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References

1. Riesz Spaces I, Luxemburg, Zaanen, 1983
2. Positive Operators, C. D. Aliprantis, O. Burkinshaw 1985
3. Banach Lattices, P. Meyer-Nieberg, Springer, 1991.

1. Basic Properties

Definition. A real vector space E is said to be an ordered vector space whenever it is equipped with an order relation $\geq$ (that is, $\geq$ is a reflexive, anti-symmetric, transitive binary relation on E) that is compatible with the algebraic structure of E in the sense that it satisfies of the set $\{x,y\}$ both exist in E.

Symbol:

$$x \vee y = \sup\{x,y\},$$

$$x \wedge y = \inf\{x,y\}.$$

Example. Let X be a set.

$$E = F(X) = \{f \mid f : X \longrightarrow \mathbb{R} \text{ is a function}\}.$$

For each pair $f, g \in E$, the functions

$$(f \vee g)(x) = \max\{f(x), g(x)\} \in E$$

$$(f \wedge g)(x) = \min\{f(x), g(x)\} \in E.$$

$f \leq g$ holds in $E \iff f(x) \leq g(x)$ for all $x \in X$.

$E = F(X)$ is a Riesz space.

Examples (1) $E = \mathbb{R}^X$ all real valued functions defined on a set X.

the following axioms:

1) If $x \geq y$, then $x+z \geq y+z$ holds for all $z \in E$.

2) If $x \geq y$, then $\alpha x \geq \alpha y$ holds for all $\alpha \geq 0$.

Definition. A vector x in an ordered vector space E is called positive whenever $x \geq 0$ holds.

The set of all positive vectors of E will be denoted by E^+.

$$E^+ = \{ x \in E : x \geq 0 \}.$$

Definition. The set E^+ of positive vectors is called the positive cone of E.

Definition. A Riesz space or a vector lattice is an ordered vector space E with the property that for each pair of vectors $x, y \in E$ the supremum and the infimum

2) $E = C(X)$ all continuous real-valued functions on a topological space X.

For example, $C[0,1]$.

3) $E = B(X)$ all bounded real-valued functions on a set X.