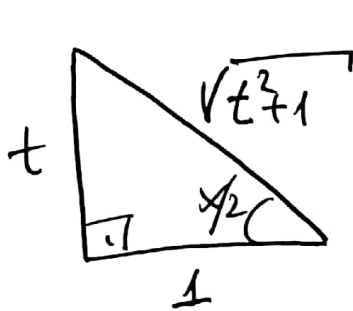


-1- (1. Hafta)

$$\boxed{\tan \frac{x}{2} = t} \quad \text{Dönüşümü}$$

$$\frac{1}{2} (1 + \tan^2 \frac{x}{2}) dx = dt$$

$$\boxed{dx = \frac{2 dt}{1 + t^2}} \quad \checkmark$$



$$\sin \frac{x}{2} = \frac{t}{\sqrt{t^2 + 1}}$$

$$\cos \frac{x}{2} = \frac{1}{\sqrt{t^2 + 1}}$$

$$\sin x = 2 \cdot \sin \frac{x}{2} \cdot \cos \frac{x}{2}, \quad \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}$$

$$\boxed{\sin x = \frac{2t}{t^2 + 1} \quad \cos x = \frac{1 - t^2}{t^2 + 1}} \quad \checkmark$$

Örnek: $\int \frac{\cos x}{1 + \cos x} dx = ?$

$$\int \frac{\frac{1-t^2}{1+t^2} \cdot \frac{2dt}{1+t^2}}{1 + \frac{1-t^2}{1+t^2}} = \frac{2}{2} \int \frac{1-t^2}{1+t^2} dt$$

Örnek: $\int \frac{\sin x}{1 + \sin x} dx = ?$

$$= \int \frac{\frac{2t}{t^2+1} \cdot \frac{2dt}{1+t^2}}{1 + \frac{2t}{t^2+1}} = 4 \int \frac{t dt}{(t^2+1)(t+1)^2}$$

Örnek: $\int \frac{dx}{1 + \sin x + \cos x} = ?$

Örnek: $\int \frac{1 + \sin x}{\cos x (1 + \cos x)} dx = ?$

Örnek: $\int \frac{dx}{\sin x + 2 \cos x} = ?$

Örnek: $\int \frac{dx}{3 + 2 \cos x} = ?$

$$\int \frac{\cos x dx}{\sin^2 x + 2 \sin x + 5} = ? \quad \text{Börs} = t \quad \cos x dx = dt$$

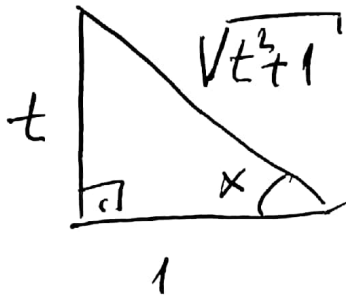
$$\int \frac{dt}{t^2 + 2t + 5}$$

-3-

$$\boxed{\tan x = t} \quad \text{dönüşümü}$$

$$(1 + \tan^2 x) dx = dt$$

$$\boxed{dx = \frac{dt}{1+t^2}} \quad \checkmark$$



$$\boxed{\begin{aligned} \sin x &= \frac{t}{\sqrt{t^2+1}} \\ \cos x &= \frac{1}{\sqrt{t^2+1}} \end{aligned}}$$

Örnek: $\int \frac{dx}{5 + \sin^2 x} = ?$

$$\int \frac{dt}{(1+t^2) \cdot \left(5 + \frac{t^2}{t^2+1}\right)} = \int \frac{dt}{6t^2+5}$$

$$= \frac{1}{6} \int \frac{dt}{t^2 + \frac{5}{6}} = \frac{1}{6} \cdot \frac{\sqrt{6}}{\sqrt{5}} \arctan \frac{\sqrt{6}t}{\sqrt{5}} + C$$

$$= \frac{1}{\sqrt{30}} \arctan \left(\frac{\sqrt{6}}{\sqrt{5}} (\tan x) \right) + C$$

-4-

Örnek:

$$\int \frac{dx}{2 + \sin x \cos x} = ?$$

Örnek:

$$\int \frac{\tan x \, dx}{3 - \cos^2 x} = ?$$

Örnek:

$$\int \frac{dx}{\sin^2 x - \tan x} = ?$$

Örnek:

$$\int \frac{dx}{\sin^2 x - \sin 2x} = ?$$

-Ekt

Örnek: $I = \int \frac{dx}{(x+1)^3 \sqrt{x^2+2x}} = ?$

$$\frac{1}{x+1} = t \text{ dersek } x = \frac{1-t}{t}$$

$$dx = -\frac{1}{t^2} dt \text{ olur}$$

$$I = \int \frac{t^3 dt}{t^2 \sqrt{\left(\frac{1-t}{t}\right)^2 + 2\left(\frac{1-t}{t}\right)}}$$

$$= - \int \frac{t^2 dt}{\sqrt{1-t^2}} \text{ olur. } \boxed{t = \sin u}$$

Örnek: $I = \int \frac{x^4 + 4x^2}{\sqrt{x^2+4}} dx = ?$

örnek: $I = \int \frac{dx}{\sqrt{x+1} + \sqrt[4]{x+1}}$

$$x+1 = t^4 \text{ dersek}$$
$$dx = 4t^3 dt \text{ olur}$$

$$I = \int \frac{4 \cdot t^3 dt}{t^2 + t}$$

EK-2

Örnek: $\int \frac{x^2 dx}{\sqrt{x^3+2}} = ?$

Örnek: $\int \sqrt{1+\sqrt{x}} dx = ?$ $1+\sqrt{x} = t^2$ dersek
 $\frac{1}{2\sqrt{x}} dx = 2t dt$
 $dx = 4t \cdot (t^2-1) dt$
 $= 4 \int t^2(t^2-1) dt$

Örnek: $\int \frac{4x^2+x+1}{x^3-1} dx = ?$

Örnek: $\int \frac{x^2}{x^2+x-6} dx = ?$

Örnek: $I = \int \frac{x e^x}{(x+1)^2} dx = ?$ "Kismi integral"
 $x e^x = u$ $\frac{dx}{(x+1)^2} = dv$
 $e^x(1+x) dx = du$ $-\frac{1}{x+1} = v$
 $I = -\frac{x e^x}{x+1} + e^x + C$

EK-3

Örnek: $\int \frac{dx}{(1+\sin x)^2} = ?$

Örnek: $\int \frac{dx}{\sin x (1+\sin x)} = ?$

Örnek: $\int \frac{2+\tan x}{1-2\tan x} dx = ?$

Örnek: $\int \frac{dx}{a^2 \cos^2 x - b^2 \sin^2 x} = ?$

Örnek: $\int \frac{\sin x - 5 \cos x}{\sin x + \cos x} dx = ?$

Örnek: $\int \frac{\cos x}{\sin x + \cos x} dx = ?$

Örnek: $\int \frac{dx}{\cos x + \sin^2 x} = ?$

EK-4

Örnek: $\int \frac{1 + \sin x}{1 - \cos x} dx = ?$

Örnek: $\int \frac{dx}{2 \cos x + 3 \sin x + 4} = ?$

Örnek: $\int \frac{x \cdot \ln(1+x^2)}{1+x^2} dx = ?$

Örnek: $\int \frac{dx}{e^{2x} + 4e^x + 5} = ?$

Örnek: $\int \frac{e^{2x} dx}{e^{2x} + 1} = ?$