Introduction to Fluid Mechanics

Chapter 7

Dimensional Analysis and Similitude

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Main Topics

- ✓ Nondimensionalizing the Basic Differential Equations
- ✓ Nature of Dimensional Analysis
- ✓ Buckingham Pi Theorem
- ✓ Significant Dimensionless Groups in Fluid Mechanics
- ✓ Flow Similarity and Model Studies

Nondimensionalizing the Basic Differential Equations

Example:

- ✓ Steady
- ✓ Incompressible
- ✓ Two-dimensional
- ✓ Newtonian Fluid

Nondimensionalizing the Basic Differential Equations

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\rho g-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Nondimensionalizing the Basic Differential Equations

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{\mu}{\rho V_\infty L}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{gL}{V_\infty^2} - \frac{\partial p^*}{\partial y^*} + \frac{\mu}{\rho V_\infty L}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Nature of Dimensional Analysis

Example: Drag on a Sphere

$$F = f(D, V, \rho, \mu)$$

- ✓ Drag depends on FOUR parameters:
  sphere size (D); speed (V); fluid density (ρ); fluid viscosity (μ)
- ✓ Difficult to know how to set up experiments to determine dependencies
- ✓ Difficult to know how to present results (four graphs?)

Nature of Dimensional Analysis

Example: Drag on a Sphere

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}\right)$$

- ✓ Only one dependent and one independent variable
- ✓ Easy to set up experiments to determine dependency
- ✓ Easy to present results (one graph)

Nature of Dimensional Analysis

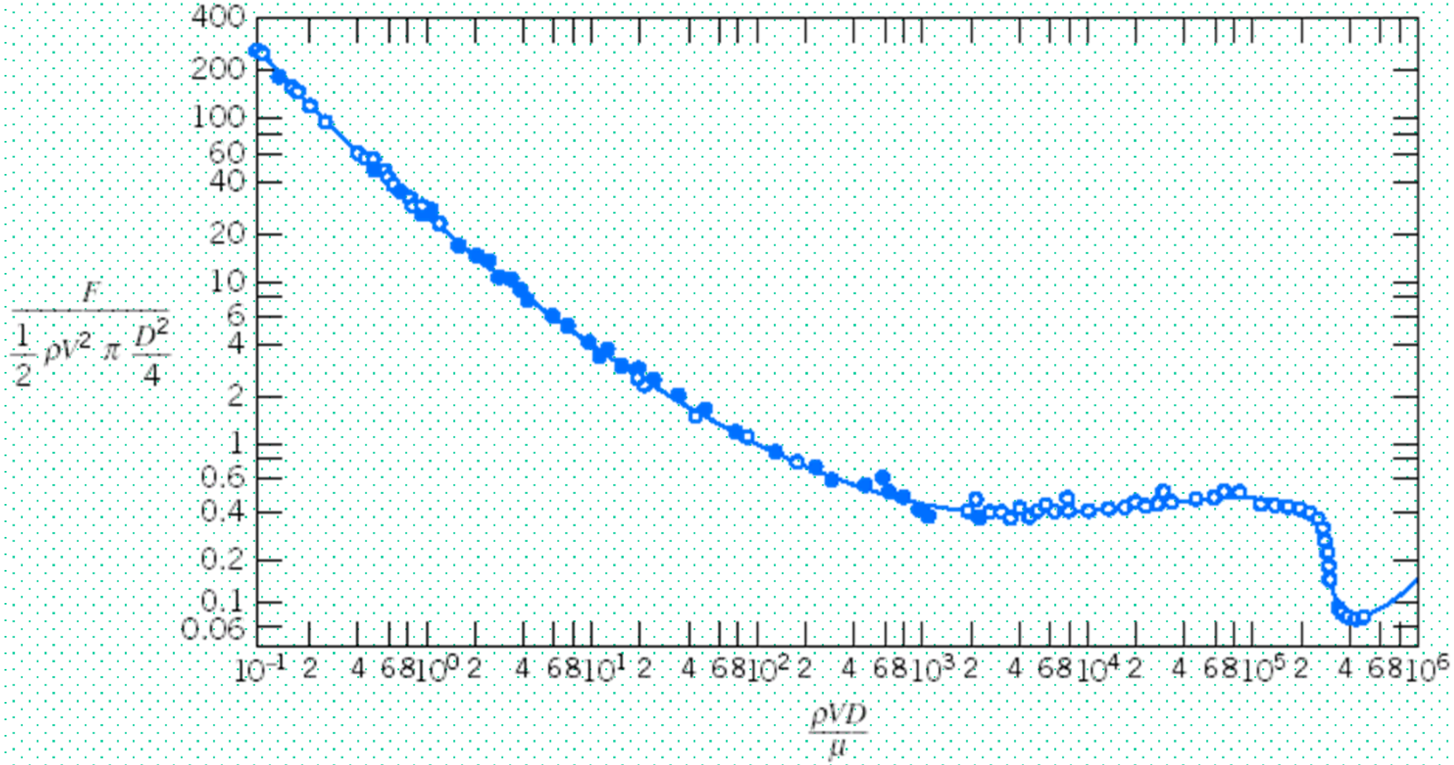

Fig. 7.1 Experimentally derived relation between the nondimensional parameters [4].

Buckingham Pi Theorem

✓ **Step 1:**

List all the dimensional parameters involved

Let n be the number of parameters

Example: For drag on a sphere, F, V, D, ρ, μ, and $n = 5$

Buckingham Pi Theorem

✓ **Step 2**

Select a set of fundamental (primary) dimensions

For example *MLt*, or *FLt*

Example: For drag on a sphere choose *MLt*

Buckingham Pi Theorem

✓ **Step 3**

List the dimensions of all parameters in terms of primary dimensions

Let r be the number of primary dimensions

Example: For drag on a sphere r = 3

$$\begin{array}{ccccc} F & V & D & \rho & \mu \\ \dfrac{ML}{t^2} & \dfrac{L}{t} & L & \dfrac{M}{L^3} & \dfrac{M}{Lt} \end{array}$$

Buckingham Pi Theorem

✓ **Step 4**

Select a set of r dimensional parameters that includes all the primary dimensions

Example: For drag on a sphere ($m = r = 3$) select ρ, V, D

Buckingham Pi Theorem

✓ **Step 5**

Set up dimensional equations, combining the parameters selected in Step 4 with each of the other parameters in turn, to form dimensionless groups

There will be *n – m* equations

Example: For drag on a sphere

$$\Pi_1 = \rho^a V^b D^c F$$

Buckingham Pi Theorem

✓ **Step 5 (Continued)**

Example: For drag on a sphere

$$\left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \left(\frac{ML}{t^2}\right) = M^0 L^0 t^0$$

$$\Pi_1 = \frac{F}{\rho V^2 D^2}$$

Buckingham Pi Theorem

✓ **Step 6**

Check to see that each group obtained is dimensionless

Example: For drag on a sphere

$$[\Pi_1] = \left[\frac{F}{\rho V^2 D^2}\right] \qquad F\,\frac{L^4}{Ft^2}\left(\frac{t}{L}\right)^2\frac{1}{L^2} = 1$$

Example 7.1 DRAG FORCE ON A SMOOTH SPHERE

The drag force, F, on a smooth sphere depends on the relative speed, V, the sphere diameter, D, the fluid density, ρ, and the fluid viscosity, μ. Obtain a set of dimensionless groups that can be used to correlate experimental data.

① $F \quad V \quad D \quad \rho \quad \mu$ $\qquad n = 5$ dimensional parameters

② Select primary dimensions M, L, and t.

③ $F \quad V \quad D \quad \rho \quad \mu$

$$\frac{ML}{t^2} \quad \frac{L}{t} \quad L \quad \frac{M}{L^3} \quad \frac{M}{Lt}$$

$r = 3$ primary dimensions

④ Select repeating parameters ρ, V, D. $\qquad m = r = 3$ repeating parameters

⑤ Then $n - m = 2$ dimensionless groups will result.

$$\Pi_1 = \rho^a V^b D^c F \quad \text{and} \quad \left(\frac{M}{L^3}\right)^a \left(\frac{L}{t}\right)^b (L)^c \left(\frac{ML}{t^2}\right) = M^0 L^0 t^0$$

$$\left.\begin{array}{lrr} M: & a+1=0 & a=-1 \\ L: & -3a+b+c+1=0 & c=-2 \\ t: & -b-2=0 & b=-2 \end{array}\right\} \quad \text{Therefore,} \quad \Pi_1 = \frac{F}{\rho V^2 D^2}$$

$$\Pi_2 = \rho^d V^e D^f \mu \quad \text{and} \quad \left(\frac{M}{L^3}\right)^d \left(\frac{L}{t}\right)^e (L)^f \left(\frac{M}{Lt}\right) = M^0 L^0 t^0$$

$$\left.\begin{array}{lrr} M: & d+1=0 & d=-1 \\ L: & -3d+e+f-1=0 & f=-1 \\ t: & -e-1=0 & e=-1 \end{array}\right\} \quad \text{Therefore,} \quad \Pi_2 = \frac{\mu}{\rho V D}$$

⑥ Check using F, L, t dimensions

$$[\Pi_1] = \left[\frac{F}{\rho V^2 D^2}\right] \quad \text{and} \quad F\frac{L^4}{Ft^2}\left(\frac{t}{L}\right)^2\frac{1}{L^2} = 1$$

$$[\Pi_2] = \left[\frac{\mu}{\rho V D}\right] \quad \text{and} \quad \frac{Ft}{L^2}\,\frac{L^4}{Ft^2}\,\frac{t}{L}\,\frac{1}{L} = 1$$

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho V D}\right)$$

Significant Dimensionless Groups in Fluid Mechanics

Viscous force ~	$\tau A = \mu \frac{du}{dy} A \propto \mu \frac{V}{L} L^2 = \mu V L$
Pressure force ~	$\Delta p A \propto \Delta p L^2$
Gravity force ~	$mg \propto g \rho L^3$
Surface tension ~	σL
Compressibility force ~	$E_v A \propto E_v L^2$

Viscous force $\sim \dfrac{\text{viscous}}{\text{inertia}} \sim \dfrac{\mu V L}{\rho V^2 L^2} = \dfrac{\mu}{\rho V L}$

Pressure force $\sim \dfrac{\text{pressure}}{\text{inertia}} \sim \dfrac{\Delta p L^2}{\rho V^2 L^2} = \dfrac{\Delta p}{\rho V^2}$

Gravity force $\sim \dfrac{\text{gravity}}{\text{inertia}} \sim \dfrac{g \rho L^3}{\rho V^2 L^2} = \dfrac{g L}{V^2}$

Surface tension $\sim \dfrac{\text{surface tension}}{\text{inertia}} \sim \dfrac{\sigma L}{\rho V^2 L^2} = \dfrac{\sigma}{\rho V^2 L}$

Compressibility force $\sim \dfrac{\text{compressibility force}}{\text{inertia}} \sim \dfrac{E_v L^2}{\rho V^2 L^2} = \dfrac{E_v}{\rho V^2}$

Significant Dimensionless Groups in Fluid Mechanics

- ✓ **Reynolds Number** $$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

- ✓ **Mach Number** $$M = \frac{V}{c}$$

Significant Dimensionless Groups in Fluid Mechanics

- ✓ **Froude Number** $$Fr = \frac{V}{\sqrt{gL}}$$

- ✓ **Weber Number** $$We = \frac{\rho V^2 L}{\sigma}$$

Significant Dimensionless Groups in Fluid Mechanics

- ✓ **Euler Number** $$Eu = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

- ✓ **Cavitation Number** $$Ca = \frac{p - p_v}{\frac{1}{2}\rho V^2}$$

Flow Similarity and Model Studies

- ✓ **Geometric Similarity**
  - Model and prototype have same shape
  - Linear dimensions on model and prototype correspond within constant scale factor
- ✓ **Kinematic Similarity**
  - Velocities at corresponding points on model and prototype differ only by a constant scale factor
- ✓ **Dynamic Similarity**
  - Forces on model and prototype differ only by a constant scale factor

Flow Similarity and Model Studies

✓ **Example: Drag on a Sphere**

$$F = f(D, V, \rho, \mu)$$

$$\frac{F}{\rho V^2 D^2} = f_1\left(\frac{\rho V D}{\mu}\right)$$

Flow Similarity and Model Studies

✓ Example: Drag on a Sphere

For dynamic similarity …

$$\left(\frac{\rho V D}{\mu}\right)_{\text{model}} = \left(\frac{\rho V D}{\mu}\right)_{\text{prototype}}$$

… then …

$$\left(\frac{F}{\rho V^2 D^2}\right)_{\text{model}} = \left(\frac{F}{\rho V^2 D^2}\right)_{\text{prototype}}$$

