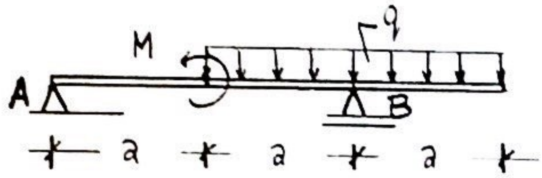
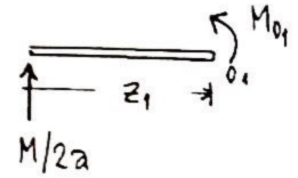


Şekildeki çıkmalı kirişin elastik eğri ifadesini Diferansiyel denklem yöntemiyle bulunuz.

Çözüm: Problemi üç bölgede incelemek uygundur. Önce mesnet tepkilerini hesaplayalım.

$\Sigma M_B = 0 \rightarrow A_y \times 2a + M = 0$ $\quad A_y = M/2a \quad B_y = \left(2qa - \frac{M}{2a}\right)$

1. Bölge $\quad 0 \leq z_1 \leq a$



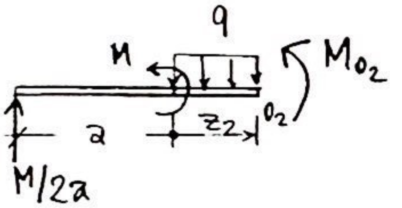
$$M_{O_1} = \frac{M}{2a} z_1$$

$$EI v_1'' = -\frac{M z_1}{2a}$$

$$EI v_1' = -\frac{M z_1^2}{4a} + C_1$$

$$EI v_1 = -\frac{M z_1^3}{12a} + C_1 z + C_2$$

2. Bölge $\quad 0 \leq z_2 \leq a$



$$M_{O_2} = \frac{M}{2a}(z_2 + a) - M - \frac{q z_2^2}{2}$$

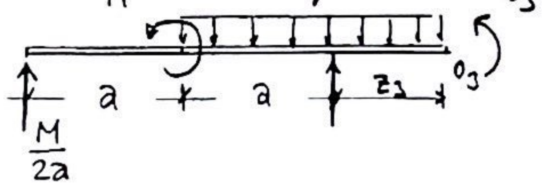
$$M_{O_2} = \frac{M z_2}{2a} - \frac{q z_2^2}{2} - \frac{M}{2}$$

$$EI v_2'' = -\frac{M}{2a} z_2 + \frac{q}{2} z_2^2 + \frac{M}{2}$$

$$EI v_2' = -\frac{M}{4a} z_2^2 + \frac{q}{6} z_2^3 + \frac{M}{2} z_2 + D_1$$

$$EI v_2 = -\frac{M}{12a} z_2^3 + \frac{q}{24} z_2^4 + \frac{M}{4} z_2^2 + D_1 z_2 + D_2$$

3. Bölge $\quad 0 \leq z_3 \leq a$



$$M_{O_3} = \frac{M}{2a}(z_3 + 2a) - M - q\frac{(z_3 + a)^2}{2} + \left(2qa - \frac{M}{2a}\right) z_3$$

$$M_{O_3} = -\frac{q}{2} z_3^2 + qa z_3 - \frac{qa^2}{2}$$

$$EI v_3'' = \frac{q}{2} z_3^2 - qa z_3 + \frac{qa^2}{2}$$

$$EI v_3' = \frac{q}{6} z_3^3 - \frac{qa}{2} z_3^2 + \frac{qa^2}{2} z_3 + E_1$$

$$EI v_3 = \frac{q}{24} z_3^4 - \frac{qa}{6} z_3^3 + \frac{qa^2}{4} z_3^2 + E_1 z_3 + E_2$$

Sınır koşulları: $v_1\Big|_{z_1=0} = 0$, $v_2\Big|_{z_2=a} = 0$, $v_3\Big|_{z_3=0} = 0$

Süreklilik koşulları $v_1\Big|_{z_1=a} = v_2\Big|_{z_2=0}$, $v_1'\Big|_{z_1=a} = v_2'\Big|_{z_2=0}$, $v_2'\Big|_{z_2=a} = v_3'\Big|_{z_3=0}$

Şimdi integrasyon sabitlerini belirleyelim.

