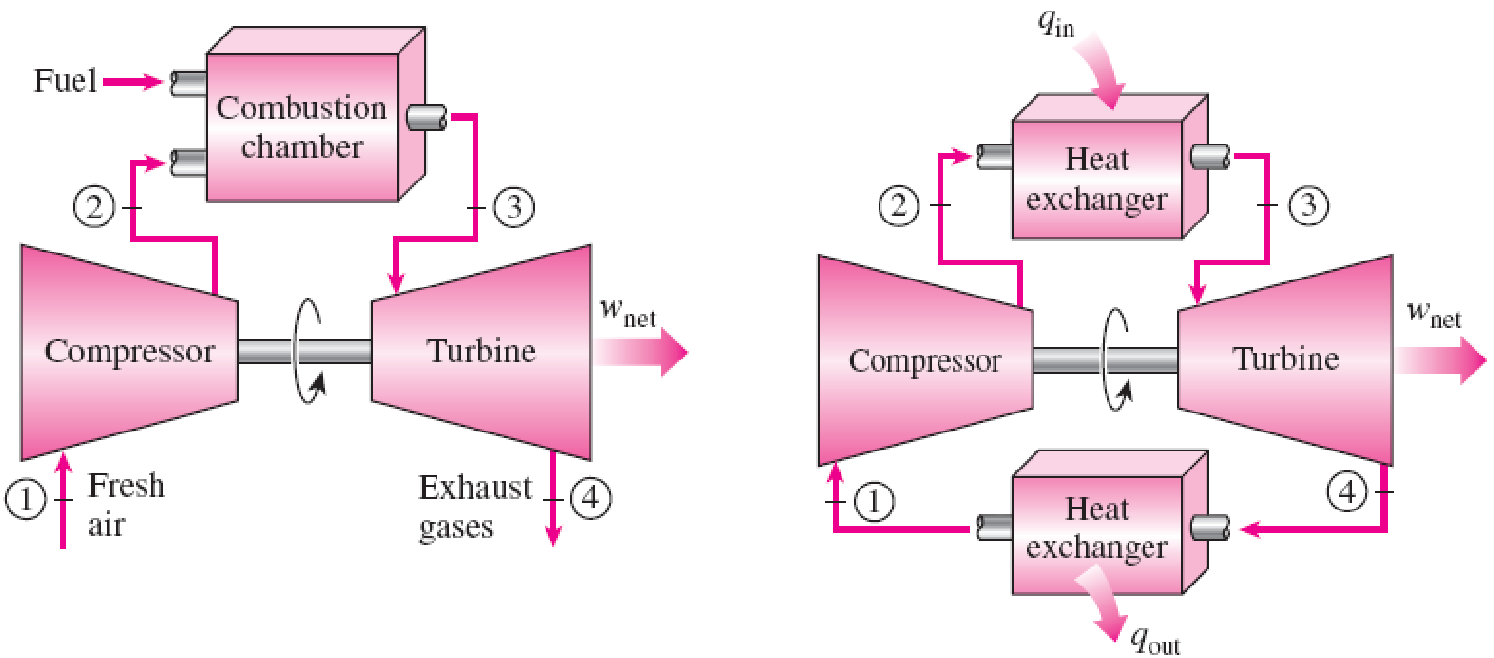
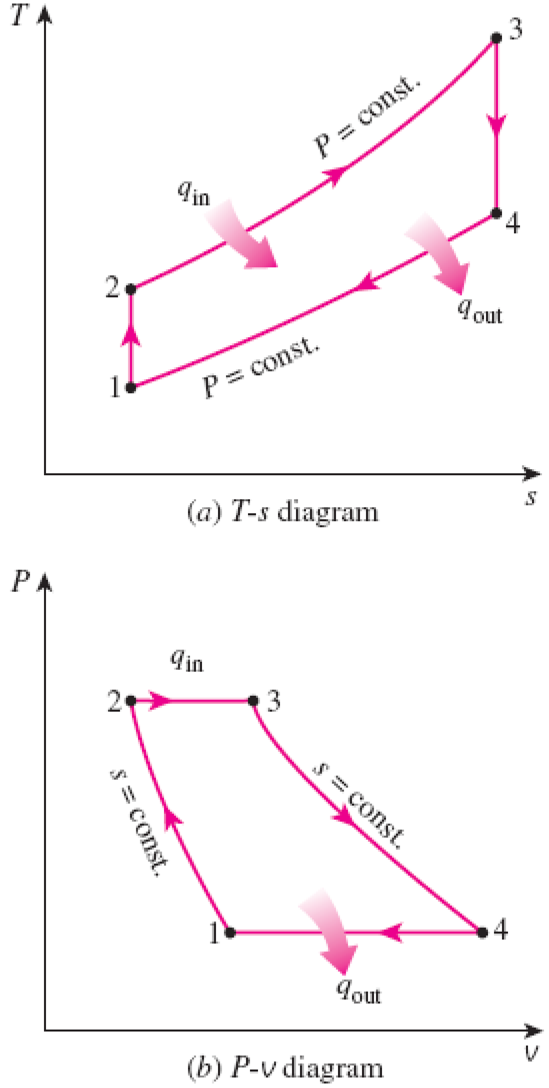
Brayton Cycle





