1) a) Under the stated conditions, the production tech. can be represented as

$$Y = A\,(X + N_r)^{1-\alpha} \cdot N_n^{\alpha} \qquad (1)$$

So the pareto-efficient allocation described in the question solves:

$$\max \left(\Pi_n U_n + \Pi_r U_r\right)$$

s.t

$$\Pi_n c_n + \Pi_r c_r + G + \phi X = A\,(X + \Pi_r)^{1-\alpha}\,\Pi_n^{\alpha}$$

since $N_r = \Pi_r$ and $N_n = \Pi_n$ (i.e., individuals inelastically supply 1 unit of labor)

The Lagrangian of this problem is

$$\mathcal{L} = \Pi_n (\ln c_n + \chi \ln G) + \Pi_r (\ln c_r + \chi \ln G) +$$
$$\lambda \cdot \left(A(X + \Pi_r)^{1-\alpha}\,\Pi_n^{\alpha} - \Pi_n c_n - \Pi_r c_r - G - \phi X\right)$$

The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial c_j} = \Pi_j / c_j - \lambda \Pi_j = 0 \quad \text{for } j = r, n. \qquad (2)$$

$$\frac{\partial \mathcal{L}}{\partial X} = \left((1-\alpha)\frac{Y}{(X + \Pi_r)} - \phi\right) \cdot \lambda = 0 \qquad (3)$$

Conclude that the Pareto-efficient allocation is

$$X^* = \pi_n \cdot \left(\frac{(1-\alpha)A}{\phi}\right)^{1/\alpha} - \pi_r$$

$$C^* = A^{\frac{1}{\alpha}} \cdot \pi_n \cdot \left(\frac{1-\alpha}{\phi}\right)^{\frac{1-\alpha}{\alpha}} - \phi X^*$$

$$G^* = \gamma \cdot C^*$$

b) $G = 0$, due to no taxation. Profit max. gives:

$$w_n = (1-\alpha) \cdot \frac{Y}{N_r} = \alpha \cdot \frac{A(X+\pi_r)^{-\alpha}}{\pi_n^{-\alpha}} \quad (1)$$

$$w_\pi = \alpha \cdot A \cdot \left(\frac{\pi_n}{\pi_r + X}\right)^{\alpha-1} \quad (2)$$

$$\phi = A\left(\frac{X+\pi_r}{\pi_n}\right)^{-\alpha} \cdot (1-\alpha) \quad (3)$$

Use (1) and (3) to conclude

$$w_n^* = \phi \quad (4)$$

Use (3) to see $X^* = \left(\frac{\phi}{(1-\alpha)A}\right)^{\frac{1}{-\alpha}} \cdot \pi_n - \pi_r \quad (5)$

Now use (5) and (2) to deduce

$$w_n^* = \alpha A \left(\frac{\Pi_n}{\Pi_r + X^*} \right)^{\alpha - 1} \qquad (6).$$

The utility max. implies that the budget constraint holds:

$$c_j^* = w_j^* \text{ for } j = n, r \qquad (7)$$

where w_n^* is in (6) and w_r^* is in (4).

Conclude (4-5-6-7) give us the perfectly competitive equilibrium.

c) What we need to do is to correct (3) and update all other equations accordingly:

Now (3) becomes

$$\phi \cdot (1+\tau_x) = (1-\alpha) A \left(\frac{X + \Pi_r}{\Pi_n} \right)^{-\alpha} \qquad (3^*)$$

which gives:

$$X^* = \left(\frac{\phi(1+\tau_x)}{(1-\alpha)A} \right)^{\frac{1}{-\alpha}} \Pi_n - \Pi_r$$

