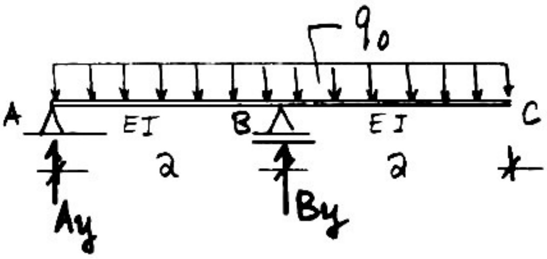


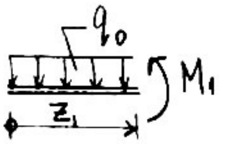

Şekilde görülmekte olan EI rijitlikli çıkmalı kirişte C ucundaki dönmeyi enerji yöntemiyle bulunuz. Kesme etkisini ihmal ediniz.

Çözüm: Öncelikle mesnet tepkilerini hesaplayalım.

$\Sigma M_A = 0 \rightarrow \quad 2q_0 a^2 - B_y a = 0 \qquad B_y = 2q_0 a$

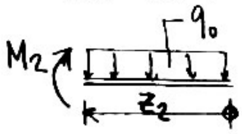
$\Sigma Y = 0 \rightarrow \quad A_y = 0$

AB arasında moment kesit tesirinin değişimi



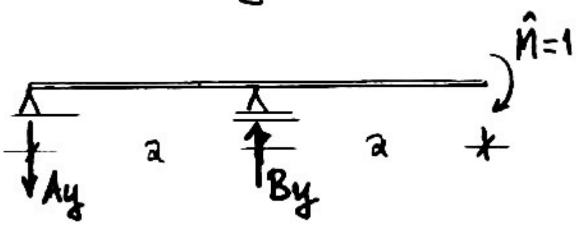

$M_1(z_1) = -q_0 \dfrac{z_1^2}{2} \qquad 0 \leqslant z_1 \leqslant a$

BC arasında moment kesit tesirinin değişimi



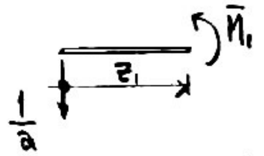

$M_2(z_2) = -q_0 \dfrac{z_2^2}{2} \qquad 0 \leqslant z_2 \leqslant a$

C noktasındaki dönme için, bu noktaya bir birimlik $\hat{M}=1$ momenti uygulanır. Bu yükleme sonucu mesnet tepkileri ve moment kesit tesirinin değişimi hesaplanır.

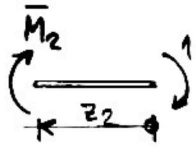



$\Sigma M_A = 0 \rightarrow \quad B_y = \hat{M}/a$

$\Sigma Y = 0 \rightarrow \quad A_y = B_y$



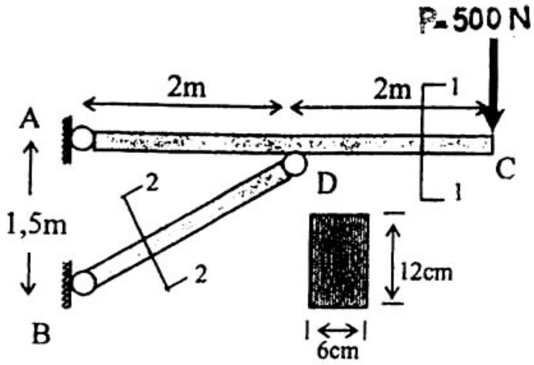

$\bar{M}_1 = -\dfrac{z_1}{a} \qquad 0 \leqslant z_1 \leqslant a$




$M_2 = -1 \qquad 0 \leqslant z_2 \leqslant a$

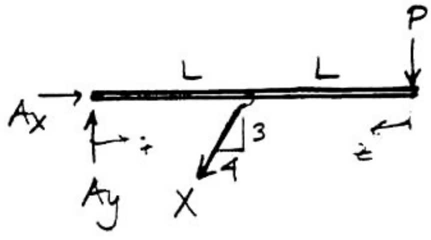
Virtüel iş denklemi: $\quad \theta_C = \displaystyle\int \frac{M\bar{M}}{EI} dz$

$$\theta_C = \frac{1}{EI}\left[\int_0^a \frac{q_0 z_1^3}{2a} dz_1 + \int_0^a \frac{q_0 z_2^2}{2} dz_2\right] = \frac{1}{EI}\left[\left|\frac{q_0 z_1^4}{8a}\right|_0^a + \left|\frac{q_0 z_2^3}{6}\right|_0^a\right] = \frac{7}{24}\frac{q_0 a^3}{EI}$$




