Yıldız Technical University
Civil Engineering Department
Construction Management Division

Engineering Economy- 3

Nominal vs. Effective Interest Rates

The concepts of nominal and effective interest rates must be used, when interest is compounded more than once each year.

Nominal Interest Rate (Annual percentage rate, APR)**:**is an interest rate that does not include any consideration of compounding. A nominal rate may be stated for any time period – 1 year, 6 months, quarter, week, day etc.

i= interest rate per period
j= nominal interest rate
m= number of periods

$$J = i^{*}m$$

*j= 1,5 % per month * 24 months = 36% per 2-year period*
*j=1,5% per month * 12 months = 18% per year*
*j=1,5% per month * 6 months = 9% per 6 months*

The format: j% per time period t

Nominal vs. Effective Interest Rates

Nominal interest	Period	m	Interest rate per period
%9 per year	year	1	9/1 = %9 per year
%6 per year	quarter	4	6/4 = %1.5 per quarter
%18 per year	month	12	18/12= %1.5 per month
%5 per 6 months	week	26	5/26= %0,192 per week

Nominal vs. Effective Interest Rates

Effective Interest Rate **:**is the actual rate that applies for a stated period of time. The compounding of interest during the time period of the corresponding nominal rate is accounted for by the effective interest rate.

An effective rate has the compounding frequency attached to the nominal rate statement.

4% per year, compounded monthly
12 % per year, compounded quarterly
9% per 6 months, compounded monthly

The format: " j per time period t, compounded m-ly"

The effective interest rate can also be expressed directly.

Effective 2,5% per year, compounded quarterly
Effective 3% per six months, compounded yearly

Example 1

- The different bank loan rates for three separate projects are listed below. Determine the effective rate on the basis of the compounding period for each quote.
  - 8% per year, compounded quarterly.
  - 4,5 % per 6 months, compounded monthly.
  - The period interest rate for 6 months is 2%.

Example 1

Nominal Interest rate	Compounding period	m	Effective rate per compounding period	Distribution over one year period
%8 per year	quarterly	4	2 %	%2 | %2 | %2 | %2
%4,5 per 6 months	monthly	6	0,75%	
%4	6 months	1	2%	%2 | %2

Nominal and effective interest rate

D = m* d

i=	j/m	j/m	j/m	j/m	j/m	...	...	j/m
	d	d	d	d	d	d	d	d

D = period
d = compounding period
m = number of compounding in a period
j = nominal interest of period
i = j/m = effective interest rate of compounding period
P = Capital

- Interest earned at the end of the period can be calculated by using effective interest rate of compounding period.

At the end of first period: $F_1=P\ (1+i)$
At the end of second period : $F_2=F_1\ (1+i)=P\ (1+i)^2$
At the end of third period : $F_3=F_2\ (1+i)=P\ (1+i)^3$

at m. period: $F_m = P\ (1+i)^m$

Bulduğumuz bu dönem sonu değer, dönem başındaki değere bileşik faiz uygulanması ile de bulunabilir;

$i_e = \frac{F-P}{P} = \frac{P*(1+i)^m-P}{P}$>>>

$$\boldsymbol{i_e = (1+i)^m - 1}$$

Nominal and effective interest rate

- Effective interest rate of the period, i_e ?

j=nominal interest rate
m = number of compounding in a period
i = effective interest rate of compounding period= j / m

$$\mathrm{i_e} = (1+i)^m - 1 = (1 + j/m)^m - 1$$

Nominal vs. Effective Interest Rates

- effective interest rate per time period t, r?

$$r = (1 + i)^m - 1$$

i = effective interest rate per compounding period.
m =compounding frequency – the number of times that m compounding occurs within the time period t.

$$r = (1 + j/m)^m - 1$$

j=nominal interest rate

Nominal vs. Effective Interest Rates

- Effective interest rate for 52% nominal interest per year;

Period	Compounding frequency	Interest rate per period	Effective interest per year
Year	1	%52	$(1+0,52)^{1}$ -1=%52
6 months	2	%26	$(1+0,26)^{2}$ -1= %58.76
3 months	4	%13	$(1+0,13)^{4}$ -1=%63.05
Month	12	%4.33	$(1+0,0433)^{12}$ -1=%66,31
Week	52	%1	$(1+0,01)^{52}$ -1=%67,77

Example 2

- The effective interest rate per month is 1,5% for a credit card. What is the nominal interest rate and effective interest rate per year?