Use this equation in (2-6-7). Also note that (4) should be updated as

$$w_p^* = \phi \cdot (1+\tau_x) \qquad (4^*)$$

So the equilibrium with taxation is:

$$X^* = \left(\frac{\phi \cdot (1+\tau_x)}{(1-\alpha)A} \right)^{-\frac{1}{\alpha}} \pi_n - \pi_r$$

$$w_p^* = \phi(1+\tau_x)$$

$$w_n^* = (1-\alpha) \cdot A \cdot \left(\frac{\pi_n}{\pi_r + X^*} \right)^{\alpha - 1}$$

$$C_j^* = w_j^* \qquad j = n, r$$

$$G^* = \tau_x X^*$$

$$N_j^* = \pi_n \qquad j = n, r$$

d) Note that, in equilibrium with taxation,

$$\pi_n U_n + \pi_r U_r = \sum \pi_j (\ln c_j^* + \chi \ln G^*) =$$

$$\left(\sum \pi_j \ln c_j^*\right) + \chi \ln G^* = \left(\sum \pi_j \ln w_j^*\right) + \chi \ln G^* =$$

$$\pi_r (\ln \phi + \ln(1+\tau_x)) + \pi_n \Big(\ln \alpha A - (1-\alpha)\ln \pi_n +$$

$$(1-\alpha)\ln(\pi_r + X^*)\Big) + \chi(\ln \tau_x + \ln X^*) = \Omega$$

where Ω is the sum of all equilibrium utilities.

We need to max Ω wrt (with respect to) τ_x:

$$\frac{\partial \Omega}{\partial \tau_x} = \frac{\pi_r}{1+\tau_x} + (1-\alpha)\pi_n \cdot \frac{dX^*/d\tau_x}{\pi_r + X^*} + \frac{\chi}{\tau_x} + \frac{\chi}{X^*} dX^*/d\tau_x = 0$$

But

$$\frac{dX^*}{d\tau_x} = -\frac{\pi_n}{\alpha} \left(\frac{\phi(1+\tau_x)}{(1-\alpha)A}\right)^{-\frac{1}{\alpha}-1} \cdot \frac{\phi}{(1-\alpha)A}$$

$$= -\frac{1}{\alpha} \frac{(X^* + \pi_r)}{1+\tau_x} \qquad (a)$$

Deduce

$$\frac{\partial \Omega}{\partial \tau_x} = \underbrace{\frac{\pi_r}{1+\tau_x} - \frac{(1-\alpha)}{\alpha}\frac{\pi_n}{1+\tau_x} \overbrace{\frac{X^* + \pi_r}{\pi_r + X^*}}^{=1}}_{=0} + \frac{\chi}{\tau_x} - \frac{\chi}{\alpha}\frac{1}{1+\tau_x} \cdot \frac{X^* + \pi_r}{X^*} = 0$$

$$\frac{\partial \Omega}{\partial \tau_x} = \chi \left(\frac{1}{\tau_x} - \frac{1}{\alpha} \frac{1}{1+\tau_x} \cdot \frac{X^* + \Pi_r}{X^*} \right) = 0$$

But

$$\frac{\partial G^*}{\partial \tau_x} = X^* + \tau_x \frac{dX^*}{d\tau_x}$$

$$= X^* - \frac{\tau_x}{\alpha} \frac{X^* + \Pi_r}{1+\tau_x}.$$

(due to (a) in the previous page)

Conclude:

$$\underbrace{\frac{\partial \Omega}{\partial \tau_x} = 0 \text{ if and only if } \frac{\partial G^*}{\partial \tau_x} = 0}$$

Maximizing social welfare in equilibrium is equivalent to Maximizing public good production

e) In that case $\tau_x = 0$ because τ_x causes distortion but labor tax does not. That is because, workers supply labor inelastically. No matter what their wage is, they work. So all tax must be imposed on labor income for efficiency purposes.