1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection

Balance Equations

1-2 Isentropic Compression

$mBE: \quad \dot{m}_1 = \dot{m}_2$

$EBE: \dot{m}_1 h_1 + \dot{W}_c = \dot{m}_2 h_2$

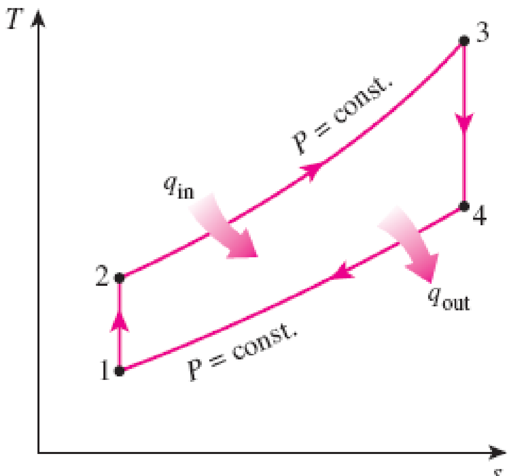
$h_1 + w_C = h_2$

$EnBE: \dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$

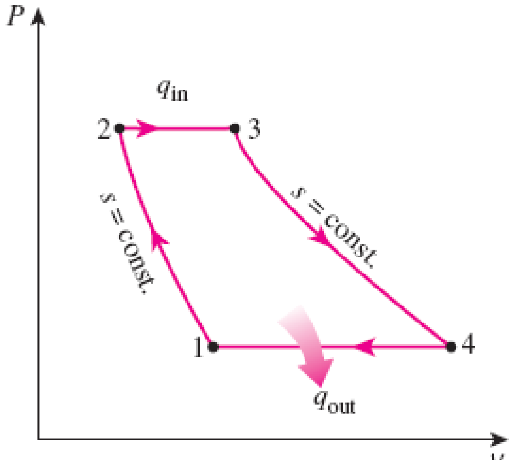
$s_1 + s_{gen} = s_2$

$ExBE: \dot{m}_1 ex_1 + \dot{W}_c = \dot{m}_2 ex_2 + \dot{E}x_D$

$ex_1 + w_C = ex_2 + ex_D$



(a) T-s diagram



(b) P-v diagram

2-3 Constant-pressure heat addition

$mBE: \quad \dot{m}_2 = \dot{m}_3$

$EBE: \dot{m}_2 h_2 + \dot{Q}_{in} = \dot{m}_3 h_3$

$h_2 + q_{in} = h_3$

$EnBE: \dot{m}_2 s_2 + \frac{Q_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_3 s_3$

