EK-3

$$\left.\begin{aligned} x_1 &= k_1 e^{-3t} + k_2 e^{2t} + \tfrac{1}{2} e^{t} \\ x_2 &= -4k_1 e^{-3t} + k_2 e^{2t} - e^{-2t} \end{aligned}\right\}$$ Genel Çözüm.

2. Yol

$$\left.\begin{aligned} c_1' e^{-3t} + c_2' e^{2t} &= e^{-2t} \\ -4c_1' e^{-3t} + c_2' e^{2t} &= -2e^{t} \end{aligned}\right\}$$

$$\begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix} \begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = \begin{pmatrix} e^{-2t} \\ -2e^{t} \end{pmatrix}$$

$\boxed{AC' = B}$ $\boxed{C' = A^{-1}B}$

$$A = \begin{bmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{bmatrix} \qquad A^{-1} = \frac{1}{|A|} \cdot \operatorname{Adj} A \qquad |A| = 5e^{-t} \neq 0$$

$$\operatorname{Adj} A = \begin{bmatrix} e^{2t} & -e^{2t} \\ 4e^{-3t} & e^{-3t} \end{bmatrix}$$

$$A^{-1} = \frac{1}{5} e^{t} \begin{bmatrix} e^{2t} & -e^{2t} \\ 4e^{-3t} & e^{-3t} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} e^{3t} & -e^{3t} \\ 4e^{-2t} & e^{-2t} \end{bmatrix}$$

$$C' = A^{-1}B$$

$$\begin{bmatrix} c_1' \\ c_2' \end{bmatrix} = \frac{1}{5} \begin{bmatrix} e^{3t} & -e^{3t} \\ 4e^{-2t} & e^{-2t} \end{bmatrix} \begin{bmatrix} e^{-2t} \\ -2e^{t} \end{bmatrix} = \frac{1}{5} \begin{bmatrix} e^{t} + 2e^{4t} \\ 4e^{-4t} - 2e^{-t} \end{bmatrix}$$

$$c_1' = \frac{1}{5}(e^{t} + 2e^{4t}) \Rightarrow c_1 = \frac{1}{5}\left(e^{t} + \frac{1}{2}e^{4t}\right) + k_1$$

$$c_2' = \frac{1}{5}(4e^{-4t} - 2e^{-t}) \Rightarrow c_2 = \frac{1}{5}\left(-e^{-4t} + 2e^{-t}\right) + k_2$$

c_1 ve c_2 homojen sistemin genel çözümünde yerine yazılarak verilen sistemin çözümü elde edilmiş olur.

<u>Not</u>: $y'' + y' - 2y = 0$

$x_1 = y \; , \; x_2 = y'$ dersek

$x_1' = x_2$ ve $y'' = x_2'$ olur

$x_2' + x_2 - 2x_1 = 0$

$$\left.\begin{aligned} x_1' &= x_2 \\ x_2' &= 2x_1 - x_2 \end{aligned}\right\}$$ olur.

Örnek:
$$\frac{dx_1}{dt} = -x_1 - x_2 + 1$$
$$\frac{dx_2}{dt} = x_1 - 3x_2 - 1$$
Sistemini çözünüz

Çözüm: $X' = AX + B \quad A = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$X' = AX$

$(A - \lambda I)K = 0$ $\qquad |A - \lambda I| = 0$

$\begin{vmatrix} -1-\lambda & -1 \\ 1 & -3-\lambda \end{vmatrix} = 0$

$$\left.\begin{aligned} (-1-\lambda)k_1 - k_2 &= 0 \\ k_1 + (-3-\lambda)k_2 &= 0 \end{aligned}\right\}$$

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda+2)^2 = 0 \quad \lambda_{1,2} = -2$ iki katlı kök.

