Parent-Child Education Game

Homework

Suppose that a child (Player 1) and a parent (Player 2) are engaged in an interaction that we denote by a game

$$Γ=\left(I,\left(u\_{i},S\_{i}\right)\_{i\in I}\right)$$

where $I=\{1,2\}$ and $S\_{1}=[0,1]$ and $S\_{2}⊂R\_{+}$.

The child (player 1) decides how much time to spend on studying, $s\_{1}\in S\_{1}$. The parent decides $s\_{2}\in S\_{2}$ which shows how much the parent invests in the child’s education.

The objective of the child is

$$u\_{1}\left(s\_{1},s\_{2}\right).$$

The objective of the parent is

$$u\_{1}\left(s\_{1},s\_{2}\right)=-s\_{2}+u\_{1}\left(s\_{1},s\_{2}\right).$$

The idea is that the parent cares about the child but dislikes spending money. The child is selfish but affected by the parent’s investment in education.

1. Assume $S\_{2}=R\_{+}$ and $u\_{1}\left(s\_{1},s\_{2}\right)=2s\_{2}$. Show that there is no Nash equilibrium.
2. Find the most general conditions on $u\_{1}\left(s\_{1},s\_{2}\right)$ and $S\_{2}$ so that the Maximum Theorem applies. Note that the best responses would be “continuous” in this case.
3. Having established continuity, find conditions that we would guarantee that the Brouwer fixed point theorem applies. Conclude that the game has a Nash equilirium.
4. Now explain why there is no equilirium in part (a)?
5. Write down an example for $u\_{1}\left(s\_{1},s\_{2}\right)$ and $S\_{2}$ in explicit form so that the sufficient conditions that you find in part (b) and (c) are satisfied. In this specific example, calculate the Nash equilibrium.