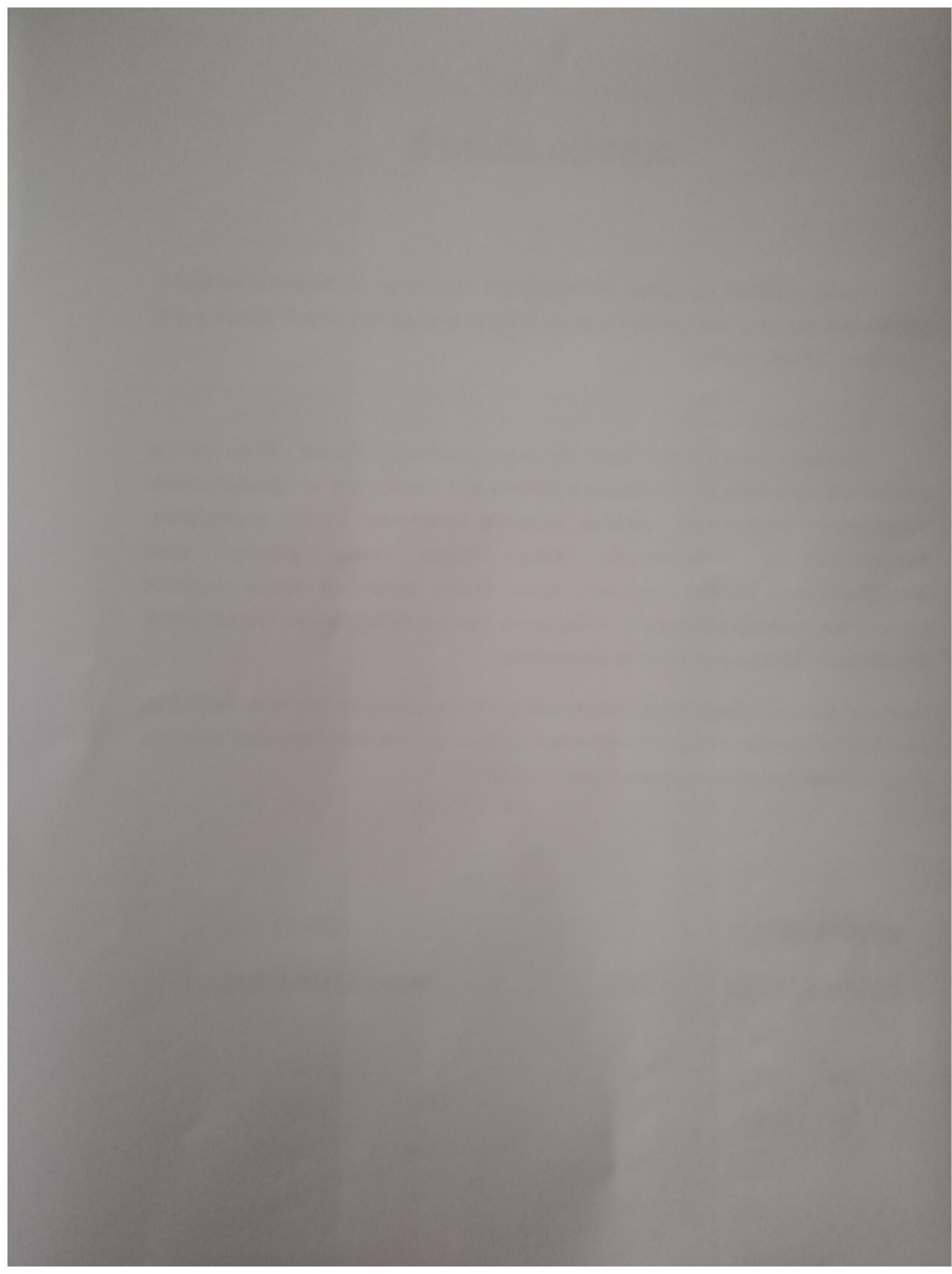




$$\boxed{\int_0^t f(z) dz = \frac{F(s)}{s}}$$

Formula Sheet for Laplace Transform $L\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$