$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$

$ExBE: \dot{m}_2 ex_2 + \dot{E}x^{Q_{in}} = \dot{m}_3 ex_3 + \dot{E}x_D$

$ex_2 + ex^{Q_{in}} = ex_3 + ex_D$

Balance Equations

3-4 Isentropic Expansion

$mBE: \dot{m}_3 = \dot{m}_4$

$EBE: \dot{m}_3 h_3 = \dot{m}_4 h_4 + \dot{W}_T$

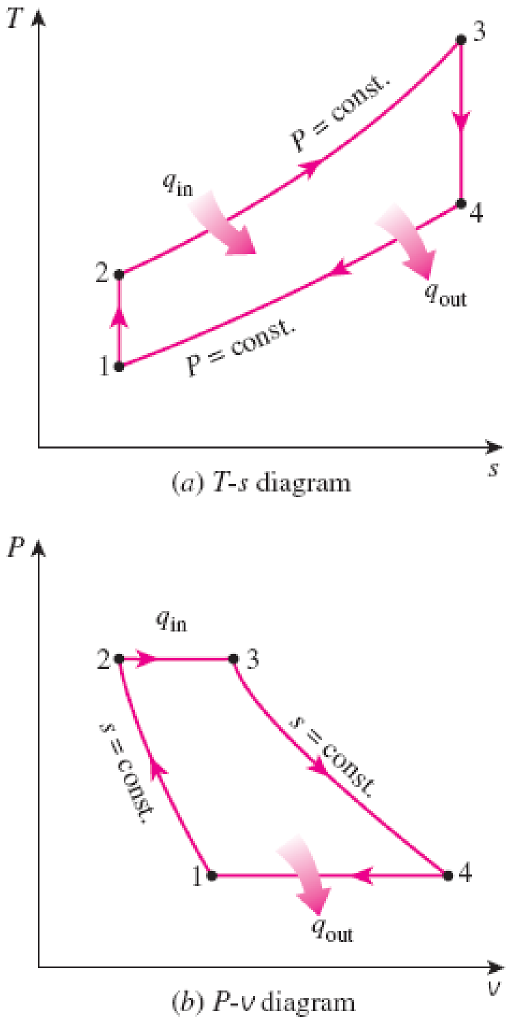
$h_3 = h_4 + w_T$

$EnBE: \dot{m}_3 s_3 + \dot{S}_{gen} = \dot{m}_4 s_4$

$s_3 + s_{gen} = s_4$

$ExBE: \dot{m}_3 ex_3 = \dot{m}_4 ex_4 + \dot{W}_T + \dot{Ex}_D$

$ex_3 = ex_4 + w_T + ex_D$



4-1 Constant-pressure heat rejection

$mBE: \dot{m}_4 = \dot{m}_1$

$EBE: \dot{m}_4 h_4 = \dot{m}_1 h_1 + \dot{Q}_{out}$

$h_4 = h_1 + q_{out}$

$EnBE: \dot{m}_4 s_4 + \dot{S}_{gen} = \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_b}$

$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$

$ExBE: \dot{m}_4 ex_4 = \dot{m}_1 ex_1 + \dot{Ex}^{Q_{out}} + \dot{Ex}_D$

$ex_4 = ex_1 + ex^{Q_{out}} + ex_D$

Problem-1

A simple Brayton cycle using air as the working fluid has a pressure ratio of 8. The minimum and maximum temperatures in the cycle are 310 and 1160 K. Assuming an isentropic efficiency of 75 percent for the compressor and 82 percent for the turbine, determine

(a) the air temperature at the turbine exit,

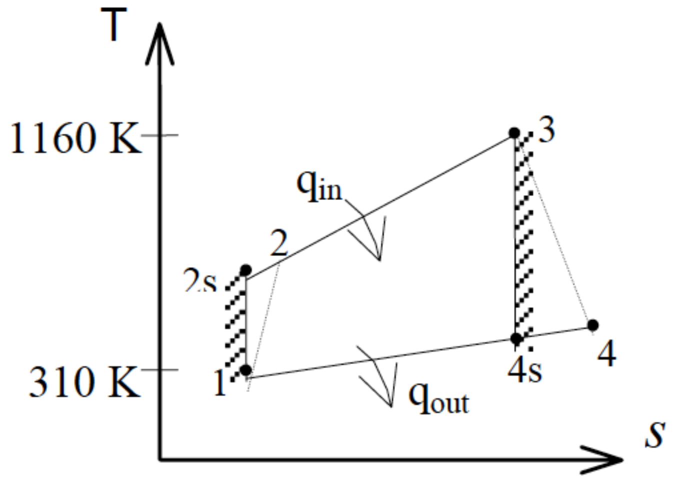
(b) the net work output,

(c) entropy generation

(d) exergy destruction

(e) the thermal efficiency, and

(f) exergy efficiency.



Assumptions Problem-1

1 Steady operating conditions exist,

2 The air-standard assumptions are applicable,

3 Kinetic and potential energy changes are negligible,

4 Air is an ideal gas with variable specific heats.

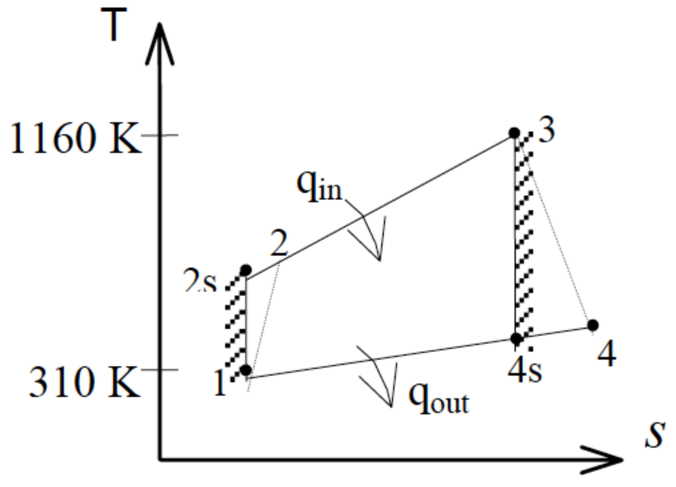
$$T_1 = 310 \text{ K} \longrightarrow \begin{matrix} h_1 = 310.24 \text{ kJ/kg} \\ P_{r_1} = 1.5546 \end{matrix}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (8)(1.5546) = 12.44 \longrightarrow h_{2s} = 562.58 \text{ kJ/kg} \text{ and } T_{2s} = 557.25 \text{ K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow h_2 = h_1 + \frac{h_{2s} - h_1}{\eta_C}$$

