**Growth**

Consider any variable which depends on time, . Then we can write . Then the rate of change of is simply is the time derivative:

Based on this notion of “change”, we can also define growth:

Why is this the definition of the growth rate?

**Example:** Suppose that GDP per capita in Turkey is $10K this year. If the GDP per capita will rise to $11K next year, then

Therefore, if the definition is correct, then we should see:

**Funny fact:** If you take the derivative of then the answer is

That is because, if you take the derivative of an expression in logarithms, then you should first take the derivative of the original expression and divide the derivative by the original expression.

**Remark:** If you want to calculate the growth rate of any variable, then you can calculate the derivative of its logarithm.

**Application:** Assume that . Then the growth rate of can be calculated as

Take the time derivative of both sides to find the growth rate of :

**Solow’s Model**

Consider an economy where the production technology is Cobb-Douglas:

where is output, is capital, is labor, and is the average (per worker) information or knowledge. can also be interpreted as “technology”. In fact, is called “effective labor”.

We assume that the growth rates of and are constant, denoted by and . This means technological growth and population growth are exogenous. Now let us see the growth rate of . The rate of change in is

where is the saving rate, and is the depreciation rate. To find the growth rate of , simply divide the rate of change by to obtain

In real life .

**Definition:** In economics, long-run is when endogenous variables are constant.

**Example:** In Solow’s model, the growth rate of is endogenous so it would be constant in the long-run.

**Theorem:** in the long-run.

**Proof**: First note that is a constant in the long-run. This means

is a constant. However, by assumption, and are constants. Deduce that

can be neither increasing nor decreasing over time. This means

Conclude . End of proof.

As a consequence, the following result ensues:

**Theorem:** in the long-run.

Proof: Recall that

Therefore, take the logarithm of this expression to see:

Now take the time derivative to find the growth rates:

Since in the long-run, this expression simplifies to

which is the desired result in the long-run. This completes the second proof.

Our final result is this.

**Theorem:** in the long run.

Proof: Note that the growth of GDP per capita is

Conclude

This completes our final result.