$$1) L\{1\} = \frac{1}{s}$$

$$L\{t^n\} = \frac{n!}{s^{n+1}}, n \text{ is a positive integer}$$

$$2) L\{\sin kt\} = \frac{k}{s^2 + k^2}$$

$$L\{\cos kt\} = \frac{s}{s^2 + k^2}$$

$$3) L\{e^{at}\} = \frac{1}{s-a}$$

$$4) L\{e^{at} f(t)\} = F(s) \Big|_{s \rightarrow s-a}$$

$$L^{-1}\{F(s) \Big|_{s \rightarrow s-a}\} = e^{at} f(t)$$

$$5) L\{g(t)U(t-a)\} = e^{-as} L\{g(t+a)\} \quad L^{-1}\{e^{-as} F(s)\} = f(t-a)U(t-a)$$

$$6) L\{U(t-a)\} = \frac{e^{-as}}{s}$$

$$L^{-1}\left\{\frac{e^{-as}}{s}\right\} = U(t-a)$$

$$7) L\{f^{(n)}(t)\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

$$8) L\{t^n f(t)\}(s) = (-1)^n \frac{d^n F(s)}{ds^n}$$

$$L^{-1}\left\{\frac{d^n F(s)}{ds^n}\right\} = (-1)^n t^n f(t)$$

$$9) L\{f(t) * g(t)\} = F(s).G(s)$$

$$f(t) * g(t) = \int_0^t f(\tau)g(t-\tau)d\tau$$

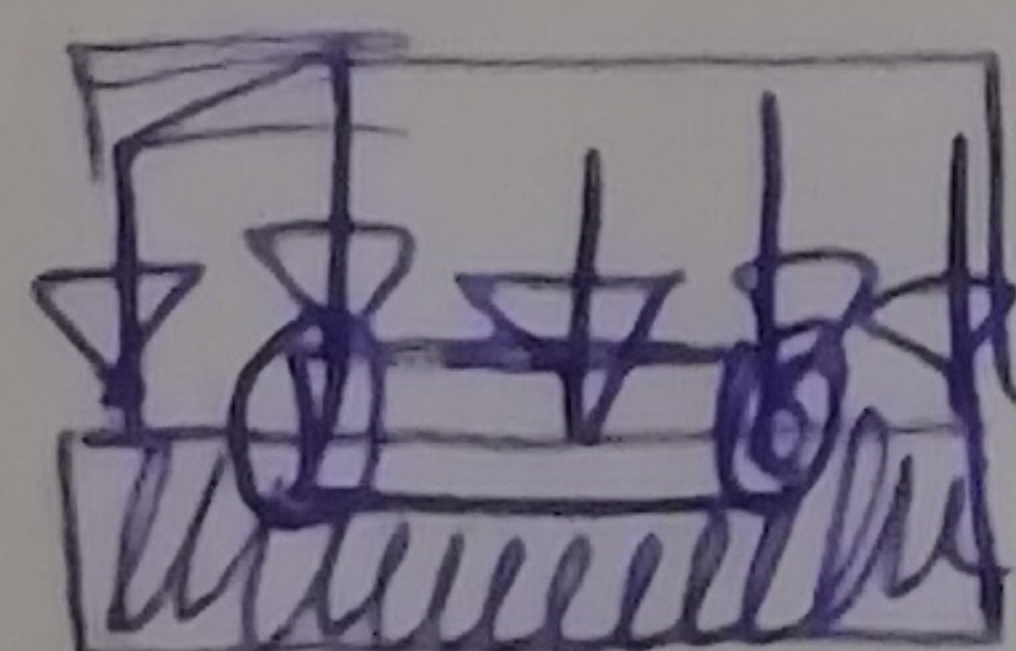
$$10) L\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$$

$$L^{-1}\{F(s).G(s)\} = f(t) * g(t)$$

$$11) L\{\delta(t)\} = 1$$

$$L\{\delta(t-t_0)\} = e^{-st_0}$$

$$12) \text{ If } f(t) \text{ is periodic with period } T \text{ then } L\{f(t)\} = \frac{1}{1-e^{-Ts}} \int_0^T e^{-st} f(t) dt.$$