Flow Similarity and Model Studies

✓ **Incomplete Similarity**

Sometimes (e.g., in aerodynamics) complete similarity cannot be obtained, but phenomena may still be successfully modelled

Flow Similarity and Model Studies

✓ **Scaling with Multiple Dependent Parameters**

Example: Centrifugal Pump

Pump Head $$h = g_1(Q,\ \rho,\ \omega,\ D,\ \mu)$$

Pump Power $$\mathscr{P} = g_2(Q,\ \rho,\ \omega,\ D,\ \mu)$$

Flow Similarity and Model Studies

✓ **Scaling with Multiple Dependent Parameters**

Example: Centrifugal Pump

Head Coefficient

$$\frac{h}{\omega^2 D^2} = f_1\left(\frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$$

Power Coefficient

$$\frac{\mathcal{P}}{\rho\omega^3 D^5} = f_2\left(\frac{Q}{\omega D^3}, \frac{\rho\omega D^2}{\mu}\right)$$

Flow Similarity and Model Studies

✓ **Scaling with Multiple Dependent Parameters**

Example: Centrifugal Pump (Negligible Viscous Effects)

If … $$\frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3}$$ **… then …**

$$\frac{h_1}{\omega_1^2 D_1^2} = \frac{h_2}{\omega_2^2 D_2^2} \qquad \frac{\mathcal{P}_1}{\rho_1 \omega_1^3 D_1^5} = \frac{\mathcal{P}_2}{\rho_2 \omega_2^3 D_2^5}$$

Flow Similarity and Model Studies

✓ **Scaling with Multiple Dependent Parameters**

Example: Centrifugal Pump

Specific Speed

$$N_s = \frac{\omega Q^{1/2}}{h^{3/4}} \qquad N_{s_{cu}} = \frac{\omega Q^{1/2}}{H^{3/4}}$$

Introduction to Fluid Mechanics

Chapter 8

Internal Incompressible Viscous Flow

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Main Topics

- ✓ Entrance Region
- ✓ Fully Developed Laminar Flow Between Infinite Parallel Plates
- ✓ Fully Developed Laminar Flow in a Pipe
- ✓ Turbulent Velocity Profiles in Fully Developed Pipe Flow
- ✓ Energy Considerations in Pipe Flow
- ✓ Calculation of Head Loss
- ✓ Solution of Pipe Flow Problems
- ✓ Flow Measurement

Laminar versus Turbulent Flow

Entrance Region

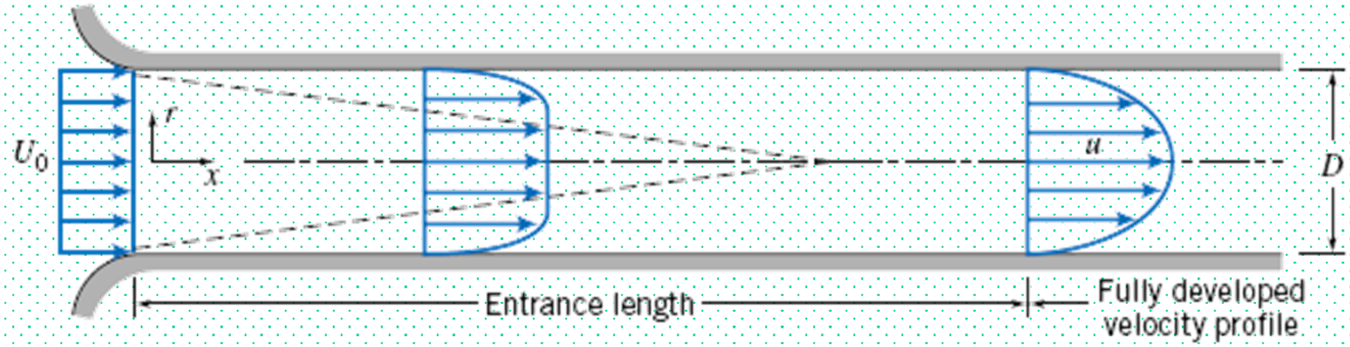


Fig. 8.1 Flow in the entrance region of a pipe.

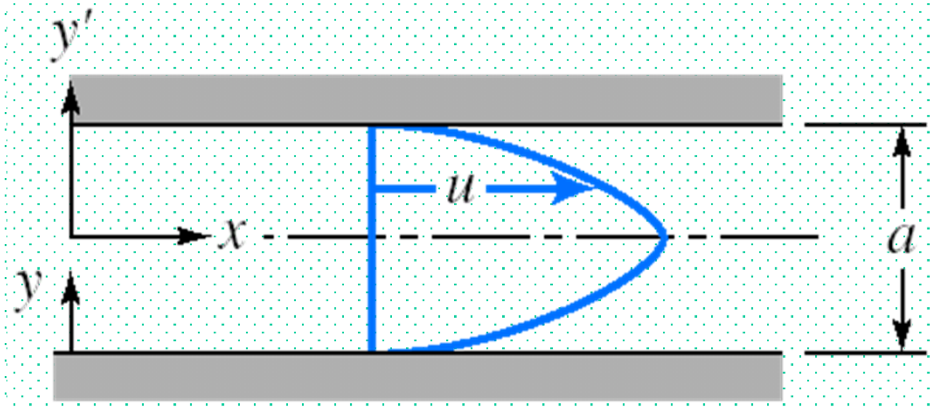
The entrance length for laminar pipe flow

$$\frac{L}{D} \simeq 0.06 \frac{\rho \bar{V} D}{\mu}$$

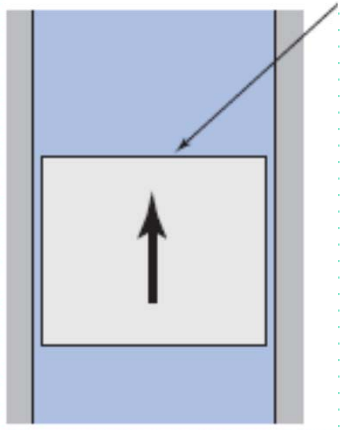
$$L \simeq 0.06\ ReD \leq (0.06)(2300)\ D = 138D$$

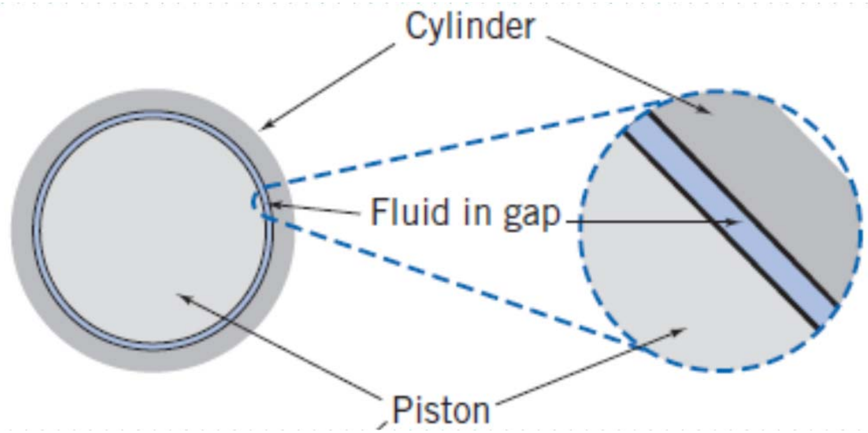
Fully Developed Laminar Flow Between Infinite Parallel Plates

✓ **Both Plates Stationary**

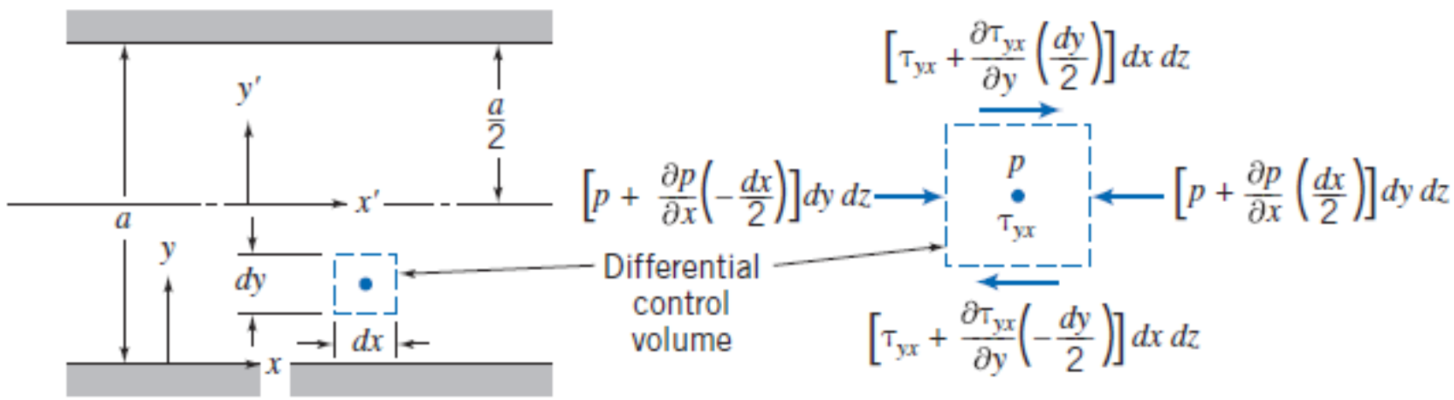


$$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$





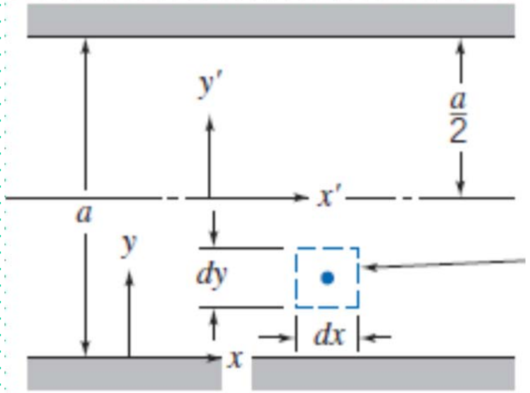
Piston-cylinder approximated as parallel plates.



(*a*) Geometry of CV (*b*) Forces acting on CV

Fig. 8.3 Control volume for analysis of laminar flow between stationary infinite parallel plates.

$$\text{at}\quad y = 0 \qquad u = 0$$
$$\text{at}\quad y = a \qquad u = 0$$



Basic equation:

$$\cancelto{=0(3)}{F_{B_x}} \qquad \cancelto{=0(1)}{\frac{\partial}{\partial t}}$$

$$F_{S_x} + \cancel{F_{B_x}} = \cancel{\frac{\partial}{\partial t}}\int_{\mathrm{CV}} u\,\rho\, d\forall + \int_{\mathrm{CS}} u\,\rho \vec{V}\cdot d\vec{A}$$

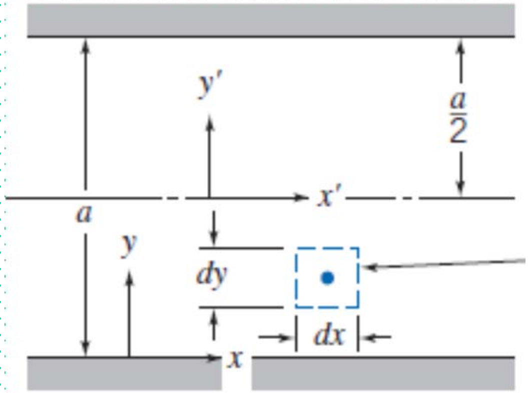
Assumptions: (1) Steady flow (given)
(2) Fully developed flow (given)
(3) $F_{B_x} = 0$ (given)

the pressure force

$$F_{S_x} = 0$$

$$dF_L = \left(p - \frac{\partial p}{\partial x}\frac{dx}{2}\right) dy\, dz \qquad dF_R = -\left(p + \frac{\partial p}{\partial x}\frac{dx}{2}\right) dy\, dz$$

the shear force



$$dF_T = \left(\tau_{yx} + \frac{d\tau_{yx}}{dy}\frac{dy}{2}\right)dx\,dz$$

$$\frac{d\tau_{yx}}{dy} = \frac{\partial p}{\partial x} = \text{constant}$$

$$\tau_{yx} = \left(\frac{\partial p}{\partial x}\right)y + c_1$$

$$\tau_{yx} = \mu\frac{du}{dy}$$

$$\mu\frac{du}{dy} = \left(\frac{\partial p}{\partial x}\right)y + c_1$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)y^2 + \frac{c_1}{\mu}y + c_2$$

$y = a,\ u = 0.$

$$0 = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)a^2 + \frac{c_1}{\mu}a \qquad c_1 = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right)a$$

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)y^2 - \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)ay = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$$

