Vector Algebra

Usefull Info

$$
\begin{aligned}
\vec{a}_R\cdot\vec{a}_x &= \sin\theta\ \cos\phi & \vec{a}_R\cdot\vec{a}_y &= \sin\theta\ \sin\phi & \vec{a}_R\cdot\vec{a}_z &= \cos\theta \\
\vec{a}_\theta\cdot\vec{a}_x &= \cos\theta\ \cos\phi & \vec{a}_\theta\cdot\vec{a}_y &= \cos\theta\ \sin\phi & \vec{a}_\theta\cdot\vec{a}_z &= -\sin\theta \\
\vec{a}_\phi\cdot\vec{a}_x &= -\sin\phi & \vec{a}_\phi\cdot\vec{a}_y &= \cos\phi & \vec{a}_\phi\cdot\vec{a}_z &= 0
\end{aligned}
$$

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\begin{aligned}
\vec{a}_r\cdot\vec{a}_x &= \cos\phi & \vec{a}_r\cdot\vec{a}_y &= \sin\phi & \vec{a}_r\cdot\vec{a}_z &= 0 \\
\vec{a}_\phi\cdot\vec{a}_x &= -\sin\phi & \vec{a}_\phi\cdot\vec{a}_y &= \cos\phi & \vec{a}_\phi\cdot\vec{a}_z &= 0 \\
\vec{a}_z\cdot\vec{a}_x &= 0 & \vec{a}_z\cdot\vec{a}_y &= 0 & \vec{a}_z\cdot\vec{a}_z &= 1
\end{aligned}
$$

$$
\begin{aligned}
x &= R\ \sin\theta\ \cos\phi & R &= \sqrt{x^2+y^2+z^2} \\
y &= R\ \sin\theta\ \sin\phi & \theta &= \cos^{-1}\big(z/R\big) \\
z &= R\ \cos\theta & \phi &= \tan^{-1}\big(y/x\big)
\end{aligned}
$$

$$
\begin{aligned}
x &= r\ \cos\phi & r &= \sqrt{x^2+y^2} \\
y &= r\ \sin\phi & \phi &= \tan^{-1}\big(y/x\big) \\
z &= z & z &= z
\end{aligned}
$$

$$
\begin{aligned}
\vec{d\ell} &= \vec{a}_x\ dx+\vec{a}_y\ dy+\vec{a}_z\ dz & dV &= dx\ dy\ dz \\
\vec{d\ell} &= \vec{a}_r\ dr+\vec{a}_\phi\ r\ d\phi+\vec{a}_z\ dz & dV &= r\ dr\ d\phi\ dz \\
\vec{d\ell} &= \vec{a}_R\ dR+\vec{a}_\theta\ R\ d\theta+\vec{a}_\phi\ R\ \sin\theta\ d\phi & dV &= R^2\ \sin\theta\ dR\ d\theta\ d\phi
\end{aligned}
$$

$$
\begin{aligned}
\vec{dS} &= \vec{a}_x\ dy\ dz & &\vec{a}_y\ dx\ dz & &\vec{a}_z\ dx\ dy \\
\vec{dS} &= \vec{a}_r\ r\ d\phi\ dz & &\vec{a}_\phi\ dr\ dz & &\vec{a}_z\ r\ dr\ d\phi \\
\vec{dS} &= \vec{a}_R\ R^2\ \sin\theta\ d\theta\ d\phi & &\vec{a}_\theta\ R\ \sin\theta\ dR\ d\phi & &\vec{a}_\phi\ R\ dR\ d\theta
\end{aligned}
$$

$$
\int_V \left[\nabla\cdot\vec{A}(\vec{r})\right]\ dV = \oint_S \vec{A}(\vec{r})\cdot d\vec{S} \qquad \int_S \left[\nabla\times\vec{A}(\vec{r})\right]\cdot d\vec{S} = \oint_C \vec{A}(\vec{r})\cdot d\vec{\ell}
$$

$$
\vec{A} = \nabla\Phi+\nabla\times\vec{F} \qquad \nabla\times\nabla\Phi \equiv 0 \qquad \nabla\cdot\nabla\times\vec{A} \equiv 0
$$

$$
\nabla\big(\Phi\Psi\big) = \Phi\ \nabla\Psi+\Psi\ \nabla\Phi \qquad \nabla\times\nabla\times\vec{A} = \nabla\big(\nabla\cdot\vec{A}\big)-\nabla^2\vec{A}
$$

$$
\nabla\times\big(\Phi\vec{A}\big) = \Phi\ \big(\nabla\times\vec{A}\big)+\nabla\Phi\times\vec{A} \qquad \nabla\cdot\big(\Phi\vec{A}\big) = \Phi\ \big(\nabla\cdot\vec{A}\big)+\nabla\Phi\cdot\vec{A}
$$

$$
\nabla\left(\frac{1}{|\vec{R}-\vec{R}'|}\right) = -\nabla'\left(\frac{1}{|\vec{R}-\vec{R}'|}\right) = -\frac{\vec{R}-\vec{R}'}{|\vec{R}-\vec{R}'|^3}
$$

$$\nabla V = \vec{a}_x \frac{\partial V}{\partial x} + \vec{a}_y \frac{\partial V}{\partial y} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla V = \vec{a}_r \frac{\partial V}{\partial r} + \vec{a}_\phi \frac{1}{r} \frac{\partial V}{\partial \phi} + \vec{a}_z \frac{\partial V}{\partial z}$$

$$\nabla V = \vec{a}_R \frac{\partial V}{\partial R} + \vec{a}_\theta \frac{1}{R} \frac{\partial V}{\partial \theta} + \vec{a}_\phi \frac{1}{R\sin\theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r}\big(rA_r\big) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2} \frac{\partial}{\partial R}\big(R^2 A_R\big) + \frac{1}{R\sin\theta} \frac{\partial}{\partial \theta}\big(\sin\theta A_\theta\big) + \frac{1}{R\sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \vec{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \vec{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \vec{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$$\nabla \times \vec{A} = \vec{a}_r \left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \vec{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \vec{a}_z \frac{1}{r}\left(\frac{\partial}{\partial r}\big[rA_\phi\big] - \frac{\partial A_r}{\partial \phi}\right)$$

$$\nabla \times \vec{A} = \vec{a}_R \frac{1}{R\sin\theta} \left(\frac{\partial}{\partial \theta}\big[\sin\theta A_\phi\big] - \frac{\partial A_\theta}{\partial \phi}\right) + \vec{a}_\theta \frac{1}{R} \left(\frac{1}{\sin\theta}\frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R}\big[RA_\phi\big]\right)$$

$$+ \vec{a}_\phi \frac{1}{R}\left(\frac{\partial}{\partial R}\big[A_\theta R\big] - \frac{\partial A_R}{\partial \theta}\right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R}\left(R^2\frac{\partial V}{\partial R}\right) + \frac{1}{R^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{R^2\sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$

$$\int \sin^2 x \; dx = \frac{x}{2} - \frac{1}{4} \sin 2x \qquad \int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\frac{x}{a}$$

$$\int \cos^2 x \; dx = \frac{x}{2} + \frac{1}{4} \sin 2x \qquad \int \log_e x \; dx = x \log_e x - x$$

$$\int \sin x \; \cos x \; dx = \frac{1}{2} \sin^2 x \qquad \int x \, e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax-1)$$