Laplace-1
" - 2.

$$L\left\{\int_0^t f(z) dz\right\} = \frac{F(s)}{s}$$

$L^{-1}\{$

TEMEL LAPLACE DÖNÜŞÜMLERİ

$$f(t) = \mathcal{L}^{-1}\{F(s)\} \implies F(s) = \mathcal{L}\{f(t)\}$$

$$1. f(t) = 1 \implies \mathcal{L}\{1\} = \frac{1}{s} \quad (s > 0)$$

$$2. f(t) = t \implies \mathcal{L}\{t\} = \frac{1}{s^2} \quad (s > 0)$$

$$3. f(t) = t^n \quad (n = 1, 2, \dots) \implies \mathcal{L}\{t^n\} = \frac{(n)!}{s^{n+1}} \quad (s > 0)$$

$$4. f(t) = t^p \quad (p > -1) \implies \mathcal{L}\{t^p\} = \frac{\Gamma(p+1)}{s^{p+1}} \quad (s > 0)$$

$$5. f(t) = e^{at} \implies \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \quad (s > a)$$

$$6. f(t) = \sin at \implies \mathcal{L}\{\sin at\} = \frac{a}{s^2 + a^2} \quad (s > 0)$$

$$7. f(t) = \cos at \implies \mathcal{L}\{\cos at\} = \frac{s}{s^2 + a^2} \quad (s > 0)$$

$$8. f(t) = \sinh at \implies \mathcal{L}\{\sinh at\} = \frac{a}{s^2 - a^2} \quad (s > |a|)$$

$$9. f(t) = \cosh at \implies \mathcal{L}\{\cosh at\} = \frac{s}{s^2 - a^2} \quad (s > |a|)$$

$$10. f(t) = e^{at} \sin bt \implies \mathcal{L}\{e^{at} \sin bt\} = \frac{b}{(s-a)^2 + b^2} \quad (s > a)$$

$$11. f(t) = e^{at} \cos bt \implies \mathcal{L}\{e^{at} \cos bt\} = \frac{s-a}{(s-a)^2 + b^2} \quad (s > a)$$

$$12. f(t) = t^n e^{at} \quad (n = 1, 2, \dots) \implies \mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}} \quad (s > a)$$

$$13. f(t) = u_c(t) \implies \mathcal{L}\{u_c(t)\} = \frac{e^{-cs}}{s} \quad (s > 0)$$

$$14. h(t) = u_c(t) f(t-c) \implies \mathcal{L}\{u_c(t) f(t-c)\} = e^{-cs} F(s)$$

$$15. h(t) = e^{ct} f(t) \implies \mathcal{L}\{e^{ct} f(t)\} = F(s-c)$$

$$16. h(t) = f(ct) \implies \mathcal{L}\{f(ct)\} = \frac{1}{c} F\left(\frac{s}{c}\right) \quad (c > 0)$$

$$17. h(t) = \int_0^t f(t-\tau) g(\tau) d\tau \implies \mathcal{L}\left\{\int_0^t f(t-\tau) g(\tau) d\tau\right\} = F(s) G(s)$$

$$18. f(t) = \delta(t-c) \implies \mathcal{L}\{\delta(t-c)\} = e^{-cs}$$

$$19. h(t) = f^{(n)}(t) \Rightarrow$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$$

$$20. h(t) = (-1)^n \underset{t^n}{f(t)} \Rightarrow \mathcal{L}\{(-1)^n \underset{t^n}{f(t)}\} = F^{(n)}(s)$$

$$\hookrightarrow \mathcal{L}\{f(t)\} = F(s) \text{ for}$$

$$\mathcal{L}\{(-1)^n t^n f(t)\} = F^{(n)}(s) \text{ der.}$$

$$\lim_{t \rightarrow 0} \frac{f(t)}{t} \text{ for } \mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} F(s) ds$$

$$\mathcal{L}\left\{\int_0^t f(z) dz\right\} = \frac{F(s)}{s}$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{smt\} + \mathcal{L}\left\{u_{\frac{\pi}{4}}(t) \cdot \cos\left(t - \frac{\pi}{4}\right)\right\}$$