1-1 ve 2-2 kesiti

Şekildeki sistemde normal kuvvet ve eğilme momentini göz önüne alarak P=500 N luk yükün altındaki çökmeyi enerji yöntemi ile bulunuz. (E=20 GPa)

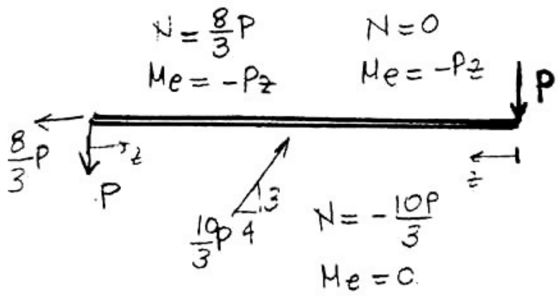



$$\Sigma M_A = 0 \Rightarrow P \cdot 2L + \frac{3}{5} X \cdot L = 0 \Rightarrow X = -\frac{10}{3} P$$

$$\Sigma Y = 0 \Rightarrow A_y - \frac{3}{5} X - P = 0 \Rightarrow A_y = P$$

$$\Sigma X = 0 \Rightarrow A_x - \frac{4}{5} X = 0 \Rightarrow A_x = -\frac{8}{3} P$$

Virtüel iş yöntemiyle çözüm:




$$v_C = \int_0^L \frac{\frac{8}{3}P \cdot \frac{8}{3}}{EA} dz + \int_0^L \frac{(-Pz)(-z)}{EI} dz + \int_0^L \frac{(-Pz)(-z)}{EI} dz + \int_0^{\frac{5L}{4}} \frac{\left(-\frac{10P}{3}\right)\left(-\frac{10}{3}\right)}{EA} dz$$

$$\Rightarrow v_C = \frac{64}{9} \frac{PL}{EA} + \frac{P}{EI} \int_0^L z^2 dz + \frac{P}{EI} \int_0^L z^2 dz + \frac{100}{9} \cdot \frac{5}{4} \cdot \frac{PL}{EA} = 21 \frac{PL}{EA} + \frac{2}{3} \frac{PL^3}{EI}$$

$E = 20 \times 10^9 \ N/m^2 \qquad L = 2m$

$P = 500 \ N$

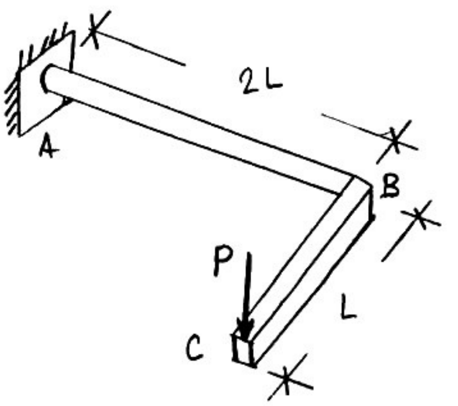
$I = \frac{6 \cdot 12^3}{12} = 864 \ cm^4 = 864 \times 10^{-8} \ m^4$

