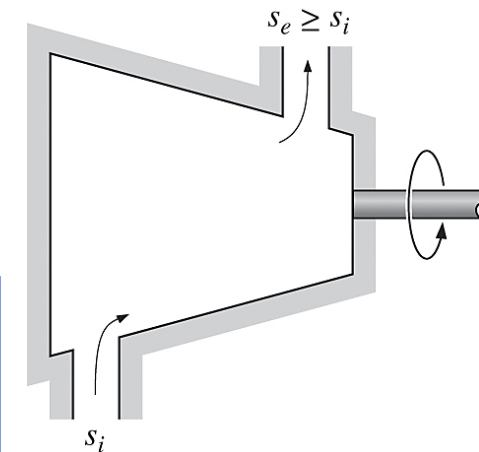
*Steady-flow, single-stream, adiabatic:*

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i)$$

The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.



The entropy of a control volume changes as a result of mass flow as well as heat transfer.

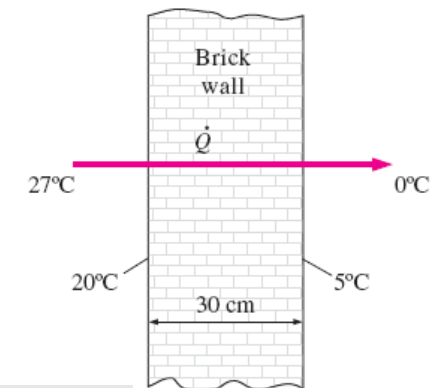


## EXAMPLE 7-17

Consider steady heat transfer through a 5-m 7-m brick wall of a house of thickness 30 cm. On a day when the temperature of the outdoors is 0°C, the house is maintained at 27°C. The temperatures of the inner and outer surfaces of the brick wall are measured to be 20°C and 5°C, respectively, and the rate of heat transfer through the wall is 1035 W. Determine the rate of entropy generation in the wall, and the rate of total entropy generation associated with this heat transfer process..

**Solution** Steady heat transfer through a wall is considered. For specified heat transfer rate, wall temperatures, and environment temperatures, the entropy generation rate within the wall and the total entropy generation rate are to be determined.

**Assumptions** 1 The process is steady, and thus the rate of heat transfer through the wall is constant. 2 Heat transfer through the wall is 1-D.



$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}^0}_{\text{Rate of change in entropy}}$$

$$\left( \frac{\dot{Q}}{T} \right)_{\text{in}} - \left( \frac{\dot{Q}}{T} \right)_{\text{out}} + \dot{S}_{\text{gen}} = 0$$

$$\frac{1035 \text{ W}}{293 \text{ K}} - \frac{1035 \text{ W}}{278 \text{ K}} + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen, wall}} = \mathbf{0.191 \text{ W/K}}$$

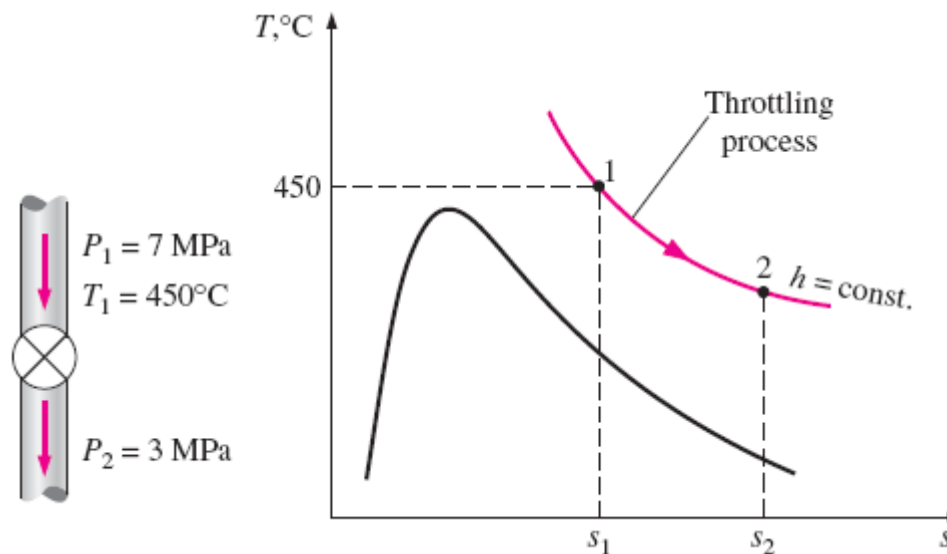
$$\frac{1035 \text{ W}}{300 \text{ K}} - \frac{1035 \text{ W}}{273 \text{ K}} + \dot{S}_{\text{gen, total}} = 0 \rightarrow \dot{S}_{\text{gen, total}} = \mathbf{0.341 \text{ W/K}}$$

## EXAMPLE 7-18

Steam at 7 MPa and 450°C is throttled in a valve to a pressure of 3 MPa during a steady-flow process. Determine the entropy generated during this process and check if the increase of entropy principle is satisfied.

**Solution** Steam is throttled to a specified pressure. The entropy generated during this process is to be determined, and the validity of the increase of entropy principle is to be shown.

**Assumptions** **1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{CV}=0$ ,  $\Delta E_{CV}=0$ , and  $\Delta S_{CV}=0$ . **2** Heat transfer to or from the valve is negligible. **3** The kinetic and potential energy changes are negligible,  $\Delta ke = \Delta pe = 0$ .



$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 7 \text{ MPa} \\ T_1 = 450^\circ\text{C} \end{array} \right\} \quad \begin{array}{l} h_1 = 3288.3 \text{ kJ/kg} \\ s_1 = 6.6353 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 3 \text{ MPa} \\ h_2 = h_1 \end{array} \right\} \quad s_2 = 7.0046 \text{ kJ/kg} \cdot \text{K}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$

$$s_{\text{gen}} = s_2 - s_1 = 7.0046 - 6.6353 = \mathbf{0.3693 \text{ kJ/kg} \cdot \text{K}}$$

This is the amount of entropy generated per unit mass of steam as it is throttled from the inlet state to the final pressure, and it is caused by unrestrained expansion. The increase of entropy principle is obviously satisfied during this process since the entropy generation is positive.

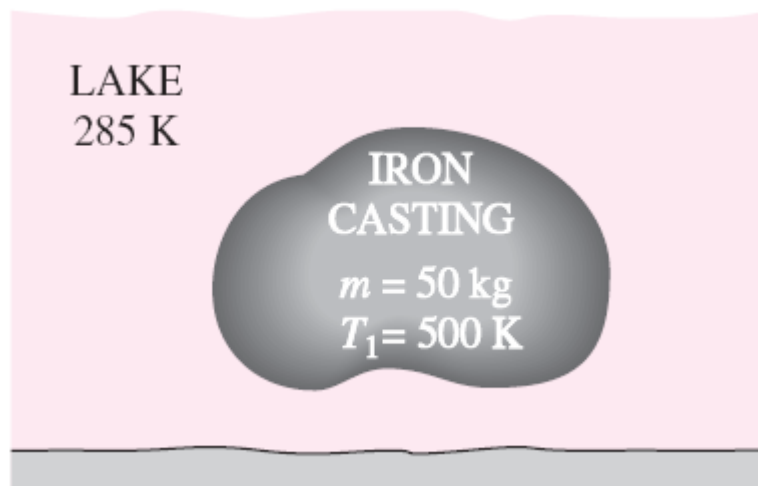
## EXAMPLE 7-19

A 50-kg block of iron casting at 500 K is thrown into a large lake that is at a temperature of 285 K. The iron block eventually reaches thermal equilibrium with the lake water. Assuming an average specific heat of 0.45 kJ/kg · K for the iron, determine (a) the entropy change of the iron block, (b) the entropy change of the lake water, and (c) the entropy generated during this process.

**Solution** A hot iron block is thrown into a lake, and cools to the lake temperature. The entropy changes of the iron and of the lake as well as the entropy generated during this process are to be determined.

**Assumptions** **1** Both the water and the iron block are incompressible substances. **2** Constant specific heats can be used for the water and the iron. **3** The kinetic and potential energy changes of the iron are negligible,  $\Delta KE = \Delta PE = 0$  and thus  $\Delta E = \Delta U$ . **4** There are no work interactions.

(a) Approximating the iron block as an incompressible substance, its entropy change can be determined from



$$\begin{aligned}\Delta S_{\text{iron}} &= m(s_2 - s_1) = mc_{\text{av}} \ln \frac{T_2}{T_1} \\ &= (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K}) \ln \frac{285 \text{ K}}{500 \text{ K}} \\ &= -12.65 \text{ kJ/K}\end{aligned}$$

(b) the entropy change of the lake water

$$-Q_{\text{out}} = \Delta U = mc_{\text{av}}(T_2 - T_1)$$

$$Q_{\text{out}} = mc_{\text{av}}(T_1 - T_2) = (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K})(500 - 285) \text{ K} = 4838 \text{ kJ}$$

$$\Delta S_{\text{lake}} = \frac{Q_{\text{lake}}}{T_{\text{lake}}} = \frac{+4838 \text{ kJ}}{285 \text{ K}} = \mathbf{16.97 \text{ kJ/K}}$$

(c) the entropy generated during this process.

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$

$$-\frac{Q_{\text{out}}}{T_b} + S_{\text{gen}} = \Delta S_{\text{system}}$$

$$S_{\text{gen}} = \frac{Q_{\text{out}}}{T_b} + \Delta S_{\text{system}} = \frac{4838 \text{ kJ}}{285 \text{ K}} - (12.65 \text{ kJ/K}) = \mathbf{4.32 \text{ kJ/K}}$$

**Discussion** The entropy generated can also be determined by taking the iron block and the entire lake as the system, which is an isolated system, and applying an entropy balance. An isolated system involves no heat or entropy transfer, and thus the entropy generation in this case becomes equal to the total entropy change,

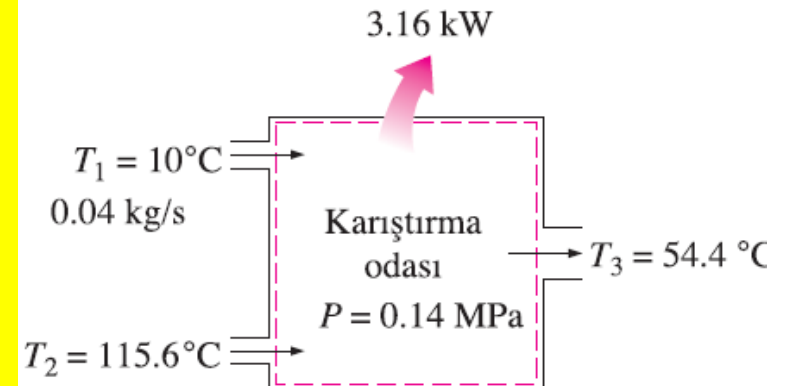
$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{lake}} = -12.65 + 16.97 = 4.32 \text{ kJ/K}$$

which is the same result obtained above.



### EXAMPLE 7-20

Water at 200 kPa and 10°C enters a mixing chamber at a rate of 150 kg/min where it is mixed steadily with steam entering at 200 kPa and 150°C. The mixture leaves the chamber at 200 kPa and 70°C, and heat is lost to the surrounding air at 20°C at a rate of 190 kJ/min. Neglecting the changes in kinetic and potential energies, determine the rate of entropy generation during this process.



*Kütle dengesi:*  $\dot{m}_g - \dot{m}_ç = \frac{dm_{\text{sistem}}}{dt} \overset{0 \text{ (sürekli)}}{=} 0 \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$

*Enerji dengesi:*  $\underbrace{\dot{E}_g - \dot{E}_ç}_{\text{Birim zamanda ısı, iş ve kütle ile gerçekleşen enerji geçişi}} = \underbrace{\frac{dE_{\text{sistem}}}{dt} \overset{0 \text{ (sürekli)}}{=}}_{\text{Birim zamanda sistemin iç, kinetik, potansiyel vb. enerjilerindeki değişim}} 0$

$$\dot{E}_g = \dot{E}_ç$$
$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{Q}_ç$$

$$(\dot{W} = 0, \text{ ke} \cong \text{pe} \cong 0 \text{ olduğundan})$$

$$\dot{Q}_ç = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\text{Hal 1:} \quad \left. \begin{array}{l} P_1 = 200 \text{ kPa} \\ T_1 = 10^\circ \text{C} \end{array} \right\} \quad \begin{array}{l} h_1 = h_{f@10^\circ \text{C}} = 42.022 \text{ kJ/kg} \\ s_1 = s_{f@10^\circ \text{C}} = 0.1511 \text{ kJ/kg.K} \end{array}$$

$$\text{Hal 2:} \quad \left. \begin{array}{l} P_2 = 200 \text{ kPa} \\ T_2 = 150^\circ \text{C} \end{array} \right\} \quad \begin{array}{l} h_2 = 2769.1 \text{ kJ/kg} \\ s_2 = 7.2810 \text{ kJ/kg.K} \end{array}$$

$$\text{Hal 3:} \quad \left. \begin{array}{l} P_3 = 200 \text{ kPa} \\ T_3 = 70^\circ \text{C} \end{array} \right\} \quad \begin{array}{l} h_3 = h_{f@70^\circ \text{C}} = 293.07 \text{ kJ/kg} \\ s_3 = s_{f@70^\circ \text{C}} = 0.9551 \text{ kJ/kg.K} \end{array}$$

$$190 \text{ kJ/dk} = [150 \times 42.022 + \dot{m}_2 \times 2769.1 - (150 + \dot{m}_2) \times 293.07] \text{ kJ/dk}$$

$$\dot{m}_2 = 15.29 \text{ kJ/dk}$$

$$\dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \frac{\dot{Q}_\xi}{T_b} + \dot{S}_{\text{üretim}} = 0$$

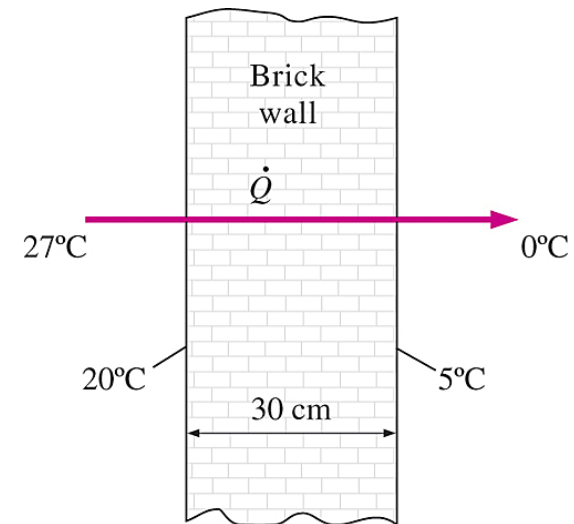
$$\begin{aligned} \dot{S}_{\text{üretim}} &= \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 + \frac{\dot{Q}_\xi}{T_b} \\ &= (165.29 \times 0.9551 - 150 \times 0.1511 - 15.29 \times 7.2810) \text{ kJ/dk} \cdot \text{K} \\ &\quad + \frac{190 \text{ kJ/dk}}{293 \text{ K}} \\ &= \mathbf{24.53 \text{ kJ/dk} \cdot \text{K}} \end{aligned}$$