$$= \frac{1}{s^2+1} + e^{-\frac{\pi s}{4}} \cdot \frac{s}{s^2+1}$$

$$\rightarrow \mathcal{L}\{smt\} + e^{-\frac{\pi s}{4}} \mathcal{L}\{\cos t\}$$

$$F(s) = \frac{1 + s \cdot e^{-\frac{\pi s}{4}}}{s^2+1}$$

Örnek: $f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$

$$f(t) = u_2(t) \cdot (t-2)^2$$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{u_2(t) \cdot (t-2)^2\} = e^{-2s} \cdot \frac{2}{s^3}$$

Örnek: $f(t) = \begin{cases} t, & 0 \leq t < 2, \\ 2, & 2 \leq t < 5, \\ 7-t, & 5 \leq t < 7 \\ 0, & t \geq 7 \end{cases}$

$$f(t) = t + (2-t)u_2(t) + (7-t-2)u_5(t) + (0-(7-t))u_7(t)$$

$$f(t) = t + (2-t)u_2(t) + (5-t)u_5(t) - (7-t)u_7(t)$$

$$\begin{aligned} \mathcal{L}\{f(t)\} &= \mathcal{L}\{t\} + \mathcal{L}\{u_2(t) \cdot (2-t)\} + \mathcal{L}\{u_5(t) \cdot (5-t)\} - \mathcal{L}\{u_7(t) \cdot (7-t)\} \\ &= \frac{1}{s^2} + e^{-2s} \cdot \left(-\frac{1}{s^2}\right) - e^{-5s} \cdot \frac{1}{s^2} + e^{-7s} \cdot \frac{1}{s^2} \end{aligned}$$

Örnek: $F(s) = \frac{3!}{(s-2)^4}$ ve $\mathcal{L}^{-1}\{F(s)\} = ?$

$$\mathcal{L}^{-1}\left\{\frac{3!}{(s-2)^4}\right\} = e^{2t} \cdot t^3$$

Örnek: $f(t) = \begin{cases} 0, & t < \pi \\ t - \pi, & \pi \leq t < 2\pi \\ 0, & t \geq 2\pi \end{cases}$

$$f(t) = u_{\pi}(t) \cdot (t - \pi) - u_{2\pi}(t) \cdot (t - \pi)$$

$$\begin{aligned} \mathcal{L}\{f(t)\} &= e^{-\pi s} \mathcal{L}\{t\} - e^{-2\pi s} \mathcal{L}\{t\} = e^{-\pi s} \frac{1}{s^2} - e^{-2\pi s} \frac{1}{s^2} \\ &= \frac{e^{-\pi s} - e^{-2\pi s}}{s^2} \end{aligned}$$

$$\rightarrow f(t) = (t - \pi) (u_{\pi}(t) - u_{2\pi}(t))$$

Örnek: $2y'' + y' + 2y = g(t)$

$$g(t) = \begin{cases} 1, & 5 \leq t < 20 \\ 0, & 0 \leq t < 5 \text{ ve } t \geq 20 \end{cases}$$

$$g(t) = u_5(t) - u_{20}(t)$$

$$y(0) = y'(0) = 0$$

$$\mathcal{L}\{2y'' + y' + 2y\} = \mathcal{L}\{u_5(t) - u_{10}(t)\}$$

$$2s^2Y(s) - 2sy(0) - 2y'(0) + sY(s) - y(0) + 2Y(s) = \frac{e^{-5s}}{s} - \frac{e^{-10s}}{s}$$

$$Y(s)(2s^2 + s + 2) = \frac{e^{-5s} - e^{-10s}}{s}$$

$$Y(s) = \frac{e^{-5s} - e^{-10s}}{s(2s^2 + s + 2)} = \frac{1}{s(2s^2 + s + 2)} \cdot (e^{-5s} - e^{-10s})$$