$v_1\Big|_{z_1=0} = 0 \rightarrow C_2 = 0$

$v_1\Big|_{z_1=a} = v_2\Big|_{z_2=0} \rightarrow -\frac{Ma^2}{12} + C_1 a = D_2$

$v_1'\Big|_{z_1=a} = v_2'\Big|_{z_2=0} \rightarrow -\frac{Ma}{4} + C_1 = D_1$

$v_2\Big|_{z_2=a} = 0 \rightarrow -\frac{Ma^2}{12} + \frac{qa^4}{24} + \frac{Ma^2}{4} + D_1 a + D_2 = 0$

Bu denklemlerden C_1, D_1 ve D_2 çözülürse

$C_1 = \frac{Ma}{12} - \frac{qa^3}{48}$ $D_1 = -\frac{Ma}{6} - \frac{qa^3}{48}$ $D_2 = -\frac{qa^4}{48}$

bulunur. $v_3\Big|_{z_3=0} = 0 \rightarrow E_2 = 0$ dır.

$v_2'\Big|_{z_2=a} = v_3'\Big|_{z_3=0} \rightarrow -\frac{Ma}{4} + \frac{qa^3}{6} + \frac{Ma}{2} - \frac{Ma}{6} - \frac{qa^3}{48} = E_1$

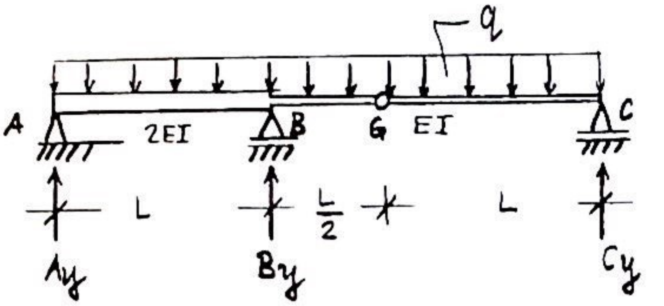
$$E_1 = \frac{Ma}{12} + \frac{7qa^3}{48}$$

Her bölge için elastik eğri ifadesi

$$EI\, v_1(z_1) = -\frac{M}{12a} z_1^3 + \left(\frac{Ma}{12} - \frac{qa^3}{48}\right) z_1$$

$$EI\, v_2(z_2) = -\frac{M}{12a} z_2^3 + \frac{q}{24} z_2^4 + \frac{M}{4} z_2^2 + \left(-\frac{Ma}{6} - \frac{qa^3}{48}\right) z_2 - \frac{qa^4}{48}$$

$$EI\, v_3(z_3) = \frac{q}{24} z_3^4 - \frac{qa}{6} z_3^3 + \frac{qa^2}{4} z_3^2 + \left(\frac{Ma}{12} + \frac{7qa^3}{48}\right) z_3$$



Şekildeki kiriş q yayılı yükü etkisindedir.

a) Elastik eğri ifadesini integrasyon yöntemi ile hesaplayınız.

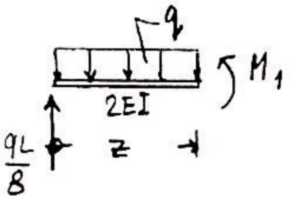
b) G mafsalındaki dönme süreksizliğini hesaplayınız.

Çözüm: Mesnet tepkileri: $\Sigma M_{G_{sol}} = 0 \rightarrow C_y = \frac{1}{2} qL$

$\Sigma M_A = 0 \rightarrow B_y \cdot L + 5\frac{L}{2} C_y - q\,\frac{5L}{2} \cdot \frac{5L}{4} = 0 \rightarrow B_y = \frac{15}{8} qL$

$\Sigma Y = 0 \rightarrow A_y = \frac{5}{2} qL - \frac{19}{8} qL \qquad A_y = \frac{qL}{8}$

