System Dynamics, Modeling and Simulation	KOM5107

Modeling & Simulation of Dynamic Systems (MSDS)

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Process Modeling: Blending Tank and fluid systems

A Systematical Approach for Modeling

1. State the modeling objectives and the end use of the model.
2. Determine the required levels of model detail and model accuracy.
3. Draw a schematic diagram of the process and label all process variables.
4. List all of the assumptions involved in developing the model. The model should not be no more complicated than necessary to meet the modeling objectives.
5. Determine if spatial variations are important. If so, a partial differential equation model will be required.

6. Write appropriate conservation equations (mass, component, energy, etc.)
7. Introduce constitutive equations, which are equilibrium relations and other algebraic equations.
8. Perform a degrees of freedom analysis to ensure that the model equations can be solved.
9. Simplify the model output = f (inputs)
10. Is this model form convenient for computer simulation and analysis?
11. Classify inputs as disturbance variables or as manipulated variables.

The input variables can be further classified as:

1. Manipulated (or adjustable) variables, if their values can be adjusted freely by the human operator or a control mechanism.
2. Disturbances, if their values are not the result of adjustment by an operator or a control system.

The output variables are also classified as:

1. Measured output variables, if their values are known by directly measuring them.
2. Unmeasured output variables, if they are not or cannot be measured directly.

A mass balance gives:

$$\left\{\begin{array}{l}\text{rate of accumulation of} \\ \text{mass in the tank}\end{array}\right\} = \left\{\begin{array}{c}\text{rate of} \\ \text{mass in}\end{array}\right\} - \left\{\begin{array}{c}\text{rate of} \\ \text{mass out}\end{array}\right\}$$

The mass of liquid in the tank can be expressed as product of the volume of the liquid and its density.

System: liquid in the tank

assumptions:

1. tank is well mixed = density of liquid is not changing
2. Volume is variable

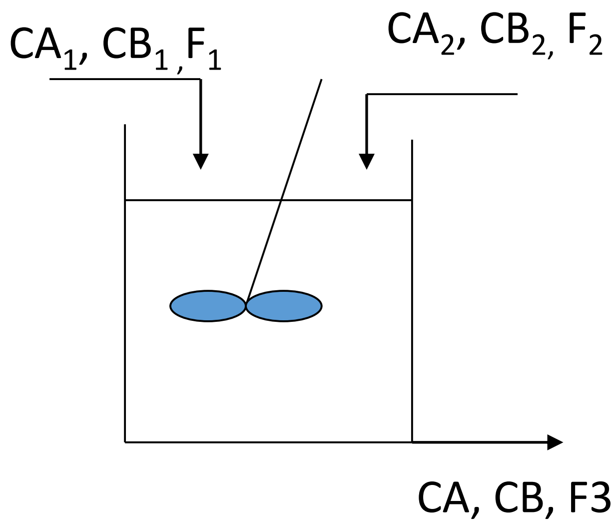
Process Modeling: Blending Tank

Consider a continuous stirred tank blending system where two input systems are blended to produce an outlet stream F3 that has desired composition CA and CB.

Stream 1 is a mass fraction composition CA1 and CB1 and its mass flow rate F1

Stream 2 consists of CA2 and CB2. The mass flow rate of stream 2 is F_2

The *objective* here is to develop a model that relates the inputs to outputs CA and CB that we wish to regulate



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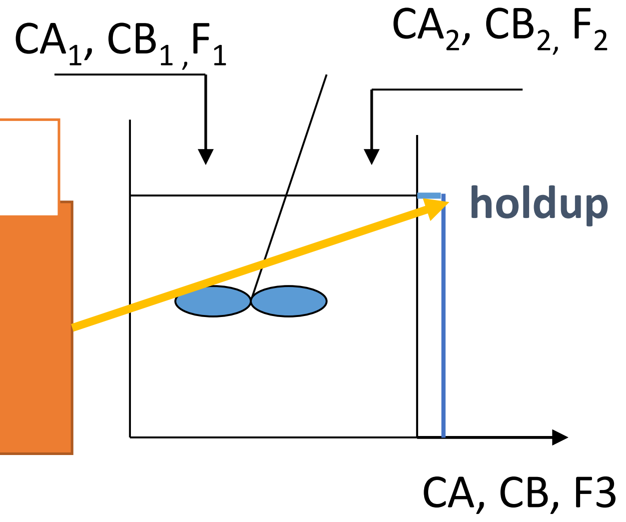
Do it yourself!

What happen with a constant Volume ???