$\underbrace{s(2s^2 + s + 2)}_{H(s)}$

$$H(s) = \frac{1}{s(2s^2 + s + 2)}$$

$$\mathcal{L}^{-1}\{H(s)\} = ?$$

$$\frac{1}{s(2s^2 + s + 2)} = \frac{A}{s} + \frac{Bs + C}{2s^2 + s + 2} \quad A = \frac{1}{2}, B = -1, C = -\frac{1}{2} \text{ obs.}$$

$$1 = (2A + B)s^2 + (A + C)s + 2A$$

$$\begin{cases} 2A + B = 0 \\ A + C = 0 \\ 2A = 1 \end{cases}$$

$$Y(s) = (e^{-5s} - e^{-10s}) \cdot \left(\frac{1}{2} \cdot \frac{1}{s} + \frac{-s - \frac{1}{2}}{2s^2 + s + 2} \right)$$

$$Y(s) = (e^{-5s} - e^{-10s}) \left(\frac{1}{2} \cdot \frac{1}{s} + \frac{1}{2} \cdot \frac{(s + \frac{1}{4}) + \frac{1}{4}}{(s + \frac{1}{4})^2 + \frac{15}{16}} \right)$$

$$\mathcal{L}^{-1}\{Y(s)\} = u_5(t) \cdot \frac{1}{2} - u_{10}(t) \cdot \frac{1}{2} - u_5(t) \cdot \frac{1}{2} e^{-\frac{\sqrt{15}}{4}(t-5)} + \frac{4}{8\sqrt{15}} u_5(t) e^{-\frac{\sqrt{15}}{4}(t-5)} \sin \frac{\sqrt{15}}{4}(t-5)$$

$\neq u_{10}(t)$

$$-5- \\ \boxed{y = c_1 \cos t + c_2 \sin t + \frac{1}{4}}$$

Ödev: $y'' + 4y = g(t)$, $y(0) = 0$, $y'(0)$

$$g(t) = \begin{cases} 0, & 0 \leq t < 5 \\ (t-5)/5, & 5 \leq t < 10 \\ 1, & t \geq 10 \end{cases}$$

$$g(t) = (u_5(t)(t-5) - u_{10}(t)(t-10))/5$$

$$Y(s) = (e^{-5s} - e^{-10s}) H(s)/5$$

$$H(s) = \frac{1}{s^2(s^2+4)}$$

$$\boxed{y = [u_5(t)h(t-5) - u_{10}(t)h(t-10)]/5}$$

$$H(s) = \frac{1/4}{s^2} - \frac{1/4}{s^2+4}$$

$$\boxed{h(t) = \frac{1}{4}t - \frac{1}{8}\sin 2t}$$

örnek: $y'' + y = f(t)$, $y(0) = 0$, $y'(0) = 1$

$$f(t) = \begin{cases} \frac{t}{2}, & 0 \leq t < 6 \\ 3, & t \geq 6 \end{cases}$$

Dirac Delta fonksiyonu

$$\delta(t) \neq 0, t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\delta(t-t_0) = 0, t \neq t_0$$

$$\int_{-\infty}^{\infty} \delta(t-t_0) dt = 1$$

$$\int_{-\infty}^{\infty} \delta(t-t_0) f(t) dt = f(t_0) \quad \text{şönde tanımlanır.}$$

$$\mathcal{L}\{\delta(t-c)\} = \int_0^{\infty} e^{-st} \cdot \delta(t-c) dt = \boxed{e^{-cs}}$$

Örnek: $2y'' + y' + 2y = \delta(t-5) \quad y(0) = y'(0) = 0$

$$\mathcal{L}\{2y'' + y' + 2y\} = \mathcal{L}\{\delta(t-5)\}$$

$$2s^2 Y(s) - 2s y(0) - 2y'(0) + s Y(s) - y(0) + 2Y(s) = e^{-5s}$$

$$\boxed{Y(s) = e^{-5s} \cdot \frac{1}{2s^2 + s + 2}}$$

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$$Y(s) = e^{-5s} \cdot \frac{1}{2} \cdot \frac{1}{(s + \frac{1}{4})^2 + \frac{15}{16}}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s + \frac{1}{4})^2 + \frac{15}{16}} \right\} = \frac{4}{\sqrt{15}} \cdot e^{-\frac{t}{4}} \cdot \sin \frac{\sqrt{15}}{4} t$$

$$y = \mathcal{L}^{-1} \{ Y(s) \} = \frac{2}{\sqrt{15}} u_5(t) \cdot e^{-\frac{(t-5)}{4}} \cdot \sin \frac{\sqrt{15}}{4} (t-5)$$