# EXAMPLES

## Entropy balance for heat transfer through a wall

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \xrightarrow{0}$$

$$\left(\frac{\dot{Q}}{T}\right)_{\text{in}} - \left(\frac{\dot{Q}}{T}\right)_{\text{out}} + \dot{S}_{\text{gen}} = 0$$

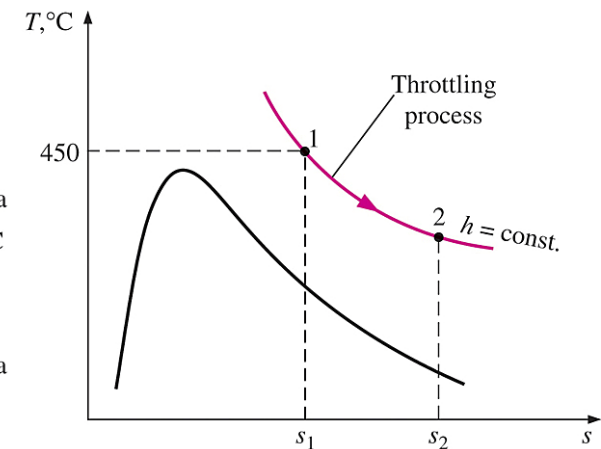
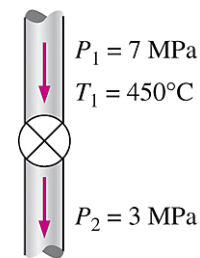


## Entropy balance for a throttling process

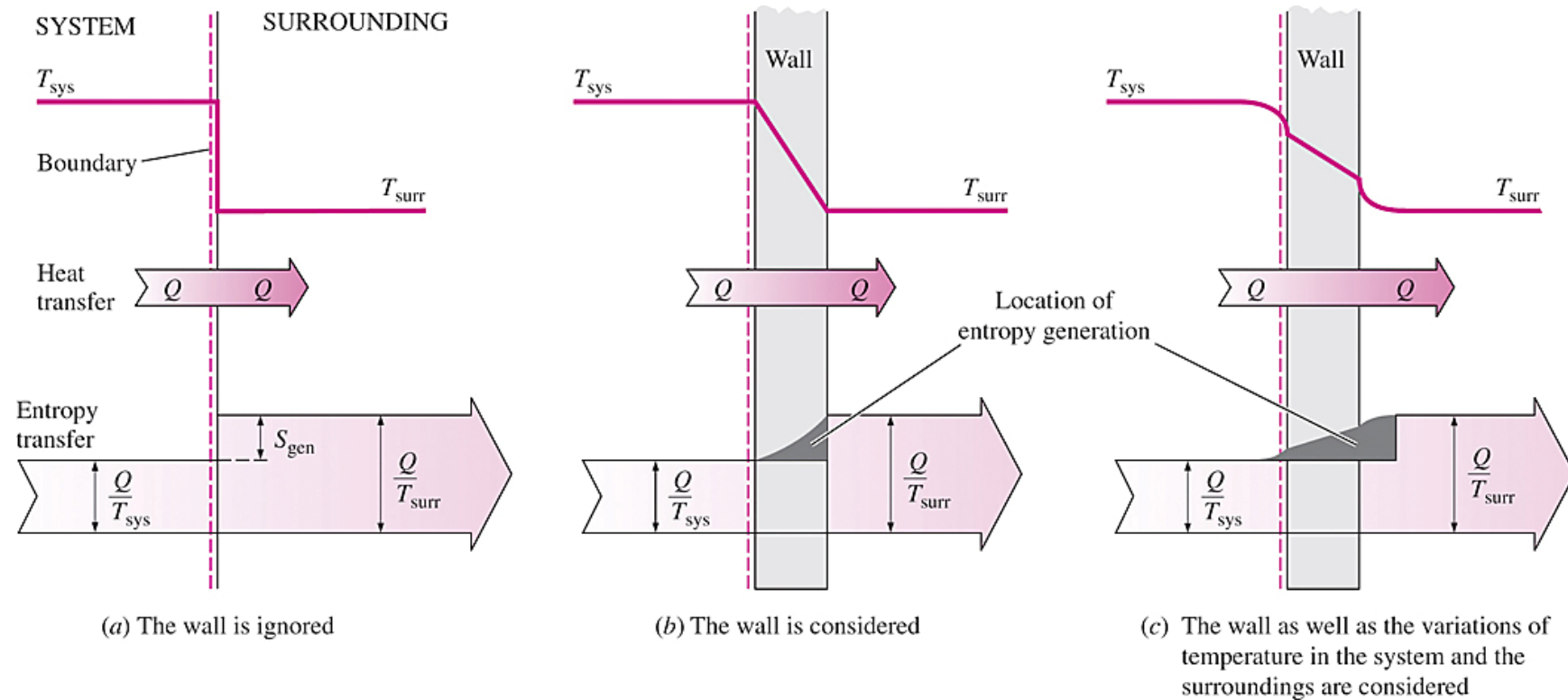
$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \xrightarrow{0 \text{ (steady)}}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$



# Entropy generation associated with a heat transfer process



Graphical representation of entropy generation during a heat transfer process through a finite temperature difference.

# SUMMARY

- Entropy
- The Increase of entropy principle
- Some remarks about entropy
- Entropy change of pure substances
- Isentropic processes
- Property diagrams involving entropy
- What is entropy?
- The  $T ds$  relations
- Entropy change of liquids and solids
- The entropy change of ideal gases
- Reversible steady-flow work
- Minimizing the compressor work
- Isentropic efficiencies of steady-flow devices
- Entropy balance

Thermodynamics: An Engineering Approach, 5<sup>th</sup> Edition  
Yunus A. Cengel, Michael A. Boles  
McGraw-Hill, 2008

# Chapter 8

## EXERGY: A MEASURE OF WORK POTENTIAL

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Prof. Dr. Ali PINARBAŞI

Yildiz Technical University  
Mechanical Engineering Department  
Yildiz, ISTANBUL

# EXERGY: A MEASURE OF WORK POTENTIAL

## 8-1 Exergy: Work Potential of Energy

Exergy (Work Potential) Associated with Kinetic and Potential Energy

## 8-2 Reversible Work and Irreversibility

## 8-3 Second-Law Efficiency, $\eta_{II}$

## 8-4 Exergy Change of a System

Exergy of a Fixed Mass: Nonflow (or Closed System) Exergy

Exergy of a Flow Stream: Flow (or Stream) Exergy

## 8-5 Exergy Transfer by Heat, Work, and Mass

Exergy by Heat Transfer,  $Q$

Exergy Transfer by Work,  $W$

Exergy Transfer by Mass,  $m$

## 8-6 The Decrease of Exergy Principle and Exergy Destruction

Exergy Destruction

## 8-7 Exergy Balance: Closed Systems

## 8-8 Exergy Balance: Control Volumes

Exergy Balance for Steady-Flow Systems

Reversible Work,  $W_{rev}$

Second-Law Efficiency of Steady-Flow Devices,  $\eta_{II}$

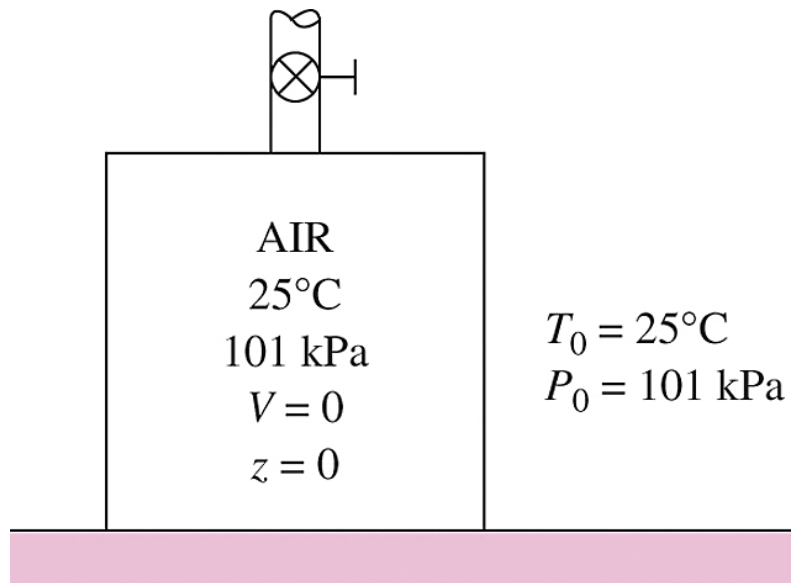


# Objectives

- Examine the performance of engineering devices in light of the second law of thermodynamics.
- Define *exergy*, which is the maximum useful work that could be obtained from the system at a given state in a specified environment.
- Define *reversible work*, which is the maximum useful work that can be obtained as a system undergoes a process between two specified states.
- Define the *exergy destruction*, which is the wasted work potential during a process as a result of irreversibilities.
- Define the *second-law efficiency*.
- Develop the *exergy balance relation*.
- Apply exergy balance to closed systems and control volumes.

# EXERGY: WORK POTENTIAL OF ENERGY

The useful work potential of a given amount of energy at some specified state is called *exergy*, which is also called the *availability* or *available energy*. A system is said to be in the *dead state* when it is in thermodynamic equilibrium with the environment it is in.



A system that is in equilibrium with its environment is said to be at the dead state.

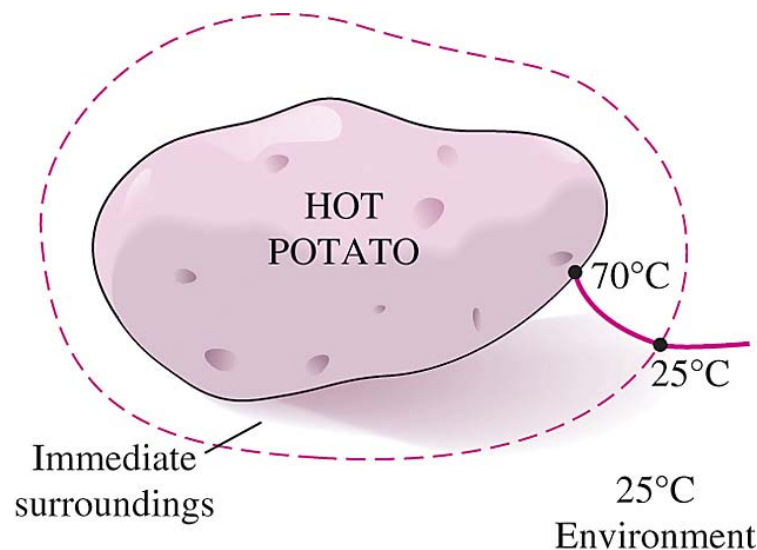


At the dead state, the useful work potential (exergy) of a system is zero.

*A system delivers the maximum possible work as it undergoes a reversible process from the specified initial state to the state of its environment, that is, the dead state.*

This represents the *useful work potential* of the system at the specified state and is called exergy.

Exergy represents the upper limit on the amount of work a device can deliver without violating any thermodynamic laws.



The immediate surroundings of a hot potato are simply the temperature gradient zone of the air next to the potato.



The atmosphere contains a tremendous amount of energy, but no exergy.

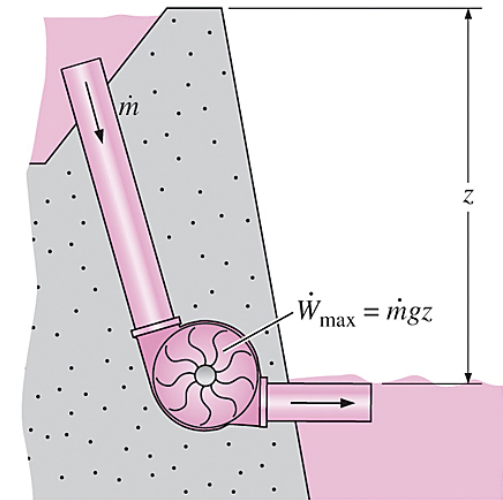
# Exergy (Work Potential) Associated with Kinetic and Potential Energy

*Exergy of kinetic energy:*

$$x_{ke} = ke = \frac{V^2}{2} \quad (\text{kJ/kg})$$

*Exergy of potential energy:*

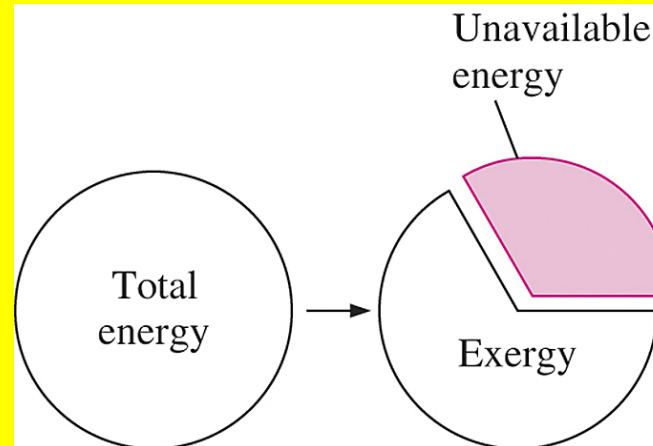
$$x_{pe} = pe = gz \quad (\text{kJ/kg})$$



The exergies of kinetic and potential energies are equal to themselves, and they are entirely available for work.

The *work potential* or *exergy* of potential energy is equal to the potential energy itself.

Unavailable energy is the portion of energy that cannot be converted to work by even a reversible heat engine.

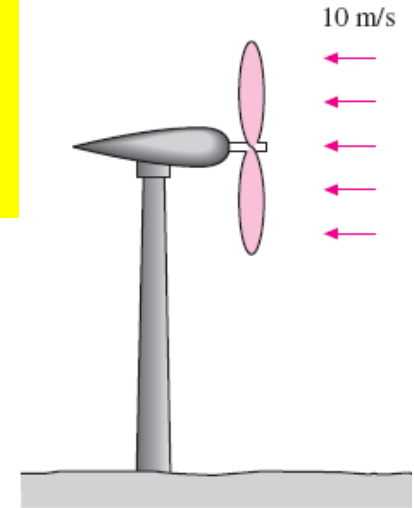


### EXAMPLE 8-1

A wind turbine with a 12-m-diameter rotor, is to be installed at a location where the wind is blowing steadily at an average velocity of 10 m/s. Determine the maximum power that can be generated by the wind turbine.

**Solution** A wind turbine is being considered for a specified location. The maximum power that can be generated by the wind turbine is to be determined.

**Assumptions** Air is at standard conditions of 1 atm and 25°C, and thus its density is 1.18 kg/m<sup>3</sup>.



$$ke_1 = \frac{V_1^2}{2} = \frac{(10 \text{ m/s})^2}{2} \left( \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} \right) = 0.05 \text{ kJ/kg}$$

$$\dot{m} = \rho AV_1 = \rho \frac{\pi D^2}{4} V_1 = (1.18 \text{ kg/m}^3) \frac{\pi (12 \text{ m})^2}{4} (10 \text{ m/s}) = 1335 \text{ kg/s}$$

$$\text{Maximum power} = \dot{m}(ke_1) = (1335 \text{ kg/s})(0.05 \text{ kJ/kg}) = \mathbf{66.7 \text{ kW}}$$

A conversion efficiency of 25 %, an actual wind turbine will convert 16.7 kW to electricity.