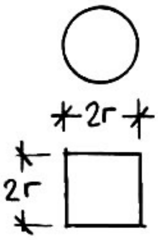
$A = 6 \cdot 12 = 72 \ cm^2 = 72 \times 10^{-4} \ m^2$

$$v_C = \frac{P}{E}\left(21 \frac{L}{A} + \frac{2}{3} \frac{L^3}{I}\right) = \frac{500}{20 \times 10^9} \left( \frac{\overset{7}{\cancel{21}} \cdot \cancel{2}}{\underset{\cancel{36}\ 12}{\cancel{72}} \times 10^{-4}} + \frac{\cancel{2 \cdot 8}}{3 \cdot \underset{54}{\cancel{864}} \times 10^{-8}} \right)$$

$$= 25 \times 10^{-5} \left( \frac{7}{12} + \frac{5000}{81} \right) = 0{,}015578 \text{ metre}$$

$$= 1{,}56 \text{ cm}$$

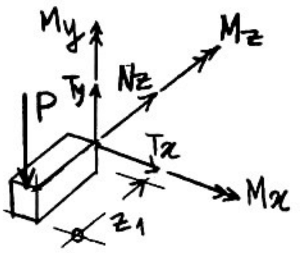



CB kolu kare kesitli, AB kolu daire kesitli olan şekildeki konsol çubuk sistemin serbest ucuna çubuk düzlemine dik doğrultuda P kuvveti etkimektedir. C ucundaki çökmeyi virtüel iş denklemini kullanarak hesaplayınız. $E=2.5G$ olup, kesme etkisi ihmal edilecektir.

Çözüm: Problemi iki bölgede incelemek uygundur.

CB arası $0 \leq z_1 \leq L$

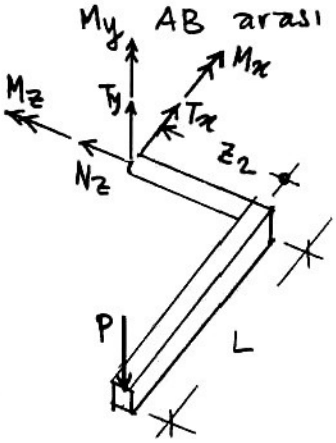



$M_x = -Pz_1$

$M_y = 0$

$M_z = 0$

AB arası $0 \leq z_2 \leq 2L$




$M_x = -Pz_2$

$M_y = 0$

$M_z = PL$

C ucunun çökmesi için bu noktaya çubuk düzlemine dik doğrultuda $\bar{P}=1$ birim yükünü uygulayalım. Kesit tesirlerinin değişimi

CB arası $0 \leq z_1 \leq L$

$\bar{M}_x = -z_1$

$\bar{M}_y = 0$

$\bar{M}_z = 0$

AB arası $0 \leq z_2 \leq L$

$\bar{M}_x = -z_2$

$\bar{M}_y = 0$

$\bar{M}_z = L$

Virtüel iş denklemi: $v_c = \int \frac{M\bar{M}}{EI} dz$

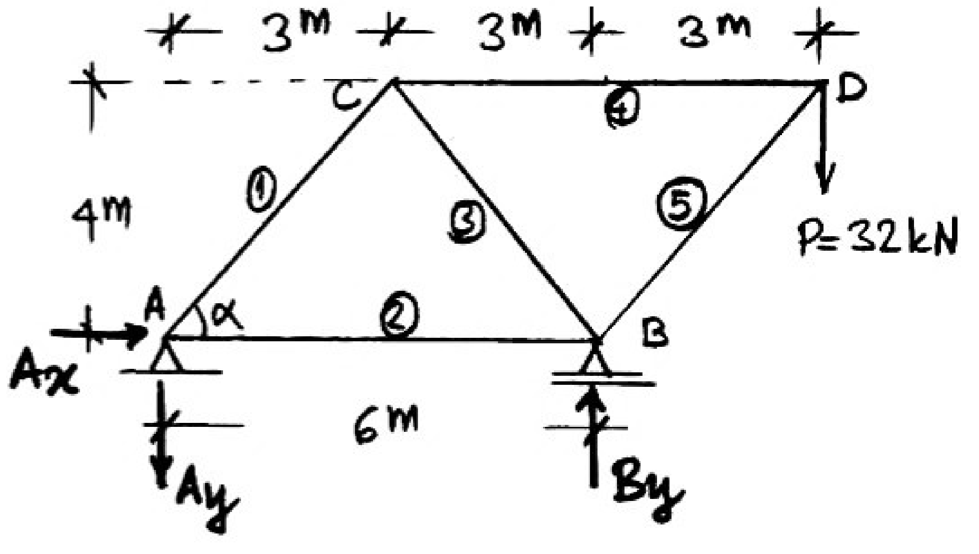
$$v_c = \int_0^L \frac{Pz_1^2}{EI_{x_1}} dz_1 + \int_0^{2L} \frac{Pz_2^2}{EI_{x_2}} dz_2 + \int_0^{2L} \frac{PL^2}{GI_b} dz_2$$