Oder: 1) $y'' + 2y' + 2y = \delta(t - \pi)$, $y(0) = 1$, $y'(0) = 0$

2) $y'' + 4y = \delta(t - \pi) - \delta(t - 2\pi)$, $y(0) = y'(0) = 0$

3) $y'' + 3y' + 2y = \delta(t - 5) + u_6(t)$, $y(0) = 0$, $y'(0) = \frac{1}{2}$

4) $y'' + 2y' + 3y = \sin t + \delta(t - 3\pi)$, $y(0) = 0$, $y'(0) = 0$

5) $2y'' + y' + 4y = \delta(t - \frac{\pi}{6}) \cdot \sin t$, $y(0) = 0$, $y'(0) = 0$

6) $y^{(4)} - y = \delta(t - 1)$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 2$, $y'''(0) = 0$

Konvolüsyon integrali

$$\mathcal{L}^{-1}\{F(s) \cdot G(s)\} = \int_0^t f(\tau) \cdot g(t-\tau) d\tau = \int_0^t f(t-\tau) \cdot g(\tau) d\tau$$

Örnek: $\mathcal{L}^{-1}\left\{\frac{4}{s^2(s^2+4)}\right\} = ?$

$$F(s) = \frac{1}{s^2}, \quad G(s) = \frac{4}{s^2+4}$$

$$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t, \quad \mathcal{L}^{-1}\left\{\frac{4}{s^2+4}\right\} = \sin 2t$$

$$\mathcal{L}^{-1}\left\{\frac{4}{s^2(s^2+4)}\right\} = \int_0^t (t-\tau) \sin(2\tau) d\tau \quad \begin{array}{l} t-\tau = u \\ d\tau = -du \\ \tau = 0 \rightarrow u = t \\ \tau = t \rightarrow u = 0 \end{array}$$

$$= -\frac{1}{2} (t-\tau) \cos 2\tau \Big|_0^t + \frac{1}{2} \int_0^t \cos 2\tau d\tau = \frac{1}{2} \sin 2\tau \Big|_0^t$$

$$= \frac{1}{2} t + \frac{1}{4} \sin 2t$$

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\cos a \sin b = \frac{1}{2} [\sin(a+b) - \sin(a-b)]$$

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Örnek: 1) $F(s) = \frac{1}{(s+1)^2(s^2+4)}$

ne $\mathcal{L}^{-1}\{F(s)\}$

2) $\mathcal{L}\left\{\int_0^t (t-\tau)^2 \cos 2\tau d\tau\right\} = ?$ $\frac{2}{s^3} \cdot \frac{s}{s^2+4}$

$$\begin{array}{cc} \downarrow & \downarrow \\ t^2 & \cos 2t \\ \downarrow & \downarrow \\ \frac{2}{s^3} & \frac{s}{s^2+4} \end{array}$$

3) $f(t) = \int_0^t \sin(t-\tau) \cdot \cos \tau d\tau$

$\mathcal{L}\{f(t)\} = ?$

$$\begin{array}{cc} \downarrow & \downarrow \\ \sin t & \cos t \\ \downarrow & \downarrow \\ \frac{1}{s^2+1} & \frac{s}{s^2+1} \end{array}$$

$\mathcal{L}\{f(t)\} = \frac{s}{(s^2+1)^2}$

-11-

Örnek: $f(t) = \int_0^t e^{-(t-\tau)} \sin \tau \, d\tau$
 $\mathcal{L}\{f(t)\} = ?$