- Obtain the system dynamics



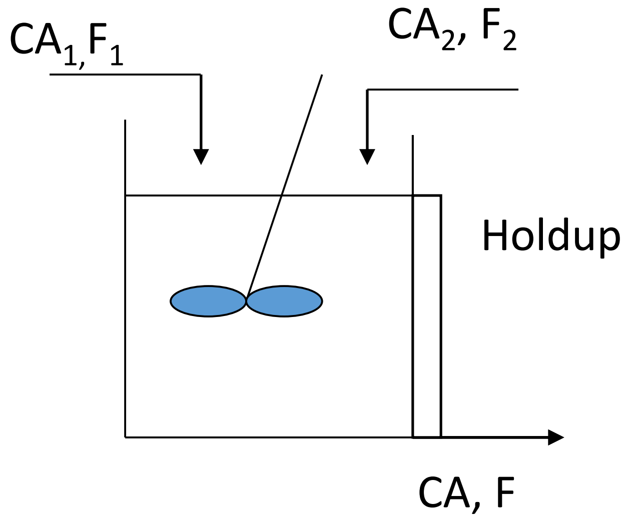
Example: Blending Tank

Consider a continuous stirred tank blending system where two input systems are blended to produce an outlet stream F that has desired composition CA.

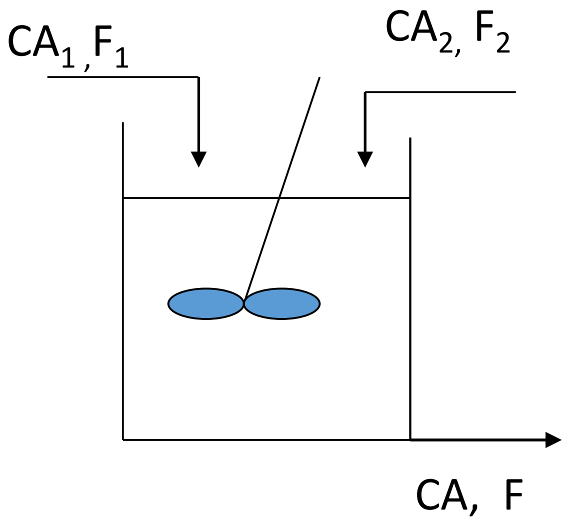
Stream 1 is a mass fraction composition CA1 and its flow rate F1

Stream 2 consists of CA2. The flow rate of stream 2 is F_2

The *objective* here is to develop a model that relates the inputs to output CA that we wish to regulate



Example: Blending Tank - Degrees of Freedom Analysis: Constant density , Variable volume



Degrees of Freedom Analysis:

Parameter(s): ρ

variables: V, CA_1, F_1, CA_2, F_2, CA, F

equations: (dV/dt and dCA/dt)

D.O.F = 7-2 = 5

outputs: V, CA

inputs: CA_1, F_1, CA_2, F_2, F

manipulated variables: F_2, F

disturbances: CA_1, F_1, CA_2

$$\boxed{\frac{d(\mathrm{V})}{dt} = F_1 + F_2 - F}$$

$$\boxed{\frac{d(CA)}{dt} = \frac{F_1}{V}(CA_1 - CA) + \frac{F_2}{V}(CA_2 - CA)}$$

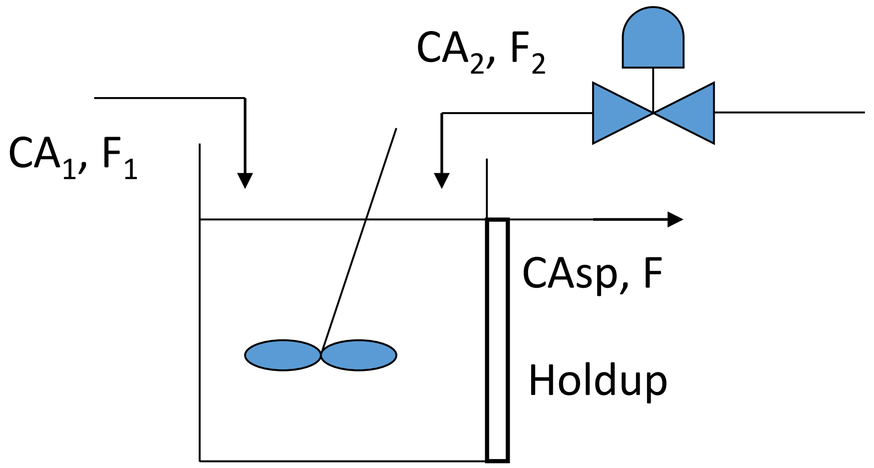
Example: Blending Tank

Consider a continuous stirred tank blending system where two input systems F1 and F2 are blended to produce an outlet stream F that has the desired composition CAsp. The tank's volume is controlled using a hold-up.

