Table 2—Section Modulus Calculation (English Units)

Moment of Inertia of Midship Section Cargo Vessel 563 ft 7¾ in. × 528 ft 6 in. × 76 ft 0 in. × 44 ft 6 in.
Assumed Neutral Axis = 20 ft 0 in. WL Design Draft = 31 ft 6 in.

Member	Scantling	a	d_n	ad_n		ad_n^2	h	i_o
Granwale angle	8 × 8 × 1	15.00	24.79	371.85		9,218.08		
Main deck plating	276 × 1.125	310.50	24.92	7,737.66		192,820.49		
Main deck strap	13.5 × 1.125	15.19	25.25	383.55		9,684.64		
Second deck plating	276 × 0.5625	155.25	15.54	2,412.59		37,491.65		
Sheer strake	60 × 0.8125	48.75	23.00	1,121.25		25,788.75	5	101
Side shell	246.5 × 0.7187	175.36	10.28	1,802.27		18,527.34	20.54	6,169
						293,530.95		6,270.00
					Σi_o	6,270.00		
Total above 20 ft 0 in. WL		720.05		13,829.17		299,800.95		
Side shell	123 × 0.7187	88.40	5.24	463.22		2,427.27	10.25	775
Bilge strake	195 × 0.8125	158.44	17.25	2,733.09		47,145.80	9.65	1228
Bottom shell	315.5 × 0.8125	256.34	20.04	5,137.05		102,946.48		
Flat keel	26.5 × 1.00	26.50	20.04	531.06		10,642.44		
I.B. margin	53 × 0.5937	31.43	15.03	472.34		7,100.02		
I.B. center strake	26.5 × 0.5937	15.71	15.03	236.12		3,548.88		
I.B. plating	365.5 × 0.50	182.75	15.06	2,752.22		41,448.43		
C.V. keel	59 × 0.5937/2	17.49	17.50	306.08		5,356.40	4.91	33
Inboard longitudinal girder	59 × 0.5312	31.34	17.50	548.45		9,597.88	4.91	67
Outboard longitudinal girder	59 × 0.4062	23.97	17.50	419.48		7,340.90	4.91	48
						237,554.55		2,051
					Σi_o	2,051.00		
Total below 20 ft 0 in. WL		830.37		13,598.12		239,605.55		

$A = \Sigma a = 1{,}550.42$ $\quad \Sigma ad_n = 231.05 \quad I_n = 539{,}406.50$

$$d_g = \frac{231.05}{1550.42} = 0.149$$

$$I/2 = 539{,}406 - 34.43 = 539{,}372$$

$$\text{Top } C = 24.59 - 0.149 = 24.44$$

$$\text{Top } \frac{I}{C} = \frac{1{,}078{,}744}{24.59} = 43{,}869$$

$$A \times dg^2 = 1550.42 \times 0.149^2 = 34.43$$

$$I = 539{,}372 \times 2 = 1{,}078{,}744$$

$$\text{Bottom } C = 20.08 + 0.149 = 20.23$$

$$\text{Bottom } \frac{I}{C} = \frac{1{,}078{,}744}{20.23} = 53{,}323$$

3.3 Calculation of Section Modulus. An important step in routine ship design is the calculation of midship section modulus. As defined in connection with Equation (27), it indicates the bending strength properties of the primary hull structure. The standard calculation is described in ABS (1987a), Section 6: "The section modulus to the deck or bottom is obtained by dividing the moment of inertia by the distance from the neutral axis to the molded deck line at side or to the base line, respectively." See Fig. 23.

"In general, the following items may be included in the calculation of the section modulus, provided they are continuous or effectively developed.

- Deck plating (strength deck and other effective decks).
- Shell and inner-bottom plating.
- Deck and bottom girders.
- Plating and longitudinal stiffeners of longitudinal bulkheads.
- All longitudinals of deck, sides, bottom and inner bottom.
- Continuous longitudinal hatch coamings."

The designation of which members should be considered as effective is subject to differences of opinion. The members of the hull girder of a ship in a seaway are stressed alternately in tension and compression, and certain of them will take compression even though deficiency in end connection makes them unable to take full tension, while other members, perhaps of light plating ineffectively stiffened, may be able to withstand tension stresses to the elastic limit, but may buckle under a moderate compressive stress. In general, however, only members which are effective in both tension and compression are assumed to act as part of the hull girder.