Ödev: 1) $y'' + 4y' + 4y = 1 - u_{\pi}(t)$, $y(0) = 1$, $y'(0) = -1$

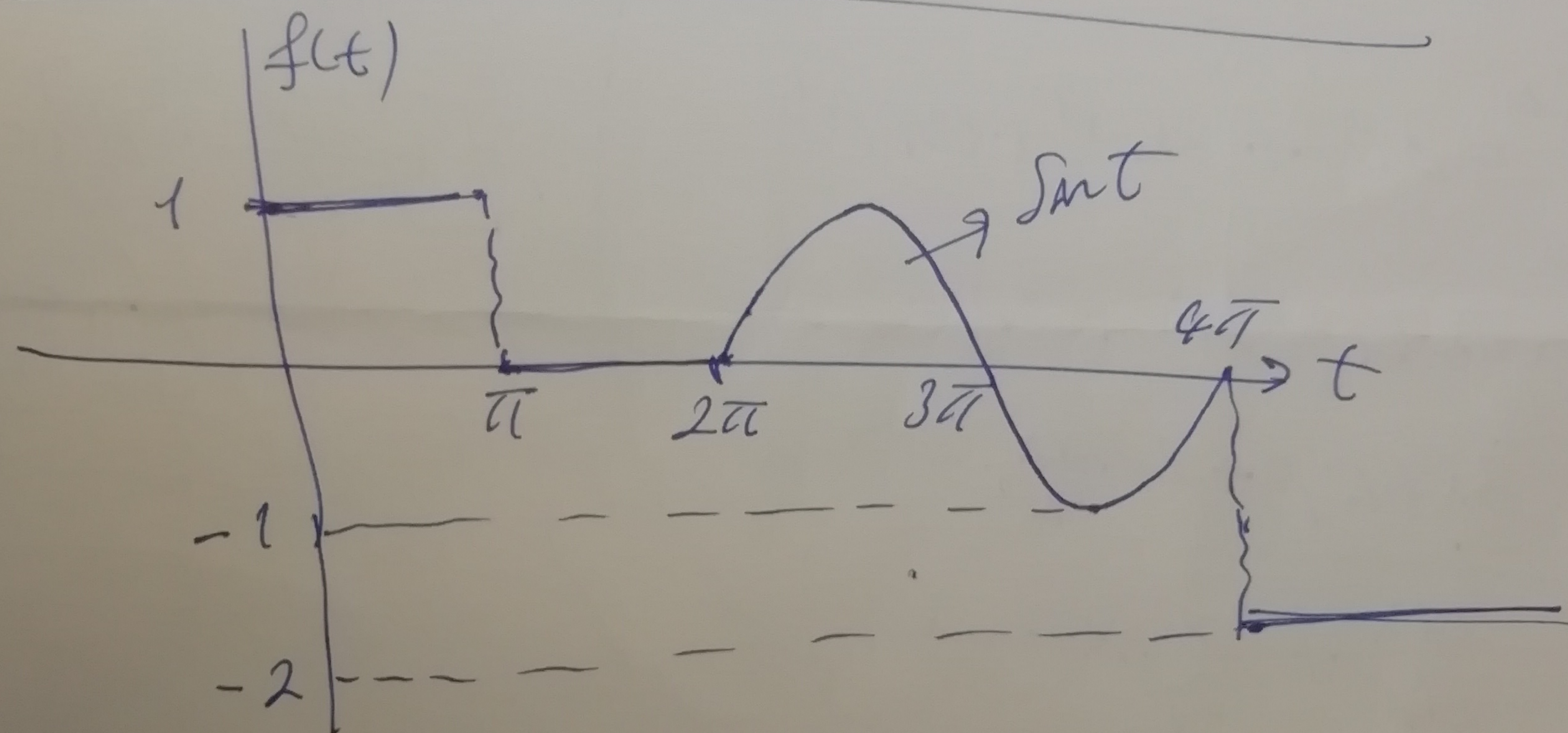
2) $y^{(4)} - y = f(t)$, $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = 0$

3) $y^{(4)} + 5y''' + 4y = f(t)$, $y(0) = 1$, $y'(0) = 0$, $y''(0) = 0$, $y'''(0) = 0$

Örnek: $x^2 y'' + 2y = 0$ $x_0 = 1$ noktası civarında kuvvet serisi
çözümünü bulunur. (Çözüm Serisi 4. derece terimlere
kadar yazarız.)

