



Serbest Ağ Dengelemesi: Kenar Ağı Uygulaması

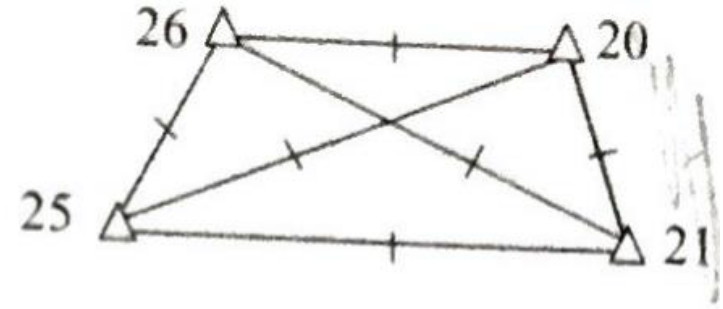
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Serbest Ağ Dengelemesi : *Uygulama-5 (Demirel H, 2009)*

Uygulama : Şekildeki kenar ağına ilişkin uzunluk ölçüleri S_{ik} ve noktaların yaklaşık koordinatları veriliyor.

j	y_{j0} (m)	x_{j0} (m)	i	k	S_{ik} (m)
20	9703.461	9333.649	20	21	3994.990
21	9016.047	5398.316	20	25	5706.900
25	5382.220	5605.971	20	26	3652.826
26	6143.636	8514.456	21	25	3639.745
			21	26	4238.070
			25	26	3006.459



$$\sigma_s = 5 + 2 \text{ ppm}$$

- Ağı tüm iz minimum yöntemiyle dengeleyiniz.
- Ağı zorlamasız klasik dengeleme yöntemiyle dengeleyiniz

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ÇÖZÜM:

Ölçü Sayısı : $n = 6$ adet kenar ölçümü

Datum Sayısı : $d = 3$

Bilinmeyen Sayısı : $u = 8$ adet nokta koordinat bileşenleri

Serbestlik Derecesi : $f = n - u + d = 6 - 8 + 3 = 1 > 0$

DENGELEME YAPILABİLİR ✓

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Katsayı ve ağırlık hesabı:

$$\sigma_0 = 1.30 \text{ cm}$$

$$P = \sigma_0^2 / \sigma_i^2$$

i	k	$x_{k0} - x_{i0}$	$y_{k0} - y_{i0}$	S_{ik0}	t_{ik0}	$a_{S_{ik}}$	$b_{S_{ik}}$	Denet.	σ_s	P
20	21	-3935.333	-687.414	3994.9198	211.00924	-0.9851	-0.1721	1.0000	1.30	1.00
	25	-3727.678	-4321.241	5706.9000	254.68623	-0.6532	-0.7572	1.0000	1.64	0.63
	26	-819.193	-3559.825	3652.8662	285.60068	-0.2243	-0.9745	1.0000	1.23	1.11
21	25	207.655	-3633.827	3639.7554	303.63401	0.0571	-0.9984	1.0000	1.23	1.12
	26	3116.140	-2872.411	4238.0507	352.58956	0.7353	-0.6778	1.0000	1.35	0.93
25	26	2908.485	761.416	3006.4995	16.30034	0.9674	0.2533	1.0000	1.10	1.39


$$\sigma_s = 5 + 2 \text{ ppm}$$

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Düzeltilme Denklemleri :

V_S	δx_{20}	δy_{20}	δx_{21}	δy_{21}	δx_{25}	δy_{25}	δx_{26}	δy_{26}	$-l \text{ (cm)}$
20-21	0.9851	0.1721	-0.9851	-0.1721	0	0	0	0	-7.02
20-25	0.6532	0.7572	0	0	-0.6532	-0.7572	0	0	0
20-26	0.2243	0.9745	0	0	0	0	-0.2243	-0.9745	4.02
21-25	0	0	-0.0571	0.9984	0.0571	-0.9984	0	0	1.04
21-26	0	0	-0.7353	0.6778	0	0	0.7353	-0.6778	-1.93
25-26	0	0	0	0	-0.9674	-0.2533	0.9674	0.2533	4.05

$$A_g = \begin{bmatrix} 0.9851 & 0.1721 & -0.9851 & -0.1721 & 0 & 0 & 0 & 0 \\ 0.6532 & 0.7572 & 0 & 0 & -0.6532 & -0.7572 & 0 & 0 \\ 0.2243 & 0.9745 & 0 & 0 & 0 & 0 & -0.2243 & -0.9745 \\ 0 & 0 & -0.0571 & 0.9984 & 0.0571 & -0.9984 & 0 & 0 \\ 0 & 0 & -0.7353 & 0.6778 & 0 & 0 & 0.7353 & -0.6778 \\ 0 & 0 & 0 & 0 & -0.9674 & -0.2533 & 0.9674 & 0.2533 \end{bmatrix}$$