2 a) To find the NE in a dynamic game, we must represent the game in normal-form: 7

		(q₁) HH	(q₂) HL	(q₃) LH	(1−q₁−q₂−q₃) LL
O	(p) R	1,0	1,0	1,0	1,0
	(1−p) N	3,3	0,10	3,3	0,10

$$EU^O = pq_1 + pq_2 + pq_3 + p(1-q_1-q_2-q_3) +$$
$$(1-p).3.(q_1+q_3) = p + (1-p)3(q_1+q_3)$$

$$\frac{\partial EU^o}{\partial p} = 1 - 3(q_1+q_3)$$

$$EU^E = (1-p)(3q_1 + 10q_2 + 3q_3 + 10(1-q_1-q_2-q_3))$$

$$\frac{\partial EU^E}{\partial q_1} = (1-p)(-7)$$

$$\frac{\partial EU^E}{\partial q_2} = (1-p).0$$

$$\frac{\partial EU^E}{\partial q_3} = (1-p)(-7)$$

$$\frac{\partial EU^E}{\partial q_4} = (1-p).0$$

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If $p=1$ and $q_1+q_3 < \frac{1}{3} \Rightarrow$ Nash Eq.

If $p<1 \Rightarrow q_1 = 0$ and $q_3 = 0$.

because $\frac{\partial EU^E}{\partial q_1} < 0$ at $p<1$

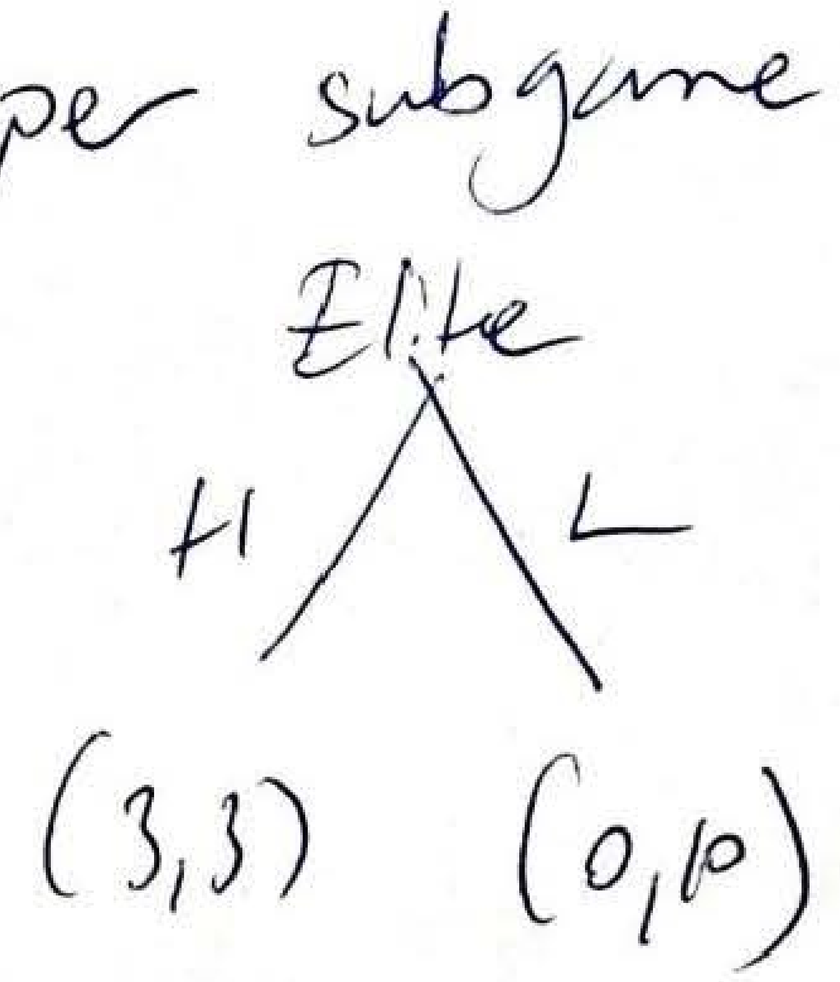
because $\frac{\partial EU^E}{\partial q_3} < 0$ at $p<1$.

