**Utility maximization and labor supply**

Consider an individual who solves

s.t.

by choosing where ) is a given tuple. Here denotes the preferences of the individual, also known as the utility function.

**Definition:** If is defined for all values of , the preference relation is “complete”.

**Definition:** Let and . Then if the preference relation is “transitive”.

**Definition:** If a preference relation is “transitive” and “complete”, then it is rational.

**Theorem:** Any continuous utility function represents a rational preference relationship.

Now let us suppose that is a differentiable function. Then we can solve the individual’s maximization problem via Lagrange formulation:

FONC:

Assuming that is concave, this set of equations yields the solution the original utility maximization problem. That is because, the constraint is linear in choice variables.

Note that we can substitute for by observing that

which gives

This can be also expressed as

In article, the same equation is expressed as

Example: Let . Then FONC reduces to

The final result would be

The most interesting feature of this solution is that the optimal working hours is independent of wages. Why? The answer is that the income effect and the substitution effect perfectly cancel out each other at Cobb-Douglas preferences.

It may be argued that does not look like Cobb-Douglas preferences. Nevertheless:

**Theorem:** If is an utlity function of a certain preference relationship, and is an increasing function, then is also a utility function representing the same preferences.

**Example:** and represent the same preferences. But note that

We can generalize the problem by introducing taxes and transfers as follows:

s.t.

Rogerson assumes that and impose This assumption can be interpreted as “real wage is unity”. In this case, the problem becomes

s.t.

Accordingly, FONC is

Rogerson assumes

The justification of this utility function is the following:

“For any infinite horizon growth model with stationary growth, should be linear in

However, there are only two classes of utility functions that satisfy this property:

1. Cobb-Douglas
2. where is a concave function.

Under this assumption, FONC is

After substituting for ,

Rogerson also imposes to obtain

Assume that and which gives

Rogerson also analyzes three additional programs:

2) Wasteful spending:

s.t.

3) Subsidy to work

s.t.

where .

4) Subsidy to leisure

s.t.

where .