Stream 1 is a mass fraction composition CA1. We assume that its mass flow rate (F1) is constant that is given, but the mass fraction of CA1 varies with time.

Stream 2 consists of pure CA2 and thus CA2=1. The mass flow rate (F_2) of stream 2 can be manipulated using a control valve.

The mass fraction in the exit stream is denoted by CA and the desired value by CA_{sp}.



Assume that the process has been operating for a long period of time

- Design Question: If the value of CA_1 is $CA_{1,s}$ a measured variable what nominal flow rate F_2 is required to produce the desired outlet concentration CA_{sp}.

With a st-st material balance,

$F_1 + F_2 - F = 0$ (overall balance)

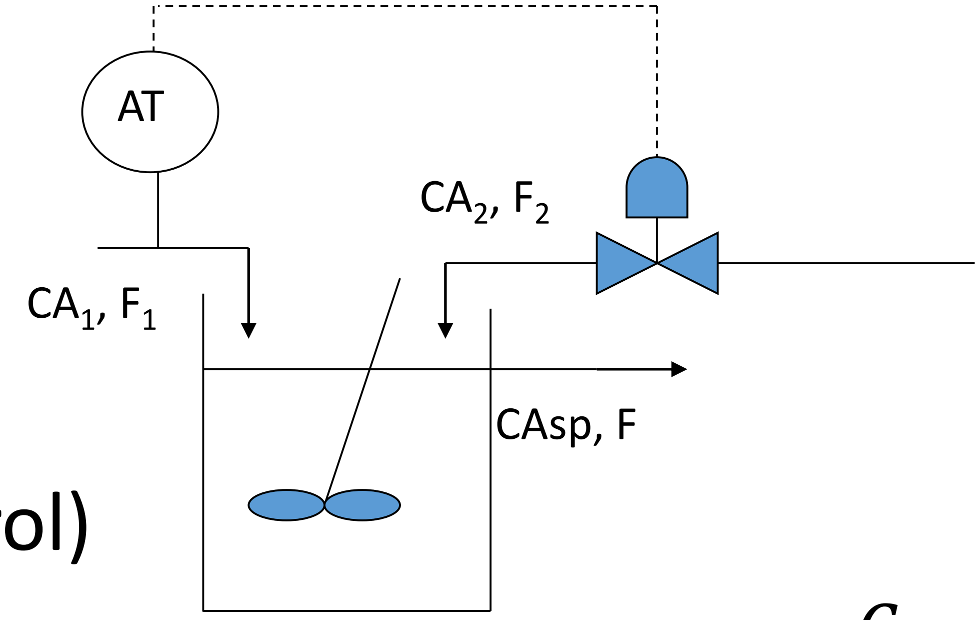
$F_1CA_1 + F_2CA_2 - FCA_{sp} = 0$ (component A balance)

$F_1CA_1+F_2(1.0)-(F_1+F_2)CA_{sp}=0$

$$F_2 = F_1 \frac{C_{A1} - C_{A-sp}}{C_{A-sp} - 1}$$

- Method 1

Measure CA_1 and adjust F_2. F_1 is a known constant value



(Feed forward Control)

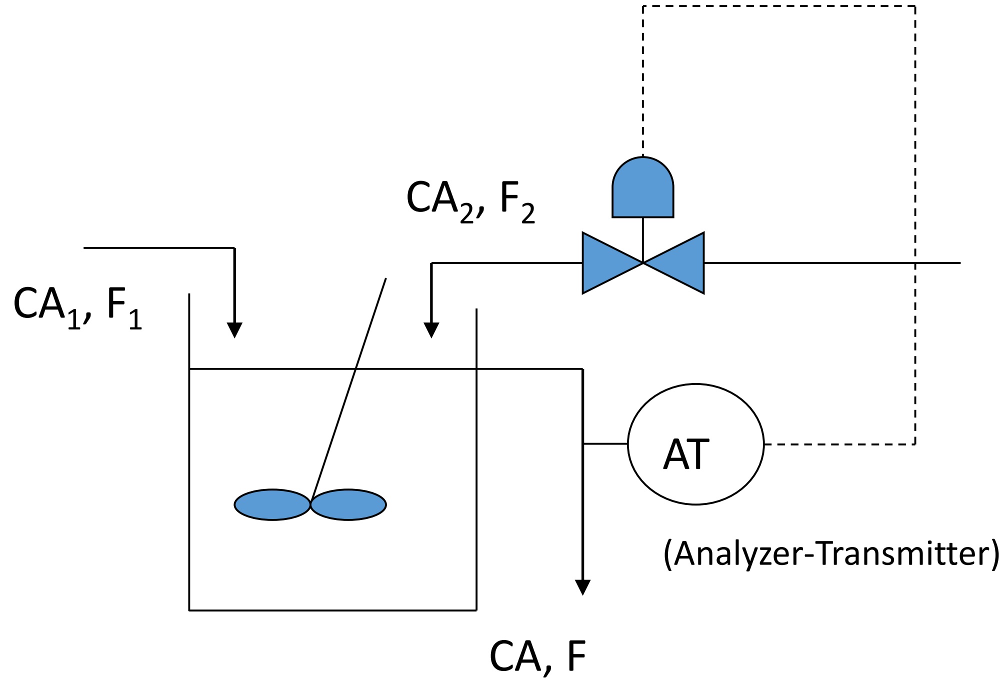
$$F_2 = F_1 \frac{C_{A1} - C_{A-sp}}{C_{A-sp} - 1}$$