See the previous page

But this would give $\frac{\partial EU^0}{\partial p} = 1 - 3(q_1+q_3) = 1 > 0 \Rightarrow p=1$

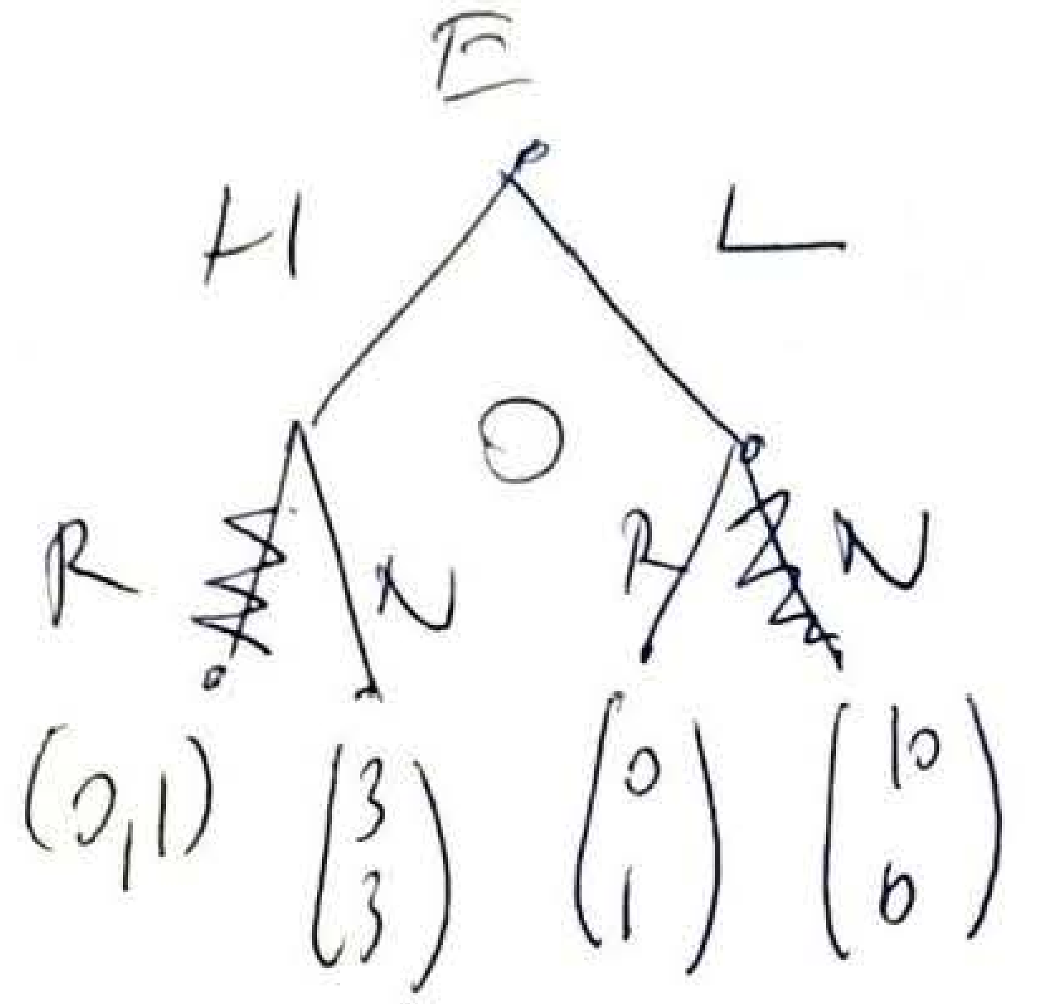
Contradiction.

b) Any SPNE is NE. What we need to check whether a particular NE is also subgame perfect.
The only proper subgame is