$\lambda_1 = -2$ için $(A + 2I)K_1 = 0$

$$\left.\begin{aligned} k_1 - k_2 &= 0 \\ k_1 - k_2 &= 0 \end{aligned}\right\}$$ $k_2 = 1$ keyfi, $k_1 = 1$ olur. $\quad K_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$u_1 = e^{-2t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ✓

$(A - \lambda I)K_2 = K_1$

$k_1 - k_2 = 1$

$k_2 = 1$ keyfi

$k_1 = 2$ olur. $\quad K_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$u_2 = (tK_1 + K_2)e^{-2t} = \begin{bmatrix} (t+2)e^{-2t} \\ (t+1)e^{-2t} \end{bmatrix}$ ✓

$$T = \begin{bmatrix} e^{-2t} & (t+2)e^{-2t} \\ e^{-2t} & (t+1)e^{-2t} \end{bmatrix}, \quad C = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$X = TC$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e^{-2t} & (t+2)e^{-2t} \\ e^{-2t} & (t+1)e^{-2t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\left.\begin{aligned} x_1 &= c_1 e^{-2t} + c_2 (t+2) e^{-2t} \\ x_2 &= c_1 e^{-2t} + c_2 (t+1) e^{-2t} \end{aligned}\right\} \quad \begin{aligned} c_1 &= c_1(t) = ? \\ c_2 &= c_2(t) = ? \end{aligned}$$

$$\left.\begin{aligned} c_1' e^{-2t} + c_2'(t+2)e^{-2t} &= 1 \\ c_1' e^{-2t} + c_2'(t+1)e^{-2t} &= -1 \end{aligned}\right\}$$

$$C' = T^{-1} \cdot B \qquad |T| = (t+1)e^{-4t} - (t+2)e^{-4t}$$

$$|T| = -e^{-4t}$$

$$T^{-1} = -e^{4t} \cdot \begin{bmatrix} (t+1)e^{-2t} & -(t+2)e^{-2t} \\ -e^{-2t} & e^{-2t} \end{bmatrix} \qquad \text{Adj}\,T = \begin{bmatrix} (t+1)e^{-2t} & -(t+2)e^{-2t} \\ -e^{-2t} & e^{-2t} \end{bmatrix}$$

$$T^{-1} = -e^{2t} \begin{bmatrix} t+1 & -(t+2) \\ -1 & 1 \end{bmatrix}. \qquad T^{-1} = \frac{1}{|T|} \cdot \text{Adj}\,T$$

$$C' = -e^{2t} \begin{bmatrix} t+1 & -t-2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} c_1' \\ c_2' \end{bmatrix}$$

$$\left.\begin{aligned} c_1' &= -e^{2t}(t+1+t+2) \\ c_2' &= -e^{2t}(-1-1) \end{aligned}\right\} \begin{aligned} c_1 &= -\int (2t+3)e^{2t}\,dt + k_1 \\ c_2 &= 2\int e^{2t}\,dt + k_2 \end{aligned} \qquad 2.$$

$$\boxed{c_2 = e^{2t} + k_2}$$

$$c_1 = -\left[\frac{1}{2}(2t+3)e^{2t} - \frac{1}{2}e^{2t}\right] + k_1$$

$$c_1 = \left[-te^{2t} - 2e^{2t}\right] + k_1$$

$$\boxed{c_1 = -(t+2)e^{2t} + k_1}$$

$$\left.\begin{aligned} x_1 &= k_1 e^{-2t} + k_2(t+2)e^{-2t} - (t+2) + t+2 \\ x_2 &= k_1 e^{-2t} + k_2(t+1)e^{-2t} - (t+2) + t+1 \end{aligned}\right\}$$

$$\left.\begin{aligned} x_1 &= k_1 e^{-2t} + k_2(t+2)e^{-2t} \\ x_2 &= k_1 e^{-2t} + k_2(t+1)e^{-2t} - 1 \end{aligned}\right\} \text{bulunur.}$$

Örnek: $X' = AX + B$

$$A = \begin{bmatrix} -1 & 2 & -4 \\ 0 & 4 & 2 \\ 0 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} e^{-2t} \\ e^{t} \\ 0 \end{bmatrix}$$

sistemini çözünüz.