- Method 2.

Measure CA and adjust F_2.

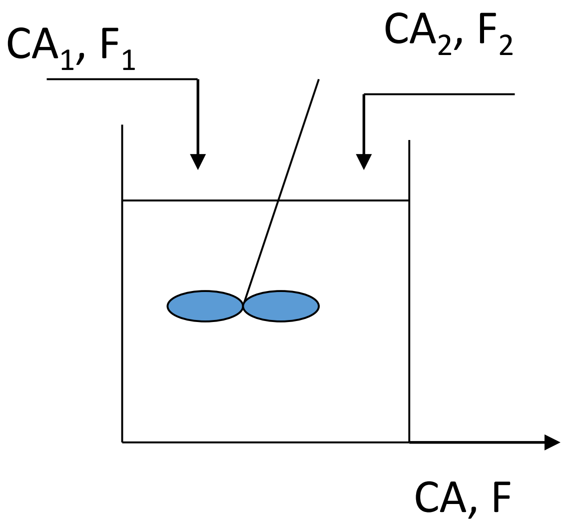
- if CA is high, F_2 should be reduced
- if CA is low, F_2 should be increased

$$F_2 = F_1 \frac{C_{A1} - C_{A-sp}}{C_{A-sp} - 1}$$



(Feedback Control)

Do it yourself!

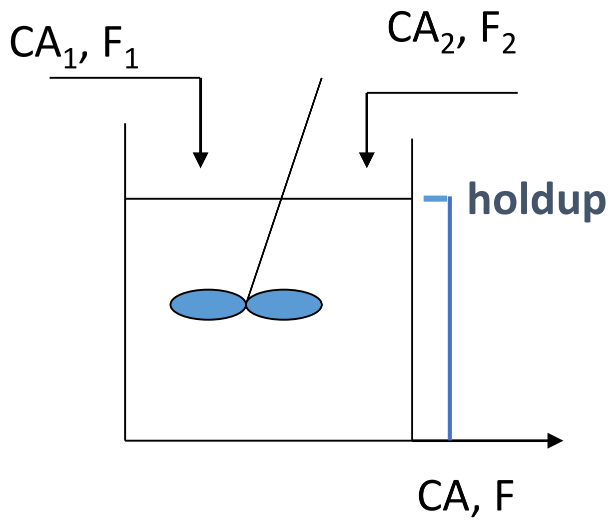


- Consider a more general version of the blending system where stream 2 is not pure and volume of the tank may vary with time.

(Not an overflow system any more but a draining system!)

Objective is again to keep CA at the desired value CA_{sp}

Example



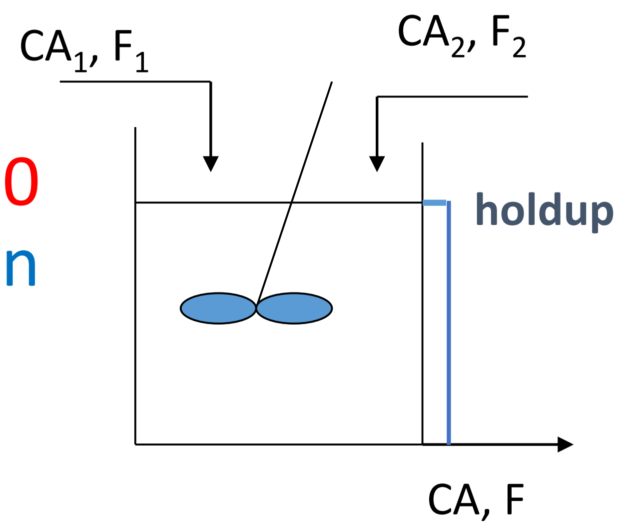
A stirred-tank blending process with a constant liquid holdup of 2 m^3 is used to blend two streams whose densities are both approximately 900 kg/m^3. The density is constant during mixing.

a) Assume that the process has been operating for a long period.

with flow rates of $F1$ = 500 kg/min and $F2$ = 200 kg/min, and feed compositions (mass fractions) of $CA1$ = 0.4 and $CA2$ = 0.75.