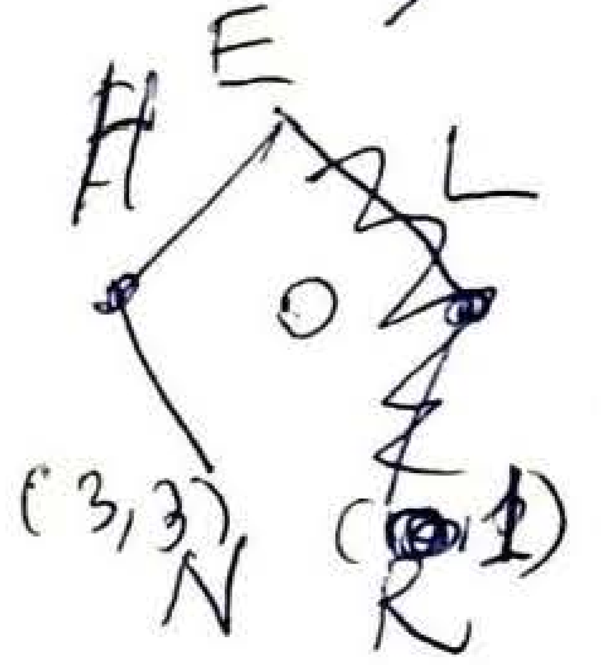


The equilibrium is unique: play L. Therefore the set of all SPNE is $p=1$ and $q_1=q_3=0$.

c) We can invoke backward induction:



eliminate actions that will not be played.



Eliminate again

d) In the first game, Ordinary People "Revolt". And the game ends. In the second game Elite compromises by playing "High taxes". The difference is due to the fact that O (ordinary) could never trust E (elites) to play H (high taxes) in the first game. So they revolt instead of being crushed under a free market system. But in the second game E can now ensure H so that O has no reason to revolt.

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e) In the first game players get (0,1).
" " second " " " (3,3).
So everyone is unambiguously better-off at the second game. Therefore, O and E would prefer to play the 2nd game. In other words, E has a first player advantage and O has a second player " in the 2nd game.
So there is no conflict of interest regarding which game should be played.