$I_{x_1} = \frac{2r.(2r)^3}{12} = \frac{4}{3} r^4$ (dikdörtgen kesit), $I_0 = \frac{\pi r^4}{2}$, $I_{x_2} = \frac{I_0}{2} = \frac{\pi r^4}{4}$ (daire kesit)

$$v_c = \frac{P}{E}\frac{3}{4r^4}\left|\frac{z_1^3}{3}\right|_0^L + \frac{P}{E}\frac{4}{\pi r^4}\left|\frac{z_2^3}{3}\right|_0^{2L} + \frac{PL^2}{E} \times 2.5 \times \frac{2}{\pi r^4}\left|z_2\right|_0^{2L}$$

$$v_c = \frac{PL^3}{4Er^4} + \frac{32PL^3}{3\pi Er^4} + \frac{10PL^3}{\pi Er^4} = \frac{PL^3}{Er^4}\left(\frac{1}{4} + \frac{32}{3\pi} + \frac{10}{\pi}\right) = \left(\frac{3\pi + 248}{12\pi}\right)\frac{PL^3}{Er^4}$$

$$= 6.828 \frac{PL^3}{Er^4}$$




Şekildeki basit kafes sistemde tüm çubukların uzama rijitliği $AE = 20\times10^3$ kN 'dur. Verilen yükleme için D noktasının yatay yerdeğiştirmesini virtüel iş denklemini kullanarak hesaplayınız.

Çözüm: Mesnet tepkileri

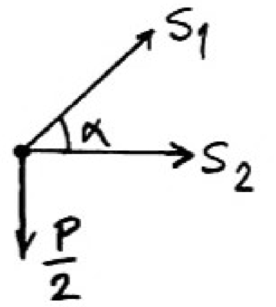
$\Sigma X=0 \rightarrow A_x = 0$

$\Sigma M_A=0 \rightarrow 6B_y - 9P = 0 \qquad B_y = \frac{3}{2}P = 48\,kN$

$\Sigma Y=0 \rightarrow A_y = \frac{3P}{2} - P = \frac{P}{2} = 16\,kN$

Şimdi gerçek sisteme ait çubuk kuvvetlerini hesaplayalım.

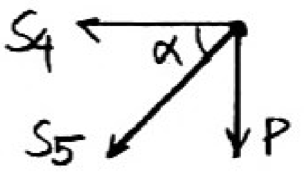
A mesnedi $\quad \cos\alpha = 3/5,\ \sin\alpha = 4/5$




$\Sigma Y=0 \rightarrow S_1\frac{4}{5} - \frac{P}{2} = 0 \qquad S_1 = \frac{5}{8}P = 20\,kN$

$\Sigma X=0 \rightarrow \frac{5}{8}P.\frac{3}{5} + S_2 = 0 \qquad S_2 = -\frac{3}{8}P = -12\,kN$

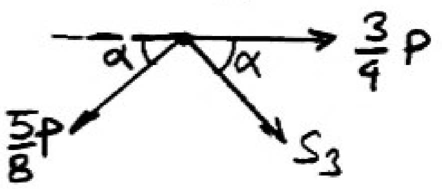
D düğümü




$\Sigma Y=0 \rightarrow S_5\frac{4}{5} + P = 0 \qquad S_5 = -\frac{5}{4}P = -40\,kN$