**Discussion** Betz's law states that the power output of a wind machine will be at maximum when the wind is slowed to one-third of its initial velocity. Therefore, for maximum power the highest efficiency of a wind turbine is about 59 %. In practice, the actual efficiency ranges between 20 and 40 % and is about 35 % for most wind turbines.

## EXAMPLE 8-2

Consider a large furnace that can transfer heat at a temperature of 1100 K at a steady rate of 3000 kW. Determine the rate of exergy flow associated with this heat transfer. Assume an environment temperature of 25°C.

**Solution** Heat is being supplied by a large furnace at a specified temperature. The rate of exergy flow is to be determined.

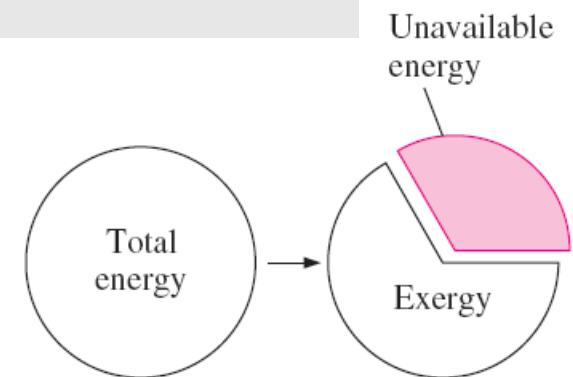
**Analysis** The furnace supplies heat indefinitely at a constant temperature. The exergy of this heat energy is its useful work potential, that is, the maximum possible amount of work that can be extracted from it. This corresponds to the amount of work that a reversible heat engine operating between the furnace and the environment can produce.

The thermal efficiency of this reversible heat engine is

$$\eta_{\text{th,max}} = \eta_{\text{th,rev}} = 1 - \frac{T_L}{T_H} = 1 - \frac{T_0}{T_H} = 1 - \frac{298 \text{ K}}{1100 \text{ K}} = 0.729 \text{ (or 72.9\%)}$$

$$\dot{W}_{\text{max}} = \dot{W}_{\text{rev}} = \eta_{\text{th,rev}} \dot{Q}_{\text{in}} = (0.729)(3000 \text{ kW}) = \mathbf{2187 \text{ kW}}$$

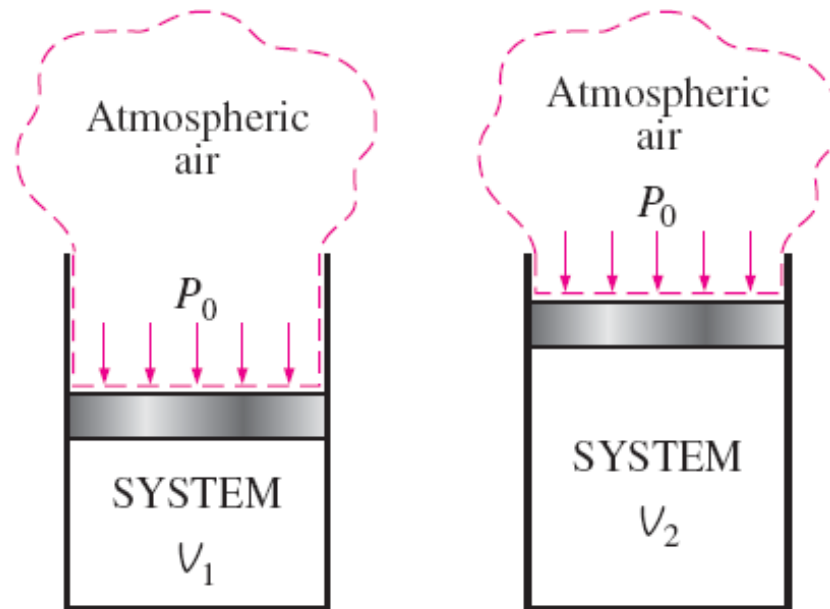
**Discussion** Notice that 26.8 percent of the heat transferred from the furnace is not available for doing work. The portion of energy that cannot be converted to work is called **unavailable energy**. Unavailable energy is simply the difference between the total energy of a system at a specified state and the exergy of that energy.



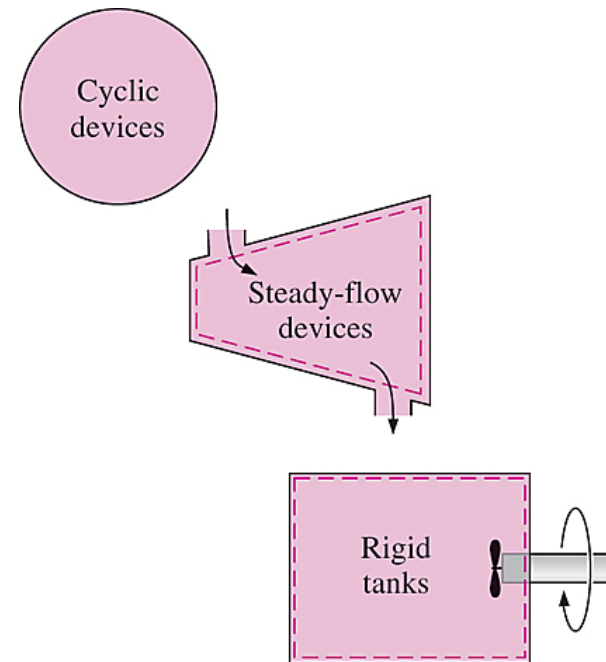


# REVERSIBLE WORK AND IRREVERSIBILITY

**Reversible work  $W_{rev}$ :** The maximum amount of useful work that can be produced (or the minimum work that needs to be supplied) as a system undergoes a process between the specified initial and final states.

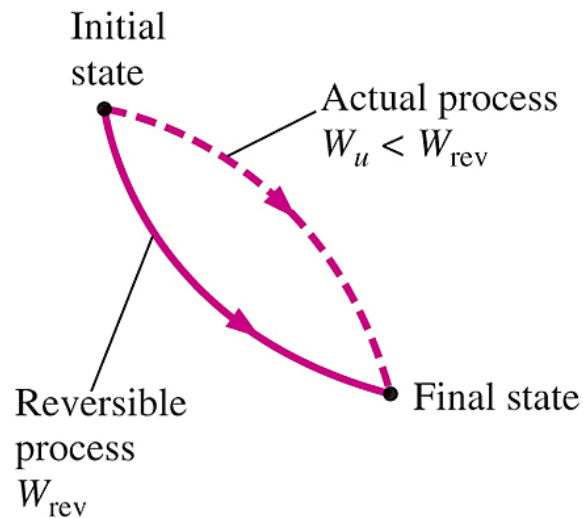


As a closed system expands, some work needs to be done to push the atmospheric air out of the way ( $W_{surr}$ ).



For constant-volume systems, the total actual and useful works are identical ( $W_u = W$ ).





$$I = W_{\text{rev}} - W_u$$

The difference between reversible work and actual useful work is the irreversibility.

**surroundings work**

$$W_{\text{surr}} = P_0(V_2 - V_1)$$

**Useful work  $W_u$**

$$W_u = W - W_{\text{surr}} = W - P_0(V_2 - V_1)$$

**irreversibility  $I$**

$$I = W_{\text{rev, out}} - W_{u, \text{out}} \quad \text{or} \quad I = W_{u, \text{in}} - W_{\text{rev, in}}$$

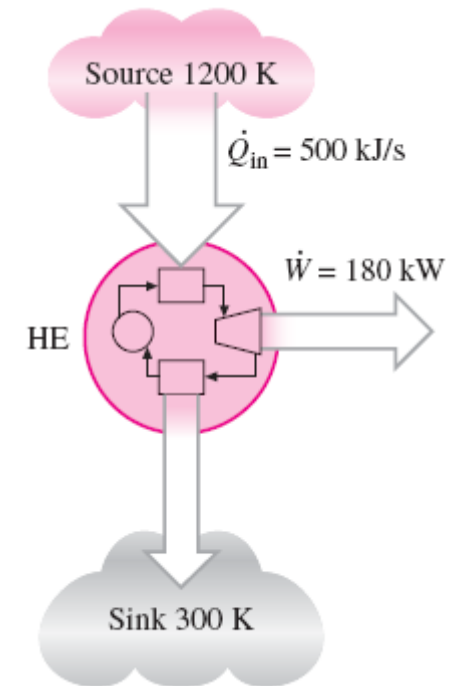
**Irreversibility** can be viewed as the *wasted work potential or the lost opportunity* to do work. It represents the energy that could have been converted to work but was not. The smaller the irreversibility associated with a process, the greater the work that will be produced (or the smaller the work that will be consumed). The performance of a system can be improved by minimizing the irreversibility associated with it.

### EXAMPLE 8-3

A heat engine receives heat from a source at 1200 K at a rate of 500 kJ/s and rejects the waste heat to a medium at 300 K. The power output of the heat engine is 180 kW. Determine the reversible power and the irreversibility rate for this process.

**Solution** A heat engine operating between a specified source and a specified sink. The reversible power and the irreversibility rate associated with this operation are to be determined.

**Analysis** The reversible power for this process is the amount of power that a reversible heat engine, such as a Carnot heat engine would produce when operating between the same temperature limits.



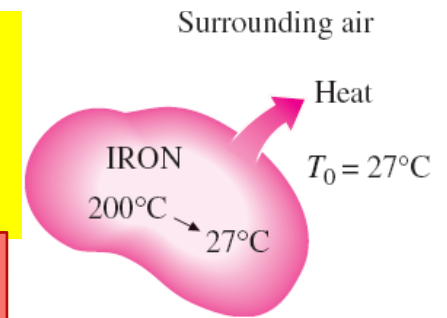
$$\dot{W}_{\text{rev}} = \eta_{\text{th, rev}} \dot{Q}_{\text{in}} = \left(1 - \frac{T_{\text{sink}}}{T_{\text{source}}}\right) \dot{Q}_{\text{in}} = \left(1 - \frac{300 \text{ K}}{1200 \text{ K}}\right) (500 \text{ kW}) = \mathbf{375 \text{ kW}}$$

$$\dot{I} = \dot{W}_{\text{rev, out}} - \dot{W}_{u, \text{out}} = 375 - 180 = \mathbf{195 \text{ kW}}$$

**Discussion** Note that 195 kW of power potential is wasted during this process as a result of irreversibilities. Also, the 500 - 375 = 125 kW of heat rejected to the sink is not available for converting to work and thus is not part of the irreversibility.

### EXAMPLE 8-4

A 500-kg iron block is initially at 200°C and is allowed to cool to 27°C by transferring heat to the surrounding air at 27°C. Determine the reversible work and the irreversibility for this process.



**Solution** A hot iron block is allowed to cool in air. The reversible work and irreversibility associated with this process are to be determined.

**Assumptions** 1 The kinetic and potential energies are negligible. 2 The process involves no work interactions.

$$\delta W_{\text{rev}} = \eta_{\text{th, rev}} \delta Q_{\text{in}} = \left(1 - \frac{T_{\text{sink}}}{T_{\text{source}}}\right) \delta Q_{\text{in}} = \left(1 - \frac{T_0}{T}\right) \delta Q_{\text{in}}$$

$$W_{\text{rev}} = \int \left(1 - \frac{T_0}{T}\right) \delta Q_{\text{in}}$$

$$-\delta Q_{\text{out}} = dU = mC_{\text{av}} dT$$

$$\delta Q_{\text{in, heat engine}} = \delta Q_{\text{out, system}} = -mC_{\text{av}} dT$$

$$\begin{aligned} W_{\text{rev}} &= \int_{T_1}^{T_0} \left(1 - \frac{T_0}{T}\right) (-mC_{\text{av}} dT) = mC_{\text{av}}(T_1 - T_0) - mC_{\text{av}}T_0 \ln \frac{T_1}{T_0} \\ &= (500 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K}) \left[ (473 - 300) \text{ K} - (300 \text{ K}) \ln \frac{473 \text{ K}}{300 \text{ K}} \right] \\ &= \mathbf{8191 \text{ kJ}} \end{aligned}$$

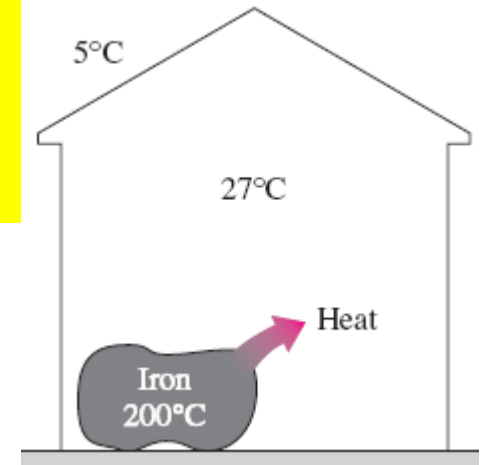
The irreversibility for this process is determined from its definition,

$$I = W_{\text{rev}} - W_u = 8191 - 0 = \mathbf{8191 \text{ kJ}}$$

### EXAMPLE 8-5

The iron block discussed in Example 8–4 is to be used to maintain a house at 27°C when the outdoor temperature is 5°C. Determine the maximum amount of heat that can be supplied to the house as the iron cools to 27°C.