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Normal Denklemler:

$$\mathbf{N}_g = \begin{bmatrix} 1.2951 & 0.7238 & -0.9704 & -0.1695 & -0.2688 & -0.3116 & -0.0558 & -0.2426 \\ & 1.4449 & -0.1695 & -0.0296 & -0.3116 & -0.3612 & -0.2426 & -1.0541 \\ & & 1.4769 & -0.3578 & -0.0037 & 0.0638 & -0.5028 & 0.4635 \\ & & & 1.5733 & 0.0638 & -1.1164 & 0.4635 & -0.4273 \\ & & & & 1.5733 & 0.5884 & -1.3008 & -0.3406 \\ & \text{Sim.} & & & & 1.5668 & -0.3406 & -0.0892 \\ & & & & & & 1.8595 & 0.1197 \\ & & & & & & & 1.5705 \end{bmatrix}$$

$$\mathbf{n}_g = \mathbf{A}_g^T \mathbf{P} \mathbf{l} = [5.9145 \quad -3.1403 \quad -8.1687 \quad -1.1545 \quad 5.3795 \quad 2.5889 \quad -3.1253 \quad 1.7059]^T$$

Serbest Ağ Dengelemesi : *Uygulama-5 (Demirel H, 2009)*

j	y_{j0} (m)	x_{j0} (m)	$y_{j0} - y_s$ (m)	$x_{j0} - x_s$ (m)
20	9703.461	9333.649	2142.120	2120.551
21	9016.047	5398.316	1454.706	-1814.782
25	5382.220	5605.971	-2179.121	-1607.127
26	6143.636	8514.456	-1417.705	1301.358
Σ	30245.364	28852.392	0.000	0.000
Σ/p	$y_s = 7561.341$	$x_s = 7213.098$		

$$\mathbf{G}^T = \begin{bmatrix} 0.5 & 0 & 0.5 & 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0.5 & 0 & 0.5 & 0 & 0.5 & 0 & 0.5 \\ -0.4240 & 0.4197 & -0.2879 & -0.3592 & 0.4313 & -0.3181 & 0.2806 & 0.2576 \end{bmatrix}$$

$$\mathbf{G}^T \mathbf{G} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \mathbf{E}$$

Serbest Ağ Dengelemesi : *Uygulama-5 (Demirel H, 2009)*

$$(Q_g = N^+ = (N + G G^T)^{-1} - G G^T)$$

$$Q_g = \begin{bmatrix} 0.2907 & 0.0260 & -0.1643 & -0.0908 & -0.0113 & -0.0921 & -0.1150 & 0.1568 \\ & 0.2553 & 0.1372 & -0.0738 & -0.1117 & -0.0455 & -0.0515 & -0.1361 \\ & & 0.3366 & -0.0408 & -0.1074 & -0.0772 & -0.0649 & -0.0192 \\ & & & 0.2599 & 0.1078 & -0.1198 & 0.0238 & -0.0663 \\ & & \text{Sim.} & & 0.2244 & 0.0319 & -0.1057 & -0.0279 \\ & & & & & 0.3126 & 0.1374 & -0.1473 \\ & & & & & & 0.2856 & -0.1096 \\ & & & & & & & 0.3497 \end{bmatrix}$$

$$\mathbf{x}_g = [\delta x_{20} \quad \delta y_{20} \quad \delta x_{21} \quad \delta y_{21} \quad \delta x_{25} \quad \delta y_{25} \quad \delta x_{26} \quad \delta y_{26}]^T$$

$$\mathbf{x}_g = Q_g \mathbf{n}_g = [3.41 \quad -2.47 \quad -4.71 \quad -0.19 \quad 2.61 \quad 0.67 \quad -1.31 \quad 2.00]^T \text{ (cm)} , \quad \mathbf{e}^T \mathbf{x}_g = \mathbf{0}$$

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Dengeli koordinatlar:

j	y_{j0} (m)	δy_i (cm)	$y_i = y_{j0} + \delta y_i$	x_{j0} (m)	δx_i (cm)	$x = x_{j0} + \delta x_i$
20	9703.461	- 2.47	9703.4363	9333.649	3.4125	9333.6831
21	9016.047	- 0.19	9016.0451	5398.316	- 4.7131	5398.2689
25	5382.220	0.67	5382.2267	5605.971	2.6095	5605.9971
26	6143.636	2.00	6143.6560	8514.456	- 1.3090	8514.4429

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Hata Hesabı:

$$\mathbf{v} = \mathbf{A}_g \mathbf{x}_g - \mathbf{l} = [0.59 \quad -1.85 \quad 0.72 \quad 0.60 \quad -0.91 \quad 0.60]^T \leftarrow \text{düzeltmeler}$$

$$\mathbf{v}^T \mathbf{P} \mathbf{v} = 4.76 \text{ cm}^2$$

Birim ağırlıklı ölçünün standart sapması; $s_0 = \sqrt{4.76 / (6 - 8 + 3)} = 2.18 \text{ cm}$