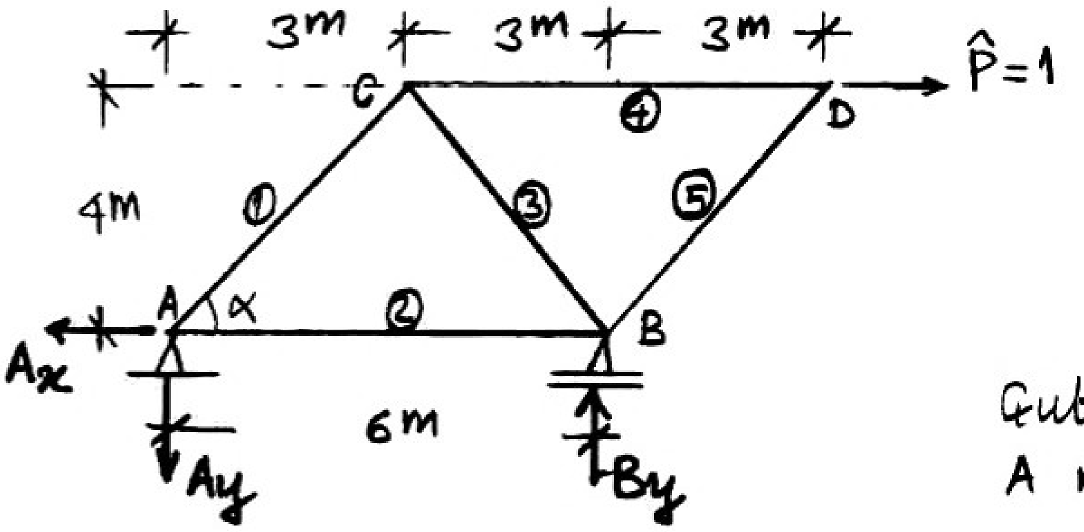
$\Sigma X=0 \rightarrow S_5\frac{3}{5} + S_4 = 0 \qquad S_4 = \frac{3}{4}P = 24\,kN$

C düğümü




$\Sigma Y=0 \qquad S_3\frac{4}{5} + \frac{5}{8}P.\frac{4}{5} = 0 \qquad S_3 = -\frac{5}{8}P = -20\,kN$

D noktasının yatay yerdeğiştirmesini hesaplamak için bu noktaya yatayda $\hat{P}=1$ birim yükünü yükleyelim.




Mesnet tepkileri

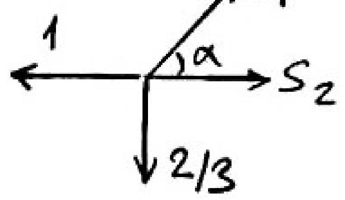
$\Sigma X=0 \rightarrow A_x = 1$

$\Sigma M_A=0 \rightarrow 4 - 6B_y = 0 \quad B_y = 2/3$

$\Sigma Y=0 \rightarrow A_y = B_y = 2/3$

Çubuk kuvvetleri

A mesnedi



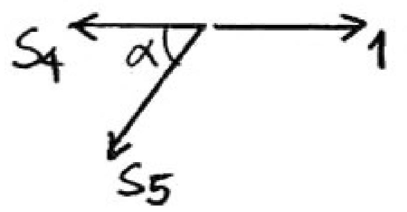

$\Sigma Y=0 \rightarrow S_1\frac{4}{5} - \frac{2}{3} = 0$

$S_1 = 5/6$

$\Sigma X=0 \rightarrow \frac{5}{6}.\frac{3}{5} - 1 + S_2 = 0$

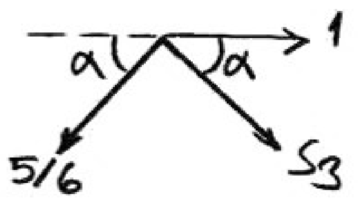
$S_2 = 1/2$

D düğümü




$$\Sigma Y = 0 \rightarrow S_5 = 0$$
$$\Sigma X = 0 \rightarrow S_4 = 1$$

C düğümü



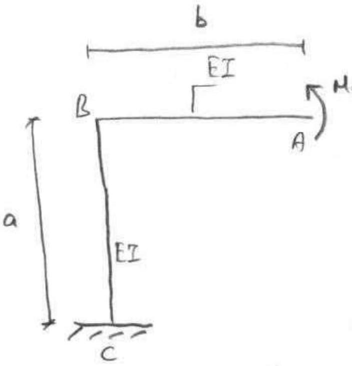

$$\Sigma Y = 0 \rightarrow \frac{5}{6} \cdot \frac{4}{5} + S_3 \cdot \frac{4}{5} = 0$$
$$S_3 = -5/6$$

