KISMİ İNTEGRAL

$u = u(x)$, $v = v(x)$ olsun.

$d(u \cdot v) = v\,du + u\,dv$ olur.

$\int d(uv) = \int v\,du + \int u\,dv$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Kısmi integral formülü

Kısmi integral genellikle iki fonksiyonun çarpımının integralinde uygunsa kullanılır.

1) Polinom ile üstel fonksiyonunun çarpımı olduğunda:

$$\underbrace{\text{Polinom}}_{u} \text{ ile } \underbrace{\text{üstel fonksiyonun}}_{dv}$$

$u \downarrow du$ $\qquad dv \downarrow v$

Örnek: $I = \int (x-2)e^{3x}dx = ?$

$x - 2 = u \qquad e^{3x}dx = dv$

$dx = du \qquad \frac{1}{3}e^{3x} = v$

$I = uv - \int v\,du$

$I = \frac{1}{3}(x-2)e^{3x} - \frac{1}{3}\int e^{3x}dx \Rightarrow \boxed{I = \frac{1}{3}(x-2)e^{3x} - \frac{1}{9}e^{3x} + c}$

Örnek: $I = \int x^2 \cdot e^x dx = ?$

$x^2 = u \qquad e^x dx = dv$

$2x dx = du \qquad e^x = v$

$I = x^2 e^x - 2\underbrace{\int x e^x dx}_{I_1} \qquad x = u \quad e^x dx = dv$

$dx = du \quad e^x = v$

$I_1 = x e^x - \int e^x dx$

$\boxed{I_1 = x e^x - e^x}$

$\boxed{I = x^2 e^x - 2(x e^x - e^x) + C}$

"Görüldüğü gibi polinomun derecesi kadar kısmi integral kuralı uygulanır."

2) $\underbrace{\text{Polinom}}_{u}$ ile trigonometrik fonk. çarpımı $\underbrace{(\sin, \cos)}_{dv}$

Örnek: $I = \int (2x+3) \cdot \cos 2x \, dx = ?$

$2x+3 = u, \quad \cos 2x \, dx = dv$

$2dx = du, \quad \frac{1}{2}\sin 2x = v$

$I = (2x+3) \cdot \frac{1}{2}\sin 2x - \int \frac{1}{\cancel{2}} \cdot \cancel{2} \cdot \sin 2x \, dx$

$\boxed{I = \frac{1}{2}(2x+3)\sin 2x + \frac{1}{2}\cos 2x + C}$

3) Üstel fonksiyon ile trigonometrik fonk. çarpımı (sin, cos)

Örnek: $I = \int e^{3x} \cdot \cos 2x \, dx = ?$

$e^{3x} = u \qquad \cos 2x \, dx = dv$

$3e^{3x} dx = du \qquad \frac{1}{2} \sin 2x = v$

$$I = \frac{1}{2} e^{3x} \cdot \sin 2x - \frac{3}{2} \underbrace{\int e^{3x} \cdot \sin 2x \, dx}_{I_1}$$

$e^{3x} = u \quad , \quad \sin 2x \, dx = dv$

$3e^{3x} dx = du \qquad -\frac{1}{2} \cos 2x = v$

$$I_1 = -\frac{1}{2} e^{3x} \cdot \cos 2x + \frac{3}{2} \underbrace{\int e^{3x} \cdot \cos 2x \, dx}_{I}$$

$$I = \frac{1}{2} e^{3x} \cdot \sin 2x - \frac{3}{2} \left[-\frac{1}{2} e^{3x} \cdot \cos 2x + \frac{3}{2} I \right]$$

$$I + \frac{9}{4} I = \frac{1}{2} e^{3x} \sin 2x + \frac{3}{4} e^{3x} \cos 2x$$

$$I = \frac{4}{13} e^{3x} \left(\frac{1}{2} \sin 2x + \frac{3}{4} \cos 2x \right) + C$$

$$\boxed{I = \frac{1}{13} e^{3x} \left(2 \sin 2x + 3 \cos 2x \right) + C}$$

4) Polinom ile Logaritma fonk. çarpımı
(dv) (u)

Örnek: $I=\int (x-1).\ln x\,dx = ?$

$(x-1)\,dx = dv$ $\quad$ $\ln x = u$

$\frac{x^2}{2}-x = v$ $\quad$ $\frac{1}{x}dx = du$

$$I=\left(\frac{x^2}{2}-x\right)\ln x - \int\left(\frac{x^2}{2}-x\right)\cdot\frac{1}{x}\,dx$$

$$\boxed{I=\left(\frac{x^2}{2}-x\right)\ln x - \frac{x^2}{4}+x+C}$$

Örnek: $I=\int \ln x\,dx = ?$

$\ln x = u$ $\quad$ $dx = dv$

$\frac{1}{x}dx = du$ $\quad$ $x = v$

$$I = x.\ln x - \int x.\frac{1}{x}\,dx$$

$$\boxed{I = x\ln x - x + C}$$

5) Polinom ile Ters trigonometrik fonk. çarpımı
(dv) (u)