**Solution** The iron block discussed before is considered for heating a house. The maximum amount of heating this block can provide is to be determined.



$$\text{COP}_{\text{HP}} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (278 \text{ K})/(300 \text{ K})} = 13.6$$

This heat pump can supply the house with 13.6 times the energy it consumes as work. In our case, it will consume the 8191 kJ of work and deliver  $8191 \times 13.6 = 111,398$  kJ of heat to the house. Therefore, the hot iron block has the potential to supply of heat to the house.

$$(30,734 + 111,398) \text{ kJ} = 142,132 \text{ kJ} \cong \mathbf{142 \text{ MJ}}$$

# SECOND-LAW EFFICIENCY, $\eta_{II}$

$$\eta_{II} = \frac{\eta_{th}}{\eta_{th, rev}} \quad (\text{heat engines})$$

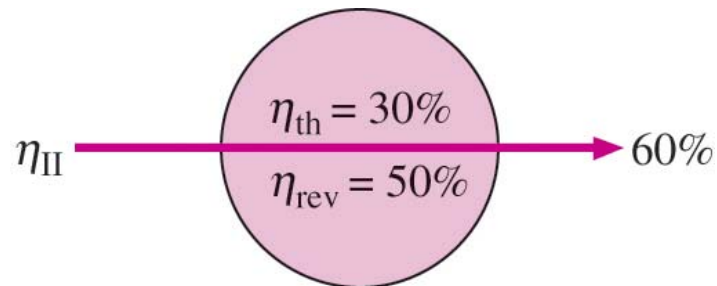
$$\eta_{II} = \frac{W_u}{W_{rev}} \quad (\text{work-producing devices})$$

$$\eta_{II} = \frac{W_{rev}}{W_u} \quad (\text{work-consuming devices})$$

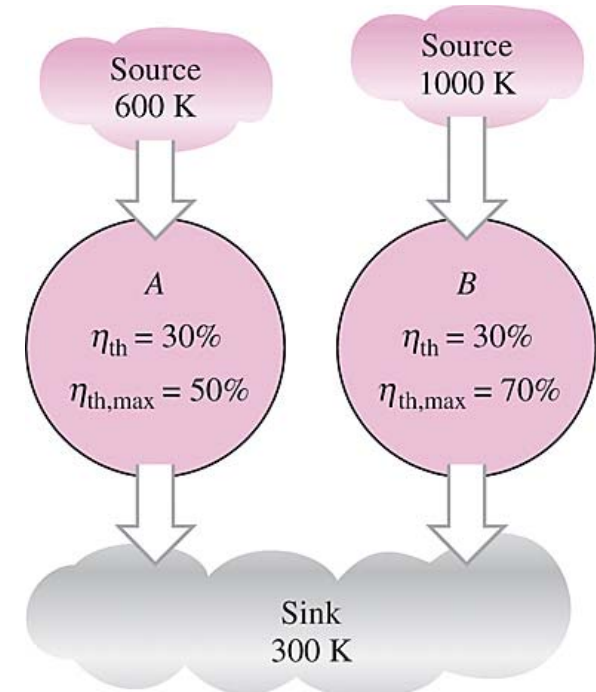
$$\eta_{II} = \frac{COP}{COP_{rev}} \quad (\text{refrigerators and heat pumps})$$

$$\eta_{rev, A} = \left( 1 - \frac{T_L}{T_H} \right)_A = 1 - \frac{300 \text{ K}}{600 \text{ K}} = 50\%$$

$$\eta_{rev, B} = \left( 1 - \frac{T_L}{T_H} \right)_B = 1 - \frac{300 \text{ K}}{1000 \text{ K}} = 70\%$$



$$\eta_{II, B} = \frac{0.30}{0.70} = 0.43$$

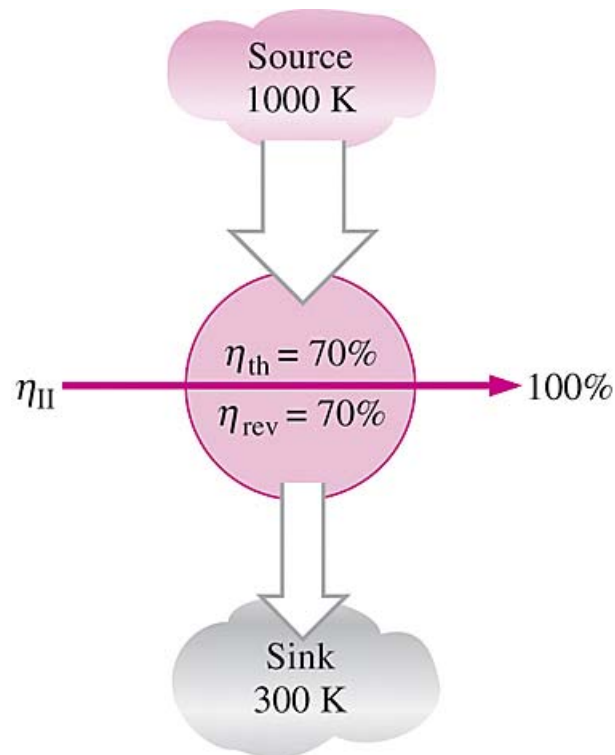


Second-law efficiency is a measure of the performance of a device relative to its performance under reversible conditions.

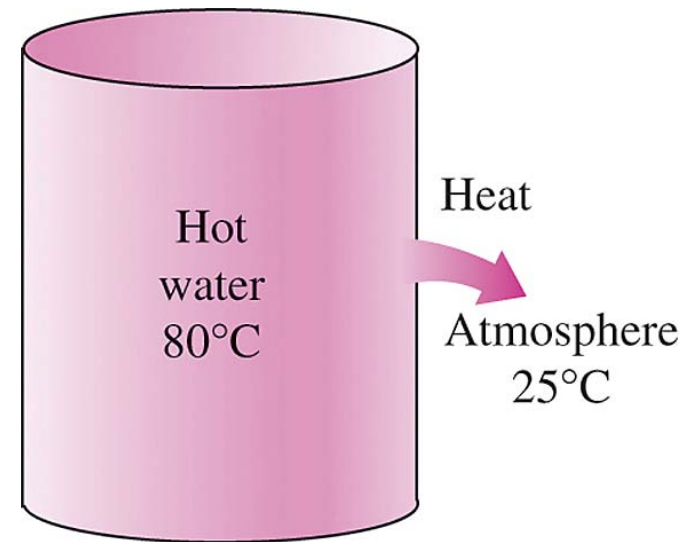
Two heat engines that have the same thermal efficiency, but different maximum thermal efficiencies.

$$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = 1 - \frac{\text{Exergy destroyed}}{\text{Exergy supplied}}$$

General definition of exergy efficiency



Second-law efficiency of all reversible devices is 100%.



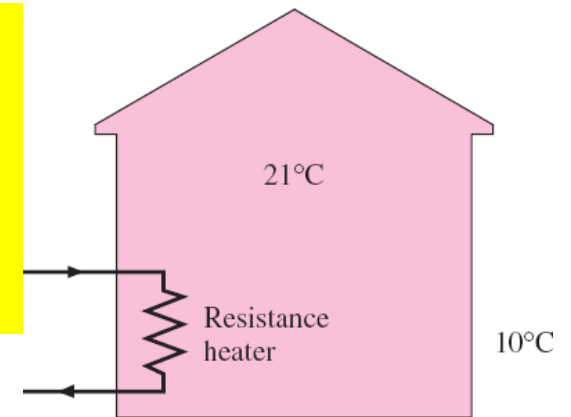
The second-law efficiency of naturally occurring processes is zero if none of the work potential is recovered.

### EXAMPLE 8-6

A dealer advertises that he has just received a shipment of electric resistance heaters for residential buildings that have an efficiency of 100 %. Assuming an indoor temperature of 21°C and outdoor temperature of 10°C, determine the second-law efficiency of these heaters.

**Solution** Electric resistance heaters are being considered for residential buildings. The second-law efficiency of these heaters is to be determined.

**Analysis** Obviously the efficiency that the dealer is referring to is the first law efficiency, meaning that for each unit of electric energy consumed, the heater will supply the house with 1 unit of energy (heat). That is, the advertised heater has a COP of 1.



A reversible heat pump would have a coefficient of the performance of

$$\text{COP}_{\text{HP, rev}} = \frac{1}{1 - T_L/T_H} = \frac{1}{1 - (283 \text{ K})/(294 \text{ K})} = 26.7$$

The second-law efficiency of this resistance heater is

$$\eta_{\text{II}} = \frac{\text{COP}}{\text{COP}_{\text{rev}}} = \frac{1.0}{26.7} = \mathbf{0.037 \text{ or } 3.7\%}$$

The dealer will not be happy to see this value. Considering the high price of electricity, a consumer will probably be better off with a “less” efficient gas heater.



# EXERGY CHANGE OF A SYSTEM

## Exergy of a Fixed Mass: Non flow (or Closed System) Exergy

$$\underbrace{\delta E_{\text{in}} - \delta E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{dE_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$
$$- \delta Q - \delta W = dU$$

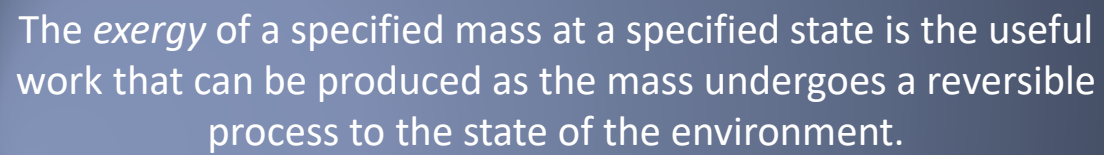
$$\delta W = P dV = (P - P_0) dV + P_0 dV = \delta W_{b, \text{useful}} + P_0 dV$$

$$\begin{aligned} \delta W_{\text{HE}} &= \left(1 - \frac{T_0}{T}\right) \delta Q = \delta Q - \frac{T_0}{T} \delta Q = \delta Q - (-T_0 dS) \rightarrow \delta Q \\ &= \delta W_{\text{HE}} - T_0 dS \end{aligned}$$

$$\delta W_{\text{total useful}} = \delta W_{\text{HE}} + \delta W_{b, \text{useful}} = -dU - P_0 dV + T_0 dS$$

$$W_{\text{total useful}} = (U - U_0) + P_0(V - V_0) - T_0(S - S_0)$$

$$X = (U - U_0) + P_0(V - V_0) - T_0(S - S_0) + m \frac{V^2}{2} + mgz$$



$$\begin{aligned}\phi &= (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \\ &= (e - e_0) + P_0(v - v_0) - T_0(s - s_0)\end{aligned}$$

Closed system  
exergy per unit mass

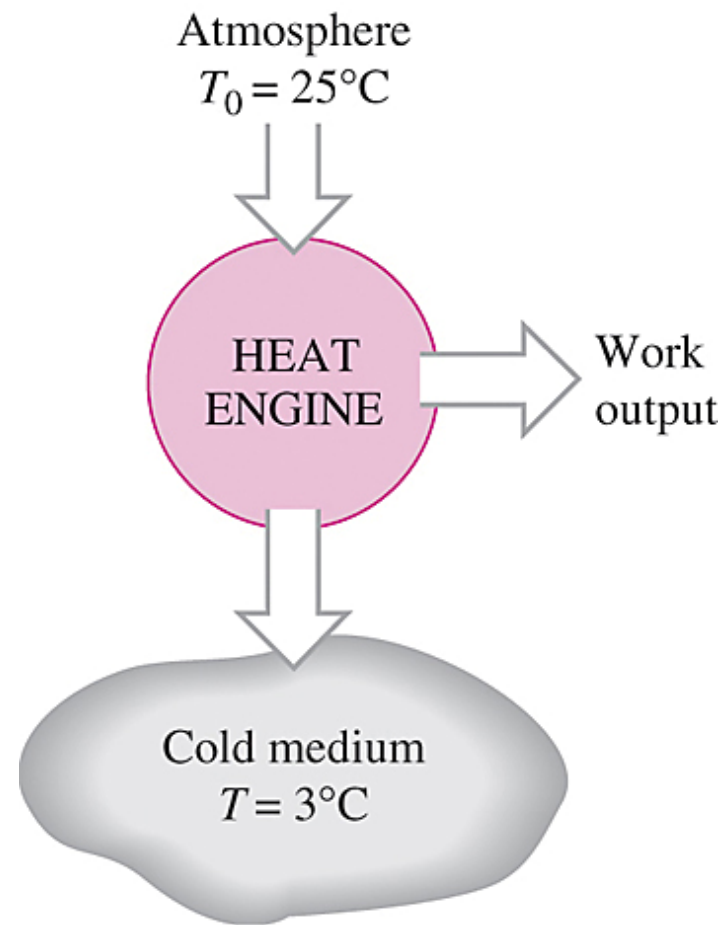
Exergy change of a closed system

$$\begin{aligned}\Delta X &= X_2 - X_1 = m(\phi_2 - \phi_1) = (E_2 - E_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1) \\ &= (U_2 - U_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1) + m\frac{V_2^2 - V_1^2}{2} + mg(z_2 - z_1)\end{aligned}$$

$$\begin{aligned}\Delta\phi &= \phi_2 - \phi_1 = (u_2 - u_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2} \\ &\quad + g(z_2 - z_1) \\ &= (e_2 - e_1) + P_0(v_2 - v_1) - T_0(s_2 - s_1)\end{aligned}$$

When the properties of a system are not uniform, the exergy of the system is

$$X_{\text{system}} = \int \phi \delta m = \int_V \rho dV$$



The *exergy* of a cold medium is also a *positive* quantity since work can be produced by transferring heat to it.

# Exergy of a Flow Stream: Flow (or Stream) Exergy

Exergy of flow energy

$$x_{\text{flow}} = P\upsilon - P_0\upsilon = (P - P_0)\upsilon$$

$$x_{\text{flowing fluid}} = x_{\text{nonflowing fluid}} + x_{\text{flow}}$$

$$= \underline{(u - u_0) + P_0(\upsilon - \upsilon_0) - T_0(s - s_0) + \frac{V^2}{2} + gz + (P - P_0)\upsilon}$$

$$= (u + P\upsilon) - (u_0 + P_0\upsilon_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

$$= (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

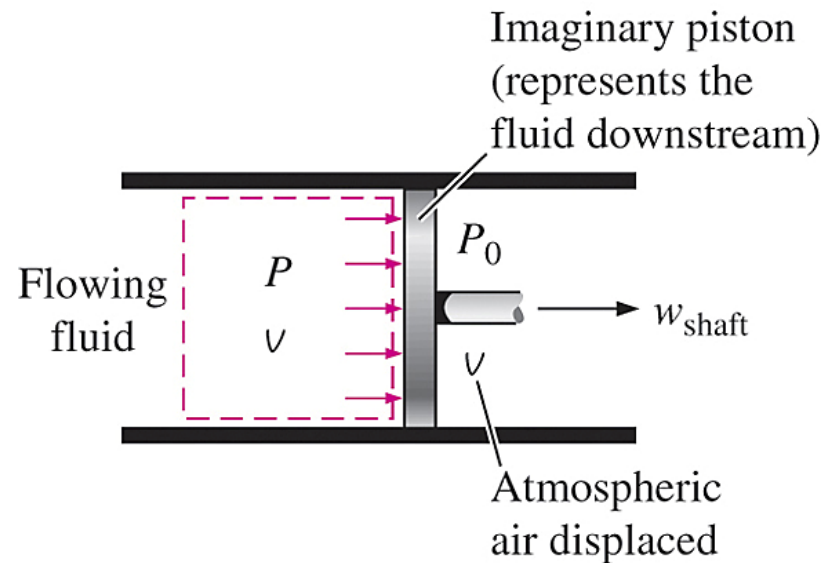
Flow exergy

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

## Exergy change of flow

$$\Delta\psi = \psi_2 - \psi_1$$

$$= (h_2 - h_1) + T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$



$$P\nu = P_0\nu + w_{\text{shaft}}$$

The *exergy* associated with *flow energy* is the useful work that would be delivered by an imaginary piston in the flow section.

**Energy:**

$$e = u + \frac{V^2}{2} + gz$$



**Exergy:**

$$\phi = (u - u_0) + P_0(v - v_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

**(a) A fixed mass (nonflowing)**

**Energy:**

$$\theta = h + \frac{V^2}{2} + gz$$



**Exergy:**

$$\psi = (h - h_0) + T_0(s - s_0) + \frac{V^2}{2} + gz$$

**(b) A fluid stream (flowing)**

The *energy* and *exergy* contents of (a) a fixed mass (b) a fluid stream.