The section-modulus calculation for the cargo ship shown in Fig. 23 is carried out in Table 2. This calculation is based on the following formula for the moment of inertia of any composite girder section:

$$I = 2[I_n - A\,d_g^2] = 2[\Sigma(i_o + ad_n^2) - A\,d_g^2] \quad (33)$$

where

I is moment of inertia of the section about a line par-

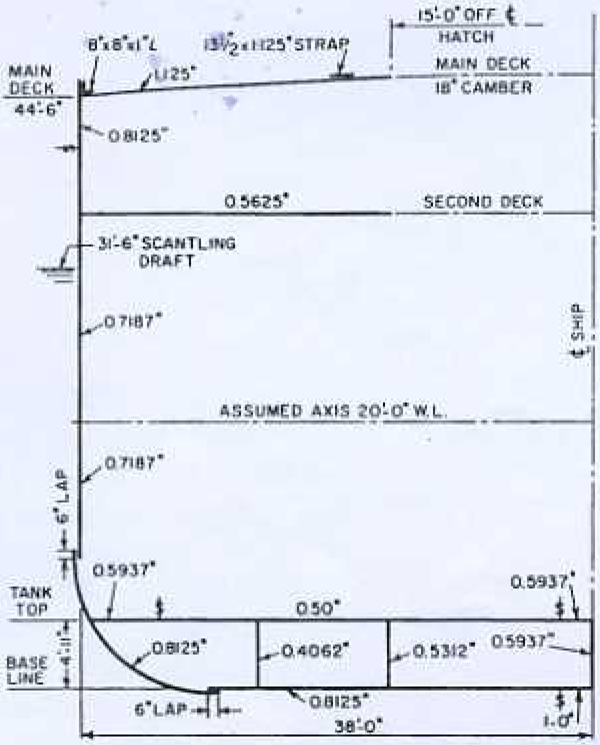

Fig. 23 Cargo ship midship section

allel to the base through the true neutral axis (center of gravity), expressed in cm²-m² (in²-ft²).

I_a is moment of inertia of the half-section about an assumed axis parallel to the true neutral axis, $= \Sigma(i_0 + ad_n^2)$.

A is total half-section area of effective longitudinal strength members, $= \Sigma a$, in cm² (in²). Generally no deduction is made for rivet holes.

d_g is distance from the assumed axis to the true axis, m (ft).

i_0 is vertical moment of inertia (about its own center of gravity) of each individual plate or shape effective for longitudinal strength.

a is area of each such plate or shape, in cm² (in)².

d_n is distance of the center of gravity of each such plate or shape from the assumed axis, in m (ft).

Owing to symmetry it is necessary to include in the calculation only the structural parts on one side of the centerline, the result of the calculation being multiplied by 2 as indicated in Equation (33). Accordingly, quantities listed in Table 2 are for one side of the ship shown in Fig. 23.

If the assumed axis be assigned an arbitrary location, the known or directly determinable values are i_0, a, and d_n; hence I_n may be obtained. The value of A is also known and $d_g = \Sigma ad_n / \Sigma a = \Sigma ad_n / A$; therefore $A\ d_g^2$ is determinable. The baseline may be used for the assumed axis. There is, however, some advantage in using an assumed axis at about mid-depth in that lever arms are decreased. In that case the assumed axis should be located at about 45 percent of the depth of the section above the baseline, the actual position of the neutral axis being normally at less than half-depth because the bottom shell plating has greater sectional area than has deck plating (except in such cases as tankers). This condition is accentuated when an inner bottom is fitted.

After I had been calculated as outlined and as indicated in Table 2, the section modulus I/c may be obtained to both top and bottom extreme fibers.

For the sake of convenience and uniformity, the following conventions are usually observed:

- Since the moments of inertia i_0 of individual horizontal members are negligible, they are omitted from the calculations.
- The top c is taken from the neutral axis to the deck at side, the bottom c to the baseline.

3.4 Distribution of Shear and Transverse Stress Components. The simple beam theory expressions given in the preceding section permit us to evaluate the longitudinal component of the primary stress, σ_x. In Fig. 24 we see that an element of shell or deck plating may, in general, be subject to two other components of stress, a direct stress in the transverse direction and a shearing stress. Fig. 24 illustrates these as the *stress resultants,* defined as the stress multiplied by plate thickness. The stress resultants have dimensions of force per unit length and are given by the following expressions:

$N_x = t\sigma_x,\ N_s = t\sigma_s$	stress resultants
$N = t\tau$	shear stress resultant or *shear flow*
$\sigma_x,\ \sigma_s$	stresses in the longitudinal and girth-wise directions
τ	shear stress
t	plate thickness

Here σ_s designates the transverse direct stress parallel to the vertical axis in the ship's side and parallel to the transverse axis in the deck and the bottom.