Örnek: $I = \int \arcsin x \, dx = ?$

$\arcsin x = u$

$\frac{1}{\sqrt{1-x^2}} dx = du$

$dx = dv$

$x = v$

$$I = x.\arcsin x - \underbrace{\int \frac{x}{\sqrt{1-x^2}} dx}_{I_1}$$

$1 - x^2 = t^2$ dersek

$-\cancel{2}x\,dx = \cancel{2}t\,dt$ olur

$$I_1 = -\int \frac{\cancel{t}\,dt}{\cancel{t}} = -\int dt = -t = -\sqrt{1-x^2}$$

$$\boxed{I = x.\arcsin x + \sqrt{1-x^2} + c}$$

CamScanner ile tarandı

Örnek: $I = \int x^2 \cdot \arctan x \, dx = ?$

$\arctan x = u \qquad x^2 dx = dv$

$\frac{1}{1+x^2} dx = du \qquad \frac{x^3}{3} = v$

$I = \frac{1}{3} x^3 \arctan x - \frac{1}{3} \underbrace{\int \frac{x^3}{1+x^2} dx}_{I_1}$

$I_1 = \int \frac{x^3}{1+x^2} dx = ?$

$$\begin{array}{r|l} x^3 & 1+x^2 \\ \cline{2-2} x^3 + x & x \\ \hline -x & \end{array}$$

$I_1 = \int \left(x - \frac{x}{1+x^2} \right) dx = \boxed{\frac{x^2}{2} - \frac{1}{2} \ln(1+x^2)}$

$$\boxed{I = \frac{1}{3} x^3 \arctan x - \frac{1}{6} x^2 + \frac{1}{6} \ln(1+x^2) + c}$$

Örnekler

1) $I = \int x \cdot (\ln x)^3 dx = ?$

$\ln x = t$ dersek

$\frac{1}{x} dx = dt$ olur.

$\rightarrow x = e^t$

$I = \int \cancel{e^{2t}} \cdot t^3 dt$

$t^3 = u \quad e^{2t} dt = dv$

$3t^2 dt = du \quad \frac{1}{2} e^{2t} = v$

$I = uv - \int v du$

$I = \frac{1}{2} t^3 e^{2t} - \frac{3}{2} \underbrace{\int t^2 e^{2t} dt}_{I_1}$

$t^2 = u \quad e^{2t} dt = dv$

$2t dt = du \quad \frac{1}{2} e^{2t} = v$

$I_1 = \frac{1}{2} t^2 e^{2t} - \underbrace{\int t e^{2t} dt}_{I_2}$

$t = u \quad e^{2t} dt = dv$

$dt = du \quad \frac{1}{2} e^{2t} = v$

$I_2 = \frac{1}{2} t e^{2t} - \frac{1}{2} \int e^{2t} dt$

$\boxed{I_2 = \frac{1}{2} t e^{2t} - \frac{1}{4} e^{2t}}$

$\boxed{I_1 = \frac{1}{2} t^2 e^{2t} - \frac{1}{2} t e^{2t} + \frac{1}{4} e^{2t}}$

$\boxed{I = \frac{1}{2} t^3 e^{2t} - \frac{3}{4} t^2 e^{2t} + \frac{3}{4} t e^{2t} - \frac{3}{8} e^{2t} + C}$

2) $I=\int x \cdot \sec^2 x\,dx = ?$

$x=u \qquad \sec^2 x\,dx = dv$

$dx=du \qquad \tan x = v$

$I = x\cdot\tan x - \int \tan x\,dx$

$$\boxed{I = x\tan x - \ln(\sec x) + C} \quad (*)$$

3) $I = \int x\cdot\tan^2 x\,dx = ?$

$I = \int x(\sec^2 x - 1)\,dx$

$I = \underbrace{\int x\sec^2 x\,dx}_{(*)} - \int x\,dx$

$$\boxed{I = x\tan x - \ln(\sec x) - \frac{x^2}{2} + C}$$

4) $I=\int \sec^3 x\,dx = ?$

$I = \int \sec x\cdot\sec^2 x\,dx \qquad \sec x = u \qquad \sec^2 x\,dx = dv$

$\sec x\cdot\tan x\,dx = du \qquad \tan x = v$

$I = \sec x\cdot\tan x - \int \tan^2 x\sec x\,dx$

$I = \sec x\tan x - \int(\sec^2 x - 1)\sec x\,dx$

$I = \sec x\tan x - \underbrace{\int \sec^3 x\,dx}_{I} + \int \sec x\,dx$

$$\boxed{I = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln(\sec x + \tan x) + C}$$