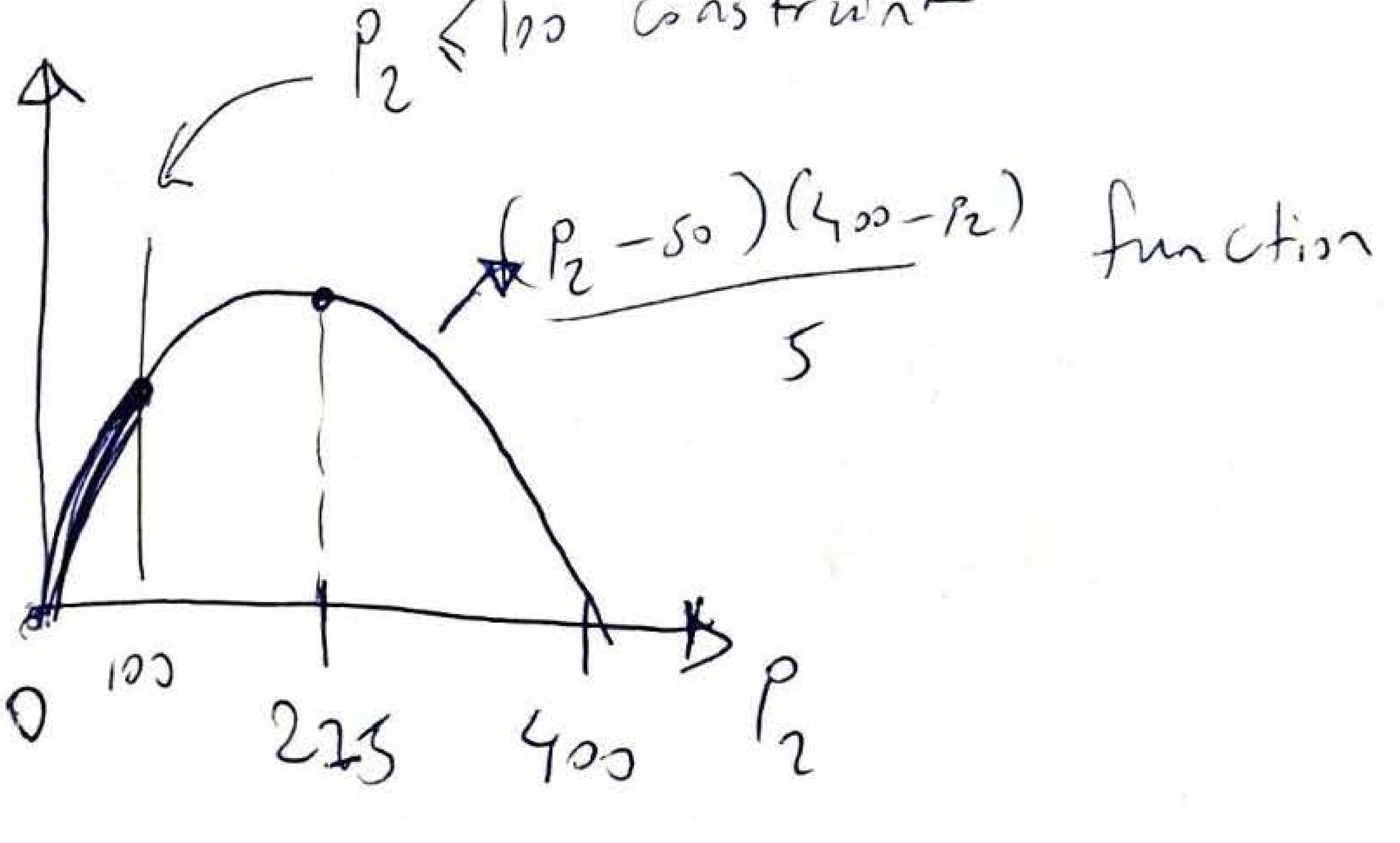
3) a) The firm which has a cost advantage is Firm 2 whose $MC = 50$. So Firm 2 would solve

$$\max \; (p_2-50)\overbrace{\left(\frac{400-p_2}{5}\right)}^{Q=\text{Demand}} \quad \text{s.t } p_2 \leq 100$$