Nominal vs. effective Faiz Oranları

- $i = \frac{j}{m}$
- $1{,}5 = \frac{j}{12}$ → j = %18 per year

$$r = (1 + j/m)^m - 1$$

- $r = (1 + 0{,}18/12)^{12} - 1$
- r = 0,1956 → %19,56

Equivalence relationships: comparing payment period and compounding period

The frequency of cash flows does not equal to the frequency of interest compounding. For example, cash flows occur monthly, and compounding occurs annually.

$$\boxed{\begin{aligned} i &= (1 + j/M)^{C} - 1 \\ &= [1 + j/CK]^{C} - 1 \end{aligned}}$$

C = number of compounding periods per payment period

K =number of payment periods

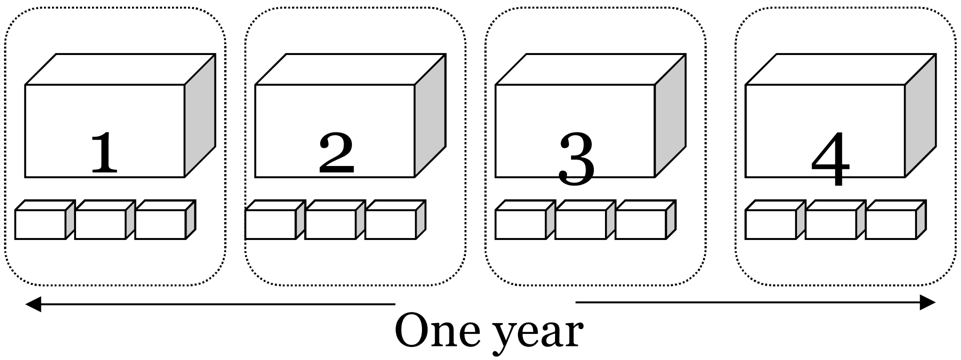
j= nominal interest rate

M = C * K : Number of compounding periods per time period

Example 3

- An amount of money is deposited quarterly into a bank account at %12 per year, compounding monthly. What is the effective interest rate quarterly?

Example



%12 per year, compounded monthly
Payment period (K) = 4
Compounding frequency (C) = 3
C.K= 12

$$i = [1 + j / CK]^{C} - 1$$

$$i = [1 + 0.12 / 12]^{3} - 1$$

i= 0.030301 → i= 3,0301 %

Example 4

- A company plans to make an investment at 18% per year, compounded daily. What effective rate is this (a) yearly and (b) semiannually?

Example

C= 365

K=1

C.K= 365

$$i = [1 + j / CK]^C - 1$$

$$i = [1 + 0.18 / 365]^{365} - 1$$

i_{year}= 0.1972 → i= 19,72 %

Example

C= 182,5

K=2

C.K= 365

$$i = [1 + j / CK]^{C} - 1$$

$$i = [1 + 0.18 / 365]^{182,5} - 1$$

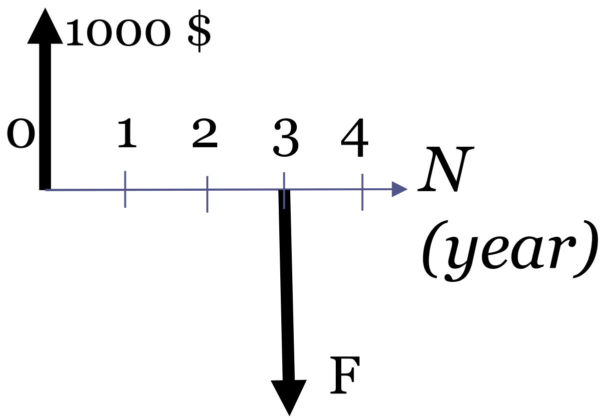
$i_{6months}$= 0. 09415 → i= 9,415 %

Example 5

- A credit amounted as \$1,000 is withdrawn at the present time at 8% per year. What is the effective interest rate per quarter if the interest is compounded:
  - (a) monthly
  - (b) weekly
  - (c) daily

What is the amount of payment will be made at the end of third year according to each interest rate?