What is the steady-state value of CA?

b) Suppose that $F1$ changes suddenly from 500 to 400 kg/min and remains at the new value. Determine an expression for $CA(t)$



$$\boxed{CA(t) = 0.5e^{-t/3} + C^*(1 - e^{-t/3})}$$

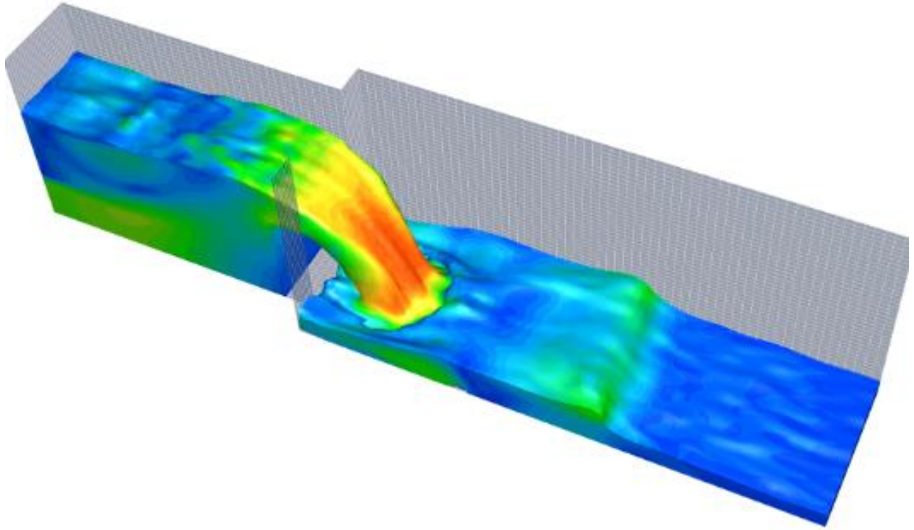
$$C^* = \frac{(400\text{ kg/min})(0.4) + (200\text{ kg/min})(0.75)}{600\text{ kg/min}} = 0.517$$

$$\bar{CA} = \frac{(500\text{ kg/min})(0.4) + (200\text{kg/min})(0.75)}{700\text{kg/min}} = 0.5$$

Hydraulic Systems

Modelling fluid systems

First order ODE
modelling fluid systems



Fluid systems

- Comprising tanks and pipes.
- Liquid systems with interaction and the system transfer function

We ignore the changes in momentum due to changes in flow rate

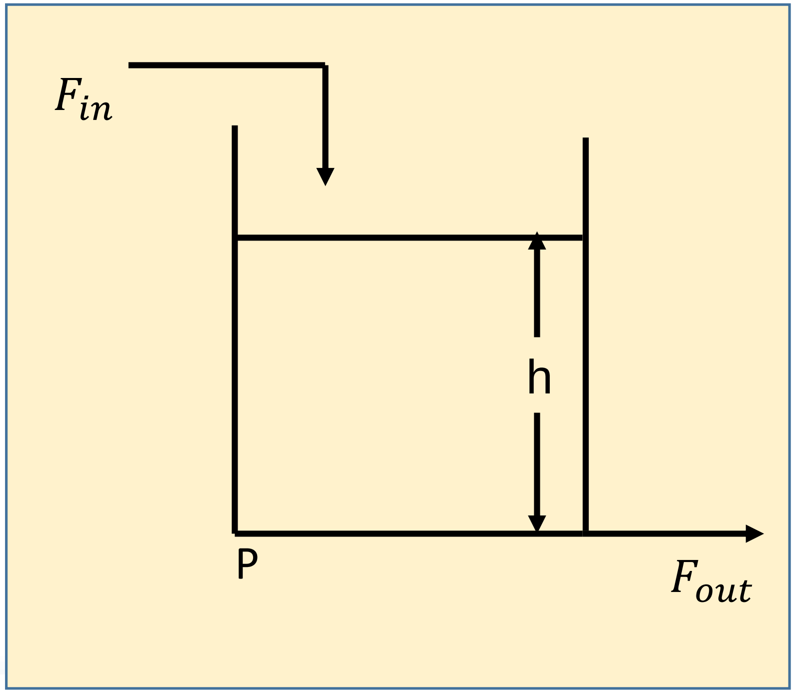
Pressure in tanks

Fluid flow is driven by differential pressure.

