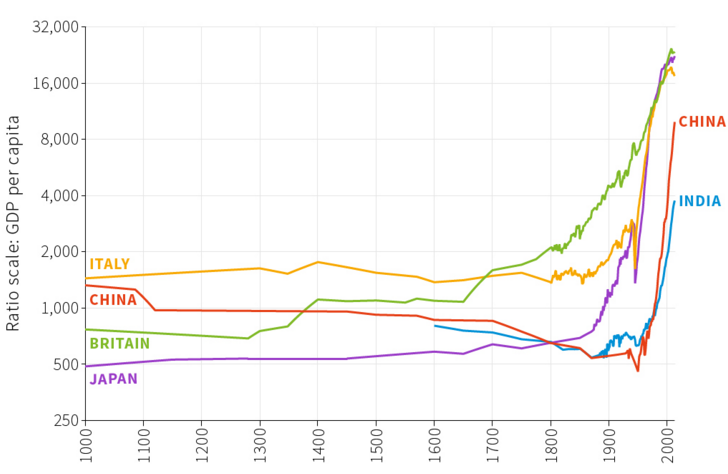
**What Is Technology?**

It is hard to believe but we know how rich people were 1000 years ago. Economists developed methods to estimate average real income (i.e. income per capita) for many countries dating back until 1000 AD. The estimates show that incomes per capita were more or less constant for 7 centuries, and suddenly started growing 3 centuries ago, a trend that continues ever since.

**Figure 1.** Income per capita between 1000-2000 AD.



Source: www.core-econ.org

The figure above shows that the income per capita in India has increased 6 fold while that in Britain increased staggering 16 times over the last 300 hundred years. The start of this explosion of income per capita starts around 1700, which is also considered to be the start of two major events: Industrial Revolution and Modern Capitalism.

Let us comment on these two major events. Industrial Revolution brought about mass production with labor saving technology. This enabled the transition from hand production methods to automated machines. Modern capitalism, on the other hand, incentivized investment in technology, new plants and new resources in pursuit of higher profits.

It is widely believed that the sudden increase in income per capita all over the world is a consequence of the interaction of **modern capitalism** and **industrial revolution**. These two very important events caused enormous improvements in all dimensions of production technology.

But what is technology? Now let us define this important term.

**Definition:** Technology is the information of how much output can be produced with certain amounts of inputs.

Assume that labor and capital are the only inputs for producing as output. Technology is simply the following information: given the available amounts of and how much can be produced.

**Example:** Consider a corn farm. Suppose there are laborers, and bushels of seed at the farm. It is the technology that tells us the amount of corn that can be produced using these inputs.

Of course, this definition can be stated in mathematical terms. Technology determines how much can be produced using as input. So technology is a function such that is the level of output.

The technology is also called the “production function”. Labor and capital are also called “factors of production” or “input”. Of course, is “production” or “output”. Technological change this means changes.

**Exercise\*:** What is wrong with the idea that technology is producing more with less labor?

**Important:** The aggregate amount of produced in a country is GDP – Gross Domestic Product. Another important notion is GDP per capita, which is average GDP per person. Therefore, dividing GDP by population gives GDP per capita. GDP per capita is also known as income per capita.

In real life, we observe – production, and inputs - for certain periods of time in many economies. For example, see Figure 2 which depicts the annual amounts of labor, capital, and output in the US from 1899 to 1922.

Note that we do not know the technology although we know In other words, we know the how much GDP is produced (Y) with certain amounts of input (K and L) at different points in time. But there is no observation of the technology as a mathematical function .

So economists try to discover the function by observing . In other words, we observe over time, and try to find out the relation between output and inputs .

At this stage, it may be hard to understand even what the question is. Let us have a look at an early example of this discussion. The results are surprising and illuminating regarding technology of our times. We shall gradually reach to modern analysis over the course of this lesson.

**Cobb-Douglas Technology – Early 20th Centruy**

In 1927, Paul Douglas, a professor of economics, computed the series of output, capital, and labor for the US. His data starts from 1899 reaching to 1922. He obtains Figure 2 below after two steps: set each variable in 1899 to 100 and compute their natural logarithms.

Paul Douglas, obviously, tried to make his observations fit in a single graph by making them start from the same point, 100. Normalizing all the variables to 100 at the initial year is only a change of units. This is totally safe since technology cannot be sensitive to our measurement units. Moreover, taking the logarithm of these series makes them less steep and more flat, which also helps to fit everything in a single graph.

Paul Douglas showed the graph above to a mathematician friend of his, Charles W. Cobb who suggested that the data seem to follow the following relation:

since seems like a linear function of and in the graph above. Now take the anti-logarithm of this linear relation (suggested by Charles Cobb) to see

**Remark:** We are using two mathematical rules here.

**Rule 1:**  so that and .

**Rule 2:** so that .

If we write , which is just a constant, then the production technology above is

which is a function of inputs, . [[1]](#footnote-1) This expression is what we know as Cobb-Douglas function, presumably the most influential functional form in the history of economics.

**Definition:** The production technology

is called the Cobb-Douglas technology.

**Remark:** Cobb-Douglas function was first developed by someone else, Philip Wicksteed, a fact that Paul Douglas openly acknowledges. Indeed, no scientific discovery is named after its original discoverer, according to Stephen Stigler, and this case is a classic example of this general phenomenon.