$F(s) = \frac{2s}{(s^2+7)^2}$ fonk. ters Laplace dönüşümü
kavayışın ile bulun.

$2y'' + 10y = -5\delta(t-4)$, $y(0) = -1$, $y'(0) = -2$.



$$f(t) = \begin{cases} 1 & ; t < \pi \\ 0 & ; \pi < t < 2\pi \\ \sin t & ; 2\pi < t < 4\pi \\ -2 & ; 4\pi < t < \infty \end{cases}$$

Yap $xy'' = \sqrt{1+y'^2}$ $3yy'' = 8y'$, $e^{y-y'x} = y'^2$

$y^{(4)} - 2y''' + 11y'' - 18y' + 18y = 0$

$$① \quad y'' + 3y' + 2y = \delta(t-5) + u_{10}(t) \quad y(0)=0 \quad y'(0) = \frac{1}{2}$$

$$\mathcal{L}\{y\} = Y(s) \quad \mathcal{L}\{y'\} = sY(s) - y(0) = sY(s) \quad (2)$$

$$\mathcal{L}\{y''\} = s^2 Y(s) - sy(0) - y'(0) = s^2 Y(s) - \frac{1}{2} \quad (2)$$

$$\mathcal{L}\{\delta(t-5)\} = e^{-5s} \quad \mathcal{L}\{u_{10}(t)\} = \frac{e^{-10s}}{s} \quad (2)$$

$$\Rightarrow s^2 Y(s) - \frac{1}{2} + 3sY(s) + 2Y(s) = e^{-5s} + \frac{e^{-10s}}{s}$$

$$Y(s) [s^2 + 3s + 2] = e^{-5s} + \frac{e^{-10s}}{s} + \frac{1}{2} \quad (2)$$

$$Y(s) = \frac{e^{-5s}}{s^2 + 3s + 2} + \frac{e^{-10s}}{s(s^2 + 3s + 2)} + \frac{1}{2(s^2 + 3s + 2)} \quad (2) \quad s^2 + 3s + 2 = (s+1)(s+2)$$

$$\frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \Rightarrow A = 1 \quad B = -1 \quad \frac{1}{s^2 + 3s + 2} = \frac{1}{s+1} - \frac{1}{s+2} \quad (2)$$

$$\frac{1}{(s+1)(s+2)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+2} \quad A = \frac{1}{2} \quad B = -1 \quad C = \frac{1}{2}$$

$$\frac{1}{(s+1)(s+2)} = \frac{1/2}{s} - \frac{1}{s+1} + \frac{1/2}{s+2} \quad (2)$$

$$Y(s) = \frac{e^{-5s}}{s+1} - \frac{e^{-5s}}{s+2} + \frac{1}{2} \frac{e^{-10s}}{s} - \frac{e^{-10s}}{s+1} + \frac{1}{2} \frac{e^{-10s}}{s+2} + \frac{1}{2} \frac{1}{s+1} - \frac{1}{2} \frac{1}{s+2}$$

$$y(t) = e^{-(t-5)} u_5(t) - e^{-2(t-5)} u_5(t) + \frac{1}{2} u_{10}(t) - e^{-(t-10)} u_{10}(t) + \frac{1}{2} e^{-2(t-10)} u_{10}(t) + \frac{1}{2} e^{-t} - \frac{1}{2} e^{-2t} \quad (4)$$

$$y' \cos y - y \sin y = \cos x \sin^2 y \quad u = y \sin y \quad \frac{du}{dx} = \frac{dy}{dx} \cos y$$

$$\frac{du}{dx} - u = \cos x u^2 \quad \text{B.V.D.} \quad \frac{1}{u^2} \frac{du}{dx} - \frac{1}{u} = \cos x \quad \frac{1}{u} = v$$

$$\frac{dv}{dx} + v = -\cos x \quad \frac{u}{u^2} = -\frac{dv}{dx} \quad -\frac{dv}{dx} - v = \cos x$$