Örnek: $y=cx^2$ parabol ailesinin diferansiyel denklemini bulunuz.

Çözüm:

$$y=cx^2 \Longrightarrow c=\frac{y}{x^2}$$

$$y'=2cx$$

$$y'=2\cdot\frac{y}{x^2}\cdot x$$

$$y'=2\frac{y}{x} \Longrightarrow \boxed{y'-\frac{2}{x}y=0}$$

$$y'-\frac{2}{x}y=0$$

$$\downarrow$$

$$\frac{dy}{dx}-\frac{2}{x}y=0$$

$$\int\frac{dy}{y}-\int\frac{2}{x}dx=0$$

$$\ln y-2\ln x=\ln c$$

$$\ln\frac{y}{x^2}=\ln c$$

$$\frac{y}{x^2}=c \Longrightarrow y=cx^2$$

Örnek: Merkezi (c_1, c_2), yarıçapı 1 olan çember ailesinin Diferansiyel denklemini bulunuz.

Çözüm: $(x-c_1)^2+(y-c_2)^2=1$ (1)

$$2(x-c_1)+2(y-c_2).y'=0$$

$$(x-c_1)+(y-c_2)y'=0 \quad (2)$$

$$1+y'^2+(y-c_2).y''=0 \quad (3)$$

(2) den $(x-c_1)=-(y-c_2)y'$

$$(x-c_1)^2=(y-c_2)^2y'^2$$

(1) den $(x-c_1)^2=1-(y-c_2)^2$

$$(y-c_2)^2y'^2=1-(y-c_2)^2$$

$$(y-c_2)^2(y'^2+1)=1 \Rightarrow \boxed{y-c_2=\frac{1}{\sqrt{y'^2+1}}}$$

(3) den $1+y'^2+\frac{y''}{\sqrt{y'^2+1}}=0$ bulunur.

Örnek: $y = C_1 e^{-x} + C_2 e^{x} - 2x + e^{2x}$ eğri ailesinin Dif. Denklemini bulunuz.

$$y' = -C_1 e^{-x} + C_2 e^{x} - 2 + 2e^{2x}$$

$$y'' = C_1 e^{-x} + C_2 e^{x} + 4e^{2x}$$

$$\begin{vmatrix} y+2x-e^{2x} & e^{-x} & e^{x} \\ y'+2-2e^{2x} & -e^{-x} & e^{x} \\ y''-4e^{2x} & e^{-x} & e^{x} \end{vmatrix} = 0$$

$$\begin{vmatrix} y+2x-e^{2x} & 1 & 1 \\ y'+2-2e^{2x} & -1 & 1 \\ y''-4e^{2x} & 1 & 1 \end{vmatrix} = \begin{vmatrix} y+2x-e^{2x} & 1 & 1 \\ y'-y+2-2x-e^{2x}-2 & 0 & 0 \\ y''-y-2x-3e^{2x} & 0 & 0 \end{vmatrix} = 0$$

$$y'' - y - 2x - 3e^{2x} = 0$$

$$\boxed{y'' - y = 2x + 3e^{2x}}$$ bulunur.

Ödevi: $y = C_1\cos 2x + C_2 \sin 2x - 1 + 2e^x$ eğri ailesinin

Dif. Denklemini bulunuz.

(Sonuç: $y'' + 4y = -4 + 10e^x$)

Ödevi: $y = C_1 e^{-x} + C_2 e^{2x} + C_3 e^{3x} + 2x + 1$

eğri ailesinin Dif. Denklemini bulunuz.

(Sonuç: $y''' - 4y'' + y' + 6y = 12x - \frac{1}{6}$)