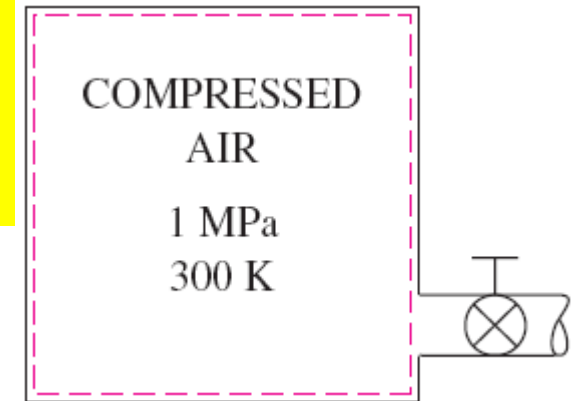


### EXAMPLE 8-7

A 200-m<sup>3</sup> rigid tank contains compressed air at 1 MPa and 300 K. Determine how much work can be obtained from this air if the environment conditions are 100 kPa and 300 K.

**Solution** Compressed air stored in a large tank is considered. The work potential of this air is to be determined.

**Assumptions** 1 Air is an ideal gas since it is at a high temperature and low pressure relative to its critical point values. 2 The kinetic and potential energies are negligible.



$$m_1 = \frac{P_1 V}{RT_1} = \frac{(1000 \text{ kPa})(200 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(300 \text{ K})} = 2323 \text{ kg}$$

The exergy content of the compressed air can be determined from

$$\begin{aligned} X_1 &= m\phi_1 \\ &= m \left[ (u_1 - u_0)^{\nearrow 0} + P_0(v_1 - v_0) - T_0(s_1 - s_0) + \frac{V_1^2}{2}^{\nearrow 0} + gz_1^{\nearrow 0} \right] \\ &= m[P_0(v_1 - v_0) - T_0(s_1 - s_0)] \end{aligned}$$

$$P_0(v_1 - v_0) = P_0 \left( \frac{RT_1}{P_1} - \frac{RT_0}{P_0} \right) = RT_0 \left( \frac{P_0}{P_1} - 1 \right) \quad (\text{since } T_1 = T_0)$$

$$T_0(s_1 - s_0) = T_0 \left( c_p \ln \frac{T_1}{T_0} - R \ln \frac{P_1}{P_0} \right) = -RT_0 \ln \frac{P_1}{P_0} \quad (\text{since } T_1 = T_0)$$

$$\begin{aligned} \phi_1 &= RT_0 \left( \frac{P_0}{P_1} - 1 \right) + RT_0 \ln \frac{P_1}{P_0} = RT_0 \left( \ln \frac{P_1}{P_0} + \frac{P_0}{P_1} - 1 \right) \\ &= (0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K}) \left( \ln \frac{1000 \text{ kPa}}{100 \text{ kPa}} + \frac{100 \text{ kPa}}{1000 \text{ kPa}} - 1 \right) \\ &= 120.76 \text{ kJ/kg} \end{aligned}$$

$$X_1 = m_1 \phi_1 = (2323 \text{ kg})(120.76 \text{ kJ/kg}) = \mathbf{280,525 \text{ kJ}}$$

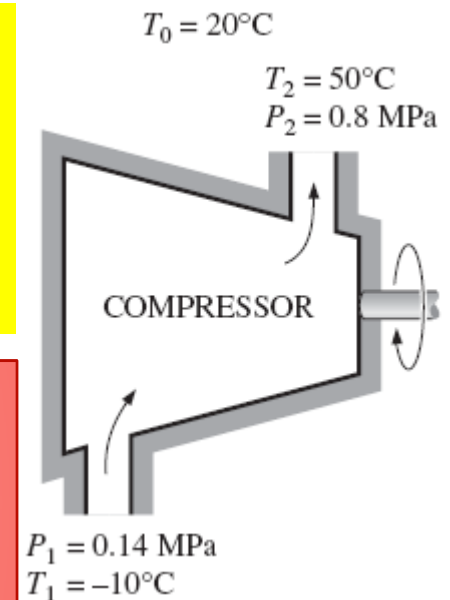
**Discussion** The work potential of the system is 280,525 kJ, and thus a maximum of 280,525 kJ of useful work can be obtained from the compressed air stored in the tank in the specified environment.

### EXAMPLE 8-8

Refrigerant-134a is to be compressed from 0.14 MPa and  $10^{\circ}\text{C}$  to 0.8 MPa and  $50^{\circ}\text{C}$  steadily by a compressor. Taking the environment conditions to be  $20^{\circ}\text{C}$  and 95 kPa, determine the exergy change of the refrigerant during this process and the minimum work input that needs to be supplied to the compressor per unit mass of the refrigerant.

**Solution** Refrigerant-134a is being compressed from a specified inlet state to a specified exit state. The exergy change of the refrigerant and the minimum compression work per unit mass are to be determined.

**Assumptions** 1 Steady operating conditions exist. 2 The kinetic and potential energies are negligible.



$$\text{Inlet state:} \quad \left. \begin{array}{l} P_1 = 0.14 \text{ MPa} \\ T_1 = -10^{\circ}\text{C} \end{array} \right\} \quad \begin{array}{l} h_1 = 246.36 \text{ kJ/kg} \\ s_1 = 0.9724 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{Exit state:} \quad \left. \begin{array}{l} P_2 = 0.8 \text{ MPa} \\ T_2 = 50^{\circ}\text{C} \end{array} \right\} \quad \begin{array}{l} h_2 = 286.69 \text{ kJ/kg} \\ s_2 = 0.9802 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\begin{aligned}
 \Delta\psi &= \psi_2 - \psi_1 = (h_2 - h_1) - T_0(s_2 - s_1) + \frac{V_2^2 - V_1^2}{2} \overset{0}{\rightarrow} + g(z_2 - z_1) \overset{0}{\rightarrow} \\
 &= (h_2 - h_1) - T_0(s_2 - s_1) \\
 &= (286.69 - 246.36) \text{ kJ/kg} - (293 \text{ K})[(0.9802 - 0.9724) \text{ kJ/kg} \cdot \text{K}] \\
 &= \mathbf{38.0 \text{ kJ/kg}}
 \end{aligned}$$

The exergy change of a system in a specified environment represents the reversible work in that environment, which is the minimum work input required for work-consuming devices such as compressors. Therefore, the increase in exergy of the refrigerant is equal to the minimum work that needs to be supplied to the compressor:

$$w_{\text{in, min}} = \psi_2 - \psi_1 = \mathbf{38.0 \text{ kJ/kg}}$$

**Discussion** Note that if the compressed refrigerant at 0.8 MPa and 50°C were to be expanded to 0.14 MPa and -10°C in a turbine in the same environment in a reversible manner, 38.0 kJ/kg of work would be produced.

# EXERGY TRANSFER BY HEAT, WORK, AND MASS

## Exergy by Heat Transfer, $Q$

$$X_{\text{heat}} = \left(1 - \frac{T_0}{T}\right)Q$$

Exergy transfer by heat

$$X_{\text{heat}} = \int \left(1 - \frac{T_0}{T}\right) \delta Q$$

When temperature is not constant

**HEAT SOURCE**

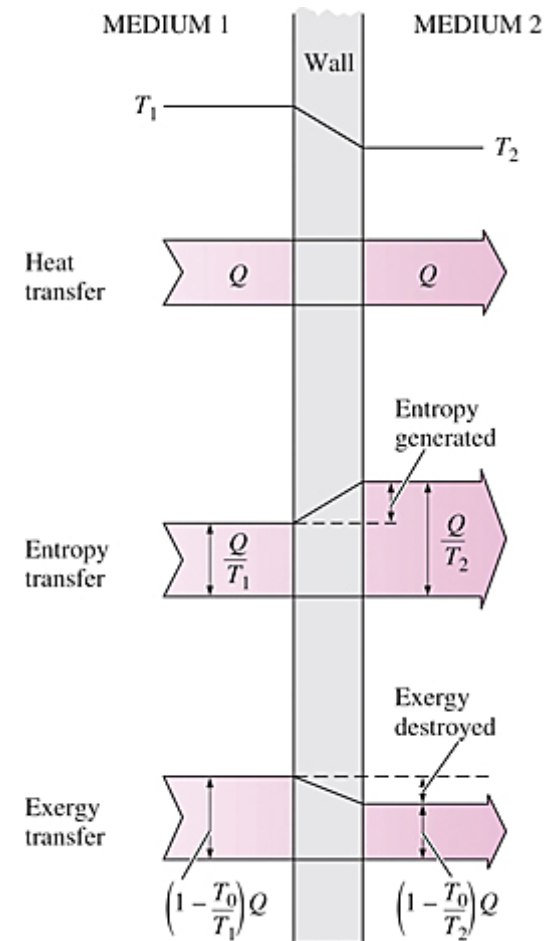
Temperature:  $T$

Energy content:  $E$

Exergy =  $\left(1 - \frac{T_0}{T}\right)E$

$T_0$

The Carnot efficiency  $\eta_c = 1 - T_0/T$  represents the fraction of the energy transferred from a heat source at temperature  $T$  that can be converted to work in an environment at temperature  $T_0$ .

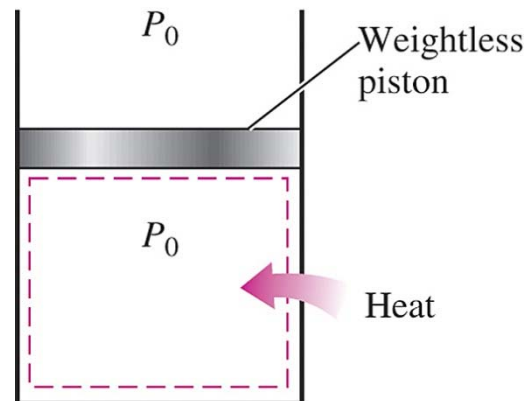


The transfer and destruction of exergy during a heat transfer process through a finite temperature difference.

# Exergy Transfer by Work, $W$

$$X_{\text{work}} = \begin{cases} W - W_{\text{surr}} & \text{(for boundary work)} \\ W & \text{(for other forms of work)} \end{cases}$$

$$W_{\text{surr}} = P_0(V_2 - V_1)$$



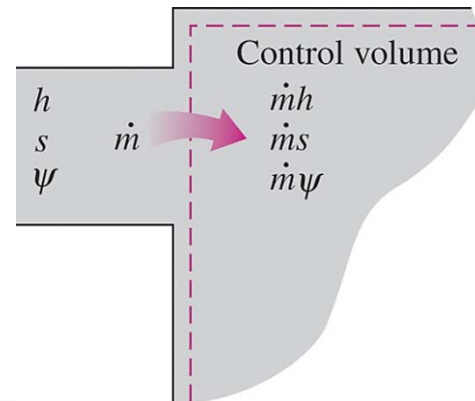
There is no useful work transfer associated with boundary work when the pressure of the system is maintained constant at atmospheric pressure.

# Exergy Transfer by Mass, $m$

$$X_{\text{mass}} = m\psi$$

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

$$\dot{X}_{\text{mass}} = \int_{A_c} \psi \rho V_n dA_c \quad \text{and} \quad X_{\text{mass}} = \int \psi \delta m = \int_{\Delta t} \dot{X}_{\text{mass}} dt$$



Mass contains energy, entropy, and exergy, and thus mass flow into or out of a system is accompanied by energy, entropy, and exergy transfer.



# THE DECREASE OF EXERGY PRINCIPLE AND EXERGY DESTRUCTION

Energy balance:  $E_{in}^0 - E_{out}^0 = \Delta E_{system} \rightarrow 0 = E_2 - E_1$

Entropy balance:  $S_{in}^0 - S_{out}^0 + S_{gen} = \Delta S_{system} \rightarrow S_{gen} = S_2 - S_1$

$$-T_0 S_{gen} = E_2 - E_1 - T_0(S_2 - S_1)$$

$$\begin{aligned} X_2 - X_1 &= (E_2 - E_1) + P_0(V_2 - V_1)^0 - T_0(S_2 - S_1) \\ &= (E_2 - E_1) - T_0(S_2 - S_1) \end{aligned}$$

$$-T_0 S_{gen} = X_2 - X_1 \leq 0$$

$$\Delta X_{isolated} = (X_2 - X_1)_{isolated} \leq 0$$

No heat, work  
or mass transfer

Isolated system

$$\Delta X_{isolated} \leq 0$$

(or  $X_{destroyed} \geq 0$ )

The isolated system considered  
in the development of the  
decrease of exergy principle.

*The exergy of an isolated system during a process always decreases or, in the limiting case of a reversible process, remains constant. In other words, it never increases and exergy is destroyed during an actual process. This is known as the **decrease of exergy principle**.*

# Exergy Destruction

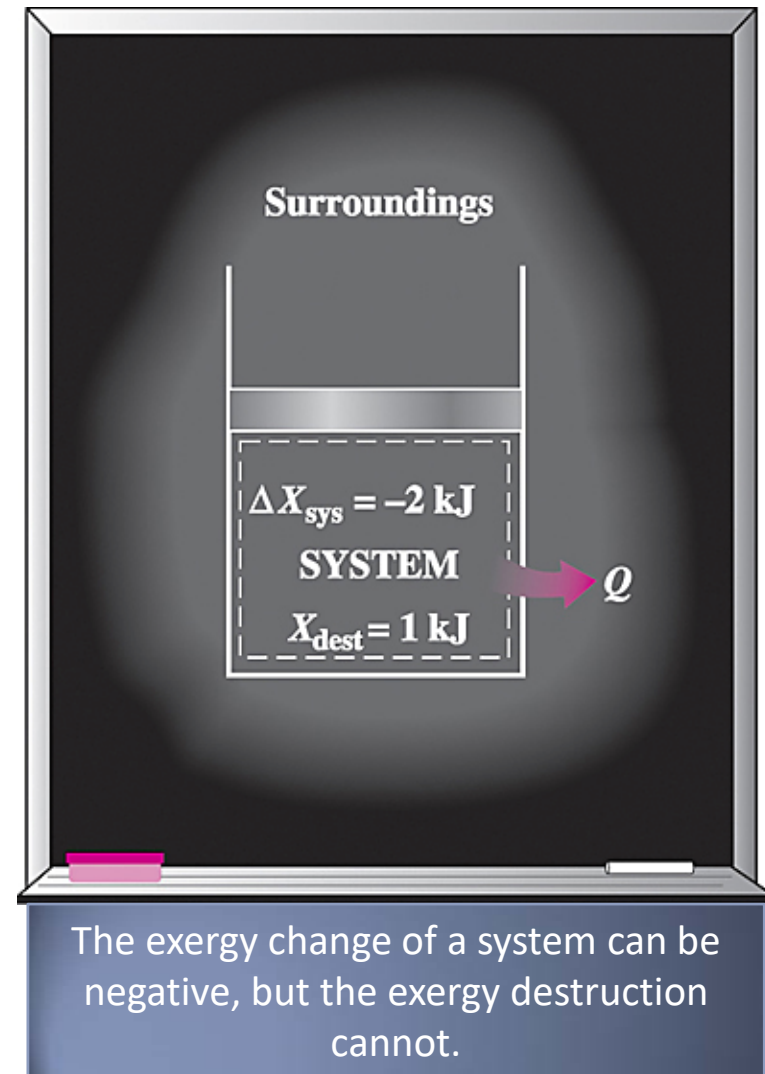
$$X_{\text{destroyed}} = T_0 S_{\text{gen}} \geq 0$$

$$X_{\text{destroyed}} = \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$

Exergy destroyed is a *positive quantity* for any actual process and becomes zero for a reversible process.