The shear stress distribution

$$\tau_{yx} = \left(\frac{\partial p}{\partial x}\right)y + c_1 = \left(\frac{\partial p}{\partial x}\right)y - \frac{1}{2}\left(\frac{\partial p}{\partial x}\right)a = a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right]$$

Volume Flow Rate

$$Q = \int_0^a ul\,dy \qquad \text{or} \qquad \frac{Q}{l} = \int_0^a \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right)(y^2 - ay)\,dy$$

$$\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$$

Flow Rate as a Function of Pressure Drop

$$\frac{\partial p}{\partial x} = \frac{p_2 - p_1}{L} = \frac{-\Delta p}{L}$$

$$\frac{Q}{l} = -\frac{1}{12\mu}\left[\frac{-\Delta p}{L}\right]a^3 = \frac{a^3 \Delta p}{12\mu L}$$

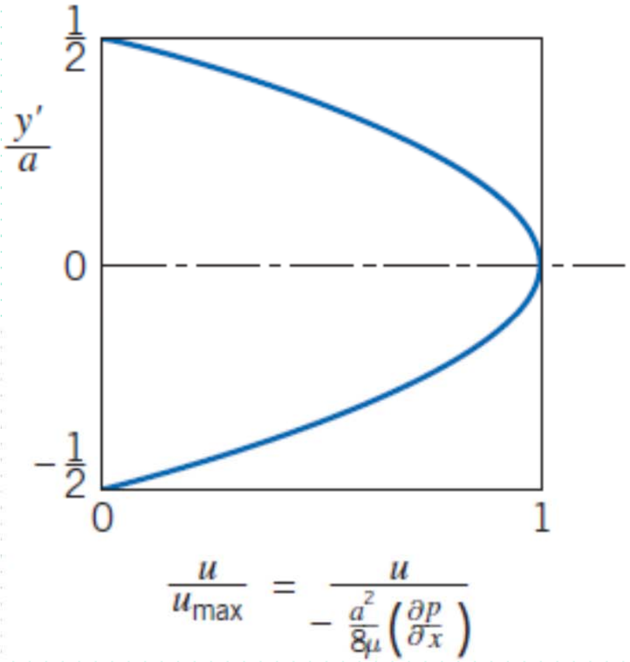
Average Velocity

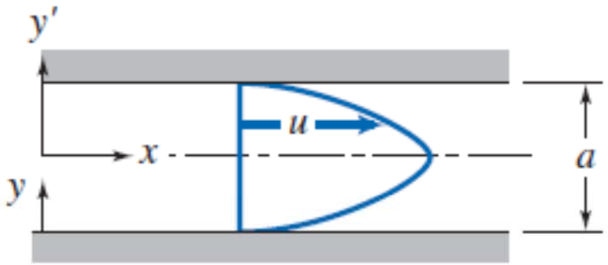
$$\overline{V} = \frac{Q}{A} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)\frac{a^3 l}{la} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^2$$

Point of Maximum Velocity

$$\frac{du}{dy} = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\frac{2y}{a^2} - \frac{1}{a}\right] \qquad \frac{du}{dy} = 0 \quad \text{at} \quad y = \frac{a}{2}$$

$$y = \frac{a}{2}, \qquad u = u_{\max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)a^2 = \frac{3}{2}\overline{V}$$



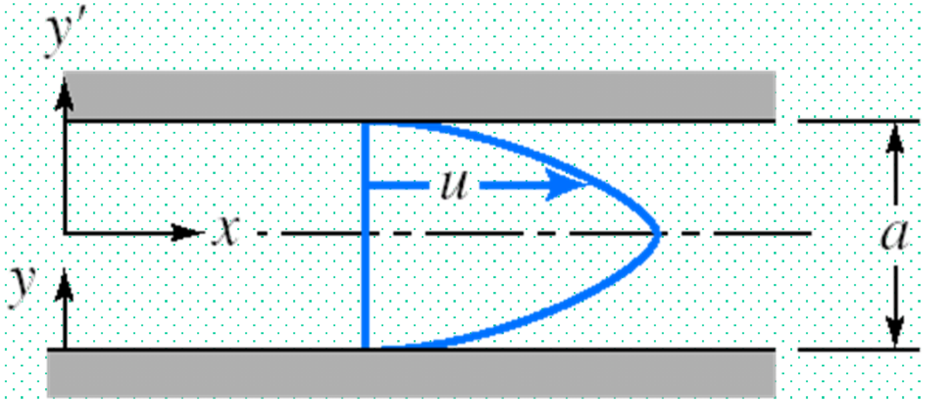


$$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y'}{a}\right)^2 - \frac{1}{4}\right]$$

Fully Developed Laminar Flow Between Infinite Parallel Plates

✓ Both Plates Stationary

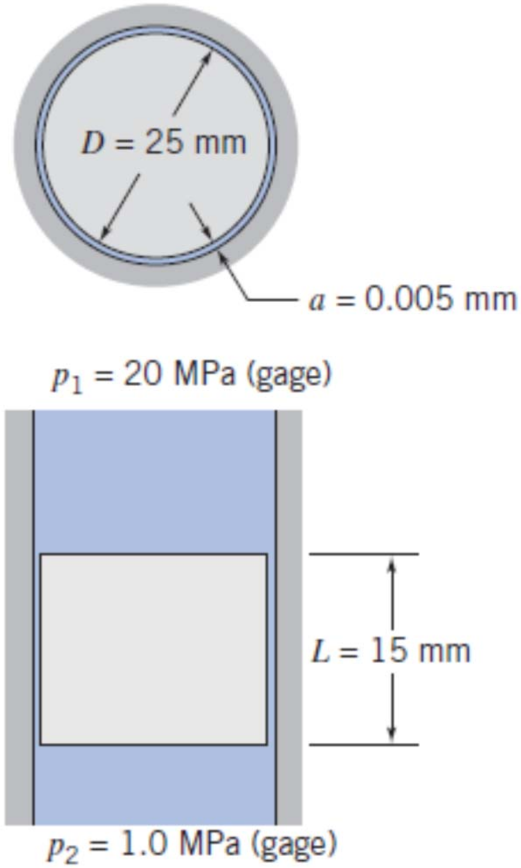
- **Transformation of Coordinates**



$$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y'}{a}\right)^2 - \frac{1}{4}\right]$$

Example 8.1 LEAKAGE FLOW PAST A PISTON

A hydraulic system operates at a gage pressure of 20 MPa and 55C. The hydraulic fluid is SAE 10W oil. A control valve consists of a piston 25 mm in diameter, fitted to a cylinder with a mean radial clearance of 0.005 mm. Determine the leakage flow rate if the gage pressure on the low-pressure side of the piston is 1.0 MPa. (The piston is 15 mm long.)



Assumptions:
(1) Laminar flow.
(2) Steady flow.
(3) Incompressible flow.
(4) Fully developed flow. (Note L=a 5 15=0:005 5 3000!)

$$Q = \frac{\pi D a^3 \Delta p}{12 \mu L}$$

For SAE 10W oil at 55°C, $\mu = 0.018\ \mathrm{kg/(m \cdot s)}$

$$Q = \frac{\pi}{12} \times 25\ \mathrm{mm} \times (0.005)^3\ \mathrm{mm}^3 \times (20-1)10^6\ \frac{\mathrm{N}}{\mathrm{m}^2} \times \frac{\mathrm{m \cdot s}}{0.018\ \mathrm{kg}} \times \frac{1}{15\ \mathrm{mm}} \times \frac{\mathrm{kg \cdot m}}{\mathrm{N \cdot s^2}}$$

$$Q = 57.6\ \mathrm{mm^3/s}$$

$$\overline{V} = \frac{Q}{A} = \frac{Q}{\pi D a} = 57.6\ \frac{\mathrm{mm}^3}{\mathrm{s}} \times \frac{1}{\pi} \times \frac{1}{25\ \mathrm{mm}} \times \frac{1}{0.005\ \mathrm{mm}} \times \frac{\mathrm{m}}{10^3\ \mathrm{mm}} = 0.147\ \mathrm{m/s}$$

$$Re = \frac{\rho \overline{V} a}{\mu} = \frac{SG \rho_{H_2O} \overline{V} a}{\mu}$$

For SAE 10W oil, $SG = 0.92$,

$$Re = 0.92 \times 1000 \frac{\mathrm{kg}}{\mathrm{m}^3} \times 0.147 \frac{\mathrm{m}}{\mathrm{s}} \times 0.005\ \mathrm{mm} \times \frac{\mathrm{m \cdot s}}{0.018\ \mathrm{kg}} \times \frac{\mathrm{m}}{10^3\ \mathrm{mm}} = 0.0375$$

Thus flow is surely laminar, since $Re \ll 1400$.

Fully Developed Laminar Flow Between Infinite Parallel Plates

✓ **Both Plates Stationary**

- **Shear Stress Distribution**

$$\tau_{yx} = a\left(\frac{\partial p}{\partial x}\right)\left[\frac{y}{a} - \frac{1}{2}\right]$$

- **Volume Flow Rate**

$$\frac{Q}{l} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$$

Fully Developed Laminar Flow Between Infinite Parallel Plates

✓ **Both Plates Stationary**

- **Flow Rate as a Function of Pressure Drop**

$$\frac{Q}{l} = \frac{a^3 \Delta p}{12 \mu L}$$

- **Average and Maximum Velocities**

$$\bar{V} = -\frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^2 \qquad u_{\max} = -\frac{1}{8\mu}\left(\frac{\partial p}{\partial x}\right)a^2 = \frac{3}{2}\bar{V}$$

Fully Developed Laminar Flow Between Infinite Parallel Plates

✓ **Upper Plate Moving with Constant Speed, *U***

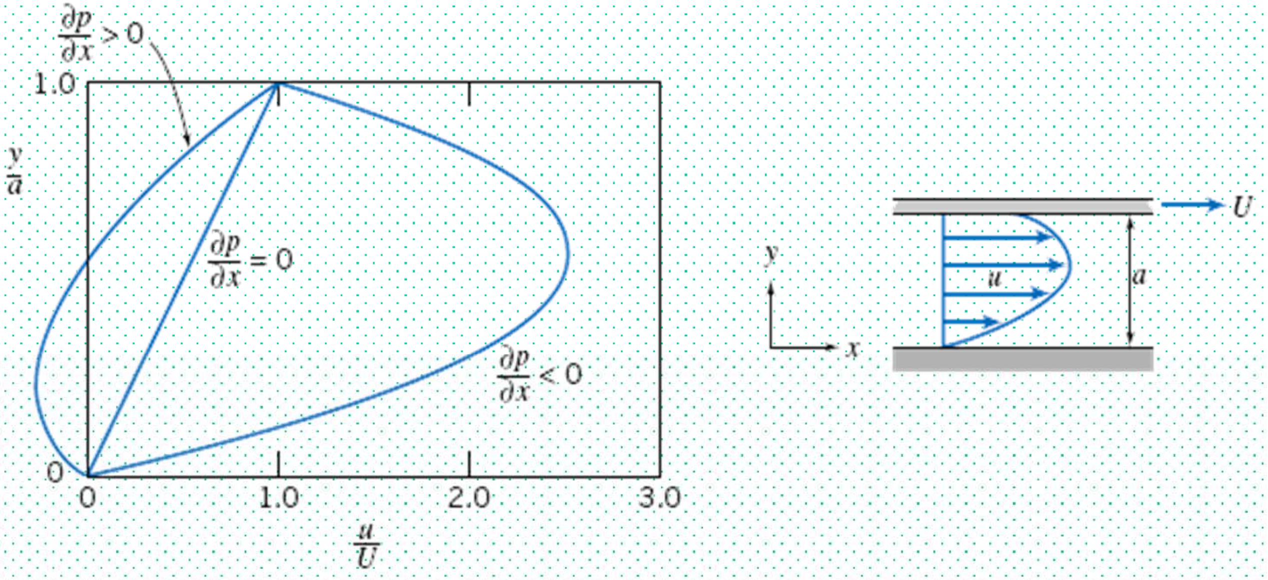


Fig. 8.6 Dimensionless velocity profile for fully developed laminar flow between infinite parallel plates: upper plate moving with constant speed, U.