Through considerations of static equilibrium of a triangular element of plating, it may be shown that the plane stress pattern described by the three component stresses σ_x, σ_s, τ may be reduced to a pair of alternative direct stresses, σ_1, σ_2. The stresses σ_1, σ_2 are called *principal stresses* and the directions of σ_1 and σ_2 are principal stress directions. The principal stresses are related to σ_x, σ_s and τ by

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_s}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_s}{2}\right)^2 + \tau^2} \qquad (34)$$

The two angles, θ, between the x-axis and the directions of σ_1, and σ_2 are

$$\tan 2\theta = -\frac{2\tau}{\sigma_x - \sigma_s} \qquad (35)$$

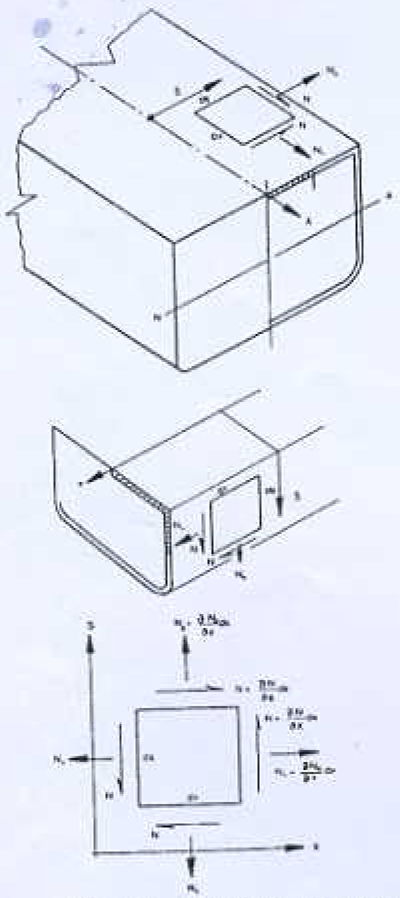

Fig. 24 Element of plate structure in deck or side shell, illustrating components of bending stress resultants

Detailed derivation of these expressions may be found, for example, in Timoshenko (1955).

In many parts of the ship, the longitudinal stress, σ_x, is the dominant component. There are, however, locations in which the shear component becomes important and under unusual circumstances the transverse component may, likewise, become important. A suitable procedure for estimating these other component stresses may be derived by considering the equations of static equilibrium of the element of plating illustrated in Fig. 24. In case the stiffeners associated with the plating support a part of the loading, this effect may also be included.

The static equilibrium conditions for the element of plate subject only to in-plane stress (i.e., no bending of the plate) are

$$\frac{\partial N_x}{\partial x} + \frac{\partial N}{\partial s} = 0$$
$$\frac{\partial N_s}{\partial s} + \frac{\partial N}{\partial x} = 0 \qquad (36)$$

In these expressions, s, is the girthwise coordinate measured on the surface of the section from the x-axis as shown in Fig. 24.

The first of Equations (36) may be integrated in the s-direction around the ship section in order to obtain the shear stress distribution. For this purpose, we assume, as a first approximation, that the longitudinal stress, σ_x, is given by the beam theory expression (27). Then, if we assume that the hull girder is prismatic (or that I changes slowly in the x-direction) so that only $M(x)$ varies with x, the derivative of N_x is given by differentiating Equation (26),

$$\frac{\partial N_x}{\partial x} = -\frac{tz}{I}\frac{dM(x)}{dx} = -\frac{tz}{I}V(x) \qquad (37)$$

where $V(x)$ = shear force in the hull at x.

The shear flow distribution around a section is given by integrating the first of Equations (36) in the s-direction,

$$N(s) - N_0 = \int_0^s \frac{\partial N}{\partial s}\, ds = -\int_0^s \frac{\partial N_x}{\partial x}\, ds = \frac{V(x)}{I}\int_0^s tz\,ds \qquad (38)$$

Here, N_0, the constant of integration, is equal to the value of the shear flow at the origin of integration, $s = 0$. By proper choice of the origin, N_0 can often be set equal to zero.

For example, in a section having transverse symmetry and subject to a bending moment in the vertical plane, the shear stress must be zero on the centerline, which therefore, is a suitable choice for the origin of the girthwise integration. The shear flow distribution around a single-walled symmetrical section is then given by

$$N(s) = \frac{V(x)}{I}m(s) \qquad (39)$$

with $N_0 = 0$ in the case of such symmetry.

The quantity $m(s) = \int_0^s tz\,ds$ is the first moment about the neutral axis of the cross sectional area of the plating between the origin at the centerline and the variable location designated by s. This is the shaded area of the section shown in Fig. 24.

If a longitudinal frame or girder that carries longitudinal stress is attached to the plate, as shown in Fig. 25, there will be a discontinuity in the shear flow, $N(s)$, at the frame corresponding to a jump in $m(s)$. This may be seen by considering the equilibrium of forces in the x-direction of the system of plate shears