Suppose that you provide consultancy service to a certain firm. When you visit the production plant, you ask the manager whether the technology that they use exhibits increasing, constant , or decreasing returns to scale. If the technology is given by:

$$Y=L^{a}$$

find the range of the parameter a according to the answers of the manager below:

1. “We have DRS because our revenues are less than doubled if we would double our output”.

Answer: Doubling output would always cause less than doubled revenues under negatively sloped demand curve. This has nothing to do with returns to scale. That is because; revenue is defined as:

$$R\left(Y\right)=P\left(Y\right)×Y.$$

The manager simply says:

$$2×(P\left(Y\right)×Y)>P\left(2Y\right)×2Y.$$

This can be reduced to:

$$P\left(Y\right)>P\left(2Y\right)$$

which means “higher output decreases prices”. This is, however, the law of demand which is irrelevant to returns to scale.

In that case, we cannot determine the value of $a$.

1. “We have DRS, because the unit cost of production would decrease if we would double our output”.

Answer: Lower unit prices due to higher production is not DRS but IRS. The manager is actually describing increasing returns to scale but reaches the verdict of decreasing returns to scale. So $a>1$.

1. “Our progress ratio is less than %100 so we have IRS”.

Answer: This is correct. Progress ratio is shows the rate of change in unit cost of production after doubling output. If the technology is

$$Y=L^{a}$$

then we can restate this relationship as

$$Y^{\frac{1}{a}}=L.$$

Then cost is

$$C=WL=WY^{\frac{1}{a}}.$$

Unit cost of production (that is, marginal cost) is

$$\frac{dC}{dY}=MC\left(Y\right)=\frac{W}{a}Y^{\frac{1}{a}-1}.$$

So the progress ratio is

$$\frac{MC\left(2Y\right)}{MC\left(Y\right)}=\frac{\frac{W}{a}2Y^{\frac{1}{a}-1}}{\frac{W}{a}Y^{\frac{1}{a}-1}}=2^{\frac{1}{a}-1}.$$

The manager says

$$2^{\frac{1}{a}-1}<1.$$

This is equivalent to

$$\frac{1}{a}-1<0.$$

Therefore, $a>1$. Conclude IRS.