**General Equilibrium (Continued)**

Suppose that there are number of individuals in an economy. Each individual is denoted by The preferences of individal is represented by

The initial endowments of is

In a competitive economy, individual solves

s.t.

where represents the profit share held by individual . For the sake of consistency,

The firm produces a consumption good using the (non-increasing returns to scale) technology

where is the level of output, is the level of capital demand, and is the level of labor demand. Therefore, the profit is

The firm solves

by choosing ).

**Definition:** The competitive equilibrium is a vector of prices

and an allocation

which solve

1. The utility maximization problem of each individual.
2. The profit maximization problme of the firm.
3. Market clearing conditions:

**Definition (Pareto-efficiency**). An allocation

is feasible if and only if

A feasible allocation

is Pareto-efficient if no other feasible allocation

satisfies the following property:

**Interpretation:** If an allocation is Pareto-efficient, then making someone better-off without hurting someone else is infeasible.

**Theorem:** Any perfectly competitive equilibrium is Pareto-efficient.

**Proof:** Suppose thatthe competitive equilibrium

is not Pareto-effcient. Therefore, there is another feasible allocation

which satisfies the following property:

Let us denote the individuals whose utility is higher at

with . In other words,

Nevertheless, this means that

due to the utility maximization. For , we have

Now sum over all and all to see

Conclude

This gives the following contradiction:

This contradicts profit maximization. ■

In practice, the Pareto-efficient allocations can be calculated by solving the standard linear welfare maximization problem:

s.t.

by choosing

for some given “welfare weight”

This is also knwon as the social planner’s problem.

**Theorem:** Any solution to the social planner’s problem-SPP (i.e. maximization of the linear welfare program) is Pareto-efficient (if for all )

**Proof:** Suppose that

solves the SPP. And assume that it is not Pareto-efficient. Therefore, there is another feasible allocation

which satisfies the following property:

This implies that

In other words, the maximum level of the linear welfare is less than the level of linear welfare at the alternative allocation:

This contradicts that

solves the social welfare maximization. ■

The linear welfare maximization problem can be equivalently expressed as

s.t.

Example: Suppose that The utility of each individual is

The production technology is

Now let us calculate the competitive equilibrium and the Pareto efficient allocations.

Competitive equilibrium:

Normalize the prices by setting Therefore,

Due to the profit maximization:

Conclude As a consequence:

In other words,

The level of consumption is

(Let us also se that the Walras’ Law holds.

Now let us solve:

s.t.

To solve this using the Lagrange technique, define:

So we need to solve:

Conclude

just as the competitive equilibrium.

Why is the social planner’s problem is so similar to the competitive equilibrium? The crucial variables in social planning and market equilibrium nicely correspond to each other. If is the market price, and is the Langrange multiplier of the budget constraint, then

|  |  |
| --- | --- |
| Linear Welfare Maximization | Perfect competition |
|  |  |
|  |  |
|  |  |

Now consider any competitive equilibrium. Define

According to the table above, this competitive equilibrium would also solve the linear welfare program.