1. The **Stone-Geary utility function** has the form

where and . The are often interpreted as ‘subsistence’ levels of the

respective commodities.

1. Derive the associated expenditure and indirect utility functions. Show that the former is *linear* in utility, whereas the latter is proportional to the amount of ‘discretionary income’,

Solution: To find the expenditure function, let us solve:

s.t.

by choosing . We know that the solution will imply

Let us remember where this result comes from. First form the Lagrangian:

FONC is:

for each . This directly gives us

So we need to compute the marginal utilities:

So the MRS rule gives us:

From this observation deduce that

Now plug into

which gives

So the Hicksian demand is

where is a given constant.

So the expenditure function is

To find the indirect utility, we must solve

which is equivalent to

and

From the first equation (MRS rule), we obtain

This gives us the Marshallian (Walrasian) demand

So the indirect utility function is

This is obviously proportional with respect to the discretionary income, .

1. Show that measures the share of this ‘discretionary income’ that will be spent on ‘discretionary’ purchases of good in excess of the subsistence level .

Answer: We already showed

Which implies

1. Let the production function be

where

1. Derive the 2nd order derivative matrix (i.e. the Hessian) of .

Solution.

where . Likewise,

Therefore,

Therefore, the Hessian is

.

1. Find a condition on to ensure that is a concave function.

Concavity requires the principal minors change signs, starting from negative. Therefore,

This is equivalent to

Note that

because

Therefore, .

You can show that

provided that

1. Assume that a competitive firm solves

by choosing . Show that the labor share of income in this economy is equal to the elasticity of .

Solution: The labor elasticity of output is defined as

However,



If the firm maximizes its profit, then

Multiply both sides by then divide by to see

In this economy, the technology is

Therefore, the labor share is

1. Average producivity of labor, , has an upward trend in almost all countries in the world, including Turkey. Labor share in Turkey has declined over the last decade. What does that imply in terms of in Turkey?

The labor share, according to our question is

We are told that this expression is getting smaller over time. Therefore,

should decrease over time. But we are also told that is increasing. Conclude .

1. According to econometric estimates, in Turkey and elsewhere. What does that imply in terms of ?

If , then the labor share (which is decreasing) would be

This can only happen if is increasing over time. In fact, should grow faster than .

1. Consider an individual who solves

s.t.

by choosing where ) is a given tuple. The parameters and denote the income tax rate and the transfers, which are exogenous for the individual. Here

1. Derive the first order conditions for this problem.

The FONC are:

In explicit terms,

Substituting for gives us

In other words,

1. Impose on the optimality conditions due to the government’s balanced budget.

In this case, the optimality condition would be

1. Now consider the graph below which plots the labor supply in Turkey from 1980 to 2017.

Find values of consistent with the trend in labor supply given that the tax-to-GDP ratio rose from 15% to 25% over the same period in Turkey.

According to these data, we can deduce that