$$= 310.24 + \frac{562.58 - 310.24}{0.75} = 646.7 \text{ kJ/kg}$$

Assuming an isentropic efficiency of 75 percent for the compressor



(a) the air temperature at the turbine exit (T_4),

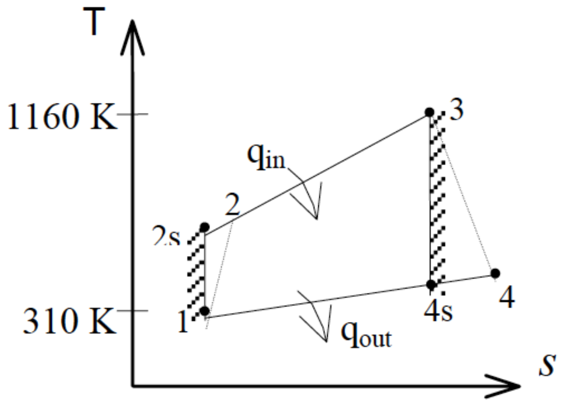
Assuming an isentropic efficiency of 82 percent for the turbine

$$T_3 = 1160\text{ K} \longrightarrow \begin{aligned} h_3 &= 1230.92\text{ kJ/kg} \\ P_{r_3} &= 207.2 \end{aligned}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{8}\right)(207.2) = 25.90 \longrightarrow h_{4s} = 692.19\text{ kJ/kg and } T_{4s} = 680.3\text{ K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow \begin{aligned} h_4 &= h_3 - \eta_T (h_3 - h_{4s}) \\ &= 1230.92 - (0.82)(1230.92 - 692.19) \\ &= 789.16\text{ kJ/kg} \end{aligned}$$

Thus, $T_4 = \mathbf{770.1\ K}$



b) the net work output (W_{net})=?

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_C$$
$$w_{net} = w_T - w_C$$

w_C=?

1-2 Isentropic Compression

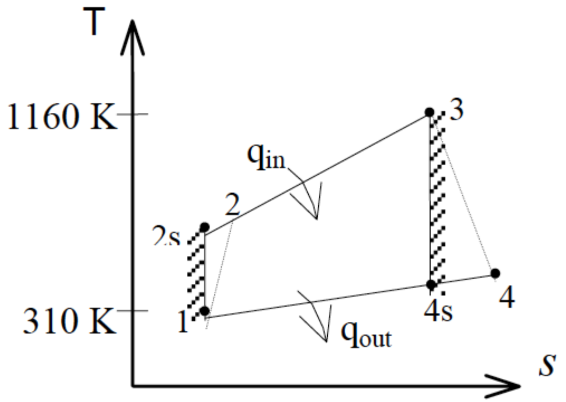
$$EBE: \dot{m}_1 h_1 + \dot{W}_c = \dot{m}_2 h_2$$
$$h_1 + w_C = h_2$$

h_1=310.24kJ/kg

h_2 =646.7kJ/kg

$$h_1 + w_C = h_2$$

w_C =336.36 kJ/kg



b) the net work output (W_{net})=?

$$\dot{W}_{net} = \dot{W}_T - \dot{W}_C$$
$$w_{net} = w_T - w_C$$

w_T=?

3-4 Isentropic Expansion

$$EBE: \dot{m}_3 h_3 = \dot{m}_4 h_4 + \dot{W}_T$$
$$h_3 = h_4 + w_T$$

h_3=1230.92kJ/kg
h_4 =789.16kJ/kg

$$h_3 = h_4 + w_T$$
w_T =441.76 kJ/kg

w_C =336.36 kJ/kg

$$w_{net} = w_T - w_C = 105.4 \text{kJ/kg}$$

Compressor

$$EnBE: \dot{m}_1 s_1 + \dot{S}_{gen} = \dot{m}_2 s_2$$
$$s_1 + s_{gen} = s_2$$

$s_1{}^o = 1.73498\ kJ/kgK$
$s_2{}^o = 2.47319\ kJ/kgK$

$$s_2 - s_1 = s_2{}^o - s_1{}^o - Rln\frac{P_2}{P_1}$$

$$s_{gen,compressor} = (2.47319 - 1.73498)kj/kgK - (0.287kJ/kgK)ln8$$

$$s_2 - s_1 = s_{gen} = 0.141\ kj/kgK$$

Heat Exchanger or Combustion Chamber

$$EnBE: \dot{m}_2 s_2 + \frac{\dot{Q}_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_3 s_3$$