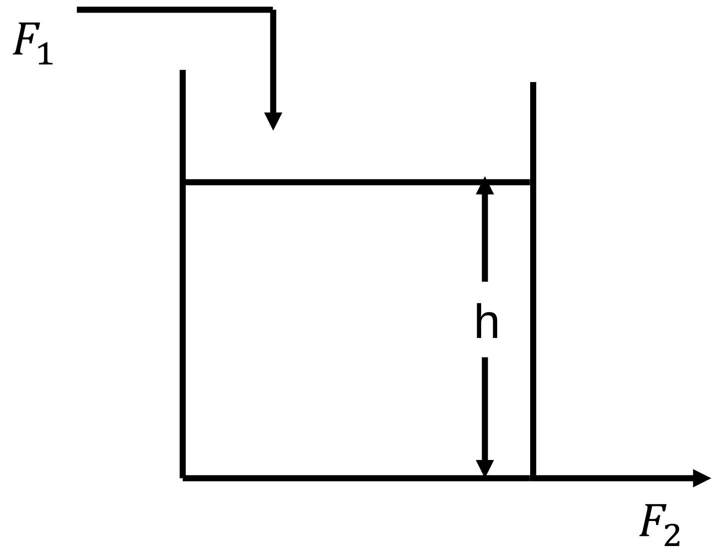
A common source of fluid pressure is fluid depth, the deeper the fluid the higher the associated pressure.

A well known formulae is:

$$\boxed{P = \rho g h}$$



Example



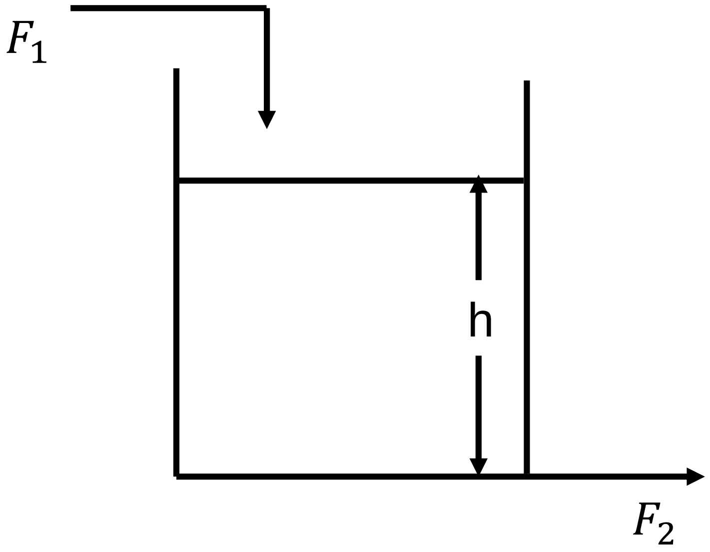
Consider the typical liquid storage process shown in the figure, where F_1 and F_2 are volumetric flow rates.

Assuming constant density and cross sectional area A, a mass balance gives:

$$A\frac{dh}{dt} = F_1 - F_2$$

There are three important variations in the liquid storage processes:

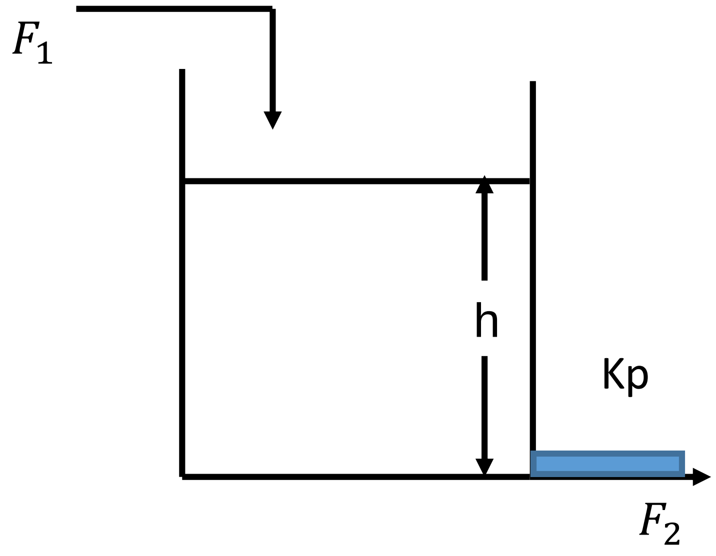
1. The inlet or outlet flow rates might be constant. In that case the exit flow rate is independent of the liquid level over a wide range of conditions. Consequently $F_1 = F_2$ at the steady state conditions.