Example



C= 3
K= 4
C.K= 12

$$i = [1 + j / CK]^{C} - 1$$

$i_{quarter}$ = [1+0,08/12]^{3} - 1
$i_{quarter}$= 0.02013 → i= 2,013 %
F= P (F/P, 2,013%, 12)
F= 1000*(1+0,02013)^{12} = 1270,183

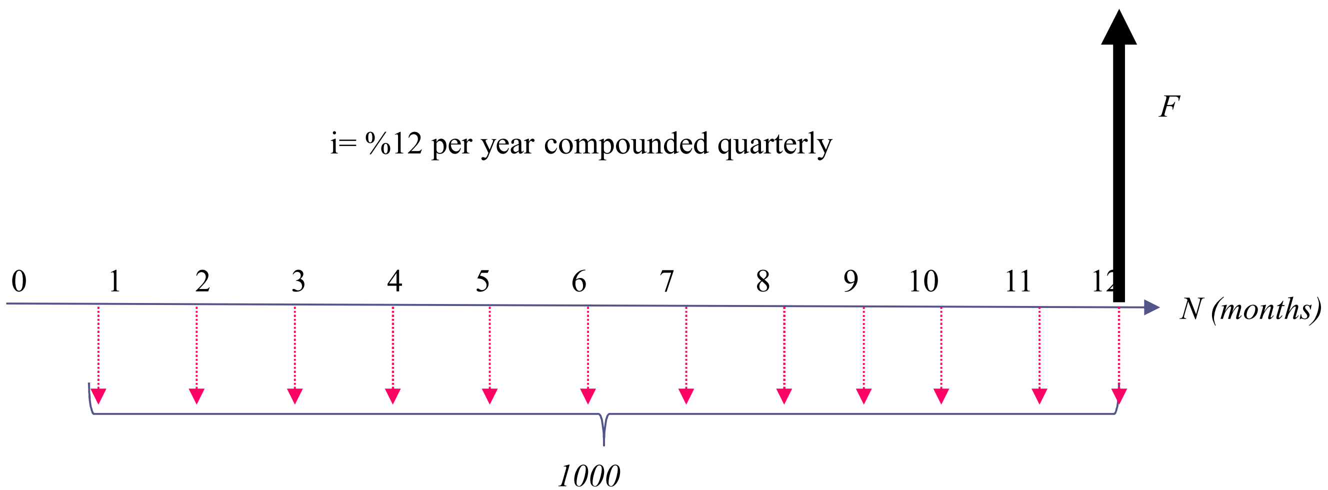
C= 52/4= 13
K= 4
C.K= 52
$i_{quarter}$ = [1+0,08/52]^{13} - 1
$i_{quarter}$= 0.020186 → i= 2,0186 %
F= P (F/P, 2,0186%, 12)
F= 1000*(1+0,020186)^{12} = 1271,020

C= 365/4= 91,25
K= 4
C.K= 365
$i_{quarter}$ = [1+0,08/365]^{91,25} - 1
$i_{quarter}$= 0.0202 → i= 2,02 %
F= P (F/P, 2,020%, 12)
F= 1000*(1+0,0202)^{12} = 1271,23

Example 6

The monthly payment is 1000 TL for a year. How much money can be withdrawn at the end of the year if the interest rate is 12% per year, compounded quarterly?

Example



C = 1/3

K = 12

C. K = 4

$$i_{month} = [1+0{,}12/4]^{1/3} - 1$$

$$i_{month} = 0{,}99\ \%$$

$$F = A\ (F/A,\ 0{,}99\%,\ 12) = 1000^{*} \frac{(1+0{,}0099)^{12}-1}{0{,}0099} = 1000^{*}12{,}6754 = 12675{,}4\ \text{TL}$$