Çözüm:

$$\left.\begin{aligned} x_1 &= c_1 e^{-t} + 2c_2 e^{2t} - 2c_3 e^{3t} \\ x_2 &= c_2 e^{2t} - 2c_3 e^{3t} \\ x_3 &= -c_2 e^{2t} + c_3 e^{3t} \end{aligned}\right\}$$

$c_1 = c_1(t) = ?$
$c_2 = c_2(t) = ?$
$c_3 = c_3(t) = ?$

Homojen Denklem Sisteminin Genel çözümü.

$$\left.\begin{aligned} c_1' e^{-t} + 2c_2' e^{2t} - 2c_3' e^{3t} &= e^{-2t} \\ c_2' e^{2t} - 2c_3' e^{3t} &= e^{t} \\ -c_2' e^{2t} + c_3' e^{3t} &= 0 \end{aligned}\right\}$$

$$\boxed{c' = T^{-1} \cdot B} \qquad T = \begin{bmatrix} e^{-t} & 2e^{2t} & -2e^{3t} \\ 0 & e^{2t} & -2e^{3t} \\ 0 & -e^{2t} & e^{3t} \end{bmatrix}$$

$$|T| = e^{-t}(e^{5t} - 2e^{5t}) = -e^{4t}$$

$$T^T = \begin{bmatrix} e^{-t} & 0 & 0 \\ 2e^{2t} & e^{2t} & -e^{2t} \\ -2e^{3t} & -2e^{3t} & e^{3t} \end{bmatrix}$$

$$\text{Adj}\, T = \begin{bmatrix} -e^{5t} & 0 & -2e^{5t} \\ 0 & e^{2t} & 2e^{2t} \\ 0 & e^{t} & e^{t} \end{bmatrix}$$

$$T^{-1} = -e^{-4t} \begin{bmatrix} -e^{5t} & 0 & -2e^{5t} \\ 0 & e^{2t} & 2e^{2t} \\ 0 & e^{t} & e^{t} \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} e^{t} & 0 & 2e^{t} \\ 0 & -e^{-2t} & -2e^{-2t} \\ 0 & -e^{-3t} & -e^{-3t} \end{bmatrix} \qquad c' = T^{-1} B$$

$$c' = \begin{bmatrix} c_1' \\ c_2' \\ c_3' \end{bmatrix} = \begin{bmatrix} e^{t} & 0 & 2e^{t} \\ 0 & -e^{-2t} & -2e^{-2t} \\ 0 & -e^{-3t} & -e^{-3t} \end{bmatrix} \begin{bmatrix} e^{-2t} \\ e^{t} \\ 0 \end{bmatrix}$$

$$c_1' = e^{-t} \Rightarrow c_1 = \int e^{-t}\,dt + k_1$$

$$c_2' = -e^{-t} \Rightarrow c_2 = -\int e^{-t}\,dt + k_2$$

$$c_3' = -e^{-2t} \Rightarrow c_3 = -\int e^{-2t}\,dt + k_3$$

$$\boxed{c_1 = -e^{-t} + k_1} \quad \boxed{c_2 = e^{-t} + k_2} \quad \boxed{c_3 = \frac{1}{2}e^{-2t} + k_3}$$

$$\left.\begin{aligned}
x_1 &= k_1 e^{-t} + 2k_2 e^{2t} - 2k_3 e^{3t} - e^{-2t} + 2e^{t} - e^{t} \\
x_2 &= k_2 e^{2t} - 2k_3 e^{3t} + \cancel{e^{t}} - \cancel{e^{t}} \\
x_3 &= -k_2 e^{2t} + k_3 e^{3t} - e^{t} + \frac{1}{2} e^{t}
\end{aligned}\right\} \text{ bulunur.}$$