1. Bölge $\quad 0 \leqslant z \leqslant L$



$M_1 = \frac{qLz}{8} - \frac{qz^2}{2}$

$2EI v_1'' = \frac{qz^2}{2} - \frac{qL}{8} z$

$2EI v_1' = \frac{qz^3}{6} - \frac{qL}{16} z^2 + C_1$

$2EI v_1 = \frac{qz^4}{24} - \frac{qL}{48} z^3 + C_1 z + C_2$

Sınır koşullarımızı uygulayacak olursak

$v_1(z)\big|_{z=0} = 0 \rightarrow C_2 = 0$

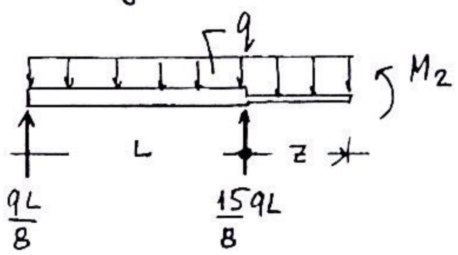
$v_1(z)\big|_{z=L} = 0 \rightarrow \frac{qL^4}{24} - \frac{qL^4}{48} + C_1 L = 0 \rightarrow C_1 = -\frac{qL^3}{48}$

1. Bölge için elastik eğri ifadesi

$2EI v_1 = \frac{qz^4}{24} - \frac{qL}{48} z^3 - \frac{qL^3}{48} z$

$2EI v_1' = \frac{qz^3}{6} - \frac{qL}{16} z^2 - \frac{qL^3}{48}$

2. Bölge $\quad 0 \leqslant z \leqslant L/2$



$M_2 = -\frac{qz^2}{2} + qLz - \frac{3}{8} qL$

$EI v_2'' = \frac{qz^2}{2} - qLz + \frac{3}{8} qL$

$EI v_2' = \frac{qz^3}{6} - \frac{qL}{2} z^2 + \frac{3}{8} qL^2 z + D_1$

$EI v_2 = \frac{qz^4}{24} - \frac{qL}{6} z^3 + \frac{3}{16} qL^2 z^2 + D_1 z + D_2$

2. bölgede sınır koşullarını yazacak olursak

$$v_2\Big|_{z=0} = 0 \quad \rightarrow \quad D_2 = 0$$

süreklilik koşulu ise $v_1'(z)\Big|_{z=L} = v_2'(z)\Big|_{z=0}$

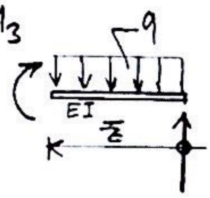
$$\frac{qL^3}{12} - \frac{qL^3}{32} - \frac{qL^3}{48} = D_1 \qquad D_1 = \frac{qL^3}{32}$$

2. bölgede elastik eğri ifadesi

$$EIv_2 = \frac{qz^4}{24} - \frac{qL}{6}z^3 + \frac{3}{16}qL^2z^2 + \frac{qL^3}{32}z$$

$$EIv_2' = \frac{qz^3}{6} - \frac{qL}{2}z^2 + \frac{3}{8}qL^2z + \frac{qL^3}{32}$$

3. Bölge $0 \leq \bar{z} \leq L$



$$M_3 = \frac{1}{2}qL\bar{z} - \frac{1}{2}q\bar{z}^2$$

$$EIv_3''(\bar{z}) = \frac{q\bar{z}^2}{2} - \frac{qL}{2}\bar{z}$$

$$EIv_3'(\bar{z}) = \frac{q\bar{z}^3}{6} - \frac{qL}{4}\bar{z}^2 + E_1$$

$$EIv_3(\bar{z}) = \frac{q\bar{z}^4}{24} - \frac{qL}{12}\bar{z}^3 + E_1\bar{z} + E_2$$

Sınır koşulu $v_3\Big|_{\bar{z}=0} = 0 \quad \rightarrow \quad E_2 = 0$

Süreklilik koşulu $v_2\Big|_{z=\frac{L}{2}} = v_3\Big|_{\bar{z}=L}$

$$\frac{qL^4}{384} - \frac{qL^4}{48} + \frac{3}{64}qL^4 + \frac{qL^4}{64} = \frac{qL^4}{24} - \frac{qL^4}{12} + E_1L \quad \rightarrow \quad E_1 = \frac{11}{128}qL^3$$

3. bölgede elastik eğri

$$EIv_3(\bar{z}) = \frac{q\bar{z}^4}{24} - \frac{qL}{12}\bar{z}^3 + \frac{11}{128}qL^3\bar{z}$$

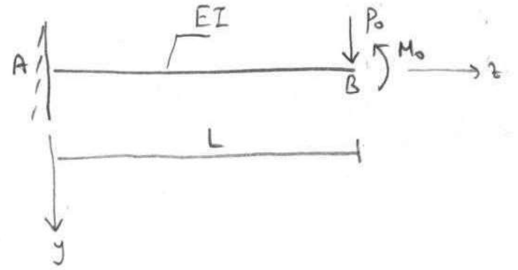
G mafsalındaki dönme

$\bar{z}$ artım yönü z'nin tersinde bu yüzden $v_3'(z) = -v_3'(\bar{z})$ olur.

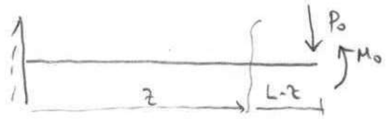
$$\Omega_G = -v_3'(\bar{z})\Big|_{\bar{z}=L} - v_2'(z)\Big|_{z=\frac{L}{2}} = -\frac{1}{384}\frac{qL^3}{EI} - \frac{11}{96}\frac{qL^3}{EI} = -\frac{15}{128}\frac{qL^3}{EI}$$

Uyg.-2:

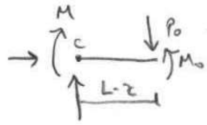
Şekildeki konsol kirişin açıklığı L, eğilme rijitliği EI sabittir. B ucundaki dönmenin sıfır olması için P_0 kuvveti ile eğilme momenti M_0 arasında sağlatılması gereken koşulu diferansiyel denklem yöntemiyle elde ediniz.



Çözüm:



$\longrightarrow$

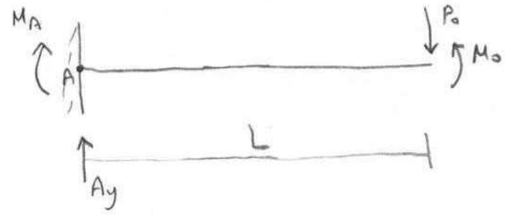


$\circlearrowleft + \sum M_c = 0$

$M_0 - M - P_0(L-z) = 0$

$\boxed{M = M_0 - P_0(L-z)}$

veya M kesit tesir değeri şu şekilde bulunabilir.



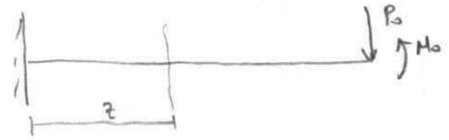
$\circlearrowleft + \sum M = 0$

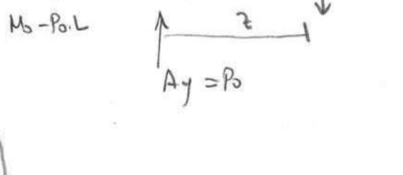
$-M_A + M_0 - P_0 . L = 0$

$\boxed{M_A = M_0 - P_0 . L}$

$+\uparrow \sum F_y = 0$

$A_y - P_0 = 0 \quad \boxed{A_y = P_0}$





$\circlearrowleft + \sum M_c = 0$

$-(M_0 - P_0 . L) - P_0 . z + M = 0$

$\boxed{M = M_0 - P_0(L-z)}$ → aynı sonuç bulunur.

$$\frac{d^2v}{dz^2} = -\frac{M}{EI}$$

$$\frac{d^2v}{dz^2} = -\frac{\left(M_0 - P_0(L-z)\right)}{EI} = -\frac{M_0}{EI} + \frac{P_0 \cdot L}{EI} - \frac{P_0 \cdot z}{EI} = \frac{1}{EI}\left(-M_0 + P_0 \cdot L - P_0 \cdot z\right)$$

$$\frac{dv}{dz} = \frac{1}{EI}\left(-M_0 \cdot z + P_0 \cdot L \cdot z - P_0 \frac{z^2}{2}\right) + C_1$$

$$V(z) = \frac{1}{EI}\left(-M_0 \cdot \frac{z^2}{2} + P_0 \cdot L \frac{z^2}{2} - P_0 \cdot \frac{z^3}{6}\right) + C_1 \cdot z + C_2$$

Sınır şartları:

$z = 0 \longrightarrow V(0) = 0 \longrightarrow \frac{1}{EI}\left(-M_0 \cdot 0 + P_0 \cdot L \cdot 0 - P_0 \cdot \frac{0}{6}\right) + C_1 \cdot 0 + C_2 = 0$ $\quad \boxed{C_2 = 0}$

$z = 0 \longrightarrow \varphi(0) = \frac{dv(0)}{dz} = V'(0) = 0 \longrightarrow \frac{1}{EI}\left(-M_0 \cdot 0 + P_0 \cdot L \cdot 0 - P_0 \cdot \frac{0}{2}\right) + C_1 = 0$ $\quad \boxed{C_1 = 0}$

$$\boxed{V(z) = \frac{1}{EI}\left(-M_0 \cdot \frac{z^2}{2} + P_0 \cdot L \frac{z^2}{2} - P_0 \cdot \frac{z^3}{6}\right)}$$ → Elastik Eğri

B ucunda → dönme ⇒ $\varphi_B(L) = 0$ olması için

$(z = L)$

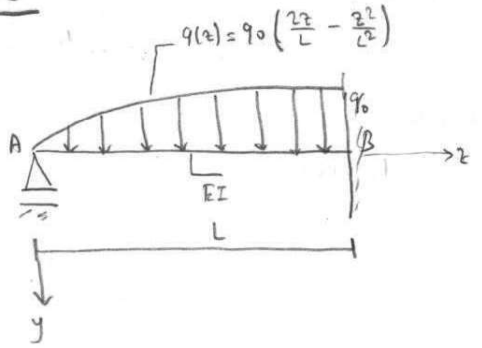
$$\frac{dv}{dz} = \varphi(z) = \frac{1}{EI}\left(-M_0 \cdot z + P_0 \cdot L \cdot z - P_0 \cdot \frac{z^2}{2}\right)$$

$$\varphi_B(L) = 0 \longrightarrow \frac{1}{EI}\left(-M_0 \cdot L + P_0 \cdot L^2 - P_0 \cdot \frac{L^2}{2}\right) = 0$$

$$-M_0 \cdot L + \frac{P_0 \cdot L^2}{2} = 0 \qquad \boxed{M_0 = \frac{1}{2} P_0 \cdot L}$$ → olmalıdır.

Uyg.-3:

Şekilde L açıklıklı kirişe parabolik bir yük etkiyor. Diferansiyel denklem yöntemiyle elde edilecek elastik eğri ifadesinden faydalanarak mesnet tepkilerini bulunuz.



Çözüm:

Dördüncü dereceden diferansiyel denge denkleminde yayılı yük ifadesi yerleştirilerek dört kez integre edilir.

$$\frac{d^4v}{dz^4} = \frac{q_z}{EI} = \frac{1}{EI} \cdot q_0\left(\frac{2z}{L} - \frac{z^2}{L^2}\right)$$

$$\frac{d^3v}{dz^3} = \frac{L}{EI}\left(\frac{q_0 z^2}{L} - \frac{q_0 z^3}{3L^2}\right) + C_1$$

$$\frac{d^2v}{dz^2} = \frac{L}{EI}\left(\frac{q_0 \cdot z^3}{3L} - \frac{q_0 z^4}{12L^2}\right) + C_1 . z + C_2$$

$$\frac{dv}{dz} = \frac{1}{EI}\left(\frac{q_0 z^4}{12L} - \frac{q_0 z^5}{60L^2}\right) + C_1 . \frac{z^2}{2} + C_2 . z + C_3$$

$$\boxed{v(z) = \frac{1}{EI}\left(\frac{q_0 \cdot z^5}{60L} - \frac{q_0 \cdot z^6}{360L^2}\right) + C_1 . \frac{z^3}{6} + C_2 . \frac{z^2}{2} + C_3 . z + C_4}$$

Sınır Şartları:

$$z=0 \rightarrow v(0) = 0 \longrightarrow \frac{1}{EI}\left(\frac{q_0 \cdot 0}{60L} - \frac{q_0 \cdot 0}{360L^2}\right) + C_1 . \frac{0}{6} + C_2 . \frac{0}{2} + C_3 . 0 + C_4 = 0 \qquad \boxed{C_4 = 0}$$

$$z=0 \rightarrow \frac{d^2v}{dz^2} = 0 \longrightarrow \frac{1}{EI}\left(\frac{q_0 \cdot 0}{3L} - \frac{q_0 \cdot 0}{12L^2}\right) + C_1 . 0 + C_2 = 0 \qquad \boxed{C_2 = 0}$$

$$z=L \rightarrow \frac{dv}{dz} = 0 \longrightarrow \frac{1}{EI}\left(\frac{q_0 . L^4}{12L} - \frac{q_0 . L^5}{60L^2}\right) + C_1 . \frac{L^2}{2} + \cancel{C_2 . L} + C_3$$

$$\frac{1}{EI}\left(\frac{q_0 . L^3}{12} - \frac{q_0 . L^3}{60}\right) + C_1 . \frac{L^2}{2} + C_3 = 0$$

$$\boxed{\frac{1}{EI}\left(\frac{q_0 L^3}{15}\right) + C_1 . \frac{L^2}{2} + C_3 = 0} \quad --- \quad \text{I}$$

$z=L \quad v(L)=0 \longrightarrow \frac{1}{EI}\left(\frac{q_0 L^5}{60L} - \frac{q_0 L^6}{360L^2}\right) + C_1 \frac{L^3}{6} + \cancel{C_2 \cdot \frac{L^2}{2}} + C_3 \cdot L + \cancel{C_4} = 0$

$$\frac{1}{EI}\left(\frac{q_0 \cdot L^4}{60} - \frac{q_0 L^4}{360}\right) + C_1 \cdot \frac{L^3}{6} + C_3 \cdot L = 0$$

$$\frac{1}{EI}\left(\frac{q_0 \cdot L^4}{72}\right) + C_1 \cdot \frac{L^3}{6} + C_3 \cdot L = 0$$

$$\boxed{\frac{1}{EI}\left(\frac{q_0 L^3}{72}\right) + C_1 \cdot \frac{L^2}{6} + C_3 = 0} \quad --- \ (II)$$

(I) ve (II) denklemleri birlikte çözülürse;

$$\frac{1}{EI}\left(\frac{q_0 \cdot L^3}{15}\right) + C_1 \frac{L^2}{2} + C_3 = 0 \quad --- \ (I)$$

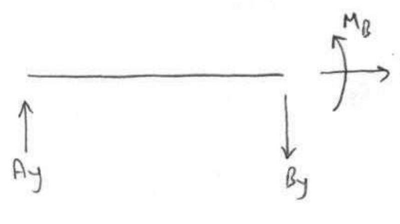
$$- \ \frac{1}{EI}\left(\frac{q_0 \cdot L^3}{72}\right) + C_1 \cdot \frac{L^2}{6} + C_3 = 0 \quad --- \ (II)$$

$$\frac{1}{EI}\left(\frac{q_0 \cdot L^3}{15} - \frac{q_0 \cdot L^3}{72}\right) + C_1\left(\frac{L^2}{2} - \frac{L^2}{6}\right) = 0 \longrightarrow \boxed{C_1 = \frac{-19}{120EI} q_0 \cdot L}$$

C_1 değeri I veya II denkleminde yerine yazılır. ve C_3 bulunur.

$$\boxed{C_3 = \frac{1}{80EI} q_0 \cdot L^3}$$

$$\boxed{V(z) = \frac{1}{EI}\left(\frac{q_0}{60L} z^5 - \frac{q_0}{360L^2} z^6 + \frac{q_0 \cdot L^3}{80} \cdot z - \frac{1.19 q_0 L}{720} z^3\right)} \rightarrow \text{Elastik Eğri}$$



$$\frac{d^3v}{dz^3} = -\frac{A_y}{EI} \longrightarrow EI \frac{d^3v}{dz^3} = -A_y$$

$$EI\left[\frac{1}{EI}\left(\frac{q_0 \cdot z^2}{L} - \frac{q_0 \cdot z^3}{3L^2} - \frac{19}{120} q_0 \cdot L\right)\right] = -A_y$$

$$z=0 \longrightarrow EI \frac{d^3v}{dz^3} = -A_y$$

$$\frac{q_0 \cdot 0}{L} - \frac{q_0 \cdot 0}{3L^2} - \frac{19}{120} q_0 \cdot L = -A_y$$

$$\boxed{A_y = \frac{19}{120} q_0 \cdot L}$$

$$z=L \longrightarrow \frac{d^2v}{dz^2} = -\frac{M_B}{EI} \Longrightarrow EI\underbrace{\frac{d^2v}{dz^2}} = -M_B$$

$$EI\left[\frac{1}{EI}\left(\frac{q_0 \cdot L^3}{3L} - \frac{q_0 \cdot L^4}{12L^2}\right) - \frac{19}{120}q_0 \cdot L^2\right] = -M_B$$

$$\boxed{M_B = -\frac{11}{120}q_0 \cdot L^2}$$

$$z=L \rightarrow \frac{d^3v}{dz^3} = -\frac{B_y}{EI} \Longrightarrow EI\frac{d^3v}{dz^3} = -B_y$$

$$EI\left[\frac{1}{EI}\left(\frac{q_0 \cdot L^2}{2} - \frac{q_0 \cdot L^3}{3L^2} - \frac{19}{120}q_0 \cdot L\right)\right] = -B_y$$

$$\boxed{B_y = -\frac{61}{120}q_0 \cdot L}$$

Yatay denge denklemi $\overset{+}{\rightarrow} \sum F_z = 0 \Rightarrow \boxed{B_z = 0}$

Tanımlar

$$\frac{dT}{dx} = -q \iff T = -\int q\,dx$$

$$\frac{dM}{dx} = T \iff M = \int T\,dx = -\iint q\,dx^2$$

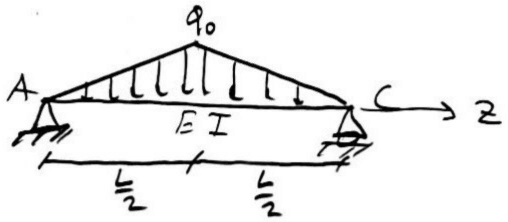
$$\frac{dv}{dx} = \theta \iff v = \int \theta\,dx$$

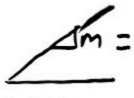
$$K = \frac{1}{\rho} = \frac{d\theta}{dx} = \frac{d^2v}{dx^2} = -\frac{M}{EI} \iff v = -\iint \frac{M}{EI}\,dx^2 \quad \text{ve} \quad \theta = -\int \frac{M}{EI}\,dx$$

$$\frac{d^3v}{dx^3} = \frac{d}{dx}\left(-\frac{M}{EI}\right) = -\frac{T}{EI} \quad ; \quad \frac{d^4v}{dx^4} = \frac{d}{dx}\left(-\frac{T}{EI}\right) = \frac{q}{EI}$$

SORU: Şekildeki basit kiriş simetrik üçgen yayılı yük etkisindedir. Diferansiyel denklem yöntemi ile elastik eğri denklemini yazınız. A noktasındaki dönmeyi ve kirişte meydana gelen en büyük çökmeyi hesaplayınız.

Çözüm: Kiriş simetrik olduğundan yarısı için hesap yapılabilir.





$m = \frac{q_0}{\frac{L}{2}} = \frac{2q_0}{L}$

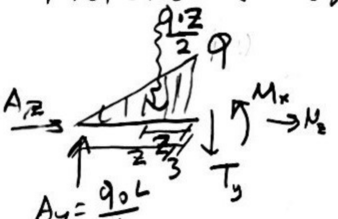
$\mapsto z$

$q = q(z) = mz + \cancel{n}_0 = \frac{2q_0 z}{L}$

(veya üçgen benzerliği $\frac{q(z)}{z} = \frac{q_0}{L/2}$

)

moment fonksiyonu:



$$M_x = \frac{q_0 L}{4} z - \frac{q z}{2} \cdot \frac{z}{3}$$

$$M_x = \frac{q_0 L}{4} z - \frac{q_0}{3L} z^3$$

diferansiyel denklem:

$$-EI_x\, v''(z) = \frac{q_0 L}{4} z - \frac{q_0}{3L} z^3$$

$$-EI_x\, v'(z) = \frac{q_0 L}{8} z^2 - \frac{q_0}{12L} z^4 + C_1$$

$$-EI_x\, v(z) = \frac{q_0 L}{24} z^3 - \frac{q_0}{60L} z^5 + C_1 z + C_2$$

Sınır koşulları:

$$-EI_x v(0) = 0 \Rightarrow C_2 = 0$$

$$-EI_x v'\left(\frac{L}{2}\right) = 0 \Rightarrow \frac{q_0 L}{8}\left(\frac{L}{2}\right)^2 - \frac{q_0}{12L}\left(\frac{L}{2}\right)^4 + C_1 = 0$$

$$C_1 = \frac{q_0 L^3}{12 \cdot 16} - \frac{q_0 L^3}{32} = -\frac{5 q_0 L^3}{192}$$

Elastik eğri ve dönme eşitlikleri $(0 \le z \le \frac{1}{2}L)$

$$v'(z) = -\frac{1}{EI_x}\left[\frac{q_0 L}{8} z^2 - \frac{q_0}{12L} z^4 - \frac{5 q_0 L^3}{192}\right]$$

$$v(z) = -\frac{1}{EI_x}\left[\frac{q_0 L}{24} z^3 - \frac{q_0}{60L} z^5 - \frac{5 q_0 L^3}{192} z\right]$$

A noktasındaki dönme;

$$\theta_A = \theta(0) = v'(0) = \frac{5 q_0 L^3}{192\, EI}$$ dönüş yönü pozitif

Maksimum sehim:

$v(z)$'nin ekstremum noktası aranmaktadır.

$$v'(z) = \theta(z) = 0$$

$$-\frac{q_0}{12L} z^4 + \frac{q_0 L}{8} z^2 - \frac{5 q_0 L^3}{192} = 0$$

$z^2 = t$ olsun

$$t_{1,2} = \frac{-\frac{q_0 L}{8} \pm \sqrt{\frac{q_0^2 L^2}{64} - \frac{5 q_0^2 L^3}{3 \cdot 192 L}}}{-\frac{q_0}{6L}} = \frac{3L^2}{4} \mp \frac{6L}{q_0}\sqrt{\frac{9 q_0^2 L^2}{576} - \frac{5 q_0^2 L^2}{576}} = \frac{3L^2}{4} \mp \frac{12 L^2}{24}$$

$$t_{1,2} = \frac{3L^2}{4} \mp \frac{2L^2}{4} = \frac{L^2}{4}(3 \mp 2)$$

$$z_{1,2,3,4} = \pm\sqrt{t_{1,2}} = \pm \frac{L\sqrt{3 \mp 2}}{2}$$

$$0 \le z \le \frac{L}{2} \Rightarrow \cancel{z_1 = \frac{L\sqrt{5}}{2}} \; (1{,}12) \; ; \; z_2 = \frac{L}{2} \; ; \; \cancel{z_3 = -\frac{L\sqrt{5}}{2}} \; ; \; \cancel{z_4 = -\frac{L}{2}}$$

$$v\left(z_2 = \frac{L}{2}\right) = -\frac{1}{EI_x}\left[\frac{q_0 L}{24}\left(\frac{L}{2}\right)^3 - \frac{q_0}{60L}\left(\frac{L}{2}\right)^5 - \frac{5 q_0 L^3}{192} \cdot \frac{L}{2}\right] = \frac{q_0 L^4}{120\, EI_x}$$