$$s_2 + \frac{q_{in}}{T_s} + s_{gen} = s_3$$

${s_2}^o$ =2.47319 kJ/kgK
${s_3}^o$=3.13916 kJ/kgK

$$h_2 + q_{in} = h_3 \Rightarrow q_{in} = 584.22 \text{ kj/kg}$$

$$s_3 - s_2 = {s_3}^o - {s_2}^o - Rln\frac{P_3}{P_2} \qquad P_3 = P_2$$

$$assuming\ T_s = 1700\ K$$

$$s_{gen} = 0.322\ kJ/kgK$$

Turbine

$$EnBE\colon \dot{m}_3 s_3 + \dot{S}_{gen} = \dot{m}_3 s_4$$
$$s_3 + s_{gen} = s_4$$

$s_3{}^o$=*3.13916* kJ/kgK
$s_4{}^o$=*2.67601* kJ/kgK

$$s_4 - s_3 = s_4{}^o - s_3{}^o - Rln\frac{P_4}{P_3}$$
$$s_4 - s_3 = s_4{}^o - s_3{}^o - 0.287\ \left(\frac{kj}{kgK}\right) ln\frac{1}{8} = 0.1339 = s_{gen}$$

Heat Exchanger (in Closed Cycle)

$$EnBE: \dot{m}_4 s_4 + \dot{S}_{gen} = \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_b}$$

$$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$$

$$s_4{}^o = 2.67601 \text{ kJ/kgK}$$

$$s_1{}^o = 1.73498\ kJ/kgK$$

$$s_1 - s_4 = s_1{}^o - s_4{}^o - R ln \frac{P_1}{P_4} \qquad P_1 = P_4$$

$$s_1 - s_4 = s_1{}^o - s_4{}^o$$

$$s_{gen} = s_1 - s_4 + \frac{q_{out}}{T_{su}}$$

Heat Exchanger (in Closed Cycle)

$$EnBE: \dot{m}_4 s_4 + \dot{S}_{gen} = \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_b}$$

$$s_4 + s_{gen} = s_1 + \frac{q_{out}}{T_b}$$

$$s_4{}^o = 2.67601 \text{ kJ/kgK}$$

$$s_1{}^o = 1.73498\ kJ/kgK$$

$$h_4 = h_1 + q_{out} \rightarrow q_{out} = 479.36 \text{kJ/kg}$$

$$T_1 = 310K$$

$$T_0 = 298K$$

$$assuming\ T_b = 306K$$

$$s_{gen} = s_1 - s_4 + \frac{q_{out}}{T_b} = 0.636 kJ/kgK$$

Exergy Destruction (Ex_D, ex_D)

$$\dot{E}x_D = T_0 \dot{S}_{gen}$$
$$ex_D = T_0 s_{gen}$$

$$ex_{D,compressor} = 298K \times 0.141kJ/kgK = 42.018kJ/kg$$
$$ex_{D,combustion\ chamber} = 298K \times 0.322kJ/kgK = 95.956kJ/kg$$
$$ex_{D,turbine} = 298K \times 0.1339kJ/kgK = 39.902kJ/kg$$
$$ex_{D,heat\ exchanger} = 298K \times 0.636kJ/kgK = 189.53kJ/kg$$

Compressor

$$ExBE: \dot{m}_1 ex_1 + \dot{W}_C = \dot{m}_2 ex_2 + \dot{Ex}_D$$
$$ex_1 + w_C = ex_2 + ex_D$$

Heat Exchanger or Combustion Chamber

$$ExBE: \dot{m}_2 ex_2 + \dot{Ex}^{Q_{in}} = \dot{m}_3 ex_3 + \dot{Ex}_D$$
$$ex_2 + ex^{Q_{in}} = ex_3 + ex_D$$

$$ex_i = (h_i - h_0) - T_0(s_i - s_0)$$
$$ex_3 - ex_2 = (h_3 - h_2) - T_0(s_3 - s_2)$$
$$ex_3 - ex_2 = 584.22 - 298 \times (0.1623) = 535.85 kJ/kg$$
$$ex^{Q_{in}} = 535.85 kJkg + 95.956 kJ/kg = 631.806 kJ/kg$$

Turbine

$$ExBE: \dot{m}_3 ex_3 = \dot{m}_4 ex_4 + \dot{W}_T + \dot{E}x_D$$
$$ex_3 = ex_4 + w_e + ex_D$$

