**Production**

Technology is the relationship between output and input. In particular, technology tells us how much output can be produced by a certain amount of inputs. In economics, this relationship is represented by a production function

where is a vector of inputs (e.g. labor, capital, energy, etc.) and is the level of output. Here we interpret as the technology and call it the “production function”.

Example: The Leontief production function is

In this formulation, all inputs are complementary.

Example: The linear production function is

In this formulation, all inputs are substitutes.

If we consider substitutes and complements as two polar cases, then Cobb-Douglas technology is the exact intermediate case.

Example: The Cobb-Douglas production function is

where

In this formulation, all inputs are neither substitutes nor complements.

All these well-known production technologies are actually special cases of a more general production technology:

**Definition:** If the production function is “constant elasticity of substitution” (CES) type, then

where is a constant parameter. We interpret

as the “elasticity of substitution”.

Moreover, is the factor-augmenting technology term for the input .

**Theorem:** If the production technology is of the CES type then the technology satisfies the following properties:

1. As , the technology converges to the linear technology.
2. As , the technology converges to the Leontief technology.
3. As , the technology converges to the Cobb-Douglas technology.

The CES formulation is typically used in the context of labor vs. capital or high vs. low skilled labor. For example,

Another example is

where is the high-skilled labor and is the low-skilled labor.

Now consider a firm which has the production technology above:

and its profit is given by

To maximize the profit, the firm solves

Let us take the ratios of wages to obtain

In general, elasticity of substitution is

To calculate the elasticity of substitution, note that

Therefore,

which means the elasticity of substitution is

Note that is the percentage change in relative wages when there is a 1% change in relative labor supply. Interestingly this value is equal to

because The impact of technology on relative wages is given by

However, at perfect substitutes and at Cobb-Douglas and at Leontief technologies. This means that the sign of the impact of technology on relative wages depend on substitubility between and .

Moreover, the income share of workers can be calculated as

By the same token,

Let us define as the “average productivity of ”.

If the average productivity of increases, does also increase? It depends on Finally note that

So the profit is zero. The reason behind result is constant returns to scale:

if . The CES technology satisfies this property: scaling up all inputs by a constant factor increases the output by the same factor.



Going back to the discussion on wage inequality (i.e. college premium) and technological change, Acemoğlu estimates

this linear model by using the following econometric model:

after assuming that grows at a constant rate. According to this formulation

Therefore, . This means

The estimation results show that

which means so and are substitutes. Moreover, which means the growth rate of the augmenting technological change is 3.3% per annum.