Fully Developed Laminar Flow in a Pipe

✓ **Velocity Distribution**

$$u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

✓ **Shear Stress Distribution**

$$\tau_{rx} = \frac{r}{2}\left(\frac{\partial p}{\partial x}\right)$$

Fully Developed Laminar Flow in a Pipe

- ✓ **Volume Flow Rate**

$$Q = -\frac{\pi R^4}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

- ✓ **Flow Rate as a Function of Pressure Drop**

$$Q = \frac{\pi \Delta p D^4}{128\mu L}$$

Fully Developed Laminar Flow in a Pipe

- **Average Velocity**

$$\bar{V} = -\frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

- **Maximum Velocity**

$$u_{\max} = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right) = 2\bar{V}$$

Turbulent Velocity Profiles in Fully Developed Pipe Flow

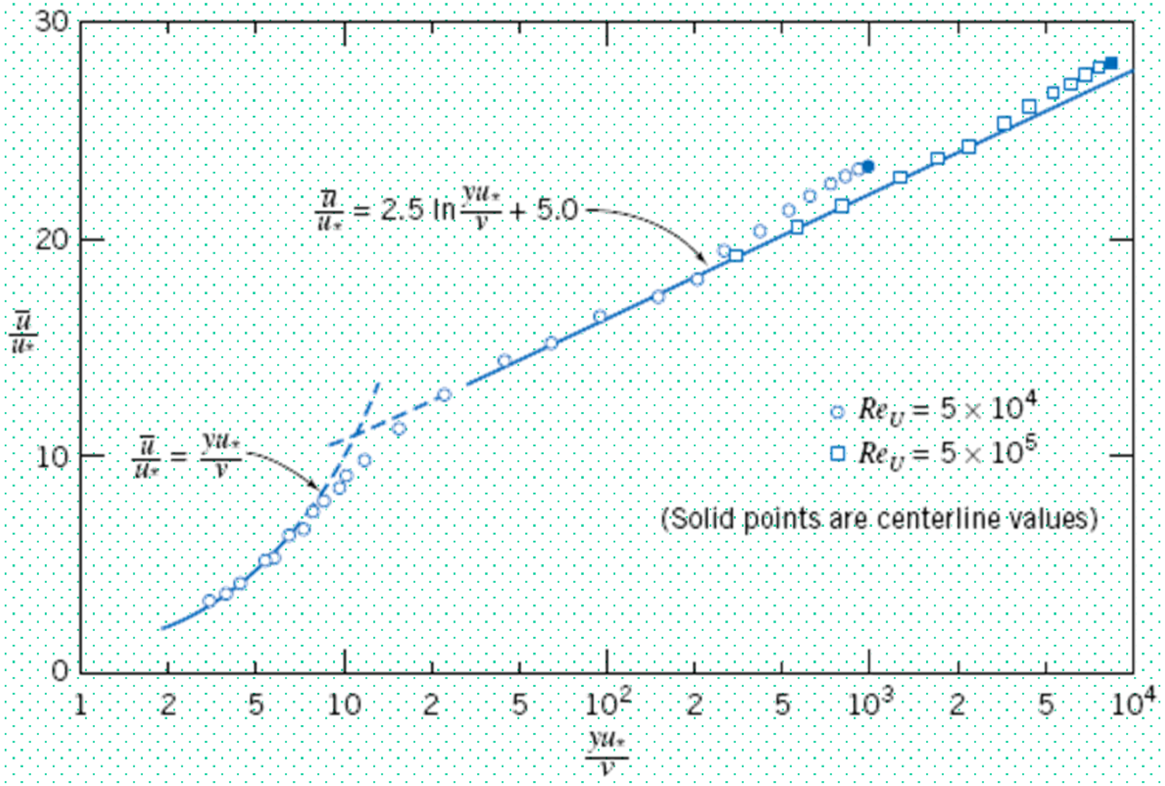


Fig. 8.9 Turbulent velocity profile for fully developed flow in a smooth pipe. (Data from [5].)

Turbulent Velocity Profiles in Fully Developed Pipe Flow

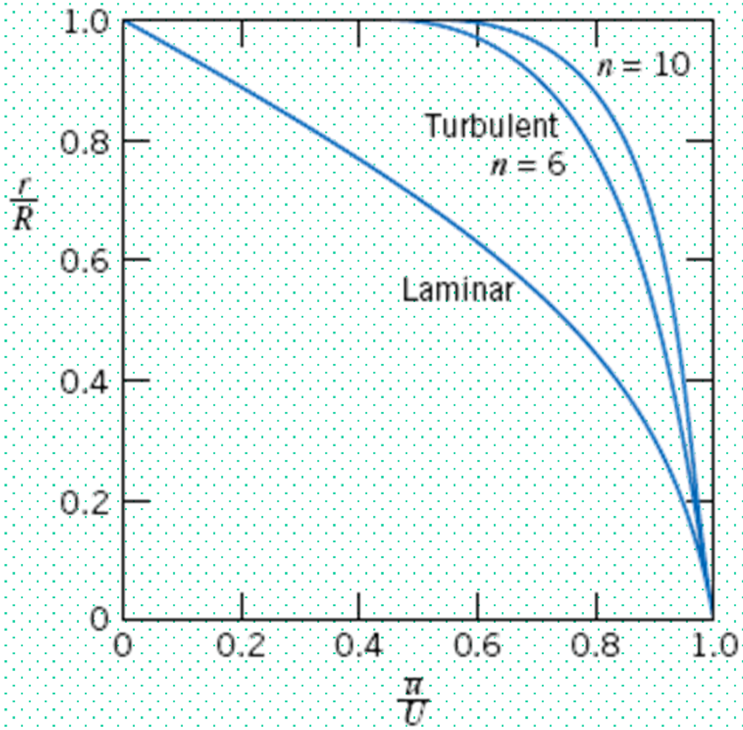


Fig. 8.11 Velocity profiles for fully developed pipe flow.

power-law equation

$$\frac{\bar{u}}{U} = \left(\frac{y}{R}\right)^{1/n} = \left(1 - \frac{r}{R}\right)^{1/n}$$

As a representative value, 7 often is used for fully developed turbulent flow:

Energy Considerations in Pipe Flow

✓Energy Equation

$$\dot{Q} = \dot{m}(u_2 - u_1) + \dot{m}\left(\frac{p_2}{\rho} - \frac{p_1}{\rho}\right) + \dot{m}g(z_2 - z_1) + \dot{m}\left(\frac{\alpha_2 \bar{V}_2^2}{2} - \frac{\alpha_1 \bar{V}_1^2}{2}\right)$$

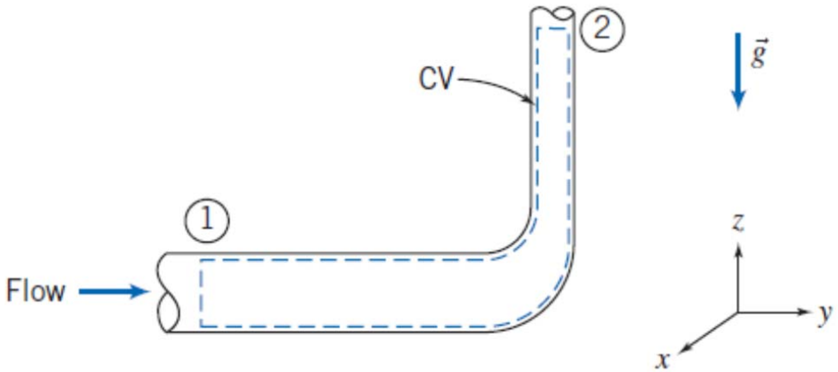


Fig. 8.12 Control volume and coordinates for energy analysis of flow through a 90° reducing elbow.

Assumptions:

(1) W_s=0; W_{other} = 0.
(2) W_{shear}=0
(3) Steady flow.
(4) Incompressible flow.
(5) Internal energy and pressure uniform across sections 1 and 2 .

Energy Considerations in Pipe Flow

✓Head Loss

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T}$$

$$\left(\frac{p_1}{\rho g} + \alpha_1 \frac{\bar{V}_1^2}{2g} + z_1\right) - \left(\frac{p_2}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} + z_2\right) = \frac{h_{l_T}}{g} = H_{l_T}$$

Calculation of Head Loss

✓Major Losses: Friction Factor

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$H_l = f \frac{L}{D} \frac{\bar{V}^2}{2g}$$

Calculation of Head Loss

✓Laminar Friction Factor

$$f = \frac{64}{Re} \qquad Re < 2300$$

✓Turbulent Friction Factor

Colebrook equation

$$\frac{1}{f^{0.5}} = -2.0 \log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\, f^{0.5}}\right) \qquad Re \geq 2300$$

Calculation of Head Loss

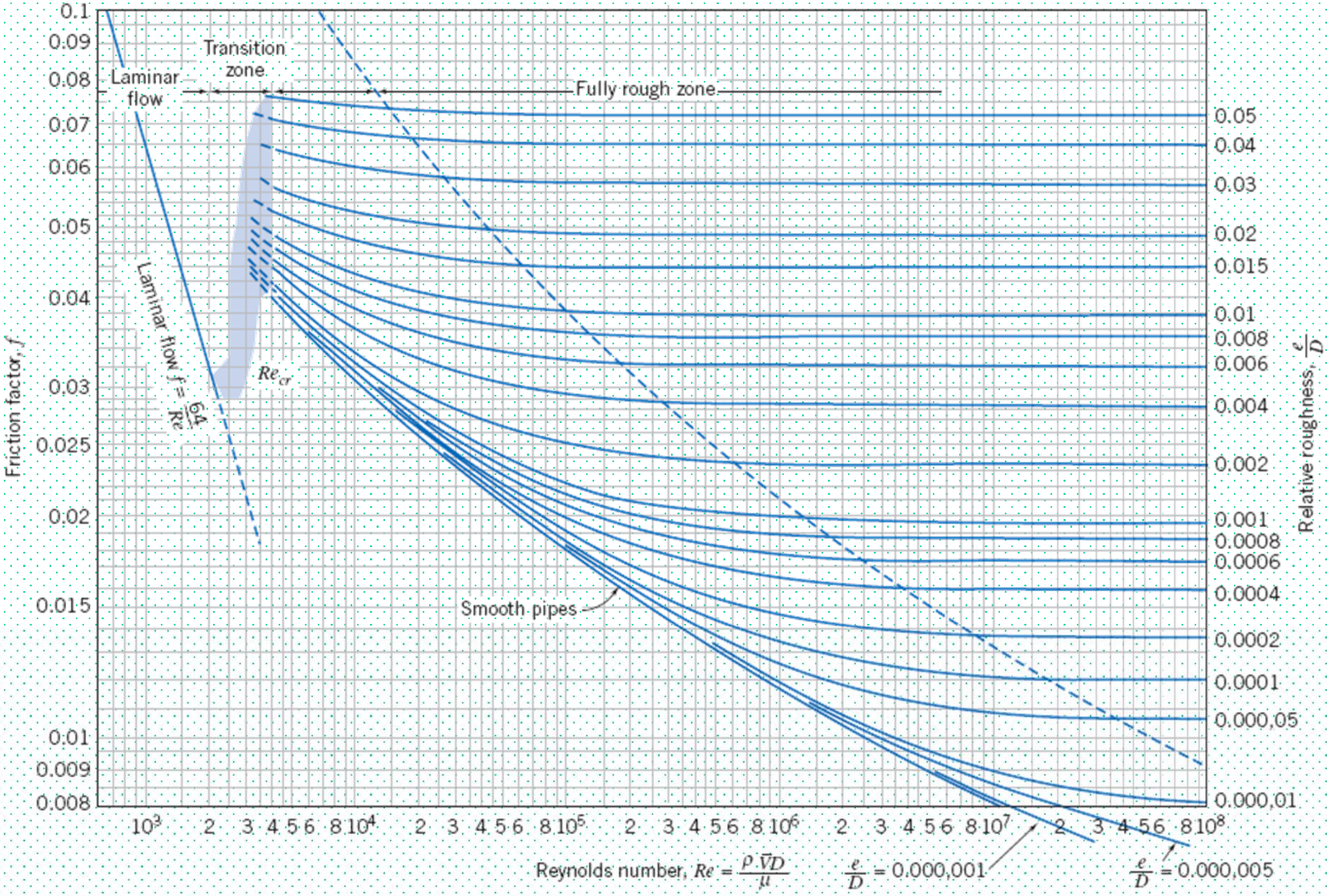


Fig. 8.13 Friction factor for fully developed flow in circular pipes. (Data from [8], used by permission.)