Exergy destroyed represents the lost work potential and is also called the *irreversibility* or *lost work*.

Can the exergy change of a system during a process be negative?

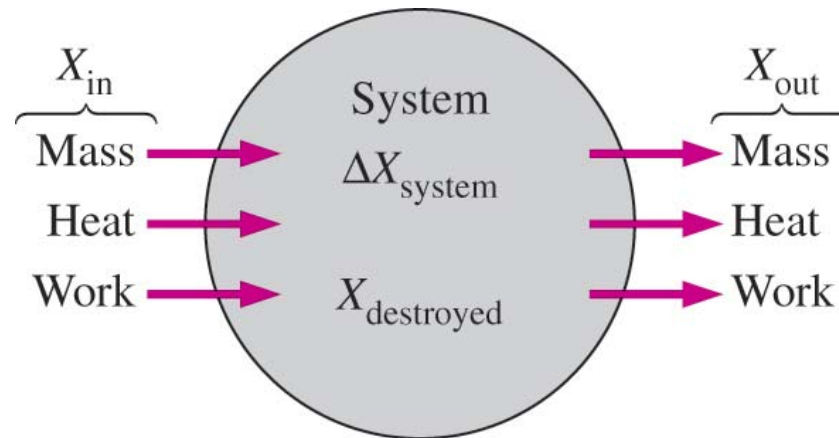


Consider heat transfer from a system to its surroundings. How do you compare exergy changes of the system and the surroundings?

# EXERGY BALANCE: CLOSED SYSTEMS

The nature of exergy is opposite to that of entropy in that exergy can be *destroyed*, but it cannot be created.

Therefore, the exergy change of a system during a process is equal to the difference between the net exergy transfer through the system boundary and the exergy destroyed within the system boundaries as a result of irreversibilities.



Mechanisms of exergy transfer.

$$\left( \begin{array}{c} \text{Total} \\ \text{exergy} \\ \text{entering} \end{array} \right) - \left( \begin{array}{c} \text{Total} \\ \text{exergy} \\ \text{leaving} \end{array} \right) - \left( \begin{array}{c} \text{Total} \\ \text{exergy} \\ \text{destroyed} \end{array} \right) = \left( \begin{array}{c} \text{Change in the} \\ \text{total exergy} \\ \text{of the system} \end{array} \right)$$

$$\text{General:} \quad \underbrace{X_{\text{in}} - X_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{X_{\text{destroyed}}}_{\text{Exergy destruction}} = \underbrace{\Delta X_{\text{system}}}_{\text{Change in exergy}} \quad (\text{kJ})$$

in the **rate form**

$$\text{General:} \quad \underbrace{\dot{X}_{\text{in}} - \dot{X}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\dot{X}_{\text{destroyed}}}_{\text{Rate of exergy destruction}} = \underbrace{\Delta \dot{X}_{\text{system}}}_{\text{Rate of change in exergy}} \quad (\text{kW})$$

$$\dot{X}_{\text{heat}} = (1 - T_0/T)\dot{Q},$$

$$\dot{X}_{\text{work}} = \dot{W}_{\text{useful}},$$

$$\dot{X}_{\text{mass}} = \dot{m}\psi$$

$$\text{General, unit-mass basis:} \quad (x_{\text{in}} - x_{\text{out}}) - x_{\text{destroyed}} = \Delta x_{\text{system}}$$

$$X_{\text{destroyed}} = T_0 S_{\text{gen}} \quad \text{or} \quad \dot{X}_{\text{destroyed}} = T_0 \dot{S}_{\text{gen}}$$

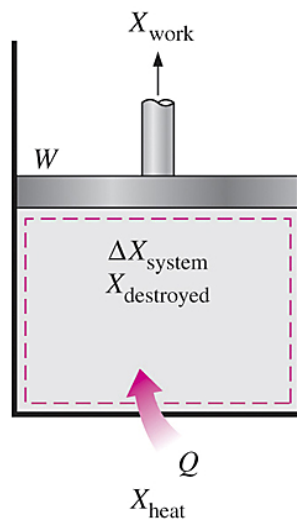
Closed system:

$$X_{\text{heat}} - X_{\text{work}} - X_{\text{destroyed}} = \Delta X_{\text{system}}$$

$$\text{Closed system: } \sum \left( 1 - \frac{T_0}{T_k} \right) Q_k - [W - P_0(V_2 - V_1)] - T_0 S_{\text{gen}} = X_2 - X_1$$

$$\text{Rate form: } \sum \left( 1 - \frac{T_0}{T_k} \right) \dot{Q}_k - \left( \dot{W} - P_0 \frac{dV_{\text{system}}}{dt} \right) - T_0 \dot{S}_{\text{gen}} = \frac{dX_{\text{system}}}{dt}$$

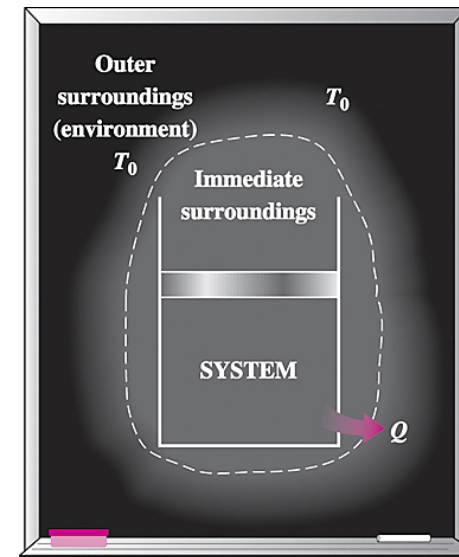
$Q_k$  is the heat transfer through the boundary at temperature  $T_k$  at location  $k$ .



$$X_{\text{heat}} - X_{\text{work}} - X_{\text{destroyed}} = \Delta X_{\text{system}}$$

Exergy balance for a closed system when heat transfer is to the system and the work is from the system.

The heat transfer to a system and work done by the system are taken to be positive quantities.



Exergy destroyed outside system boundaries can be accounted for by writing an exergy balance on the extended system that includes the system and its immediate surroundings.

### EXAMPLE 8-9

Starting with energy and entropy balances, derive the general exergy balance relation for a closed system

$$\sum \left(1 - \frac{T_0}{T_k}\right) Q_k - [W - P_0(V_2 - V_1)] - T_0 S_{\text{gen}} = X_2 - X_1$$

*Energy balance:*  $E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}} \rightarrow Q - W = E_2 - E_1$

*Entropy balance:*  $S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}} \rightarrow \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{boundary}} + S_{\text{gen}} = S_2 - S_1$

$$Q - T_0 \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{boundary}} - W - T_0 S_{\text{gen}} = E_2 - E_1 - T_0(S_2 - S_1)$$

$$\int_1^2 \delta Q - T_0 \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{boundary}} - W - T_0 S_{\text{gen}} = X_2 - X_1 - P_0(V_2 - V_1)$$

$$\int_1^2 \left(1 - \frac{T_0}{T_b}\right) \delta Q - [W - P_0(V_2 - V_1)] - T_0 S_{\text{gen}} = X_2 - X_1$$

**Discussion** The exergy balance relation above is obtained by adding the energy and entropy balance relations, and thus it is not an independent equation. However, it can be used in place of the entropy balance relation as an alternative second law expression in exergy analysis.

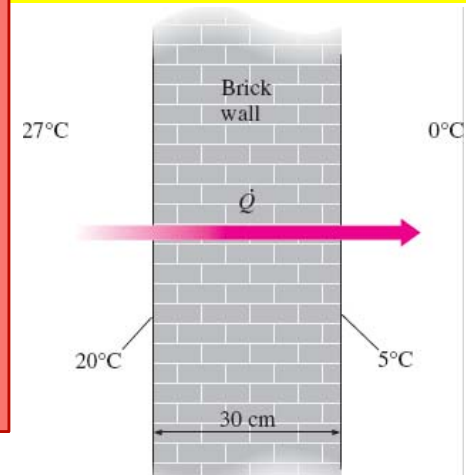


## EXAMPLE 8-10

Consider steady heat transfer through a 5-m x 6-m brick wall of a house of thickness 30 cm. On a day when the temperature of the outdoors is 0°C, the house is maintained at 27°C. The temperatures of the inner and outer surfaces of the brick wall are measured to be 20°C and 5°C, respectively, and the rate of heat transfer through the wall is 1035 W. Determine the rate of exergy destruction in the wall, and the rate of total exergy destruction associated with this heat transfer process.

**Solution** Steady heat transfer through a wall is considered. For specified heat transfer rate, wall surface temperatures, and environment conditions, the rate of exergy destruction within the wall and the rate of total exergy destruction are to be determined.

**Assumptions** **1** The process is steady, and thus the rate of heat transfer through the wall is constant. **2** The exergy change of the wall is zero during this process since the state and thus the exergy of the wall do not change anywhere in the wall. **3** Heat transfer through the wall is 1-D.



$$\underbrace{\dot{X}_{\text{in}} - \dot{X}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} - \underbrace{\dot{X}_{\text{destroyed}}}_{\text{Rate of exergy destruction}} = \underbrace{\Delta \dot{X}_{\text{system}}}_{\text{Rate of change in exergy}} \overset{0 \text{ (steady)}}{=} 0$$

$$\dot{Q} \left( 1 - \frac{T_0}{T} \right)_{\text{in}} - \dot{Q} \left( 1 - \frac{T_0}{T} \right)_{\text{out}} - \dot{X}_{\text{destroyed}} = 0$$

$$(1035 \text{ W}) \left( 1 - \frac{273 \text{ K}}{293 \text{ K}} \right) - (1035 \text{ W}) \left( 1 - \frac{273 \text{ K}}{278 \text{ K}} \right) - \dot{X}_{\text{destroyed}} = 0 \quad \dot{X}_{\text{destroyed}} = \mathbf{52.0 \text{ W}}$$



the rate of total exergy destruction

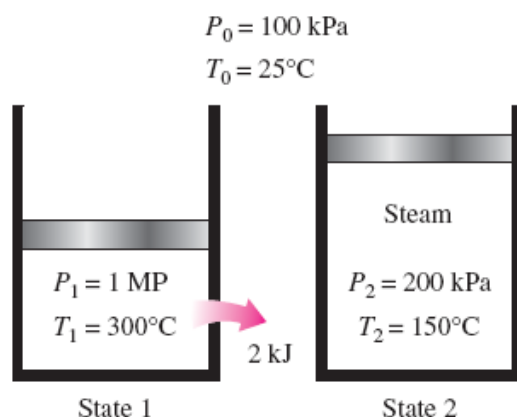
$$\dot{X}_{\text{destroyed, total}} = (1035 \text{ W}) \left( 1 - \frac{273 \text{ K}}{300 \text{ K}} \right) - (1035 \text{ W}) \left( 1 - \frac{273 \text{ K}}{273 \text{ K}} \right) = \mathbf{93.2 \text{ W}}$$

The difference between the two exergy destructions is 41.2 W and represents the exergy destroyed in the air layers on both sides of the wall. The exergy destruction in this case is entirely due to irreversible heat transfer through a finite temperature difference.

**Discussion** We could have determined the exergy destroyed by simply multiplying the entropy generations by the environment temperature of  $T_0=273 \text{ K}$ .

## EXAMPLE 8-11

A piston–cylinder device contains 0.05 kg of steam at 1 MPa and 300°C. Steam now expands to a final state of 200 kPa and 150°C, doing work. Heat losses from the system to the surroundings are estimated to be 2 kJ during this process. Assuming the surroundings to be at  $T_0=25^\circ\text{C}$  and  $P_0=100$  kPa, determine (a) the exergy of the steam at the initial and the final states, (b) the exergy change of the steam, (c) the exergy destroyed, and (d) the second-law efficiency for the process.



**Solution** Steam in a piston–cylinder device expands to a specified state. The exergies of steam at the initial and final states, the exergy change, the exergy destroyed, and the second-law efficiency for this process are to be determined.

**Assumptions** The kinetic and potential energies are negligible.

(a the exergy of the steam at the initial and the final states, :

$$\text{State 1: } \left. \begin{array}{l} P_1 = 1 \text{ MPa} \\ T_1 = 300^\circ\text{C} \end{array} \right\} \begin{array}{l} u_1 = 2793.7 \text{ kJ/kg} \\ v_1 = 0.25799 \text{ m}^3/\text{kg} \\ s_1 = 7.1246 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{State 2: } \left. \begin{array}{l} P_2 = 200 \text{ kPa} \\ T_2 = 150^\circ\text{C} \end{array} \right\} \begin{array}{l} u_2 = 2577.1 \text{ kJ/kg} \\ v_2 = 0.95986 \text{ m}^3/\text{kg} \\ s_2 = 7.2810 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{Dead state: } \left. \begin{array}{l} P_0 = 100 \text{ kPa} \\ T_0 = 25^\circ\text{C} \end{array} \right\} \begin{array}{l} u_0 \cong u_{f@25^\circ\text{C}} = 104.83 \text{ kJ/kg} \\ v_0 \cong v_{f@25^\circ\text{C}} = 0.00103 \text{ m}^3/\text{kg} \\ s_0 \cong s_{f@25^\circ\text{C}} = 0.3672 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\begin{aligned}
 X_1 &= m[(u_1 - u_0) - T_0(s_1 - s_0) + P_0(v_1 - v_0)] \\
 &= (0.05 \text{ kg})\{(2793.7 - 104.83) \text{ kJ/kg} \\
 &\quad - (298 \text{ K})[(7.1246 - 0.3672) \text{ kJ/kg} \cdot \text{K}] \\
 &\quad + (100 \text{ kPa})[(0.25799 - 0.00103) \text{ m}^3/\text{kg}]\}(\text{kJ/kPa} \cdot \text{m}^3) \\
 &= \mathbf{35.0 \text{ kJ}}
 \end{aligned}$$

$$\begin{aligned}
 X_2 &= m[(u_2 - u_0) - T_0(s_2 - s_0) + P_0(v_2 - v_0)] \\
 &= (0.05 \text{ kg})\{(2577.1 - 104.83) \text{ kJ/kg} \\
 &\quad - (298 \text{ K})[(7.2810 - 0.3672) \text{ kJ/kg} \cdot \text{K}] \\
 &\quad + (100 \text{ kPa})[(0.95986 - 0.00103) \text{ m}^3/\text{kg}]\}(\text{kJ/kPa} \cdot \text{m}^3) \\
 &= \mathbf{25.4 \text{ kJ}}
 \end{aligned}$$

(b) The exergy change for a process is simply the difference between the exergy at the initial and final states of the process,

$$\Delta X = X_2 - X_1 = 25.4 - 35.0 = \mathbf{-9.6 \text{ kJ}}$$

That is, if the process between states 1 and 2 were executed in a reversible manner, the system would deliver 9.6 kJ of useful work.

