**Mathematical Economics – Fake Exam 2020**

**Questions**

1) Assume that you want to sell your house in the real estate market but the offers by the customers are uncertain. The house prices (per m2) are uniformly distributed over [4000TL, 8000TL]. Suppose you will receive offers from two randomly chosen customers. The first customer offers 5500TL per m2.

1. What is the probability that you will get a lower offer from the second customer?
2. If you plan to sell your house to the highest bidder, what is the expected price that you will get?

Answer: Since prices are uniformly distributed, by definition, it follows that

Answer: The question asks

where is the second price offer. Note that

Now we need to calculate . The easy way to calculate this expression is to take into account the fact that is uniformly distributed. Therefore,

because the expected value of any uniformly distributed random variable is the average of its upper and lower bounds. Moreover, we know that the lower bound of is simply 5.5 if So the expression above follows. Now let us plug what we know into the question to obtain

2) Consider a group of drivers that have different probabilities of having an accident. Individuals of type H have a probability of having an accident of pH = 0.1. These individuals form 3/4 of the population. Individual of type L, the remaining 1/4 of the population, have a probability of having an accident of pL =0.05. Any individual who has an accident suffers an income loss of 8000TL. For each type the maximal insurance premium that they are willing to pay is 1/3 above their expected losses. Assume that there is asymmetric information: insurance ﬁrms cannot distinguish between the types of drivers. The same insurance contract must be offered to all individuals.

1. If the contract is to earn an expected proﬁt of zero, what insurance premium must be charged?
2. Is this competitive outcome efficient?

Answer: For this question, observe that the insurance companies can offer a premium with only two possible outcomes: 1) both H and L types would purchase the policy, 2) only H types would purchase the policy.

(Remark: Why is there no possibility of a policy where only L would purchase it? Because, if L is in the premium would be cheap which ensures that H is also in)

Let us consider the first scenario:

Note that

So the policy which everyone buys costs under the condition of zero profit. But everyone actually buy it? H types would pay at most

This tells us that H types would buy. L types would pay at most

So L types would not buy. The result is that “a policy that everyone buys in competitive market does not exist”.

Second scenario: Only H types buy the policy.

Note that

The equilibrium is and H types would buy this policy. Why? Because they are willing to pay at most

1. The outcome is not efficient. Because L types are not insured even though they are willing to pay more than their expected losses. In other words, there is beneficial trade between L types and the insurance companies that do not take place at the equilibrium. This is inefficient.

3) Suppose that the prevalence of European ancestry in Turkey is 30%. 23andMe is a private company that offers genetic tests which determine ancestry with 95% accuracy. If your 23andMe test tells that you have an European lineage, what is the probability that your grandparents come from Europe?

Answer: To solve this question we should use the Bayes’ Rule:

In this question, we are asked Event is European lineage and event is positive test result. Therefore,

As a consequence,

4) Consider an economy where Ahmet and Hakan consume two goods, x and y. Assume that the utility function of Ahmet is

and the utility function of Hakan is

The initial endowments of Ahmet is . The initial endowments of Hakan is . The prices of x and y are and .

1. Find the competitive equilibrium.
2. Draw the Edgeworth box of this economy and show the competitive equilibrium and the initial endowments.
3. The equilibrium is efficient. Why? Explain.

Answer:

1. If you solve “MRS=px/py and Income=Expenditure”, then the result for Ahmet would be

The answer for Hakan is

So let us take (or equivalently, and equate demand=supply ( which mean

1. The equilibrium levels of consumption are
2. Based on the previous answers, the Edgeworth box is as follows:



1. The equilibrium is efficient because MRS of Ahmet and Hakan are equal. MRS represents how much would an individual be willing to give up for a small extra unit of . Therefore, there is no room for furthering beneficial trade for Ahmet and Hakan.

**Extra questions for the fake exam:**

1. Consider the growth model of Solow. Assume that the production technology is Cobb-Douglas:

The law of motion for capital is

Assume that the growth rate of is and the growth rate of is . What is GDP per capita

in the long-run?

Answer: As we discussed in the class Growth(Y)=Growth(K) in the long-run. What is the growth rate of K? Divide by to obtain the growth rate:

However, I would like solve

This means

Therefore,

As we discussed in the class Growth(Y)- Growth(L)= Growth(K)-l. Therefore, the Growth(K)=. Deduce that

This is equivalent to

1. Consider an economy where consumers can be represented by three individuals whose incomes are

Assume that the income tax rate is so private consumption by consumer is

Tax revenues are used to finance a public good

The utilities of consumers from private and public goods are

1. Calculate the most preferred tax rates for individual
2. Suppose that two political parties and compete in the elections. Each political party proposes a tax rate to maximize its votes. Consumers vote for their most preferred tax. What is the election winning tax rate? Explain.
3. Now let us maximize with respect to by solving

The solution is which is the same for all individuals.

1. Since everyone loves , all political parties would propose the same policy.