Table 8.1

Roughness for Pipes of Common Engineering Materials

Pipe	Roughness, *e* Millimeters
Riveted steel	0.9–9
Concrete	0.3–3
Wood stave	0.2–0.9
Cast iron	0.26
Galvanized iron	0.15
Asphalted cast iron	0.12
Commercial steel or wrought iron	0.046
Drawn tubing	0.0015

Calculation of Head Loss

✓ **Minor Loss: Loss Coefficient, *K***

$$h_{l_m} = K \frac{\bar{V}^2}{2}$$

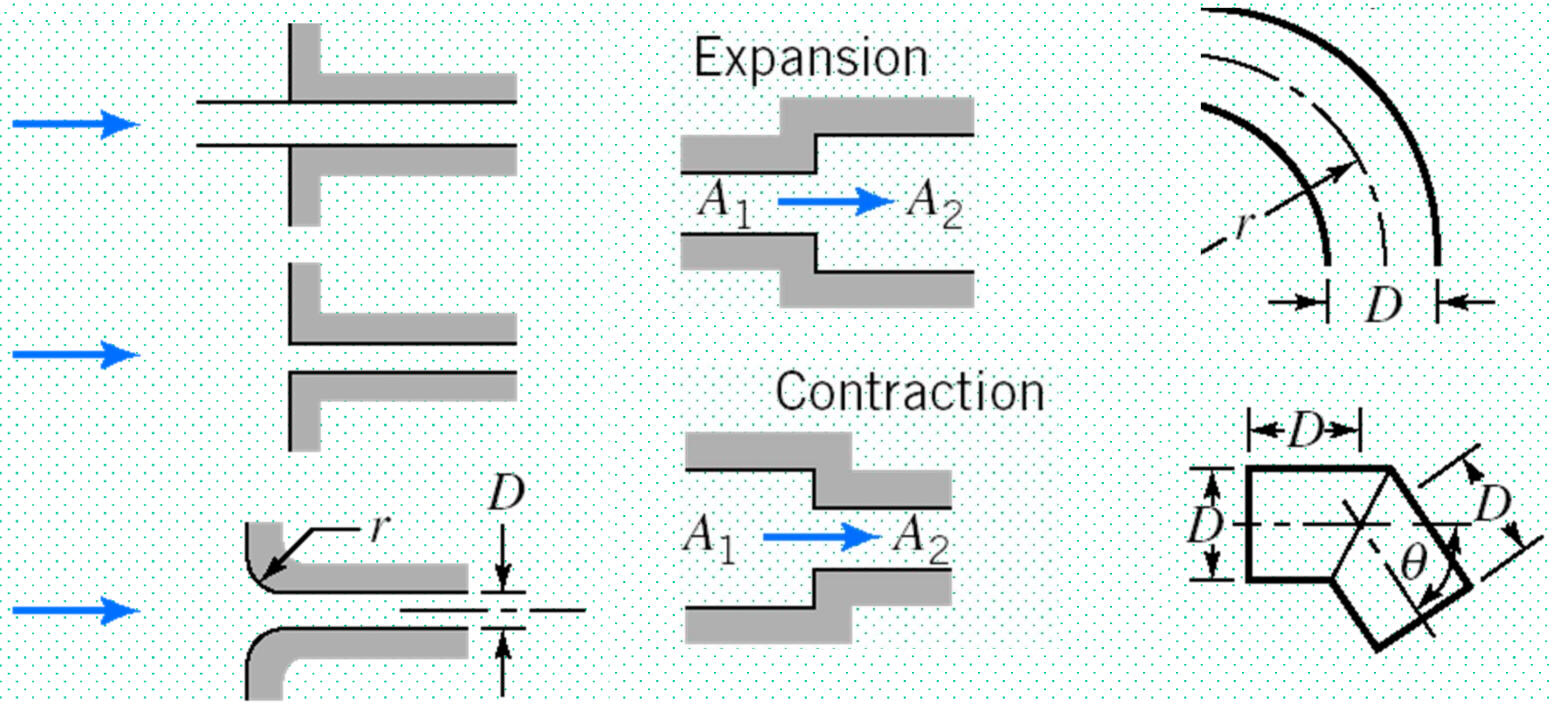
✓ **Minor Loss: Equivalent Length, L_e**

$$h_{l_m} = f \frac{L_e}{D} \frac{\bar{V}^2}{2}$$

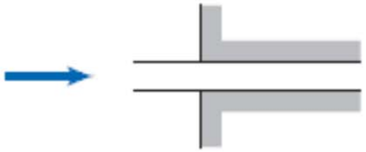
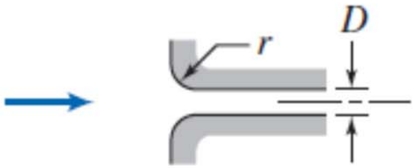
Calculation of Head Loss

✓ Minor Losses

- Examples: Inlets and Exits; Enlargements and Contractions; Pipe Bends; Valves and Fittings



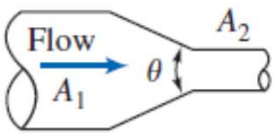
Minor Loss Coefficients for Pipe Entrances

Entrance Type		Minor Loss Coefficient, K^a
Reentrant		0.78
Square-edged		0.5
Rounded		r/D: 0.02, 0.06, ≥ 0.15 K: 0.28, 0.15, 0.04

Pressure recovery coefficient, C_p

$$C_p \equiv \frac{p_2 - p_1}{\frac{1}{2}\rho \bar{V}_1^2}$$

Loss Coefficients (K) for Gradual Contractions: Round and Rectangular Ducts

		Included Angle, θ, Degrees						
	A_2/A_1	10	15–40	50–60	90	120	150	180
	0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
	0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
	0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

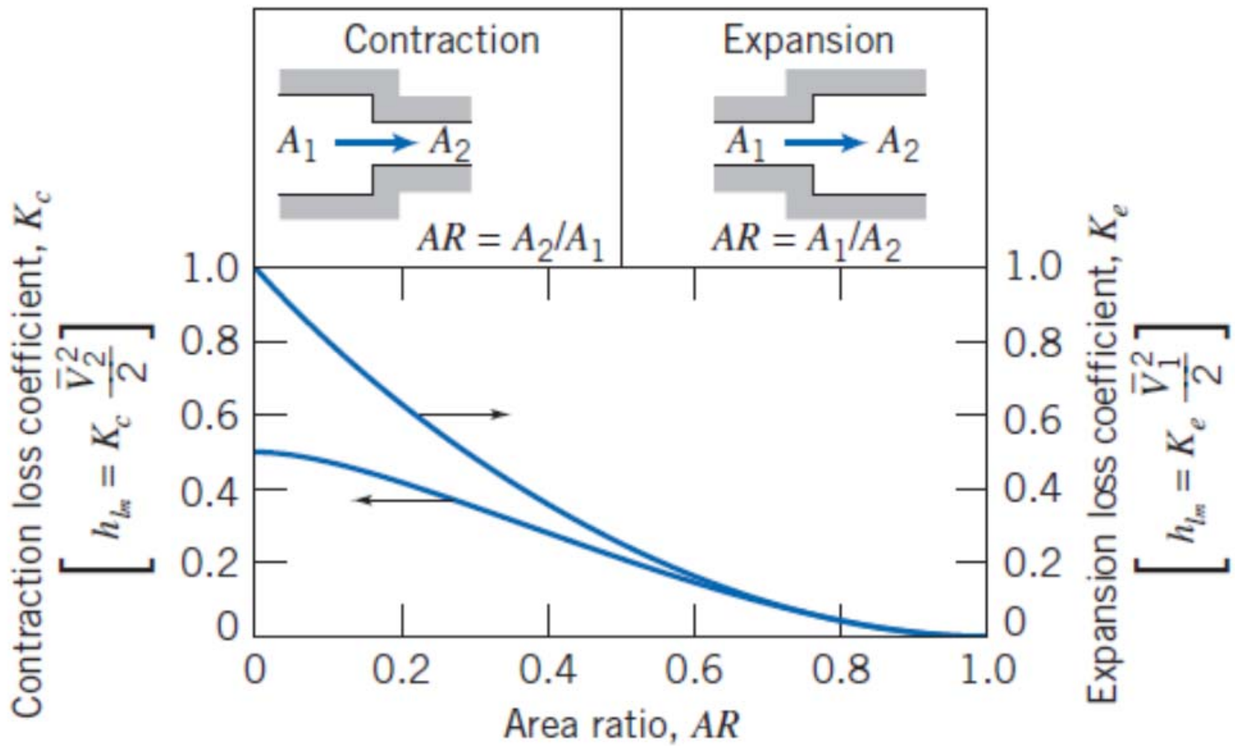


Fig. 8.15 Loss coefficients for flow through sudden area changes.

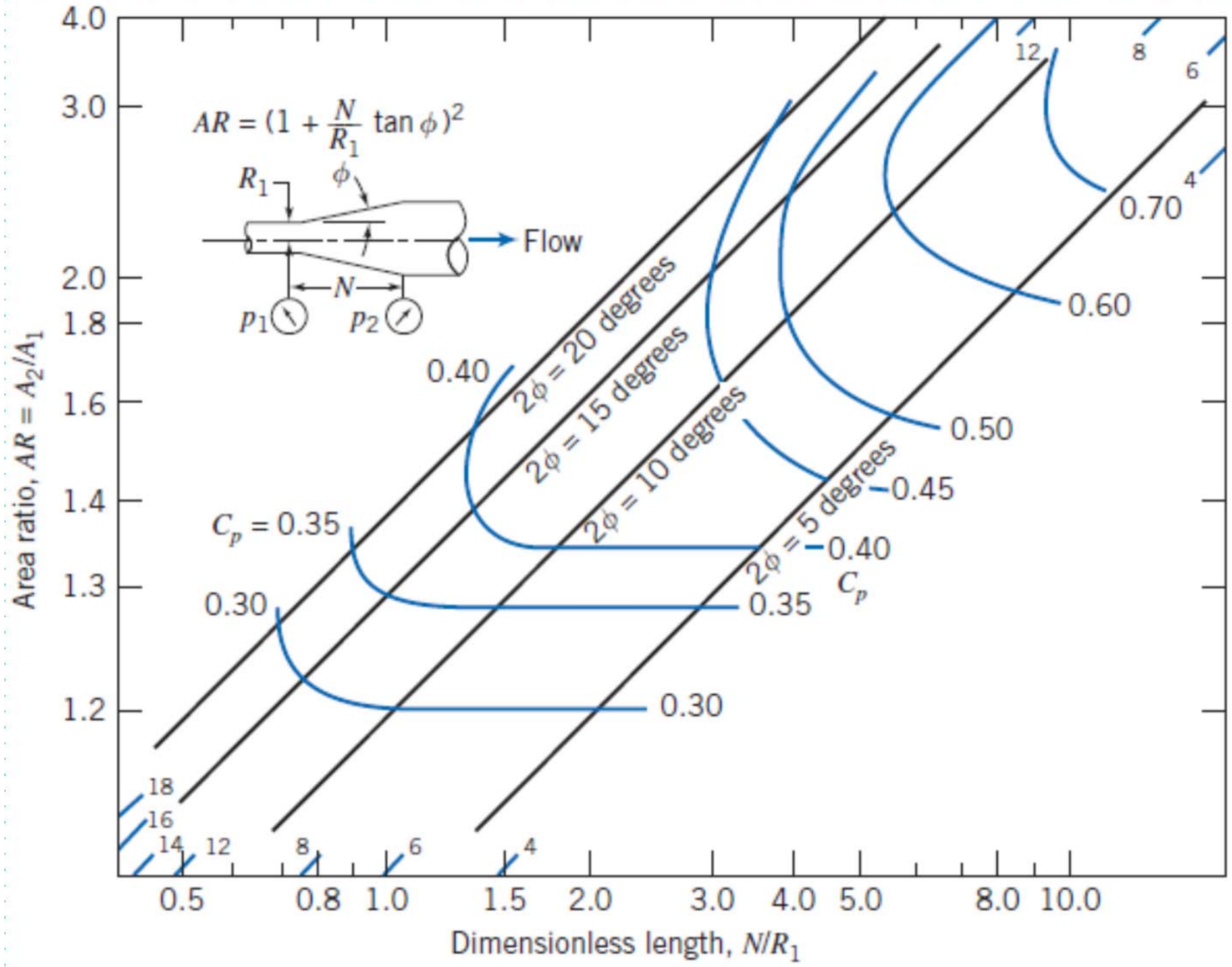


Fig. 8.16 Pressure recovery for conical diffusers with fully developed turbulent pipe flow at inlet. (Data from Cockrell and Bradley [13].)

$$h_{l_m} = \frac{\bar{V}_1^2}{2}\left[\left(1 - \frac{1}{(AR)^2}\right) - C_p\right]$$

Calculation of Head Loss

✓Pumps, Fans, and Blowers

$$\boxed{\Delta h_{\text{pump}} = \frac{\Delta p_{\text{pump}}}{\rho}}$$

$$\boxed{\dot{W}_{\text{pump}} = Q\Delta p_{\text{pump}}}$$

$$\boxed{\eta = \frac{\dot{W}_{\text{pump}}}{\dot{W}_{\text{in}}}}$$

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{\text{pump}}$$

Calculation of Head Loss

✓Noncircular Ducts

$$D_h \equiv \frac{4A}{P}$$

Example: Rectangular Duct

$$D_h = \frac{4bh}{2(b+h)}$$

Solution of Pipe Flow Problems

✓Energy Equation

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$= h_{l_T} = \sum h_l + \sum h_{l_m}$$

