Exercises

1) Find and classify the critical points of the functions:

a) $f(x,y) = x^2 + 2y^2 - 4x + 4y$.

b) $f(x,y) = x^3 + y^3 - 3xy$.

c) $f(x,y) = x^4 + y^4 - 4xy$.

d) $f(x,y) = x\, e^{-x^2+y^3}$.

2) Find the maximum and minimum values of $f(x,y) = x - x^2 + y^2$ on the rectangle $0 \le x \le 2$, $0 \le y \le 1$.

3) Find the maximum and minimum values of $f(x,y) = xy - y^2$ on the disk $x^2 + y^2 \le 1$.

4) Find the maximum and minimum values of $f(x,y) = xy(1-x-y)$ over the triangle with vertices $(0,0)$, $(1,0)$, $(0,1)$.

5) Find the shortest distance from the point $(3,0)$ to the parabola $y = x^2$.

6) Find the distance from the origin to the plane $x+2y+2z=3$ by using Lagrange multipliers.

7) Find the maximum and minimum values of the function $f(x,y,z)=x+y+z$ over the sphere $x^2+y^2+z^2=1$.

8) Find the maximum and minimum values of $x+2y-3z$ over the ellipsoid $x^2+4y^2+9z^2 \le 108$.

9) Find div F, curl F for the following vector fields:

a) $F = yz\,i + xz\,j + xy\,k$.

b) $F = xy^2\,i - yz^2\,j + zx^2\,k$.

c) Let $F = x\,i + y\,j + z\,k$,
$G = xy\,i + yz\,j + zx\,k$.
Calculate $\nabla \times (F \times G) = ?$

11) Let $F = \cos x^2\,i + \sin y\,j + \tan(xyz)\,k$.
Calculate div F, curl F.

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12) Let $\phi(x,y,z) = xy+z$; $F = x^2 i - y^2 j + z^2 k$.

i) Calculate curl(grad ϕ) = ?

ii) Calculate $\nabla \cdot (\phi F)$ = ?

iii) Calculate $\nabla \times (\phi F)$ = ?

iv) Calculate div curl(F) = ?

v) Calculate $\nabla \times (\phi F)$ = ?

Exercises 4

1) Find the largest value of the product of three positive numbers whose sum is 60.

2) Find the points of the hyperbola $x^2 - y^2 = 1$ which are closest to the point $(0, 4)$.

3) Find the directional derivative of the function $f(x,y,z) = \cos(x+y+z)$ at the point $(\ln 2, 1, -1)$ in the direction of the vector $u = i - 4j + 8k$.

4) Calculate the directional derivative of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ at the point $P = (9, -6, 2)$ in the direction of the vector $u = i - 2j + 2k$.

5) Calculate the directional derivative of $f(x,y) = \arctan\left(\frac{x}{y}\right)$ at the pont $P = (3, 4)$ in the direction of the vector $u = -6i - 8j$.

6) What is the maximum rate of increase of the function $f(x,y,z) = xy^2z^3$ at the point $P=(2,1,-1)$ and in which direction does it occur?

7) By using Lagrange multipliers investigate the extrema of the function $f(x,y) = 3x - 2y + 1$ subject to the constraint $9x^2 + 4y^2 = 18$.

8) By using Lagrange multipliers investigate the extrema of the function $f(x,y,z) = 2x - 3y + z - 1$ subject to the constraint $x^2 + y^2 + z^2 = 14$.

9) Find $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial u}$ if $w = 1 - x^2 - y^2$, $x = t\cos u$, $y = t\sin u$

10) Let $u = u(r,s,t)$, where $r = x - y$, $s = y - z$, $t = z - s$ and u is a differentiable function.

Calculate $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = ?$

11) Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ if x, y, z satisfy the equation

$$\sin(xz) + \cos(yz) = 0.$$

12) Find dz if $\sin^2 x + \sin^2 y + \sin^2 z = 1$

13) Given that $2x^2 + y^2 + 3z^2 + xy - z - 9 = 0$ find $\frac{\partial^2 z}{\partial x^2}$ at the point $(-2, 1, 1)$.

14) Assuming that the equation $F(x, y, z) = 0$ defines each of the three variables x, y and z as an implicit function of the other two, calculate $\frac{\partial x}{\partial y} \frac{\partial y}{\partial z} \frac{\partial z}{\partial x} = ?$

15) Find the tangent plane equation and normal line equation to the graph of the equation $x^3 + y^3 + z^3 = 1$ at the point $P = (-2, 1, 2)$.

16) Find equations for the tangent plane and normal line to the graph of the equation $z = \ln(x^2 + y^2)$ at the point $P = (e, 0, 2)$.