Şimdi işlemlerimizi aşağıdaki tabloda düzenleyelim.

| Çubuk No | $S_i$ [kN] | $\bar{S}_i$ | $L_i$ [m] | $S_i \bar{S}_i L_i$ |
|---|---|---|---|---|
| 1 | 20 | 5/6 | 5 | 83.33 |
| 2 | -12 | 1/2 | 6 | -36.00 |
| 3 | -20 | -5/6 | 5 | 83.33 |
| 4 | 24 | 1 | 6 | 144.00 |
| 5 | -40 | 0 | 5 | 0.00 |
| | | | $\Sigma$ | 274.67 |

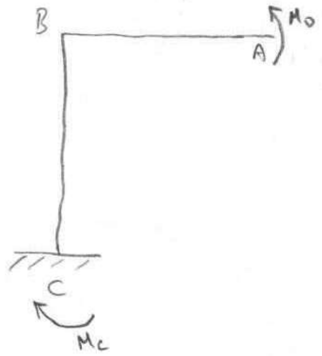
$$\Delta D_x = \frac{1}{AE} \sum_{i=1}^{5} S_i \bar{S}_i L_i = \frac{274.67}{20 \times 10^3} = 0.0137\,m \quad \text{bulunur.}$$

## Uyg.—4:




Şekildeki taşıyıcı sisteme etkiyen eğilme momenti $M_0$ olup, çubuklarda eğilme rijitliği $EI$'dır. A ucunda $V_A$ çökmesini, kayma etkisini ihmal ederek virtüel iş denklemini kullanarak elde ediniz.

## Çözüm:




$\circlearrowleft + \sum M_C = 0$

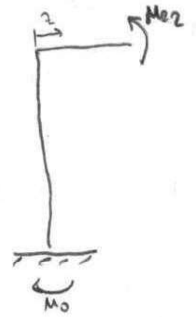
$M_C - M_0 = 0$

$\boxed{M_C = M_0}$

Gerçek yükleme için kesit tesirleri;


$M_0 - M_{e1} = 0$

$\boxed{M_{e1} = M_0}$ $\quad 0 \leq z \leq a$

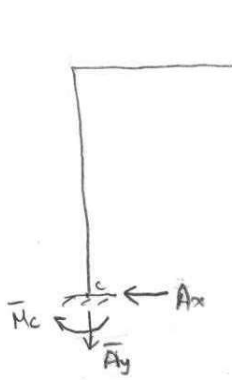



$M_0 - M_{e2} = 0$

$\boxed{M_{e2} = M_0}$

$0 \leq z \leq b$

Birim yükleme için kesit tesirleri:

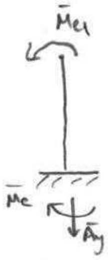



$\circlearrowleft + \sum \bar{M}_C = 0$

$1 \cdot b + \bar{M}_C = 0$

$\boxed{\bar{M}_C = -b}$

$\boxed{\bar{A}_y = -1}$




$\bar{M}_C - \bar{M}_{e1} = 0$

$\boxed{\bar{M}_{e1} = -b}$

$0 \leq z \leq a$


$\bar{M}_C - \bar{A}_y \cdot z - \bar{M}_{e2} = 0$

$\boxed{\bar{M}_{e2} = z - b}$

$0 \leq z \leq b$

$$V_A = \int M_e \cdot \frac{\bar{M}_e}{EI}\, dz$$

$$V_A = \frac{1}{EI}\left[\int_0^a M_0 \cdot (-b)\, dz + \int_0^b M_0 \cdot (z-b)\, dz\right]$$

$$V_A = \frac{1}{EI}\left\{\left[-M_0 b z\right]\Big|_0^a + \left[M_0\left(\frac{z^2}{2} - bz\right)\right]\Big|_0^b\right\}$$

$$V_A = \frac{1}{EI}\left[-M_0 ab + M_0\frac{b^2}{2} - M_0 b^2\right] = \frac{1}{EI}\left[-M_0 ab - \frac{M_0 b^2}{2}\right] = \boxed{\frac{-b\,(2a+b)\,M_0}{2EI}}$$