**General Equilibrium**

Consider an economy where and is used to produce a final consumption good using the production technology

If the prices are given by

then the profit function is

Suppose that represents the objective of a representative firm. The representative firm would solve

to maximize profits.

**Theorem:** If is homogenous of degree 1, then the maximum profit is

**Proof:** If is homogenous of degree 1, then

Now take the derivative of both sides to observe,

Multiply both sides by to see

This is equivalent to

Therefore, QED.

Assume that a representative consumer in the economy solves

s.t.

where is consumption, is working hours (labor supply), is capital stock, is the profit share.

The utility maximization yields

Let us assume that there are number of representative individuals, and number of firms.

**Definition:** A perfectly competitive general equilibrium prices is a vector

which satisfies

and solve the utility maximization

and solve the profit maximization

To normalize prices, we set . The price normalization is a consequence of the Walras’ law: if budget constraints are satisfied and all markets except 1 is in equilibrium then all markets are in equilibrium.

Let us see an example:

Let . Assume that

Now let us compute the perfectly competitive general equilibrium.

|  |  |  |
| --- | --- | --- |
|  | 1 | Utility maximization |
|  | 2 |
|  | 3 | Profit maximization |
|  | 4 |
|  | 5 | Market clearing  |
|  | 6 |
|  | 7 | Technology |

Take (6) and put it into (3)

Note that (4) implies

Due to (1 and 5)

If we put this into (2)

Let us recap: We have three unknowns and three equations

Now let us focus on the first two equations:

Put this into the third equation:

We can solve this equation in closed form for

Or example, if , then

Recall that

Moreover,

And finally,

However,

This demonstrates the Walras’ law: