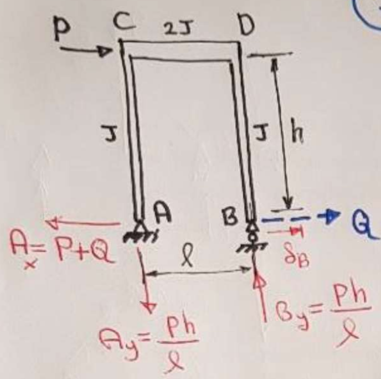
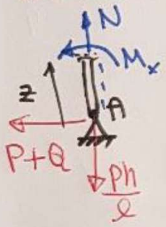
③

Şekildeki çerçevenin B mesnedinin yatay yer değiştirmesini Castigliano teoremiyle hesaplayınız. (Kesme kuvvetinin etkisini ihmal ediniz. Tüm çubuklarda kesit alanı "A" dır)



. Yer d. istenen noktada, ilgili doğrultuda kuvvet olmadığından hayali bir "Q" kuvveti alalım.

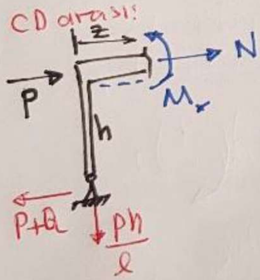
AC arası:



$N = \frac{Ph}{\ell}$, $\frac{\partial N}{\partial Q} = 0$

$M_x = (P+Q)z$, $\frac{\partial M_x}{\partial Q} = z$

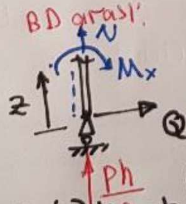
CD arası:



$N = Q$, $\frac{\partial N}{\partial Q} = 1$

$M_x = (P+Q)h - \frac{Ph}{\ell}z$, $\frac{\partial M_x}{\partial Q} = h$

BD arası:



$N = -\frac{Ph}{\ell}$, $\frac{\partial N}{\partial Q} = 0$

$M = Qz$, $\frac{\partial M}{\partial Q} = z$

$$\delta_B = \int_0^h \left(\frac{Ph}{\ell}\right)(0) + \int_0^h \frac{(P+Q)z\,(z)}{EJ}dz + \int_0^{\ell} \frac{(Q)(1)}{EA}dz + \frac{1}{2EJ}\int_0^{\ell}\left[(P+Q)h - \frac{Ph}{\ell}z\right](h)\,dz$$

$$+ \int_0^h \left(-\frac{Ph}{\ell}\right)\frac{(0)}{EA} + \int_0^h \frac{(Qz)(z)}{EJ}dz$$

$$\boxed{Q = 0 \Rightarrow \delta_B = \frac{Ph^3}{3EJ} + \frac{Ph^2\ell}{4EJ}}$$

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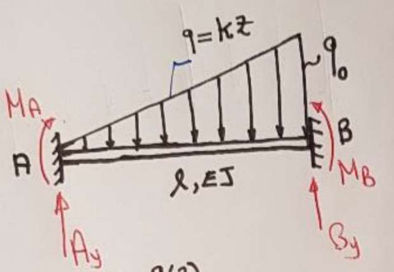
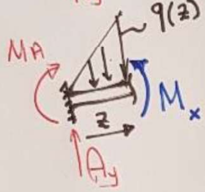
Castigliano teoremini kullanarak şekildeki sistemin mesnet tepkilerini hesaplayınız. (EJ: sabit)





$$M_x = A_y z + M_A - \frac{1}{2}(qz)\cdot\frac{z}{3}$$

$q = kz$ ile:

$$\boxed{M_x = A_y z + M_A - \frac{qz^3}{6}}$$

$$\frac{\partial M_x}{\partial M_A} = 1, \quad \frac{\partial M_x}{\partial A_y} = z$$

A noktasında çökme ve dönme sıfırdır.

$$\delta_A = \int_0^l \frac{M_x}{EJ}\cdot\frac{\partial M_x}{\partial A_y}\,dz = \frac{1}{EJ}\int_0^l \left(A_y z + M_A - \frac{qz^3}{6}\right)(z)\,dz = 0$$

$$\theta_A = \int_0^l \frac{M_x}{EJ_x}\cdot\frac{\partial M_x}{\partial M_A}\,dz = \frac{1}{EJ}\int_0^l \left(A_y z + M_A - \frac{qz^3}{6}\right)(1)\,dz = 0$$

$$\frac{M_A}{2} + A_y\frac{l}{3} = \frac{kl^3}{30} \quad (1)$$

$$M_A + A_y\frac{l}{2} = \frac{kl^3}{24} \quad (2)$$

$$\therefore \boxed{A_y = \frac{3}{20}kl^2, \quad M_A = -\frac{kl^3}{30}}$$

Not: $q = kz$

$z = 0 \rightarrow q = 0$

$z = l \rightarrow q = q_0 = kl$

$\boxed{K = \frac{q_0}{l}}$

$q = q_0\frac{z}{l}$ ✓

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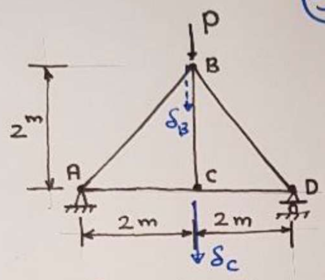


Şekildeki kafes kirişin C ve B düğüm noktalarının düşey yer değiştirmesini Castigliano teoremi ile hesaplayınız.

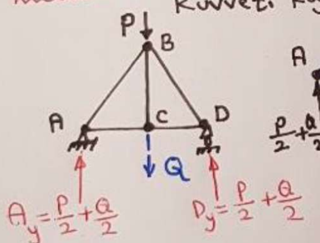
$P = 50\,kN$

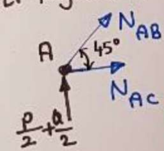
Tüm çubuklarda

$A = 5\,cm^2$, $E = 210\,GPa$

$$\delta_k = \sum_{i=1}^{n} \frac{N_i l_i}{E_i A_i} \cdot \frac{\partial N_i}{\partial P_k}$$

δ_c için: C noktasında kuvvet olmadığından bir "Q" kuvveti koyalım.





$\uparrow \sum F_y = 0:$

$$N_{AB} \frac{\sqrt{2}}{2} = \frac{P}{2} + \frac{Q}{2}$$

$$\boxed{N_{AB} = -\left(\frac{P\sqrt{2}}{2} + \frac{Q\sqrt{2}}{2}\right)}$$

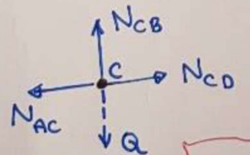
$$N_{DB} = N_{AB}$$

$\overrightarrow{\sum} F_x = 0:$

$$N_{AC} + N_{AB} \frac{\sqrt{2}}{2} = 0$$

$$\boxed{N_{AC} = \frac{P}{2} + \frac{Q}{2}}$$

$$N_{DC} = N_{AC}$$



$\uparrow \sum F_y = 0:$ $\boxed{N_{CB} = Q}$

$$\frac{\partial N_{AB}}{\partial Q} = -\frac{\sqrt{2}}{2} = \frac{\partial N_{DB}}{\partial Q}, \quad \frac{\partial N_{AC}}{\partial Q} = \frac{1}{2} = \frac{\partial N_{DC}}{\partial Q},$$

$$\frac{\partial N_{CB}}{\partial Q} = 1$$

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$$\ell_{AB} = \sqrt{2^2+2^2} = 2\sqrt{2}\ m = \ell_{DB}$$ 6

$$\ell_{AC} = \ell_{DC} = 2m$$

$(N_{AB} = N_{BD})$: 2 adet

$$\delta_C = \frac{1}{EA}\Big\{\left(-\frac{P\sqrt{2}}{2} - \frac{Q\sqrt{2}}{2}\right)(2\sqrt{2})\left(-\frac{\sqrt{2}}{2}\right)\times 2$$

$$+\left(\frac{P}{2}+\frac{Q}{2}\right)(2)\left(\frac{1}{2}\right)\times 2 \rightarrow (N_{AC} = N_{DC}) : 2 \text{ adet}$$

$$+(Q)(2)(1)\Big\}$$

$$\boxed{Q=0} \rightarrow \delta_C = \frac{1}{EA}\left\{2\sqrt{2}P + P\right\} = \frac{(1+2\sqrt{2})P}{EA}$$

$$\delta_C = \frac{(1+2\sqrt{2})\,50^{kN}}{(\underbrace{210\times 10^6}_{\frac{kN}{m^2}})(\underbrace{5\times 10^{-4}}_{m^2})} = 1,823\times 10^{-3}\ m \ \boxed{= 1,823\ mm}$$

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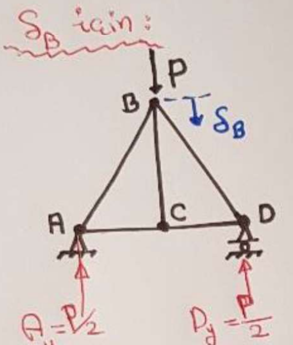
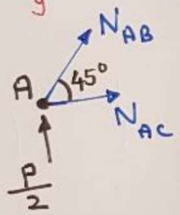
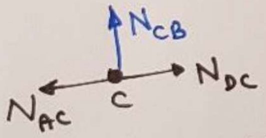
δ_B için:





$$N_{AB}\frac{\sqrt{2}}{2} = -\frac{P}{2} \rightarrow \boxed{N_{AB} = -\frac{P\sqrt{2}}{2} = N_{DB}}$$

$$\boxed{N_{AC} = -N_{AB}\frac{\sqrt{2}}{2} = \frac{P}{2} = N_{DC}}$$

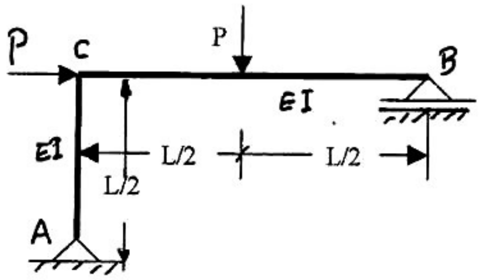


$$\boxed{N_{CB} = 0}$$

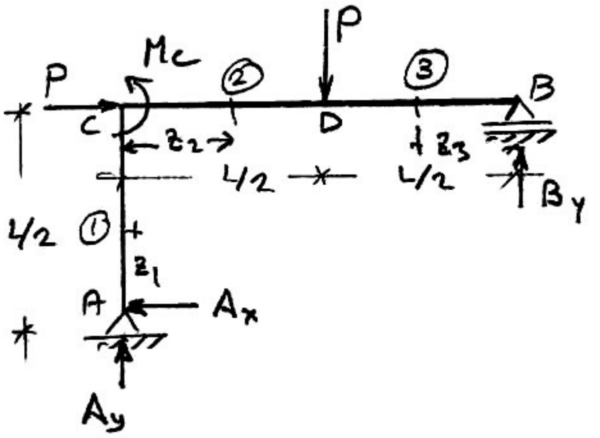
$$\frac{\partial N_{AB}}{\partial P} = -\frac{\sqrt{2}}{2} = \frac{\partial N_{DB}}{\partial P}, \quad \frac{\partial N_{AC}}{\partial P} = \frac{1}{2} = \frac{\partial N_{DC}}{\partial P}, \quad \frac{\partial N_{CB}}{\partial P} = 0$$

$$\delta_B = \frac{1}{EA}\Big\{\left(-\frac{P\sqrt{2}}{2}\right)\overset{(mm)}{\left(2000\sqrt{2}\right)}\left(-\frac{\sqrt{2}}{2}\right)\times 2 + \left(\frac{P}{2}\right)(2000)\left(\frac{1}{2}\right)\times 2 + 0\Big\}$$

$$= \frac{1}{\underset{(210)(500)}{EA}}\Big\{2000\sqrt{2}\,P + 1000P\Big\} = \boxed{1,823\ mm}\ !$$



Şekildeki yarım çerçevede C köşesindeki dönmeyi Castigliano Teoremi yardımı ile hesaplayınız.



C köşesine şekildeki gibi bir M_c momenti yükleyelim.

Mesnet tepkileri.

$\Sigma X_i = 0 \rightarrow \quad A_x = P$

$\Sigma M_A = 0 \rightarrow \quad B_y \cdot L - P \cdot \frac{L}{2} - P \cdot \frac{L}{2} + M_c = 0$

$$B_y = P - M_c/L$$

$\Sigma Y_i = 0 \rightarrow A_y + B_y - P = 0 \rightarrow A_y = \frac{M_c}{L}$

Çerçevede A-C; C-D, D-B arası olmak üzere üç bölge söz konusu olacaktır.

Moment ifadeleri ve türevleri:

$$M_1 = A_x \cdot z_1 = P \cdot z_1 \quad ; \quad \frac{\partial M_1}{\partial M_c} = 0$$

$$M_2 = A_y \cdot z_2 + A_x \cdot \frac{L}{2} - M_c = \frac{M_c}{L} z_2 + P\frac{L}{2} - M_c ; \quad \frac{\partial M_2}{\partial M_c} = \frac{z_2}{L} - 1$$

$$M_3 = B_y \cdot z_3 = \left(P - \frac{M_c}{L}\right) z_3 ; \quad \frac{\partial M_3}{\partial M_c} = -\frac{z_3}{L}$$

$$\theta_c = \int \frac{M \frac{\partial M}{\partial M_c}}{EI} \Bigg|_{M_c=0} dz = 0 + \int_0^{L/2} \frac{1}{EI} \cdot P \cdot \frac{L}{2} \cdot \left(\frac{z_2}{L} - 1\right) dz_2 + \int_0^{L/2} \frac{1}{EI} \cdot P \cdot z_3 \left(-\frac{z_3}{L}\right) dz_3$$

$$\theta_c = \frac{P}{EI} \left[\cdot \frac{L}{2} \left(\frac{1}{2L} z_2^2 - z_2 \right) \Bigg|_0^{L/2} - \frac{1}{L} \frac{z_3^3}{3} \Bigg|_0^{L/2} \right] = -\frac{11}{48} \cdot \frac{PL^2}{EI}$$