2. The tank exit line may function simply as a resistance to flow from the tank or it may contain a valve that provides significant resistance to flow at a single point. In the simplest case, the flow may be assumed to be linearly related to the driving force, the liquid level.

$$F_2 = \frac{1}{R} h$$

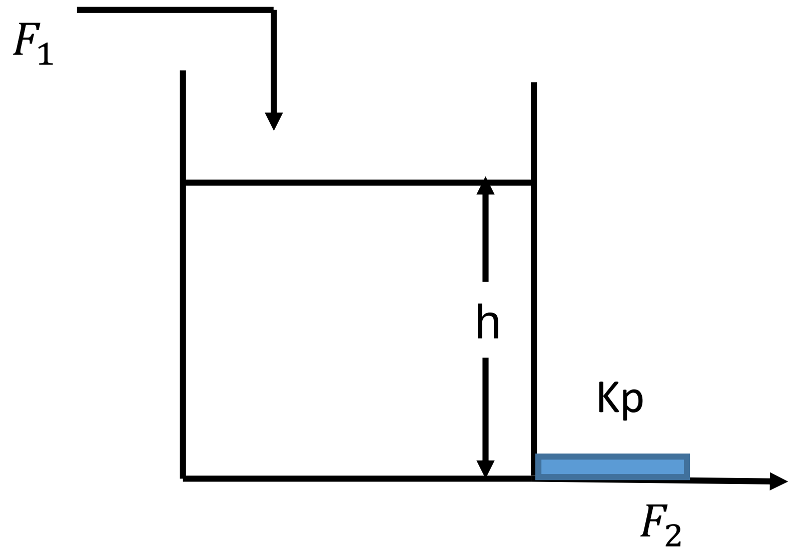
$$A\frac{dh}{dt} = F_1 - \frac{1}{R} h$$

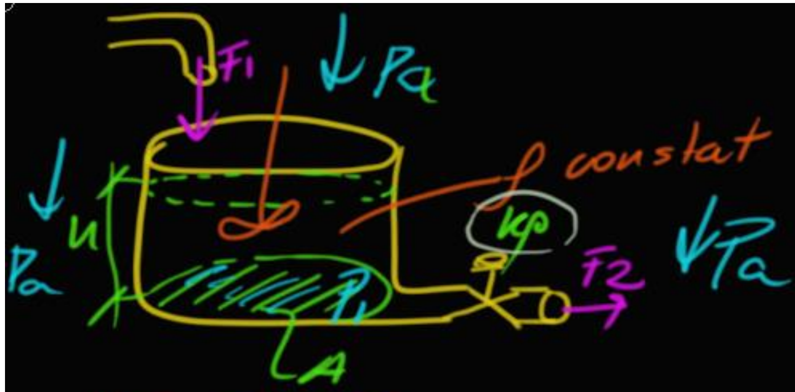


3. A more realistic expression for flow rate F_2 can be obtained when a fixed valve has been placed in the exit line and turbulent flow can be assumed. The driving force for flow through the valve is the pressure drop ΔP, ΔP=P-Pa where P is pressure at the bottom of the tank and Pa is pressure at the end of the exit line.

$$F_2 = C_v * \sqrt{\frac{\Delta P}{\rho}}$$

C_v is the valve constant





Dynamic Equation of the syste that includes a valve

P_a Atmospheric pressure

K_p : Value constant of the valve

ρ density

g gravity constant

h level of the tank

R : hydraulic resistance

$$\Delta P = P_1 - P_a$$

$$\rightarrow P_1 = \rho g h + P_a$$

$$* \ \Delta P = \rho g h + \cancel{P_a} - \cancel{P_a}$$

$$\Delta P = \rho g h = K_p F_2$$

$$F_2 = \frac{\rho g h}{K_p} = \frac{h}{R} \ ; \quad \frac{1}{R} = \frac{\rho g}{K_p}$$

$$A\frac{dh}{dt} = F_1 - F_2 \rightarrow A\frac{dh}{dt} = F_1 - \frac{h}{R}$$

$$\rightarrow A\frac{dh}{dt} + \frac{h}{R} = F_1$$

Linear differential equation

Cv = valve flow constant

$$F_2 = C_v \sqrt{\frac{\Delta P}{\rho}} = C_v \sqrt{\frac{\cancel{\rho} g h}{\cancel{\rho}}}$$

$$F_2 = C_v \sqrt{gh} = \underbrace{C_v \sqrt{g}}_{k} \sqrt{h} = k\sqrt{h}$$

The new dynamic equation

$$A\frac{dh}{dt} = F_1 - k\sqrt{h}$$

Non-linear !! differential equation.