Heat Exchanger (in Closed Cycle)

$$ExBE: \dot{m}_4 ex_4 = \dot{m}_1 ex_1 + \dot{E}x^{Q_{out}} + \dot{E}x_D$$
$$ex_4 = ex_1 + ex^{Q_{out}} + ex_D$$

$$ex_i = (h_i - h_0) - T_0(s_i - s_0)$$
$$ex_4 - ex_1 = (h_4 - h_1) - T_0(s_4 - s_1)$$
$$ex_4 - ex_1 = 479.36 - 298 \times (0.94103) = 198.85 kJ/kg$$
$$ex^{Q_{out}} = 198.85 kJkg + 189.53 kJ/kg = 388.38 kJ/kg$$

d) the thermal efficiency, and
e) exergy efficiency.

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{105.4}{584.22} = 18\%$$

$$\eta_{ex} = \frac{w_{net}}{ex^{Q_{in}}} = \frac{105.4}{631.806} = 16.68\%$$

Problem-2

Repeat first problem using constant specific heats at room temperature.

Assumptions Problem-1

1 Steady operating conditions exist,

2 The air-standard assumptions are applicable,

3 Kinetic and potential energy changes are negligible,

4 Air is an ideal gas with variable specific heats.

Assumptions; Problem-2

1 Steady operating conditions exist,

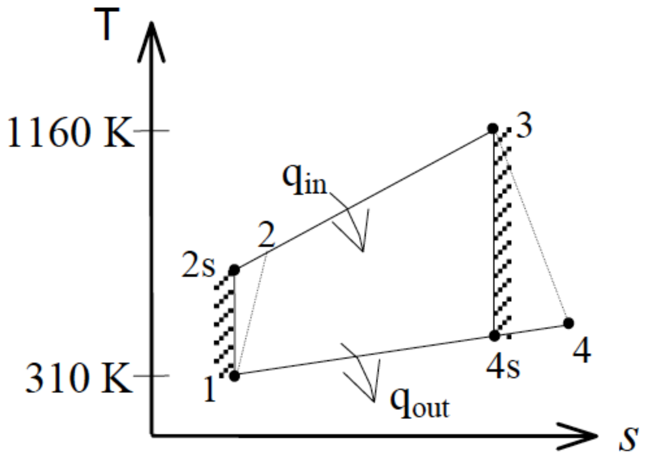
2 The air-standard assumptions are applicable,

3 Kinetic and potential energy changes are negligible,

4 Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are c_p = 1.005 kJ/kg·K and k = 1.4 (Table A-2).

Analysis (a) Using the compressor and turbine efficiency relations,



$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (310\text{ K})(8)^{0.4/1.4} = 561.5\text{ K}$$

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1160\text{ K})\left(\frac{1}{8}\right)^{0.4/1.4} = 640.4\text{ K}$$

$$\eta_C = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \longrightarrow T_2 = T_1 + \frac{T_{2s}-T_1}{\eta_C}$$

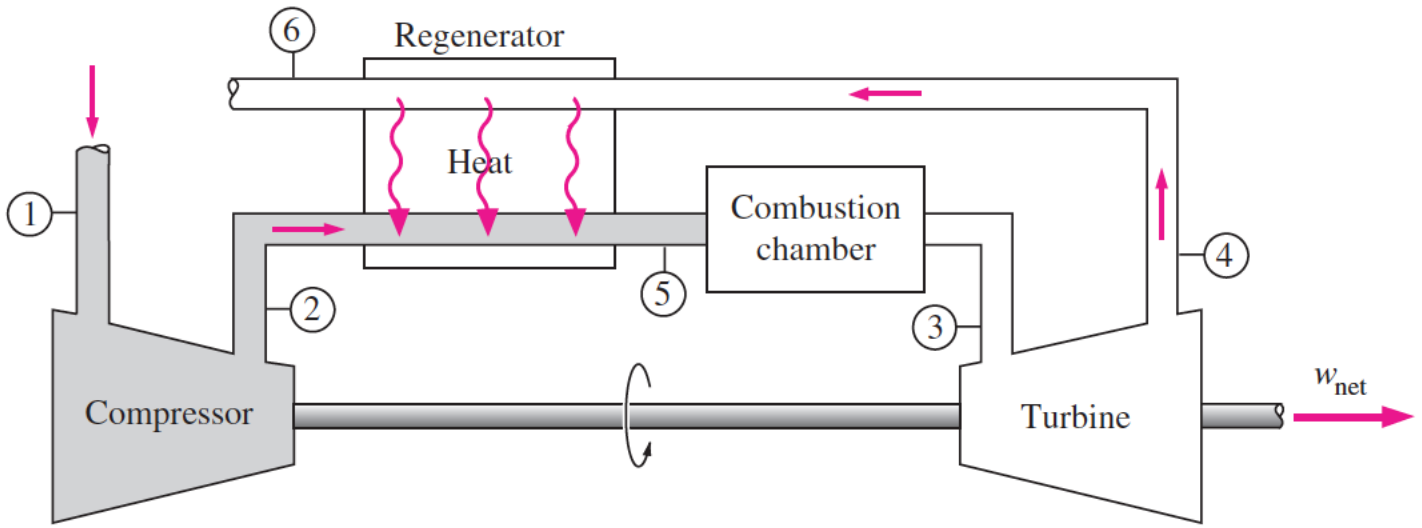
$$= 310 + \frac{561.5-310}{0.75} = 645.3\text{ K}$$