Bilinmeyen koordinatların standart sapmaları ($s_{x_j} = s_0 \sqrt{Q_{x_j x_j}}$, $s_{y_j} = s_0 \sqrt{Q_{y_j y_j}}$) ve

nokta konum hataları ($s_{p_j} = \sqrt{s_{x_j}^2 + s_{y_j}^2}$):

j	$Q_{x_j x_j}$	$Q_{y_j y_j}$	$s_{x_j} \text{ (cm)}$	$s_{y_j} \text{ (cm)}$	$s_{p_j} \text{ (cm)}$
20	0.2907	0.2553	1.18	1.10	1.61
21	0.3366	0.2599	1.27	1.11	1.69
25	0.2244	0.3126	1.03	1.22	1.60
26	0.2856	0.3497	1.17	1.29	1.74

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b) Datum sayısı $d = 3'$ tür. y_{25} , x_{26} , y_{26} koordinatları sabit olsun. Tüm iz minimum çözümünden elde edilen A_g katsayılar matrisinden bu koordinatlara ilişkin sondaki 3 sütün silinerek elde edilen A katsayılar matrisi elde edilir:

$$A = \begin{bmatrix} 0.9851 & 0.1721 & -0.9851 & -0.1721 & 0 \\ 0.6532 & 0.7572 & 0 & 0 & -0.6532 \\ 0.2243 & 0.9745 & 0 & 0 & 0 \\ 0 & 0 & -0.0571 & 0.9984 & 0.0571 \\ 0 & 0 & -0.7353 & 0.6778 & 0 \\ 0 & 0 & 0 & 0 & -0.9674 \end{bmatrix}$$

← Zorlamasız klasik çözüme ilişkin
A katsayılar matrisi

$$N = A^T P A = \begin{bmatrix} 1.2921 & 0.7238 & -0.9704 & -0.1695 & -0.2688 \\ & 1.4449 & -0.1695 & -0.0296 & -0.3116 \\ & & 1.4769 & -0.3578 & -0.0037 \\ & \text{Sim.} & & 1.5733 & 0.0638 \\ & & & & 1.5733 \end{bmatrix}, \quad n = A^T P l = \begin{bmatrix} 5.9145 \\ -3.1403 \\ -8.1687 \\ -1.1545 \\ 5.3795 \end{bmatrix}$$

Normal denklem
elemanları

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$$Q = N^{-1} = \begin{bmatrix} 3.4543 & -1.3782 & 2.3212 & 0.8625 & 0.2876 \\ & 1.2844 & -0.8344 & -0.3153 & 0.0298 \\ & & 2.2878 & 0.7464 & 0.2063 \\ & \text{Sim.} & & 0.8903 & 0.0505 \\ & & & & 0.6891 \end{bmatrix}, \quad \text{iz } Q = 8.6059$$

Bilinmeyenler:

$$x = Q n = \begin{bmatrix} \delta x_{20} \\ \delta y_{20} \\ \delta x_{21} \\ \delta y_{21} \\ \delta x_{25} \end{bmatrix} = \begin{bmatrix} 6.35 \\ -4.84 \\ -2.09 \\ -0.76 \\ 3.57 \end{bmatrix} \text{ (cm)},$$

$$x_{20} = x_{20,0} + \delta x_{20} = 9333.7125$$

$$y_{20} = y_{20,0} + \delta y_{20} = 9703.4126$$

$$x_{21} = x_{21,0} + \delta x_{21} = 5398.2951$$

$$y_{21} = y_{21,0} + \delta y_{21} = 9016.0394$$

$$x_{25} = x_{25,0} + \delta x_{25} = 5606.0067$$

Dengeli koordinatlar

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Düzeltilmeler:

$$\mathbf{v} = \mathbf{A} \mathbf{x} - \mathbf{l} = [0.59 \quad -1.85 \quad 0.72 \quad 0.60 \quad -0.91 \quad 0.60]^T, \mathbf{v}^T \mathbf{P} \mathbf{v} = 4.76 \text{ cm}^2 \quad (= \text{Çözüm (a)})$$

Dengeli ölçüler: (= Çözüm (a))

Birim ağırlıklı ölçünün standart sapması; $s_0 = \sqrt{4.76 / (6 - 5)} = 2.18 \text{ cm} \quad (= \text{Çözüm (a)})$

Koordinat ve nokta konum standart sapmaları:

$$\mathbf{f} = \mathbf{n} - \mathbf{u} = 6 - 5$$

j	$Q_{x_j x_j}$	$Q_{y_j y_j}$	$s_{x_j} \text{ (cm)}$	$s_{y_j} \text{ (cm)}$	$s_{p_j} \text{ (cm)}$
20	3.4543	1.2844	4.06	2.47	4.75
21	2.2878	0.8903	3.30	2.06	3.89
25	0.6891	—	1.81	—	1.81

Ders duyuruları, soru ve görüşleriniz için:



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