Solution of Pipe Flow Problems

✓Major Losses

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$f = \frac{64}{Re} \qquad \text{for laminar flow } (Re < 2300)$$

$$\frac{1}{f^{0.5}} = -2.0 \log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\, f^{0.5}}\right) \qquad \text{for turbulent flow } (Re \geq 2300)$$

Solution of Pipe Flow Problems

✓Minor Losses

$$h_{l_m} = K \frac{\bar{V}^2}{2}$$

$$h_{l_m} = f \frac{L_e}{D} \frac{\bar{V}^2}{2}$$

Solution of Pipe Flow Problems

✓ Single Path

- **Find Δp for a given L, D, and Q**

  Use energy equation directly

- **Find L for a given Δp, D, and Q**

  Use energy equation directly

Solution of Pipe Flow Problems

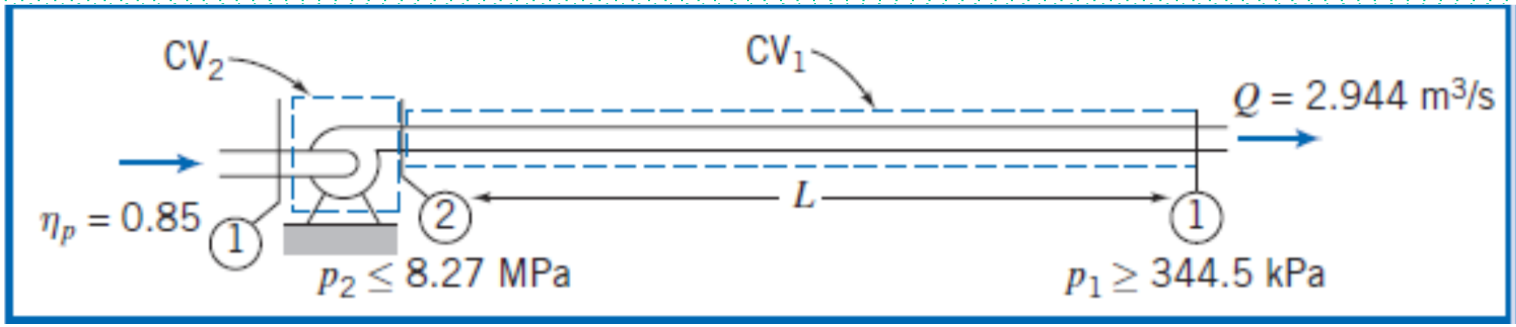
✓ Single Path (Continued)

- **Find *Q* for a given Δp, *L*, and *D***
  1. Manually iterate energy equation and friction factor formula to find *V* (or *Q*), or
  2. Directly solve, simultaneously, energy equation and friction factor formula using (for example) *Excel*

- **Find *D* for a given Δp, *L*, and *Q***
  1. Manually iterate energy equation and friction factor formula to find *D*, or
  2. Directly solve, simultaneously, energy equation and friction factor formula using (for example) *Excel*

Example 8.6 FLOW IN A PIPELINE: LENGTH UNKNOWN

Crude oil flows through a level section of the Alaskan pipeline at a rate of 2.944 m^3/s. The pipe inside diameter is 1.22 m; its roughness is equivalent to galvanized iron. The maximum allowable pressure is 8.27 MPa; the minimum pressure required to keep dissolved gases in solution in the crude oil is 344.5 kPa. The crude oil has SG=0.93; its viscosity at the pumping temperature of 60°C is μ=10.0168 $N.s/m^2$. For these conditions, determine the maximum possible spacing between pumping stations. If the pump efficiency is 85 percent, determine the power that must be supplied at each pumping station.



Assumptions: (1) $\alpha_1=\alpha_2$
(2) Horizontal pipe, $z_1=z_2$.
(3) Neglect minor losses.
(4) Constant viscosity.

$$\left(\frac{p_2}{\rho} + \alpha_2 \cancel{\frac{\bar{V}_2^2}{2}} + \cancel{g z_2}\right) - \left(\frac{p_1}{\rho} + \alpha_1 \cancel{\frac{\bar{V}_1^2}{2}} + \cancel{g z_1}\right) = h_{l_T} = h_l + h_{l_m}$$

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$h_{l_m} = K \frac{\bar{V}^2}{2}$$

$$\Delta p = p_2 - p_1 = f \frac{L}{D} \rho \frac{\bar{V}^2}{2}$$

$$L = \frac{2D}{f} \frac{\Delta p}{\rho \bar{V}^2} \quad \text{where } f = f(Re, e/D)$$

$$\overline{V} = \frac{Q}{A} = 2.944\frac{\text{m}^3}{\text{s}} \times \frac{4}{\pi(1.22)^2\text{m}^2} = 2.52 \ \text{m/s}$$

$$Re = \frac{\rho \overline{V} D}{\mu} = 0.93 \times 1000\frac{\text{kg}}{\text{m}^3} \times 2.52\frac{\text{m}}{\text{s}} \times 1.22 \text{ m} \times \frac{1}{0.0168 \text{ N·s/m}^2} \times \frac{\text{N.s}^2}{\text{kg} \cdot \text{m}}$$

$$Re = 1.71 \times 10^5$$

Table 8.1, $e = 0.00015$ m and hence $e/D = 0.00012$.

$f = 0.017$

$$L = \frac{2}{0.017} \times 1.22 \text{ m} \times (8.27 \times 10^6 - 3.445 \times 10^5)\text{Pa} \times \frac{1}{0.93 \times 1000 \times \text{kg/m}^3}$$

$$\times \frac{1}{(2.52)^2}\frac{\text{s}^2}{\text{m}^2} \times \frac{\text{N}}{\text{m}^2 \cdot \text{Pa}} \times \frac{\text{kg} \cdot \text{m}}{\text{N·s}^2} = 192,612 \text{ m}$$

$$L = 192,612 \text{ m}$$

$$\dot{W}_{\text{pump}} = Q\Delta p_{\text{pump}}$$

$$\eta = \frac{\dot{W}_{\text{pump}}}{\dot{W}_{\text{in}}}$$

$$\Delta p_{\text{pump}} = \Delta p$$

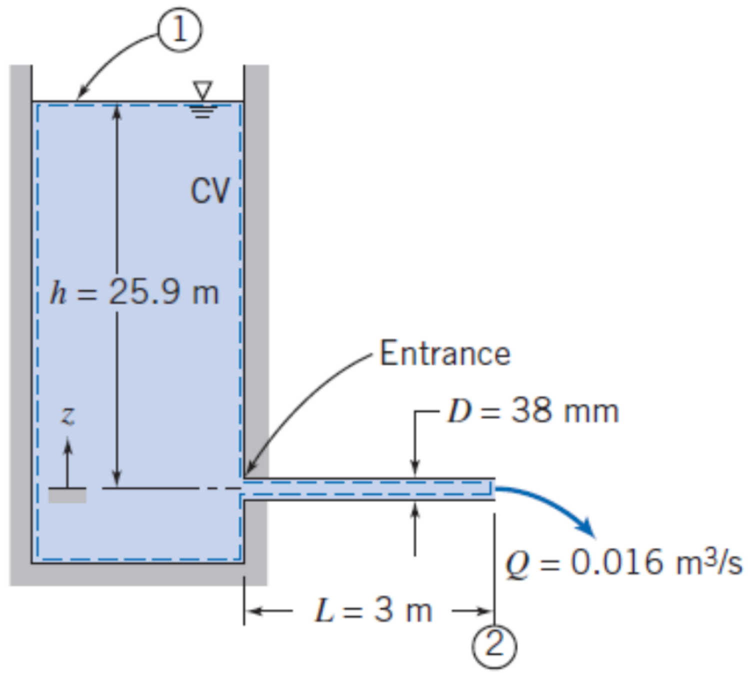
$$\dot{W}_{\text{pump}} = Q\Delta p_{\text{pump}} = 2.944\,\frac{\text{m}^3}{\text{s}} \times (8.27 \times 10^6 - 3.445 \times 10^5)\text{Pa} \times \frac{\text{N}}{\text{m}^2 \cdot \text{Pa}} \times \frac{\text{j}}{\text{N} \cdot \text{m}} \times \frac{\text{W} \cdot \text{S}}{\text{j}} \approx 23.13\ \text{MW}$$

the required power input

$$\dot{W}_{\text{in.}} = \frac{\dot{W}_{\text{pump}}}{\eta} = \frac{23.13}{0.85} = 27.21\ \text{MW}$$

Example 8.9 CALCULATION OF ENTRANCE LOSS COEFFICIENT

Hamilton reports results of measurements made to determine entrance losses for flow from a reservoir to a pipe with various degrees of entrance rounding. A copper pipe 3 m long, with 38 mm i.d., was used for the tests. The pipe discharged to atmosphere. For a square-edged entrance, a discharge of 0.016m^3/s was measured when the reservoir level was 25.9 m above the pipe centerline. From these data, evaluate the loss coefficient for a square-edged entrance.



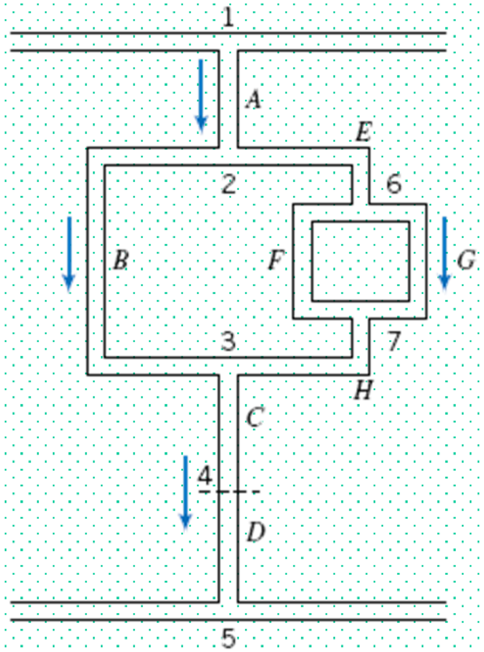
$$\frac{\cancel{p_1}}{\rho} + \alpha_1 \underbrace{\frac{\bar{V}_1^2}{2}}_{\approx 0(2)} + g z_1 = \frac{\cancel{p_2}}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + \underbrace{g z_2}_{=0} + h_{l_T}$$

$$h_{l_T} = f \frac{L}{D} \frac{\bar{V}_2^2}{2} + K_{\text{entrance}} \frac{\bar{V}_2^2}{2}$$

Solution of Pipe Flow Problems

✓ Multiple-Path Systems

Example:



A: $L = 10$ ft, $D = 1.5$ in.
B: $L = 20$ ft, $D = 1.5$ in.
C: $L = 10$ ft, $D = 2$ in.
D: $L = 10$ ft, $D = 1.5$ in.
E: $L = 5$ ft, $D = 1.5$ in.
F: $L = 10$ ft, $D = 1$ in.
G: $L = 10$ ft, $D = 1.5$ in.
H: $L = 5$ ft, $D = 2$ in.