Problem 1. A process tank has two input streams—Stream 1 at mass flow rate F1 and Stream 2 at mass flow rate F2. The tank's effluent stream, at flow rate F, discharges through a fixed valve to atmospheric pressure. The cross-sectional area of the tank, A is $5 m^2$, and the mass density of all streams is $940\ kg/m^3$.

(a) Draw a schematic diagram of the process and write an appropriate dynamic model for the tank level. What is the corresponding steady-state model?

(b) At initial steady-state conditions, with F1 =2.0 kg/s and F2 =1.2 kg/s, the tank level is 2.25 m. What is the value of the valve constant (give units)?

(c) A process control engineer decides to use a feed-forward controller to hold the level approximately constant at the set-point value (hsp=2.25 m) by measuring F1 and manipulating F2. What is the mathematical relation that will be used in the controller? If the F1 measurement is not very accurate and always supplies a value that is 1.1 times the actual flow rate, what can you conclude about the resulting level control?

(Hint: Consider the process initially at the desired steady-state level and with the feed-forward controller turned on. Because the controller output is slightly in error, F2 ≠1.2, so the process will come to a new steady state. What is it?)

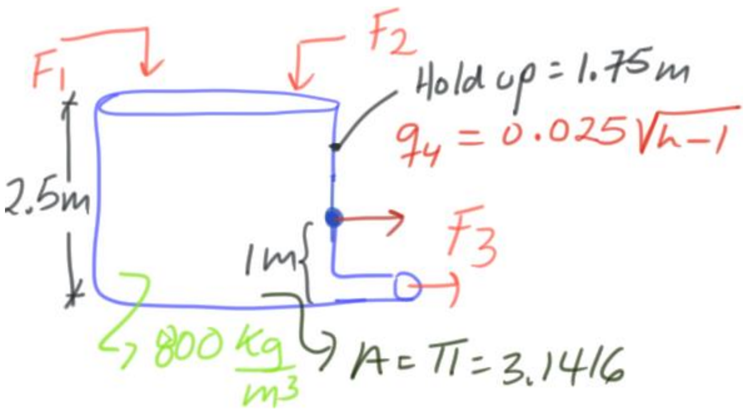
(d) What conclusions can you draw concerning the need for accuracy in a steady-state model? For the accuracy of the measurement device? For the accuracy of the control valve? Consider all of these with respect to their use in a feed-forward control system.

Do it yourself!

Problem 2. The liquid storage tank shown in the figure has two inlet streams with mass flow rates F1 and F2 and an exit stream with flow rate F3. The cylindrical tank is 2.5 m tall and 2 m in diameter. The liquid has a density of 800 kg/m^3.
Normal operating procedure is to fill the tank until the liquid level reaches a nominal value of 1.75 m using constant flow rates: F1 =120 kg/min, F2 =100 kg/min, and F3 =200 kg/min. At that point, inlet flow rate F1 is adjusted so that the level remains constant. However, on this particular day, corrosion of the tank has opened up a hole in the wall at a height of 1 m, producing a leak whose volumetric flow rate q4 (m^3/min) can be approximated by q4 where h is height in meters.

(a) If the tank was initially empty, how long did it take for the liquid level to reach the corrosion point?

(b) If mass flow rates F1, F2, and F3 are kept constant indefinitely, will the tank eventually overflow? Justify your answer.



$$q_4 = 0.025\sqrt{h-1}$$

Do it yourself!

Problem 3. Consider a blending tank that has the same dimensions and nominal flow rates as the storage tank in problem 2 but that incorporates a valve on the outflow line that is used to establish flow rate F3= $Cv\sqrt{h}$. In addition, the nominal inlet stream mass fractions of component A are CA1 =CA2 =0.5.

The process has been operating for a long time with constant flow rates and inlet concentrations. Under these conditions, it has come to steady state with exit mass fraction CA=0.5 and level h=1.75 m. Using the information below, answer the following questions:

(a) What is the value of F3 and the constant Cv?

(b) If CA1 suddenly changed from 0.5 to 0.6 without changing the inlet flow rates, what is the final value of **CA3**? How long does it take to come within 1% of this final value?

(c) If F1 is changed from 120 kg/min to 100 kg/min without changing the inlet concentrations, what will be the final value of the tank level? How long will it take to come within 1% of this final value?

Useful information: The tank is perfectly stirred.

Do it yourself!