(c) The total exergy destroyed during this process can be determined from the exergy balance applied on the *extended system* (system + immediate surroundings) whose boundary is at the environment temperature of  $T_0$  (so that there is no exergy transfer accompanying heat transfer to or from the environment),

$$-X_{\text{work, out}} - X_{\text{heat, out}} \xrightarrow{0} -X_{\text{destroyed}} = X_2 - X_1$$

$$X_{\text{destroyed}} = X_1 - X_2 - W_{u, \text{out}}$$

By writing an energy balance on the system, the total boundary work done during the process is determined to be

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$-Q_{\text{out}} - W_{b, \text{out}} = \Delta U$$

$$W_{b, \text{out}} = -Q_{\text{out}} - \Delta U = -Q_{\text{out}} - m(u_2 - u_1)$$

$$= -(2 \text{ kJ}) - (0.05 \text{ kg})(2577.1 - 2793.7) \text{ kJ/kg}$$

$$= 8.8 \text{ kJ}$$

The useful work is the difference between the two:

$$\begin{aligned} W_u &= W - W_{\text{surr}} = W_{b, \text{out}} - P_0(V_2 - V_1) = W_{b, \text{out}} - P_0 m(v_2 - v_1) \\ &= 8.8 \text{ kJ} - (100 \text{ kPa})(0.05 \text{ kg})[(0.9599 - 0.25799) \text{ m}^3/\text{kg}] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = 5.3 \text{ kJ} \end{aligned}$$

The exergy destroyed is determined to be

$$X_{\text{destroyed}} = X_1 - X_2 - W_{u, \text{out}} = 35.0 - 25.4 - 5.3 = \mathbf{4.3 \text{ kJ}}$$

The exergy destroyed could also be determined from

$$\begin{aligned} X_{\text{destroyed}} &= T_0 S_{\text{gen}} = T_0 \left[ m(s_2 - s_1) + \frac{Q_{\text{surr}}}{T_0} \right] \\ &= (298 \text{ K}) \left\{ (0.05 \text{ kg})[(7.2810 - 7.1246) \text{ kJ/kg} \cdot \text{K}] + \frac{2 \text{ kJ}}{298 \text{ K}} \right\} = 4.3 \text{ kJ} \end{aligned}$$

(d) The second-law efficiency for this process can be determined from

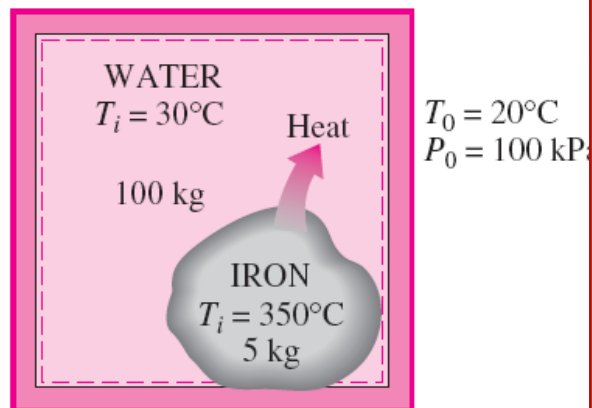
$$\eta_{\text{II}} = \frac{\text{Exergy recovered}}{\text{Exergy supplied}} = \frac{W_u}{X_1 - X_2} = \frac{5.3}{35.0 - 25.4} = \mathbf{0.552 \text{ or } 55.2\%}$$

That is, 44.8 percent of the work potential of the steam is wasted during this process.

### EXAMPLE 8-13

A 5-kg block initially at 350°C is quenched in an insulated tank that contains 100 kg of water at 30°C. Assuming the water that vaporizes during the process condenses back in the tank and the surroundings are at 20°C and 100 kPa, determine

- (a) the final equilibrium temperature,
- (b) the exergy of the combined system at the initial and the final states, and
- (c) the wasted work potential during this process.



**Solution** A hot iron block is quenched in an insulated tank by water. The final equilibrium temperature, the initial and final exergies, and the wasted work potential are to be determined.

**Assumptions** **1** Both water and the iron block are incompressible substances. **2** Constant specific heats at room temperature can be used for both the water and the iron. **3** The system is stationary and thus the kinetic and potential energy changes are zero,  $\Delta KE = \Delta PE = 0$ . **4** There are no electrical, shaft, or other forms of work involved. **5** The system is well-insulated and thus there is no heat transfer.

(a) Noting that no energy enters or leaves the system during the process, the application of the energy balance gives

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$0 = \Delta U$$

$$0 = (\Delta U)_{\text{iron}} + (\Delta U)_{\text{water}}$$

$$0 = [mc(T_f - T_i)]_{\text{iron}} + [mc(T_f - T_i)]_{\text{water}}$$

By using the specific-heat values for water and iron at room temperature (from Table A–3), the final equilibrium temperature  $T_f$  becomes

$$0 = (5 \text{ kg})(0.45 \text{ kJ/kg} \cdot ^\circ\text{C})(T_f - 350^\circ\text{C}) \\ + (100 \text{ kg})(4.18 \text{ kJ/kg} \cdot ^\circ\text{C})(T_f - 30^\circ\text{C})$$

$$T_f = \mathbf{31.7^\circ\text{C}}$$

(b) the exergy of the combined system at the initial and the final states,

$$X = (U - U_0) - T_0(S - S_0) + P_0(\check{V} - V_0) \\ = mc(T - T_0) - T_0 mc \ln \frac{T}{T_0} + 0 = mc \left( T - T_0 - T_0 \ln \frac{T}{T_0} \right)$$

$$X_{1, \text{iron}} = (5 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K}) \left[ (623 - 293) \text{ K} - (293 \text{ K}) \ln \frac{623 \text{ K}}{293 \text{ K}} \right] \\ = 245.2 \text{ kJ}$$

$$X_{1, \text{water}} = (100 \text{ kg})(4.18 \text{ kJ/kg} \cdot \text{K}) \left[ (303 - 293) \text{ K} - (293 \text{ K}) \ln \frac{303 \text{ K}}{293 \text{ K}} \right] \\ = 69.8 \text{ kJ}$$

$$X_{1, \text{total}} = X_{1, \text{iron}} + X_{1, \text{water}} = (245.2 + 69.8) \text{ kJ} = \mathbf{315 \text{ kJ}}$$



the exergy at the final state is

$$X_{2, \text{iron}} = 0.5 \text{ kJ}$$

$$X_{2, \text{water}} = 95.2 \text{ kJ}$$

$$X_{2, \text{total}} = X_{2, \text{iron}} + X_{2, \text{water}} = 0.5 + 95.2 = \mathbf{95.7 \text{ kJ}}$$

the exergy of the combined system (water + iron) decreased from 315 to 95.7 kJ as a result of this irreversible heat transfer process.

(c) the wasted work potential during this process.

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} - \underbrace{X_{\text{destroyed}}}_{\text{Exergy destruction}} = \underbrace{\Delta X_{\text{system}}}_{\text{Change in exergy}}$$

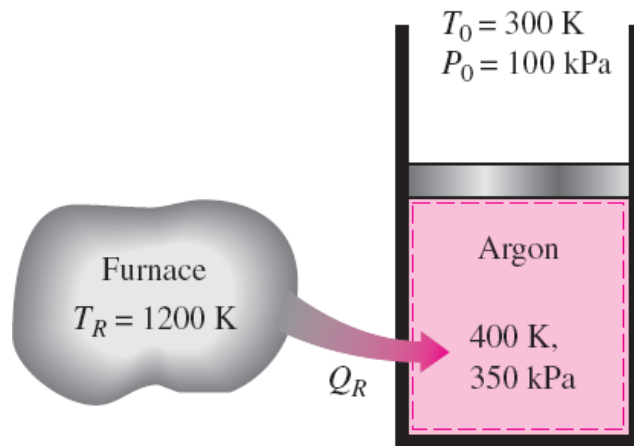
$$0 - X_{\text{destroyed}} = X_2 - X_1$$

$$X_{\text{destroyed}} = X_1 - X_2 = 315 - 9.7 = \mathbf{219.3 \text{ kJ}}$$

**Discussion** Note that 219.3 kJ of work could have been produced as the iron was cooled from 350 to 31.7°C and water was heated from 30 to 31.7°C, but was not.

## EXAMPLE 8-14

A frictionless piston–cylinder device, initially contains  $0.01 \text{ m}^3$  of argon gas at  $400 \text{ K}$  and  $350 \text{ kPa}$ . Heat is now transferred to the argon from a furnace at  $1200 \text{ K}$ , and the argon expands isothermally until its volume is doubled. No heat transfer takes place between the argon and the surrounding atmospheric air, which is at  $T_0 = 300 \text{ K}$  and  $P_0 = 100 \text{ kPa}$ . Determine (a) the useful work output, (b) the exergy destroyed, and (c) the reversible work for this process.



**Solution** Argon gas in a piston–cylinder device expands isothermally as a result of heat transfer from a furnace. The useful work output, the exergy destroyed, and the reversible work are to be determined.

**Assumptions** **1** Argon at specified conditions can be treated as an ideal gas since it is well above its critical temperature of  $151 \text{ K}$ . **2** The kinetic and potential energies are negligible.

(a) The only work interaction involved during this isothermal process is the quasi-equilibrium boundary work, which is determined from

$$W = W_b = \int_1^2 P dV = P_1 V_1 \ln \frac{V_2}{V_1} = (350 \text{ kPa})(0.01 \text{ m}^3) \ln \frac{0.02 \text{ m}^3}{0.01 \text{ m}^3} \\ = 2.43 \text{ kPa} \cdot \text{m}^3 = 2.43 \text{ kJ}$$

$$W_{\text{surr}} = P_0(V_2 - V_1) = (100 \text{ kPa})[(0.02 - 0.01) \text{ m}^3] \left( \frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) = 1 \text{ kJ}$$

The useful work is the difference between these two:

$$W_u = W - W_{\text{surr}} = 2.43 - 1 = \mathbf{1.43 \text{ kJ}}$$

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$Q_{\text{in}} - W_{b, \text{out}} = \Delta U = mc_V \Delta T \overset{0}{=} 0$$

$$Q_{\text{in}} = W_{b, \text{out}} = 2.43 \text{ kJ}$$

(b) the exergy destroyed

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$

$$\frac{Q}{T_R} + S_{\text{gen}} = \Delta S_{\text{system}} = \frac{Q}{T_{\text{sys}}}$$

$$S_{\text{gen}} = \frac{Q}{T_{\text{sys}}} - \frac{Q}{T_R} = \frac{2.43 \text{ kJ}}{400 \text{ K}} - \frac{2.43 \text{ kJ}}{1200 \text{ K}} = 0.00405 \text{ kJ/K}$$

$$X_{\text{destroyed}} = T_0 S_{\text{gen}} = (300 \text{ K})(0.00405 \text{ kJ/K}) = \mathbf{1.22 \text{ kJ/K}}$$

(c) the reversible work for this process

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} - \underbrace{X_{\text{destroyed}}}_{\text{Exergy destruction}} \overset{0 \text{ (reversible)}}{=} \underbrace{\Delta X_{\text{system}}}_{\text{Change in exergy}}$$

$$\begin{aligned}
 \left(1 - \frac{T_0}{T_b}\right)Q - W_{\text{rev, out}} &= X_2 - X_1 \\
 &= (E_2 - E_1) + P_0(V_2 - V_1) - T_0(S_2 - S_1) \\
 &= 0 + W_{\text{surr}} - T_0 \frac{Q}{T_{\text{sys}}}
 \end{aligned}$$

$$\begin{aligned}
 W_{\text{rev, out}} &= T_0 \frac{Q}{T_{\text{sys}}} - W_{\text{surr}} + \left(1 - \frac{T_0}{T_R}\right)Q \\
 &= (300 \text{ K}) \frac{2.43 \text{ kJ}}{400 \text{ K}} - (1 \text{ kJ}) + \left(1 - \frac{300 \text{ K}}{1200 \text{ K}}\right)(2.43 \text{ kJ}) \\
 &= \mathbf{2.65 \text{ kJ}}
 \end{aligned}$$

Therefore, the useful work output would be 2.65 kJ instead of 1.43 kJ if the process were executed in a totally reversible manner.

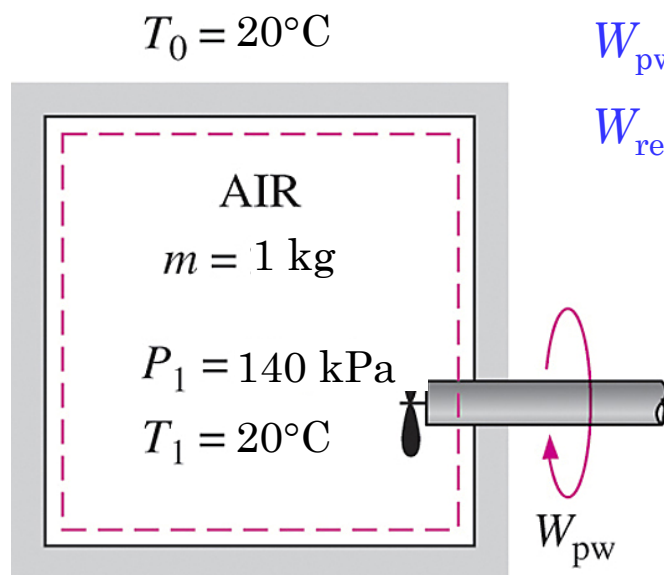
# Exergy balance for an air tank

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\text{Net exergy transfer by heat, work, and mass}} - \underbrace{X_{\text{destroyed}}}_{\text{Exergy destruction}} \overset{0 \text{ (reversible)}}{=} \underbrace{\Delta X_{\text{system}}}_{\text{Change in exergy}}$$

$$W_{\text{rev,in}} = X_2 - X_1$$

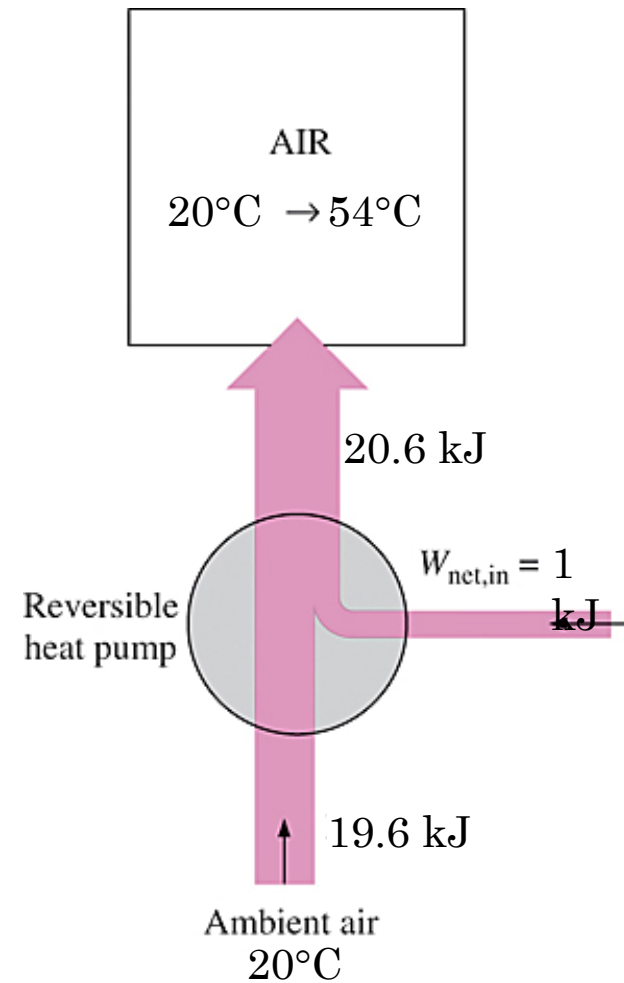
$$= (E_2 - E_1) + P_0(V_2 - V_1) \overset{0}{-} T_0(S_2 - S_1)$$

$$= (U_2 - U_1) - T_0(S_2 - S_1)$$



$$W_{\text{pw,in}} = \Delta U = 20.6 \text{ kJ}$$

$$W_{\text{rev,in}} = 1 \text{ kJ}$$



The same effect on the insulated tank system can be accomplished by a reversible heat pump that consumes only 1 kJ of work.