Fig. 8.18 Schematic of part of a pipe network.

$$z_1 = h = \alpha_2 \frac{\overline{V}_2^2}{2g} + f\frac{L}{D}\frac{\overline{V}_2^2}{2g} + K_{\text{entrance}}\frac{\overline{V}_2^2}{2g}$$

$$K_{\text{entrance}} = \frac{2gh}{\overline{V}_2^2} - f\frac{L}{D} - \alpha_2$$

$$\overline{V}_2 = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

$$\overline{V}_2 = \frac{4}{\pi} \times 0.016\frac{\text{m}^3}{\text{s}} \times \frac{1}{(38)^2\ \text{mm}^2} \times 10^6\frac{\text{mm}^2}{\text{m}^2} = 14.1\ \text{m/s}$$

Assume $T = 21°\text{C}$, so $\nu = 9.75 \times 10^{-7}\ \text{m}^2/\text{s}$

$$Re = \frac{\overline{V}D}{\nu} = 14.1\frac{\text{m}}{\text{s}} \times 38\ \text{mm} \times \frac{\text{m}}{1000\ \text{mm}} \times \frac{\text{s}}{9.75 \times 10^{-7}\text{m}^2}$$

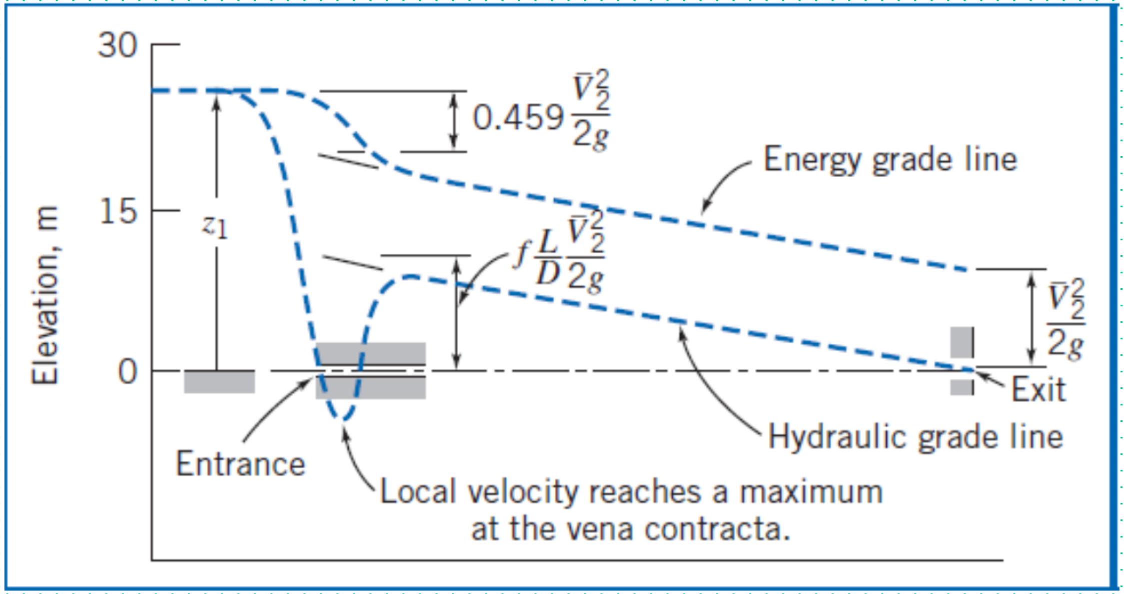
For drawn tubing, e=0.0015mm, so e/D=0.000,04 and f=0.0135.

$$\alpha = \left(\frac{U}{\overline{V}}\right)^3 \frac{2n^2}{(3+n)(3+2n)}$$

$$n = -1.7 + 1.8 \log(Re_U) \approx 8.63$$

$$\frac{\overline{V}}{U} = \frac{2n^2}{(n+1)(2n+1)} = 0.847$$

$$K_{\text{entrance}} = 2 \times 9.81 \frac{\text{m}}{\text{s}^2} \times 25.9 \text{ m} \times \frac{\text{s}^2}{(14.1)^2 \text{m}^2} - 0.0135 \frac{3\text{m}}{38 \text{ mm}} \times 1000 \frac{\text{mm}}{\text{m}} - 1.04$$



$$K_{\text{entrance}} = 0.45$$

Solution of Pipe Flow Problems

✓ Multiple-Path Systems

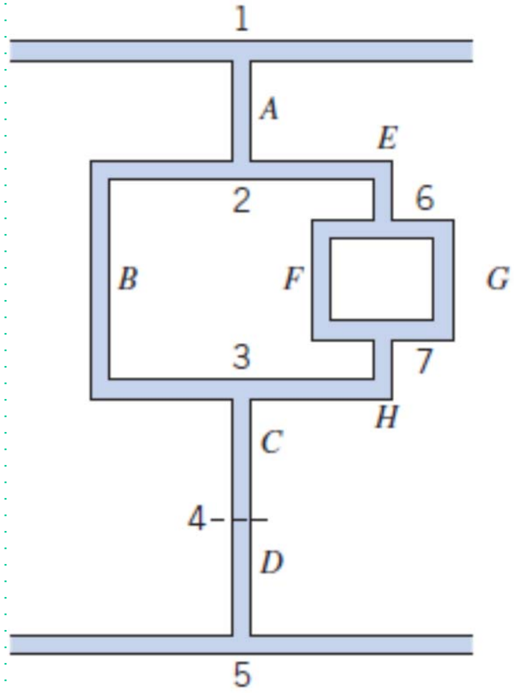
- Solve each branch as for single path

- Two additional rules
  1. The net flow out of any node (junction) is zero
  2. Each node has a unique pressure head (HGL)

- To complete solution of problem
  1. Manually iterate energy equation and friction factor for each branch to satisfy all constraints, or
  2. Directly solve, simultaneously, complete set of equations using (for example) *Excel*

Example 8.11 FLOW RATES IN A PIPE NETWORK

In the section of a cast-iron water pipe network shown in Figure, the static pressure head (gage) available at point 1 is 30 m of water, and point 5 is a drain (atmospheric pressure). Find the flow rates (L/min) in each pipe.



A: $L = 3$ m, $D = 38$ mm.
B: $L = 6$ m, $D = 38$ mm.
C: $L = 3$ m, $D = 50$ mm.
D: $L = 3$ m, $D = 38$ mm.
E: $L = 1.5$ m, $D = 38$ mm.
F: $L = 3$ m, $D = 25$ mm.
G: $L = 3$ m, $D = 38$ mm.
H: $L = 1.5$ m, $D = 50$ mm.

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + \overset{=0(1)}{\cancel{gz_1}}\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + \overset{=0(1)}{\cancel{gz_2}}\right) = h_{l_T} = h_l + \overset{=0(2)}{\cancel{\sum h_{l_m}}}$$

$$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$$

$$\text{Node 2}: \quad Q_A = Q_B + Q_E$$

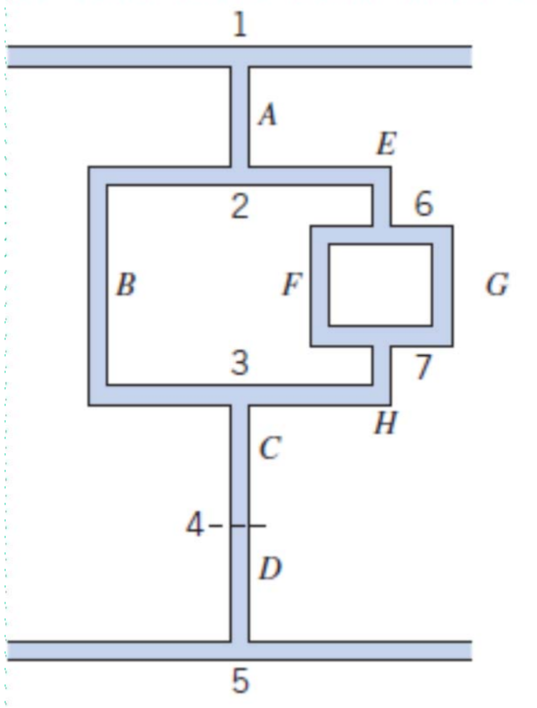
$$\text{Node 6}: \quad Q_E = Q_F + Q_G$$

$$Q_A = Q_C \qquad Q_A = Q_D \qquad Q_E = Q_H$$

$$h_{1-5}: \ h = h_A + h_B + h_C + h_D$$

$$h_{2-3}: h_B = h_E + h_F + h_H$$

$$h_{6-7}: h_F = h_G$$



$$\begin{aligned} Q_A &= Q_C = Q_D = 625.6 \text{ L/min} \\ Q_B(\text{L/min}) &= 272.0 \text{ L/min} \\ Q_E(\text{L/min}) &= Q_H(\text{L/min}) = 353.6 \text{ L/min} \\ Q_F(\text{L/min}) &= 87.1 \text{ L/min} \\ Q_G(\text{L/min}) &= 266.5 \text{ L/min} \end{aligned}$$

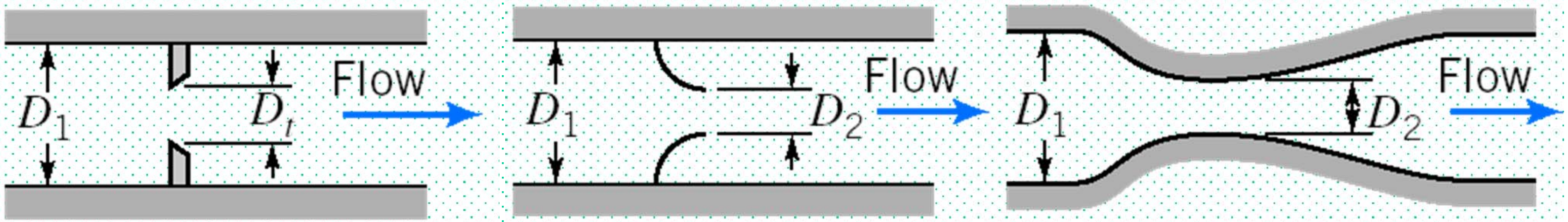
Flow Measurement

✓ **Direct Methods**

- **Examples: Accumulation in a Container; Positive Displacement Flowmeter**

✓ **Restriction Flow Meters for Internal Flows**

- **Examples: Orifice Plate; Flow Nozzle; Venturi; Laminar Flow Element**



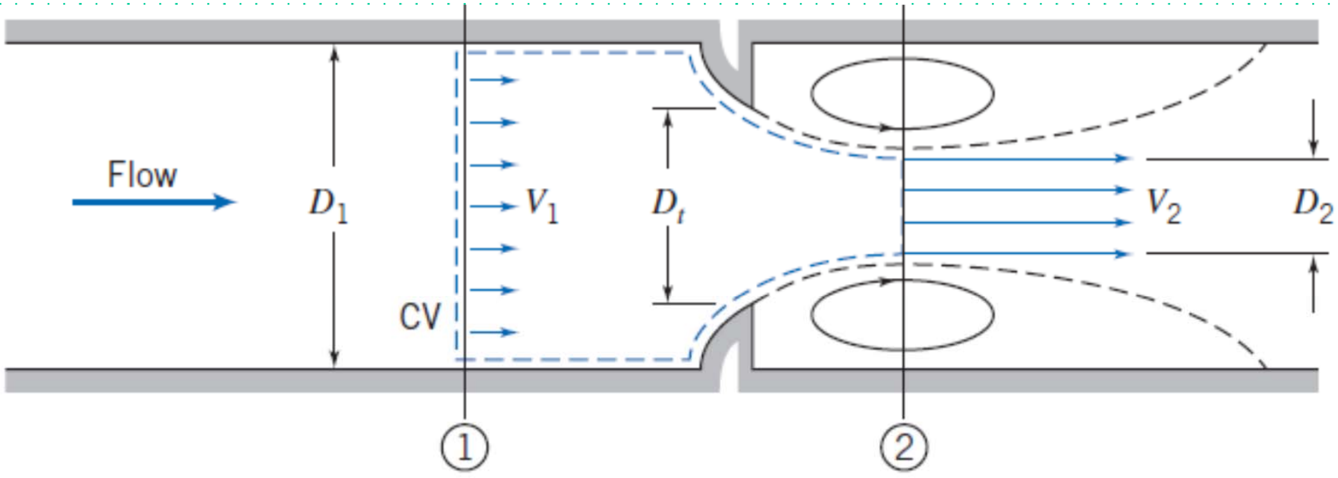


Fig. 8.19 Internal flow through a generalized nozzle, showing control volume used for analysis.

mass-conservation,

$$\sum_{CS} \vec{V} \cdot \vec{A} = 0$$

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + g\cancel{z_1} = \frac{p_2}{\rho} + \frac{V_2^2}{2} + g\cancel{z_2}$$

Assumptions:

(1) Steady flow.
(2) Incompressible flow.
(3) Flow along a streamline.
(4) No friction.
(5) Uniform velocity at sections 1 and 2 .
(6) No streamline curvature at sections 1 or 2 , so pressure is uniform across those sections.
(7) $z_1=z_2$.

$$p_1 - p_2 = \frac{\rho}{2}\left(V_2^2 - V_1^2\right) = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{V_1}{V_2}\right)^2\right]$$

$$(-\rho V_1 A_1) + (\rho V_2 A_2) = 0$$

$$V_1 A_1 = V_2 A_2 \qquad \text{so} \qquad \left(\frac{V_1}{V_2}\right)^2 = \left(\frac{A_2}{A_1}\right)^2$$

$$p_1 - p_2 = \frac{\rho V_2^2}{2}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right]$$

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

$$\begin{aligned} \dot{m}_{\text{theoretical}} &= \rho V_2 A_2 \\ &= \rho \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}\, A_2 \end{aligned}$$

$$\dot{m}_{\text{theoretical}} = \frac{A_2}{\sqrt{1-(A_2/A_1)^2}}\sqrt{2\rho(p_1-p_2)}$$

$$\dot{m}_{\text{theoretical}} \propto \sqrt{\Delta p}$$

$$\dot{m}_{\text{actual}} = \frac{CA_t}{\sqrt{1-(A_t/A_1)^2}}\sqrt{2\rho(p_1-p_2)}$$

$\beta = D_t/D_1$, then $(A_t/A_1)^2 = (D_t/D_1)^4 = \beta^4$.

$$\dot{m}_{\text{actual}} = \frac{CA_t}{\sqrt{1-\beta^4}}\sqrt{2\rho(p_1-p_2)}$$

flow coefficient

$$K \equiv \frac{C}{\sqrt{1-\beta^4}}$$

$$\dot{m}_{\text{actual}} = KA_t\sqrt{2\rho(p_1-p_2)}$$

Flow Measurement

✓ **Linear Flow Meters**

- **Examples: Float Meter (Rotameter); Turbine; Vortex; Electromagnetic; Magnetic; Ultrasonic**

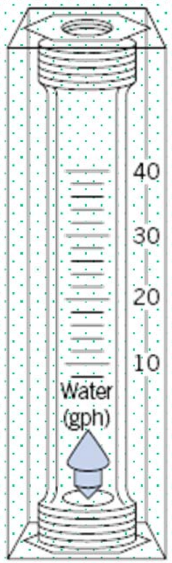


Fig. 8.25 Float-type variable-area flow meter. (Courtesy of Dwyer Instrument Co., Michigan City, Indiana.)

