**Homework Assignment 1**

**(Due date: 25th of October 2021)**

1. Assume that the utility function of a consumer is

where .

1. Derive the Marshallian and Hicksian demand functions, denoted by and , respectively.

(Hint: We already solved this in the class.)

1. Derive the indirect utility and expenditure functions, denoted by and , respectively.

(Hint: We already solved this in the class.)

1. Verify the duality relationships

(Hint: We already solved one of the second item in the class.)

1. Verify the Roy’s Identity and the Shephard’s Lemma.
2. Show that the Slutsky matrix is negative definite.
3. Solve the same question above with the CES utility function:
4. Derive the Hicksian and Marshalian demand assuming:

(Remark: It is sufficient to derive the demand functions.)

1. Solve the question above assuming
2. First assume that

Now prove that

In other words, the CES utility converges to the Cobb-Douglas utility as approaches to 0. So Cobb-Douglas is a special case of CES.

(Hint: Google for a solution. This is presumably the most difficult question in this assignment.)

1. Draw a representative indifference curve for the following utility function:
2. Let be a utility function. Assume is a monotonic increasing function (i.e., ). a) Show that the MRS of and are identical.

b) For example, let

and . Show that the Marshallian demand for and are identical.

(Hint: Utility is invariant with respect to monotonic transformation. Or utility is ordinal. This question asks you to prove this and show it with an example.)

1. Assume that denotes consumption and leisure and the preferences over of a representative worker is

where . If the price of is and the wage rate is denoted by .

1. Write down the budget constraint for this worker.
2. Derive the Marshallian demand for .
3. Derive the optimal labor supply.