If we plot the objective function over $p_2 \in [50, 400]$

$$(p_2-50)\left(\frac{400-p_2}{5}\right)$$

we see that

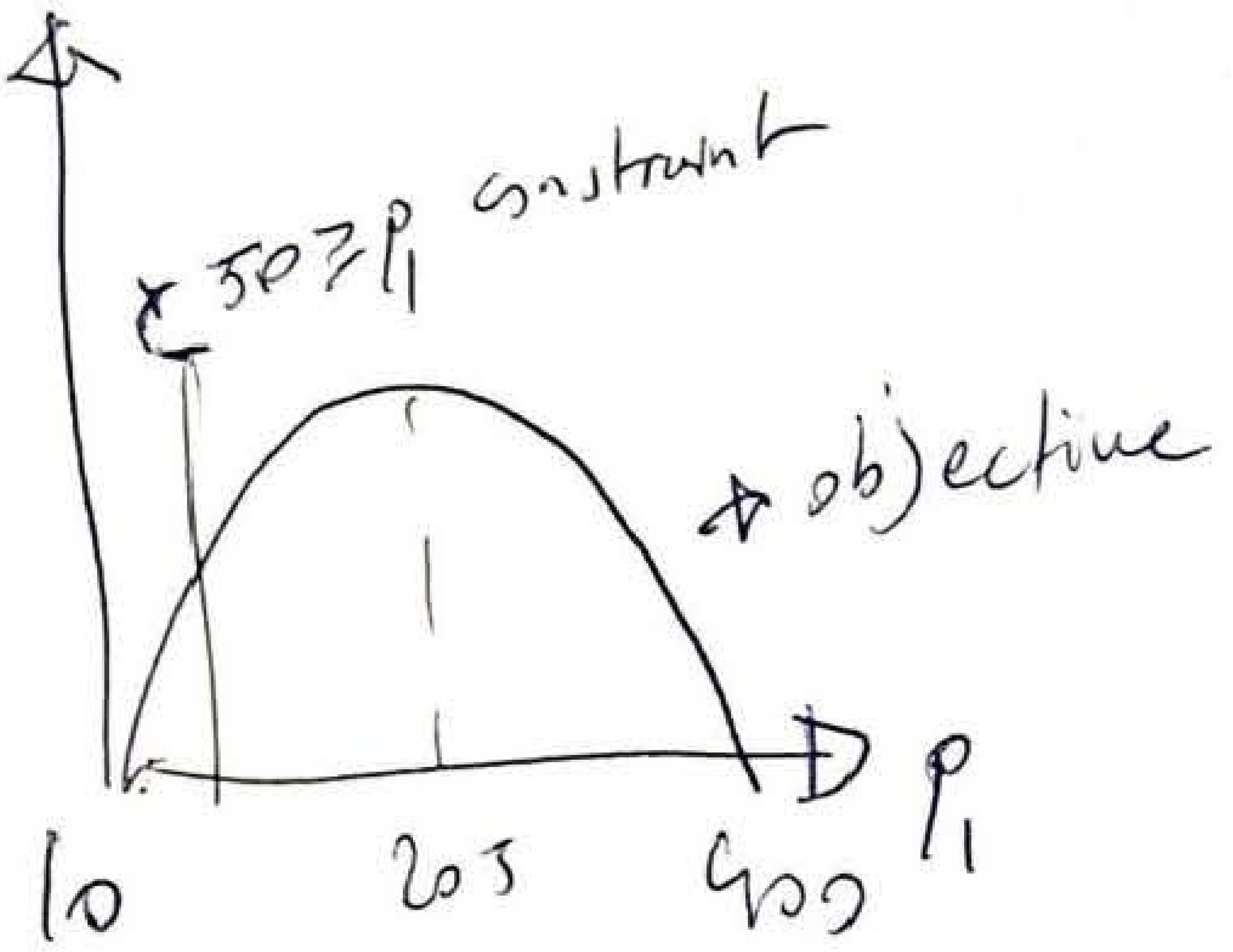


So the constraint $p_2 \leq 100$ is binding to max. profit

Nash Eqlm:
$$\begin{cases} p_2^* = 100, & q_2^* = 60 \\ p_1^* > 100, & q_1^* = 0 \end{cases}$$

b) If Firm 1 obtains the tech. with $mC = 10$, then it would have the cost adv. So it would solve

$$\max \ (p_1 - 10)\left(\frac{400 - p_1}{5}\right) \quad s.t \quad p_1 \leq 50$$



So the Nash equilibrium is

$$p_2^* > 50, \quad q_2^* = 0$$

$$p_1^* = 50, \quad q_1^* = 70$$

This would ensure $\pi_1^* = (p_1^* - 10)\left(\frac{400 - p_1^*}{5}\right)$

$$= 40 \times 70 = 2800$$

So Firm 1 can raise its profit from 0 to 2800.

This is the max. amount that Firm 1 can pay.

c) With $MC = 10$, firm 2 would earn

$$\pi_2^* = (p_2^* - 10)\left(\frac{400 - p_2^*}{5}\right) = 90.00 \times 7 = 6300$$

Firm 2 would pay 6300 at most.

d) The scientist would sell this new tech. to Firm 2 for 6300. The price of the good would still be 100 (no reduction in price despite tech. change). Profits would be zero and all surplus would be transfeoeed to the scientist.