**Income Distribution in a Democratic Competitive Capitalist Economy**

In this fake exam, we consider a competitive capitalist economy with three types of individuals: a worker, a rentier, and an entrepreneur. The government uses income taxes to redistribute income. We shall consider 3 different scenarios depending on whether taxes are distortionary or not and whether tax policy is democratically determined or not.

We start the question with specifying the firm and its production technology. There is a single firm with the production technology given by

Here is fixed technology parameter and is capital and is labor. So the firm solves

by choosing where is the wage and is the rental price of capital.

There are three individuals denoted by . Each individual solves

s.t.

by optimally choosing her consumption and labor supply . Therefore, the preferences are quasi-linear (this information is not crucial but a note in case you would be interested).

The income tax rate is and is the universal lump-sum transfer financed by tax revenues so that the government’s budget constraint is:

Individual is the worker so that and individual is the rentier so that but . Individual 3 is the entrepreneur so that but .

1. Does exhibit increasing, decreasing, or constant returns to scale? What does this imply regarding income distribution? (See the chapter titled “Technology” from your textbook if you need any explanation for returns to scale.)
2. Compute the perfectly competitive Arrow-Debreu (i.e., Walrasian) equilibrium as a function of the policy tuple .
3. Argue that the competitive equilibrium is Pareto-inefficient for any .
4. Find the policy tuple that would maximize the sum of all utilities at the equilibrium:

where for each . How is it possible that a Pareto-inefficient allocation (i..e., competitive equilibrium with taxation) maximizes total sum of utilities?

1. Find the policy tuple that would win the elections. To be more precise, find the Condorcet winner which is defined as the policy tuple that would get the most votes against any of its alternatives. According to the median voter theorem, the Condorcet winner is always the most preferred policy of the median voter. The median voter is the individual whose most desired policy divides the society into equal halves.
2. Replace each individual ’s budget constraint with

where is a lump-sum tax/transfer with . Now argue that (without any actual computation) there is a lump-sum tax and transfer policy such that everyone gets the same level of utility at the perfectly competitive equilibrium and it is Pareto-efficient.

1. Is equality of income possible according to your answer in part (e)? Which scenario can be generalized beyond this example: Equality of income or equality of utility in a competitive economy via lump-sum policies?
2. Now find the actual that would ensure everyone gets the same level of utility at the competitive equilibrium.

Bonus: What is the price of the produced good?