$$\eta_T = \frac{h_3-h_4}{h_3-h_{4s}} = \frac{c_p(T_3-T_4)}{c_p(T_3-T_{4s})} \longrightarrow T_4 = T_3 - \eta_T(T_3-T_{4s})$$

$$= 1160 - (0.82)(1160-640.4)$$

$$= \mathbf{733.9\ K}$$

Brayton Cycle with Regeneration



Problem-4

An ideal Brayton cycle with regeneration has a pressure ratio of 10. Air enters the compressor at 300 K and the turbine at 1200 K. If the effectiveness of the regenerator is 100 percent, determine

a) the net work output,

b) the thermal efficiency of the cycle,

c) exergy efficiency of the cycle. Account for the variation of specific

heats with temperature.

Problem-3

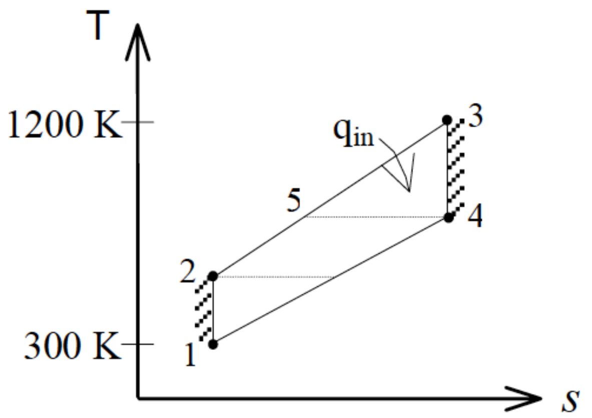
- ***Assumptions***
  1 The air standard assumptions are applicable.
  2 Air is an ideal gas with variable specific heats.
  3 Kinetic and potential energy changes are negligible.

$$T_1 = 300 \text{ K} \longrightarrow \begin{array}{l} h_1 = 300.19 \text{ kJ/kg} \\ P_{r_1} = 1.386 \end{array}$$

$$P_{r_2} = \frac{P_2}{P_1} P_{r_1} = (10)(1.386) = 13.86 \longrightarrow h_2 = 579.87 \text{ kJ/kg}$$

$$T_3 = 1200 \text{ K} \longrightarrow \begin{array}{l} h_3 = 1277.79 \text{ kJ/kg} \\ P_{r_3} = 238 \end{array}$$

$$P_{r_4} = \frac{P_4}{P_3} P_{r_3} = \left(\frac{1}{10}\right)(238) = 23.8 \longrightarrow h_4 = 675.85 \text{ kJ/kg}$$



1-2 Isentropic Compression

$mBE\colon\ m_1 = m_2$

$EBE\colon\ m_1 h_1 + W_C = m_2 h_2$

$$h_1 + w_C = h_2$$

$$w_C = h_2 - h_1$$

3-4 Isentropic Expansion

$mBE\colon\ m_3 = m_4$

$EBE\colon\ m_3 h_3 = m_4 h_4 + W_T$

$$h_3 = h_4 + w_T$$

$$h_3 - h_4 = w_T$$

$$w_{C,\text{in}} = h_2 - h_1 = 579.87 - 300.19 = 279.68 \text{ kJ/kg}$$

$$w_{T,\text{out}} = h_3 - h_4 = 1277.79 - 675.85 = 601.94 \text{ kJ/kg}$$

Thus,

$$w_{\text{net}} = w_{T,\text{out}} - w_{C,\text{in}} = 601.94 - 279.68 = \mathbf{322.26\ kJ/kg}$$