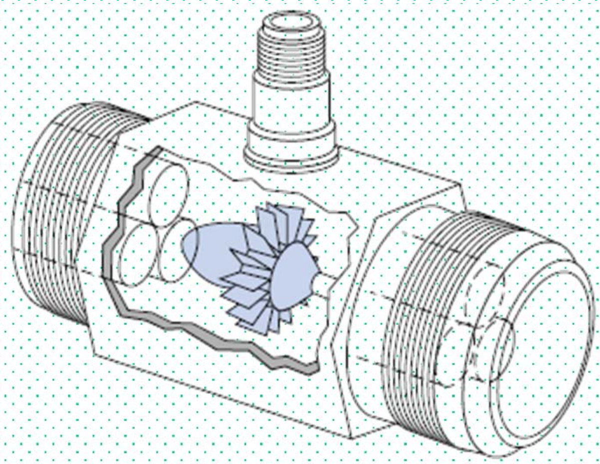
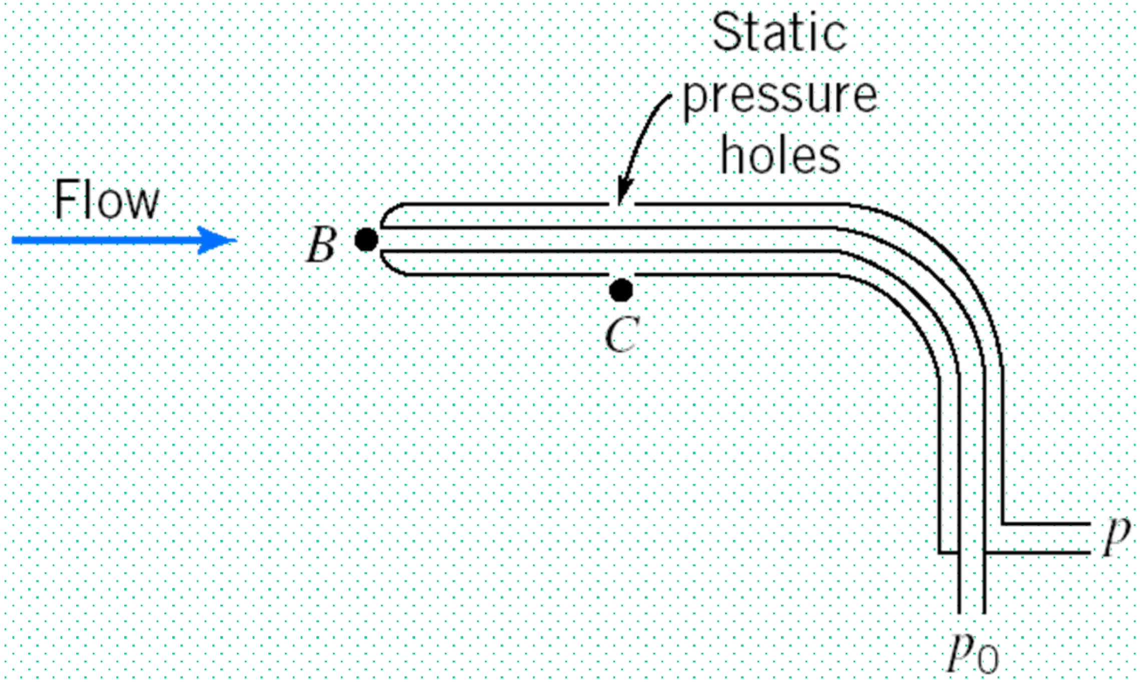


Fig. 8.26 Turbine flow meter. (Courtesy of Potter Aeronautical Corp., Union, New Jersey.)

Flow Measurement

✓ **Traversing Methods**

- **Examples: Pitot (or Pitot Static) Tube; Laser Doppler Anemometer**



$$V = \sqrt{\frac{2(p_0 - p)}{\rho}}$$

Useful Equations

Velocity profile for pressure-driven laminar flow between stationary parallel plates:	$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$
Flow rate for pressure-driven laminar flow between stationary parallel plates:	$\frac{Q}{l} = -\frac{1}{12\mu}\left[\frac{-\Delta p}{L}\right]a^3 = \frac{a^3\,\Delta p}{12\mu\,L}$
Velocity profile for pressure-driven laminar flow between stationary parallel plates (centered coordinates):	$u = \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y'}{a}\right)^2 - \frac{1}{4}\right]$
Velocity profile for pressure-driven laminar flow between parallel plates (upper plate moving):	$u = \frac{Uy}{a} + \frac{a^2}{2\mu}\left(\frac{\partial p}{\partial x}\right)\left[\left(\frac{y}{a}\right)^2 - \left(\frac{y}{a}\right)\right]$
Flow rate for pressure-driven laminar flow between parallel plates (upper plate moving):	$\frac{Q}{l} = \frac{Ua}{2} - \frac{1}{12\mu}\left(\frac{\partial p}{\partial x}\right)a^3$
Velocity profile for laminar flow in a pipe:	$u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$

Flow rate for laminar flow in a pipe:	$Q = -\frac{\pi R^4}{8\mu}\left[\frac{-\Delta p}{L}\right] = \frac{\pi \Delta p R^4}{8\mu L} = \frac{\pi \Delta p D^4}{128\mu L}$
Velocity profile for laminar flow in a pipe (normalized form):	$\frac{u}{U} = 1 - \left(\frac{r}{R}\right)^2$
Velocity profile for turbulent flow in a smooth pipe (power-law equation):	$\frac{\bar{u}}{U} = \left(\frac{y}{R}\right)^{1/n} = \left(1 - \frac{r}{R}\right)^{1/n}$
Head loss equation:	$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + g z_2\right) = h_{l_T}$
Major head loss equation:	$h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}$
Friction factor (laminar flow):	$f_{\text{laminar}} = \frac{64}{Re}$
Friction factor (turbulent flow—Colebrook equation):	$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$

Minor loss using loss coefficient K:	$h_{l_m} = K\frac{\bar{V}^2}{2}$
Minor loss using equivalent length L_e:	$h_{l_m} = f\frac{L_e}{D}\frac{\bar{V}^2}{2}$
Diffuser pressure recovery coefficient:	$C_p \equiv \frac{p_2 - p_1}{\frac{1}{2}\rho\bar{V}_1^2}$
Ideal diffuser pressure recovery coefficient:	$C_{p_i} = 1 - \frac{1}{AR^2}$
Head loss in diffuser in terms of pressure recovery coefficients:	$h_{l_m} = (C_{p_i} - C_p)\frac{\bar{V}_1^2}{2}$
Pump work:	$\dot{W}_{\text{pump}} = Q\Delta p_{\text{pump}}$

Pump efficiency:	$$\eta = \frac{\dot{W}_{\text{pump}}}{\dot{W}_{\text{in}}}$$
Hydraulic diameter:	$$D_h \equiv \frac{4A}{P}$$
Mass flow rate equation for a flow meter (in terms of discharge coefficient C):	$$\dot{m}_{\text{actual}} = \frac{CA_t}{\sqrt{1-\beta^4}}\sqrt{2\rho(p_1 - p_2)}$$
Mass flow rate equation for a flow meter (in terms of flow coefficient K):	$$\dot{m}_{\text{actual}} = KA_t\sqrt{2\rho(p_1 - p_2)}$$
Discharge coefficient (as a function of Re):	$$C = C_\infty + \frac{b}{Re_{D_1}^n}$$
Flow coefficient (as a function of Re):	$$K = K_\infty + \frac{1}{\sqrt{1-\beta^4}}\frac{b}{Re_{D_1}^n}$$

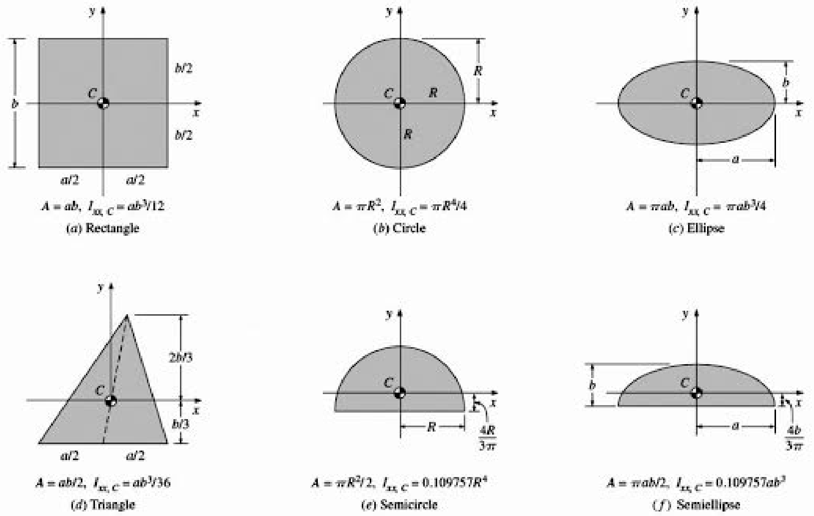
$$\tau_w = \mu \frac{du}{dr} \quad F_{shear} = \tau_w A_s \qquad \mathrm{T} = FR \qquad \boxed{SG = \rho / \rho_{H2O}} \quad \boxed{\rho = SG \times \rho_{H2O}} \quad \nu = \mu / \rho.$$

$$\tau = \mu \frac{du}{dy} \quad (\mathrm{N/m^2}) \qquad \boxed{F = \tau A = \mu A \frac{du}{dy} \quad (\mathrm{N})} \qquad F = \mu A \frac{V}{\ell} \quad (\mathrm{N})$$

$$P_{\text{gage}} = P_{\text{abs}} - P_{\text{atm}} \qquad \Delta P = P_2 - P_1 = \rho g\, \Delta z = \gamma_s\, \Delta z$$

$$P_{\text{vac}} = P_{\text{atm}} - P_{\text{abs}} \qquad P = P_{\text{atm}} + \rho g h \quad P_{\text{gage}} = \rho g h$$

$$I_{xx,O} = I_{xx,C} + y_C^2 A \qquad y_P = y_C + \frac{I_{xx,C}}{y_C A} \qquad y_P = y_C + \frac{I_{xx,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$



$A = ab,\ I_{xx,C} = ab^3/12$ (*a*) Rectangle

$A = \pi R^2,\ I_{xx,C} = \pi R^4/4$ (*b*) Circle

$A = \pi ab,\ I_{xx,C} = \pi ab^3/4$ (*c*) Ellipse

$A = ab/2,\ I_{xx,C} = ab^3/36$ (*d*) Triangle

$A = \pi R^2/2,\ I_{xx,C} = 0.109757R^4$ (*e*) Semicircle

$A = \pi ab/2,\ I_{xx,C} = 0.109757ab^3$ (*f*) Semiellipse

$$P_2 - P_1 = -\rho a_x (x_2 - x_1) - \rho(g + a_z)(z_2 - z_1) \qquad \Delta z_s = z_{s2} - z_{s1} = -\frac{a_x}{g + a_z}(x_2 - x_1)$$

$$-\frac{a_x}{g + a_z} = -\tan\theta \qquad z_s = \frac{\omega^2}{2g} r^2 + h_c \qquad z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2) \qquad \Delta z_{s,\max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g} R^2$$

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1) \qquad \sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \qquad \vec{F}_{\text{body}} = m_{\text{body}} \vec{a} = \sum_{\text{in}} \beta \dot{m} \vec{V} - \sum_{\text{out}} \beta \dot{m} \vec{V}$$