In this Cobb-Douglas formulation are fixed coefficients, called the parameters of technology. We shall soon see the economic meanings of these parameters, and the roles that they play in our analysis. It is noteworthy that Cobb and Douglas did not treat as an arbitrary couple, and imposed the following restriction:

which gives us . This is slightly more specific than the general expression . The parameter is technically called the exponential parameter.

Now let us briefly discuss what means. This discussion requires the following definition:

**Definition:** Let be an arbitrary production technology. Assume . Then exhibits

* Increasing returns to scale when
* Constant returns to scale when
* Decreasing returns to scale when .

Increasing returns to scale says that if we double all inputs then output more than doubles. Constant returns to scale says that if we double all inputs then output exactly doubles. Decreasing returns to scale says that if we double all inputs then output less than doubles.

Let us check if the production technology increasing/constant/decreasing returns to scale. Note that

Therefore, the assumption of

by Cobb and Douglas means constant returns to scale. In other words, exhibits constant returns to scale.

**Exercise\*:** Determine the returns to scale for the following technology:

**Exercise\*:** Determine the returns to scale for the following technology:

Now the next obvious thing to do finding the numerical values of and . Cobb and Douglas estimate these parameters using linear least squares. Least squares is a technique that is used to estimate parameters with the objective of minimum error in prediction. The details of the procedure is not our concern. Yet this exercise yields the estimates below:

and .

Now let us see this estimate as a graph.[[2]](#footnote-2)

The figure above plots actual output against

for each year from 1899 until 1922. This compares the estimation of Cobb-Douglas with the real life output.

It seems that the least squares estimation of the Cobb-Douglas production function performs pretty well. This means, given the data of inputs (capital K and labor L), one can use to predict quite accurately.

**Basic concepts in technology**

Although the Cobb-Douglas technology is one of the most influential mathematical expressions in economic analysis, it is actually a very special member of a large family. So Cobb-Douglas technology is actually a specific case. Other well-known members of the same family are perfect substitutes technology, and perfect complements technology.

**Definition:** If using as inputs produces

as output, then the technology is perfect substitutes (linear).

Perfect substitutes technology means capital and labor can replace each other at an exact certain proportion. In this case, we say “labor and capital are substitutes”.

**Exercise:** A particular bank teller can serve 8 customers per hour. Instead, an ATM (automated bank teller) can serve 20 customers per hour. What is the production technology of this bank if is the number of ATM, and is the number of bank tellers? The output of the bank is measured by the number of customers that it serves.

**Answer:** The service provided by ATMs and bank tellers can replace each other perfectly. So they are perfect substitutes:

Now let us find the numerical values of . According to the question if and , then which means

If and , then which means

Conclude and , implying

Now let us see the definition of another technology, perfect complements:

**Definition:** If using as inputs produces

as output, then the technology is perfect complements (Leontief).

Perfect complements technology means capital and labor should be mixed at an exact proportion. Each unit of labor needs units of capital since they are complements. In this case, we say “labor and capital are complements”.

**Exercise:** In a slaughter house 1 butcher can slaughter 12 cattle a day, which gives 1 ton of meat. If produced meat is denoted by , and number of butchers is , and number of cattle is what is the production technology ?

**Answer:** 1 ton of meatproduction needs exact proportions of the inputs, butchers and cattle. These inputs cannot replace each other in meat production at any rate so they are perfect complements:

Now let us find . According to the question

so and implying

Interestingly, Cobb-Douglas technology is exactly in the middle of these two polar cases: perfect substitution and perfect complements. In Cobb-Douglas technology, labor and capital are neither complements nor substitutes.

There is a very good reason why these technologies can be easily compared to each other, and are aligned perfectly relative to each other. The reason is that they are polar cases of a more general technology:

**Definition:** If is the choice of capital and labor, then

is the CES (constant elasticity of substitution) technology where is a fixed parameter.

Perfect substitutes, perfect complements, and Cobb-Douglas technologies are special cases of the CES technology. In particular, assume . Then we have the following table.

|  |  |  |
| --- | --- | --- |
| **Polar cases of the CES Technology** | | |
|  |  |  |
|  |  |  |
| Linear  (Perfect Substitute) | Cobb-Douglas | Leontief  (Perfect complement) |

This table clearly demonstrates that is a fundamental threshold point in technology:

A common property that all CES technologies satisfy is the constant returns to scale.

**Exercise\*:** Show that the CES technology exhibits constant returns to scale if .

**Exercise\*:**  The number of passengers that travel from Izmir to Istanbul is . A single bus can carry 100 passengers a day, and a plane can carry 600 passengers a day. If is the number of busses, and is the number of passengers, then what is ?

where

**Technology in graphs**

The standard method to draw a production function is to fix a value of such as or . Then the next step is to draw all couples that satisfy or on the 2-dimensional plane. In other words, plotting the **level curves** of is the standard approach to draw its graph. If the fixed value of is denoted by , then the solution to

is called the isoquant of the production technology at . An isoquant is just a level curve.

**Exercise\*:** Plot the isoquants of the perfect complements technology

s at and .

**Exercise\*:** Plot the isoquants of the perfect substitutes technology

at and .

Interestingly, Cobb-Douglas technology is exactly in the middle of these two polar cases: perfect substitution and perfect complements. In Cobb-Douglas technology, labor and capital are neither complements nor substitutes.

**Exercise\*:** Repeat the previous exercise for the Cobb-Douglas technology.

1. This is an innocent renaming of variables. [↑](#footnote-ref-1)
2. The actual estimation is A=1.05. [↑](#footnote-ref-2)