Combustion Chamber

$$mBE: \dot{m}_5 = \dot{m}_3$$

$$EBE: \dot{m}_5 h_5 + \dot{Q}_{in} = \dot{m}_3 h_3$$

$$h_5 + q_{in} = h_3$$

$$q_{in} = h_3 - h_5$$

$$EnBE: \dot{m}_5 s_5 + \frac{\dot{Q}_{in}}{T_s} + \dot{S}_{gen} = \dot{m}_3 s_3$$

$$s_5 + \frac{q_{in}}{T_s} + s_{gen} = s_3$$

$$ExBE: \dot{m}_5 ex_5 + \dot{Ex}^{Q_{in}} = \dot{m}_3 ex_3 + \dot{Ex}_D$$

$$ex_5 + ex^{Q_{in}} = ex_3 + ex_D$$

Also, $\varepsilon = 100\% \longrightarrow h_5 = h_4 = 675.85 \text{ kJ/kg}$

$$q_{in} = h_3 - h_5 = 1277.79 - 675.85 = 601.94 \text{ kJ/kg}$$

and

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{322.26 \text{ kJ/kg}}{601.94 \text{ kJ/kg}} = \mathbf{53.5\%}$$

$$\eta_{ex} = \frac{w_{net}}{ex^{Q_{in}}} = ?$$

Problem-4

Repeat Problem-3 using constant specific heats at room temperature.

Assumptions

1 The air standard assumptions are applicable.

2 Air is an ideal gas with constant specific heats at room temperature.

3 Kinetic and potential energy changes are negligible.

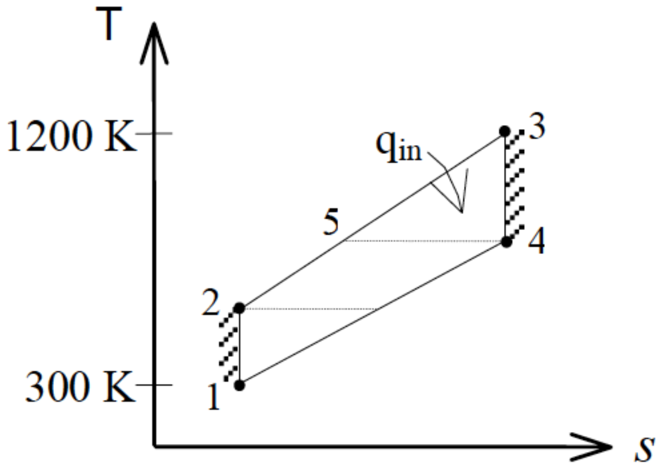
Properties

The properties of air at room temperature are c_p = 1.005 kJ/kg.K and k = 1.4 (Table A-2a).

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\ \text{K})(10)^{0.4/1.4} = 579.2\ \text{K}$$

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (1200\ \text{K})\left(\frac{1}{10}\right)^{0.4/1.4} = 621.5\ \text{K}$$

$$\varepsilon = 100\% \longrightarrow T_5 = T_4 = 621.5\ \text{K and } T_6 = T_2 = 579.2\ \text{K}$$



$$\varepsilon = 100\% \longrightarrow T_5 = T_4 = 621.5\ \text{K and } T_6 = T_2 = 579.2\ \text{K}$$

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{c_p(T_6 - T_1)}{c_p(T_3 - T_5)} = 1 - \frac{T_6 - T_1}{T_3 - T_5} = 1 - \frac{579.2 - 300}{1200 - 621.5} = \mathbf{0.517}$$

or $$\eta_{\text{th}} = 1 - \left(\frac{T_1}{T_3}\right) r_p^{(k-1)/k} = 1 - \left(\frac{300}{1200}\right)(10)^{(1.4-1)/1.4} = 0.517$$

1-2 Isentropic Compression

$$mBE: \dot{m}_1 = \dot{m}_2$$

$$EBE: \dot{m}_1 h_1 + \dot{W}_C = \dot{m}_2 h_2$$

$$h_1 + w_C = h_2$$

$$w_C = h_2 - h_1 = c_p(T_2 - T_1)$$

3-4 Isentropic Expansion

$$mBE: \dot{m}_3 = \dot{m}_4$$

$$EBE: \dot{m}_3 h_3 = \dot{m}_4 h_4 + \dot{W}_T$$

$$h_3 = h_4 + w_T$$

$$w_T = h_3 - h_4 = c_p(T_3 - T_4)$$

Then,

$$\begin{aligned} w_{\text{net}} &= w_{\text{turb, out}} - w_{\text{comp, in}} = (h_3 - h_4) - (h_2 - h_1) \\ &= c_p[(T_3 - T_4) - (T_2 - T_1)] \\ &= (1.005 \text{ kJ/kg.K})[(1200 - 621.5) - (579.2 - 300)]\text{K} \\ &= \mathbf{300.8 \text{ kJ/kg}} \end{aligned}$$