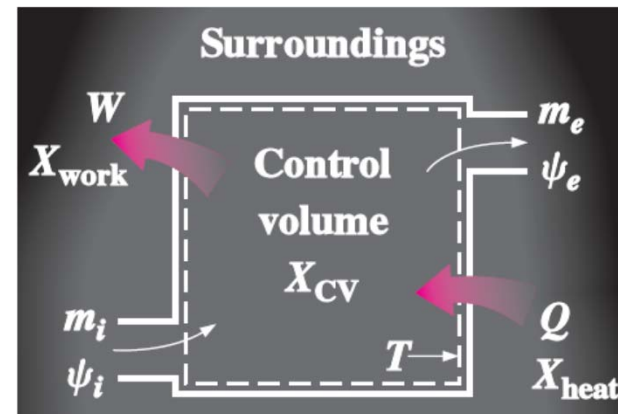
# EXERGY BALANCE: CONTROL VOLUMES

$$X_{\text{heat}} - X_{\text{work}} + X_{\text{mass, in}} - X_{\text{mass, out}} - X_{\text{destroyed}} = (X_2 - X_1)_{\text{CV}}$$

$$\sum \left(1 - \frac{T_0}{T_k}\right) Q_k - [W - P_0(V_2 - V_1)] + \sum m_i \psi_i - \sum m_e \psi_e - X_{\text{destroyed}} = (X_2 - X_1)_{\text{CV}}$$

$$\sum \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k - \left(\dot{W} - P_0 \frac{dV_{\text{CV}}}{dt}\right) + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - \dot{X}_{\text{destroyed}} = \frac{dX_{\text{CV}}}{dt}$$

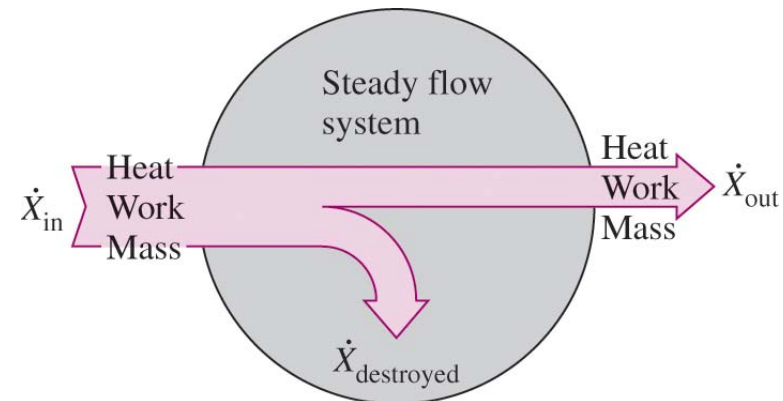
The rate of exergy change within the control volume during a process is equal to the rate of net exergy transfer through the control volume boundary by heat, work, and mass flow minus the rate of exergy destruction within the boundaries of the control volume.



Exergy is transferred into or out of a control volume by mass as well as heat and work transfer.

# Exergy Balance for Steady-Flow Systems

Most control volumes encountered in practice such as turbines, compressors, nozzles, diffusers, heat exchangers, pipes, and ducts operate steadily, and thus they experience no changes in their mass, energy, entropy, and exergy contents as well as their volumes. Therefore,  $dV_{CV}/dt = 0$  and  $dX_{CV}/dt = 0$  for such systems.



The exergy transfer to a steady-flow system is equal to the exergy transfer from it plus the exergy destruction within the system.

$$\text{Steady-flow: } \sum \left( 1 - \frac{T_0}{T_k} \right) \dot{Q}_k - \dot{W} + \sum \dot{m}_i \psi_i - \sum \dot{m}_e \psi_e - \dot{X}_{\text{destroyed}} = 0$$

$$\text{Single-stream: } \sum \left( 1 - \frac{T_0}{T_k} \right) \dot{Q}_k - \dot{W} + \dot{m}(\psi_1 - \psi_2) - \dot{X}_{\text{destroyed}} = 0$$

$$\psi_1 - \psi_2 = (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} + g(z_1 - z_2)$$

$$\text{Per-unit mass: } \sum \left( 1 - \frac{T_0}{T_k} \right) q_k - w + (\psi_1 - \psi_2) - x_{\text{destroyed}} = 0 \quad (\text{kJ/kg})$$



## Reversible Work, $W_{\text{rev}}$

The exergy balance relations presented above can be used to determine the reversible work  $W_{\text{rev}}$  by setting the exergy destroyed equal to zero. The work  $W$  in that case becomes the reversible work.

*General:*  $W = W_{\text{rev}} \quad \text{when } X_{\text{destroyed}} = 0$

*Single stream:*  $\dot{W}_{\text{rev}} = \dot{m}(\psi_1 - \psi_2) + \sum \left(1 - \frac{T_0}{T_k}\right) \dot{Q}_k \quad (\text{kW})$

*Adiabatic, single stream:*  $\dot{W}_{\text{rev}} = \dot{m}(\psi_1 - \psi_2)$

The exergy destroyed is zero only for a reversible process, and reversible work represents the maximum work output for work-producing devices such as turbines and the minimum work input for work-consuming devices such as compressors.

## Second-Law Efficiency of Steady-Flow Devices, $\eta_{II}$

The *second-law efficiency* of various steady-flow devices can be determined from its general definition,  $\eta_{II} = (\text{Exergy recovered})/(\text{Exergy supplied})$ . When the changes in kinetic and potential energies are negligible and the devices are adiabatic:

### Turbine

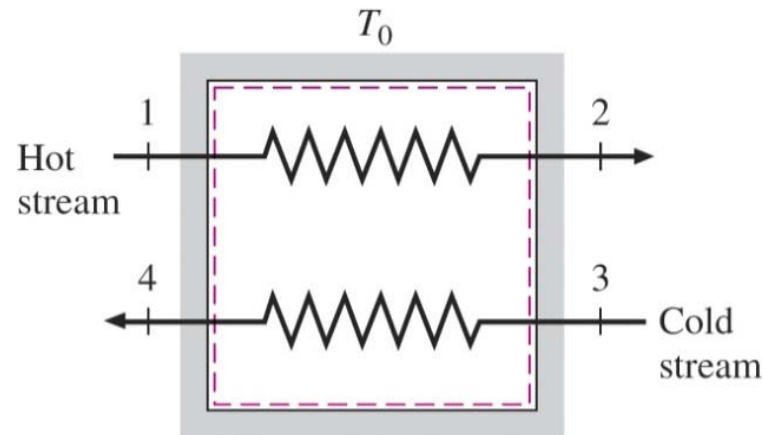
$$\eta_{II, \text{ turb}} = \frac{w}{w_{\text{rev}}} = \frac{h_1 - h_2}{\psi_1 - \psi_2} \quad \text{or} \quad \eta_{II, \text{ turb}} = 1 - \frac{T_0 s_{\text{gen}}}{\psi_1 - \psi_2}$$

### Compressor

$$\eta_{II, \text{ comp}} = \frac{w_{\text{rev, in}}}{w_{\text{in}}} = \frac{\psi_2 - \psi_1}{h_2 - h_1} \quad \text{or} \quad \eta_{II, \text{ comp}} = 1 - \frac{T_0 s_{\text{gen}}}{h_2 - h_1}$$

### Heat exchanger

$$\eta_{II, \text{ HX}} = \frac{\dot{m}_{\text{cold}}(\psi_4 - \psi_3)}{\dot{m}_{\text{hot}}(\psi_1 - \psi_2)} \quad \text{or} \quad \eta_{II, \text{ HX}} = 1 - \frac{T_0 \dot{S}_{\text{gen}}}{\dot{m}_{\text{hot}}(\psi_1 - \psi_2)}$$



A heat exchanger with two unmixed fluid streams.

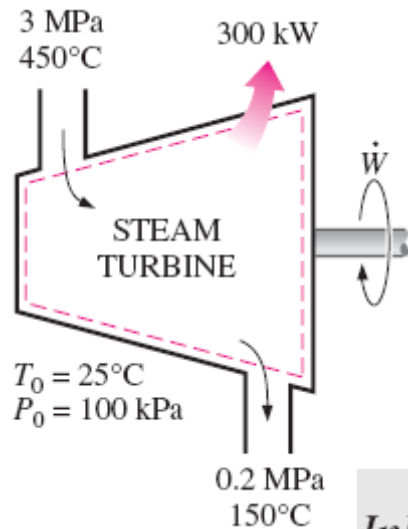
Mixing chamber

$$\eta_{\text{II, mix}} = \frac{\dot{m}_3 \psi_3}{\dot{m}_1 \psi_1 + \dot{m}_2 \psi_2} \quad \text{or} \quad \eta_{\text{II, mix}} = 1 - \frac{T_0 \dot{S}_{\text{gen}}}{\dot{m}_1 \psi_1 + \dot{m}_2 \psi_2}$$

where  $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$  and  $\dot{S}_{\text{gen}} = \dot{m}_3 s_3 - \dot{m}_2 s_2 - \dot{m}_1 s_1$ .

## EXAMPLE 8-15

Steam enters a turbine steadily at 3 MPa and 450°C at a rate of 8 kg/s and exits at 0.2 MPa and 150°C. The steam is losing heat to the surrounding air at 100 kPa and 25°C at a rate of 300 kW, and the kinetic and potential energy changes are negligible. Determine (a) the actual power output, (b) the maximum possible power output, (c) the second-law efficiency, (d) the exergy destroyed, and (e) the exergy of the steam at the inlet conditions.



**Solution** A steam turbine operating steadily between specified inlet and exit states is considered. The actual and maximum power outputs, the second-law efficiency, the exergy destroyed, and the inlet exergy are to be determined.

**Assumptions 1** This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{CV}=0$ ,  $\Delta E_{CV}=0$ , and  $\Delta X_{CV}=0$ . **2** The kinetic and potential energies are negligible.

$$\text{Inlet state: } \left. \begin{array}{l} P_1 = 3 \text{ MPa} \\ T_1 = 450^\circ\text{C} \end{array} \right\} \begin{array}{l} h_1 = 3344.9 \text{ kJ/kg} \\ s_1 = 7.0856 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{Exit state: } \left. \begin{array}{l} P_2 = 0.2 \text{ MPa} \\ T_2 = 150^\circ\text{C} \end{array} \right\} \begin{array}{l} h_2 = 2769.1 \text{ kJ/kg} \\ s_2 = 7.2810 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{Dead state: } \left. \begin{array}{l} P_0 = 100 \text{ kPa} \\ T_0 = 25^\circ\text{C} \end{array} \right\} \begin{array}{l} h_0 \cong h_f @ 25^\circ\text{C} = 104.83 \text{ kJ/kg} \\ s_0 \cong s_f @ 25^\circ\text{C} = 0.3672 \text{ kJ/kg} \cdot \text{K} \end{array}$$

(a) The actual power output of the turbine;

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{Q}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \text{ke} \cong \text{pe} \cong 0)$$

$$\begin{aligned}\dot{W}_{\text{out}} &= \dot{m}(h_1 - h_2) - \dot{Q}_{\text{out}} \\ &= (8 \text{ kg/s})[(3344.9 - 2769.1) \text{ kJ/kg}] - 300 \text{ kW} \\ &= \mathbf{4306 \text{ kW}}\end{aligned}$$

(b) the maximum possible power output

$$\dot{X}_{\text{in}} = \dot{X}_{\text{out}}$$

$$\dot{m}\psi_1 = \dot{W}_{\text{rev, out}} + \dot{X}_{\text{heat}} + \dot{m}\psi_2$$

$$\begin{aligned}\dot{W}_{\text{rev, out}} &= \dot{m}(\psi_1 - \psi_2) \\ &= \dot{m}[(h_1 - h_2) - T_0(s_1 - s_2) - \Delta \text{ke}^0 - \Delta \text{pe}^0]\end{aligned}$$

$$\begin{aligned}\dot{W}_{\text{rev, out}} &= (8 \text{ kg/s})[(3344.9 - 2769.1) \text{ kJ/kg} \\ &\quad - (298 \text{ K})(7.0856 - 7.2810) \text{ kJ/kg} \cdot \text{K}] = \mathbf{5072 \text{ kW}}\end{aligned}$$

(c) The second-law efficiency of a turbine is the ratio of the actual work delivered to the reversible work,

$$\eta_{\text{II}} = \frac{\dot{W}_{\text{out}}}{\dot{W}_{\text{rev, out}}} = \frac{4306 \text{ kW}}{5072 \text{ kW}} = \mathbf{0.849 \text{ or } 84.9\%}$$

That is, 15.1 percent of the work potential is wasted during this process.

(d) The difference between the reversible work and the actual useful work is the exergy destroyed, which is determined to be

$$\dot{X}_{\text{destroyed}} = \dot{W}_{\text{rev, out}} - \dot{W}_{\text{out}} = 5072 - 4306 = \mathbf{766 \text{ kW}}$$

(e) The exergy (maximum work potential) of the steam at the inlet conditions is simply the stream exergy, and is determined from

$$\begin{aligned}\psi_1 &= (h_1 - h_0) - T_0(s_1 - s_0) + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1} \\ &= (h_1 - h_0) - T_0(s_1 - s_0) \\ &= (3344.9 - 104.83) \text{ kJ/kg} - (298 \text{ K})(7.0856 - 0.3672 \text{ kJ/kg} \cdot \text{K}) \\ &= \mathbf{1238 \text{ kJ/kg}}\end{aligned}$$

That is, not counting the kinetic and potential energies, every kilogram of the steam entering the turbine has a work potential of 1238 kJ. This corresponds to a power potential of  $(8 \text{ kg/s})(1238 \text{ kJ/kg}) = 9904 \text{ kW}$ . Obviously, the turbine is converting  $4306/9904 = 43.5 \%$  of the available work potential of the steam to work.

# SUMMARY

- Exergy: Work potential of energy
  - Exergy (work potential) associated with kinetic and potential energy
- Reversible work and irreversibility
- Second-law efficiency
- Exergy change of a system
  - Exergy of a fixed mass: Nonflow (or closed system) exergy
  - Exergy of a flow stream: Flow (or stream) exergy
- Exergy transfer by heat, work, and mass
- The decrease of exergy principle and exergy destruction
- Exergy balance: Closed systems
- Exergy balance: Control volumes
  - Exergy balance for steady-flow systems
  - Reversible work
  - Second-law efficiency of steady-flow devices