# Yıldız Technical University
# Civil Engineering Department
# Construction Management Division

## Engineering Economy

# Time value of money

# Time value of money

- Money makes money which is known as earning power of money.
- The change in the amount of money over a given time period is called the time value of money; it is the most important concept in engineering economy.
- Interest is the manifestation of the time value of money.
- Computationally, interest is the difference between an ending amount of money and the beginning amount.
- Interest is paid when a person or organization borrowed money. Interest is earned when a person or organization lent money.

  Interest = amount owed now- original amount
- Interest, interest period and interest rate

# Simple and compound interest

- Simple interest: is calculated using the principal only, ignoring any interest accrued in preceding interest periods.
- Compound interest: the interest accrued for each interest period is calculated on the principal plus the total amount of interest accumulated in all previous periods. Thus, compound interest means interest on top of interest.

| Period | Initial amount | Interest | Final amount |
|---|---|---|---|
| 0 | | | $1,000 |
| 1 | $1,000 | $80 | $1,080 |
| 2 | $1,080 | $80 | $1,160 |
| 3 | $1,160 | $80 | $1,240 |

| Period | Initial amount | Interest | Final amount |
|---|---|---|---|
| 0 | | | $1,000 |
| 1 | $1,000 | $80 | $1,080 |
| 2 | $1,080 | $86.40 | $1,166.40 |
| 3 | $1,166.40 | $93.31 | $1,259.71 |

# Simple Interest

F = Future Value

P = Present Value

i = interest ratio

n = Interest period

$$\mathbf{F = P + [(n.i.P) / 100]}$$

# Compound Interest

F = Future Value

P = Present Value

i = interest ratio

n = Interest period

$$F = P * (1+i/100)^n$$

# Example 1

- Mr. A borrows 100 000 Turkish Lira at %40 per year for 5 year period,
  - If the interest is calculated by using simple interest, how much will he pay at the end of 5 year interest period?
  - If the interest is compounded, how much will he pay at the end of 5 year interest period?

# Example 1

- $F = P + P.i.n / 100$

$F = 100000 + 100000 * 0,4 * 5$

$F = 300000$ TL

- $F = P(1 + i/100)^n$

$F = 100000\ (1 + 0,4)^5$

$F = 100000 * 5,37824$

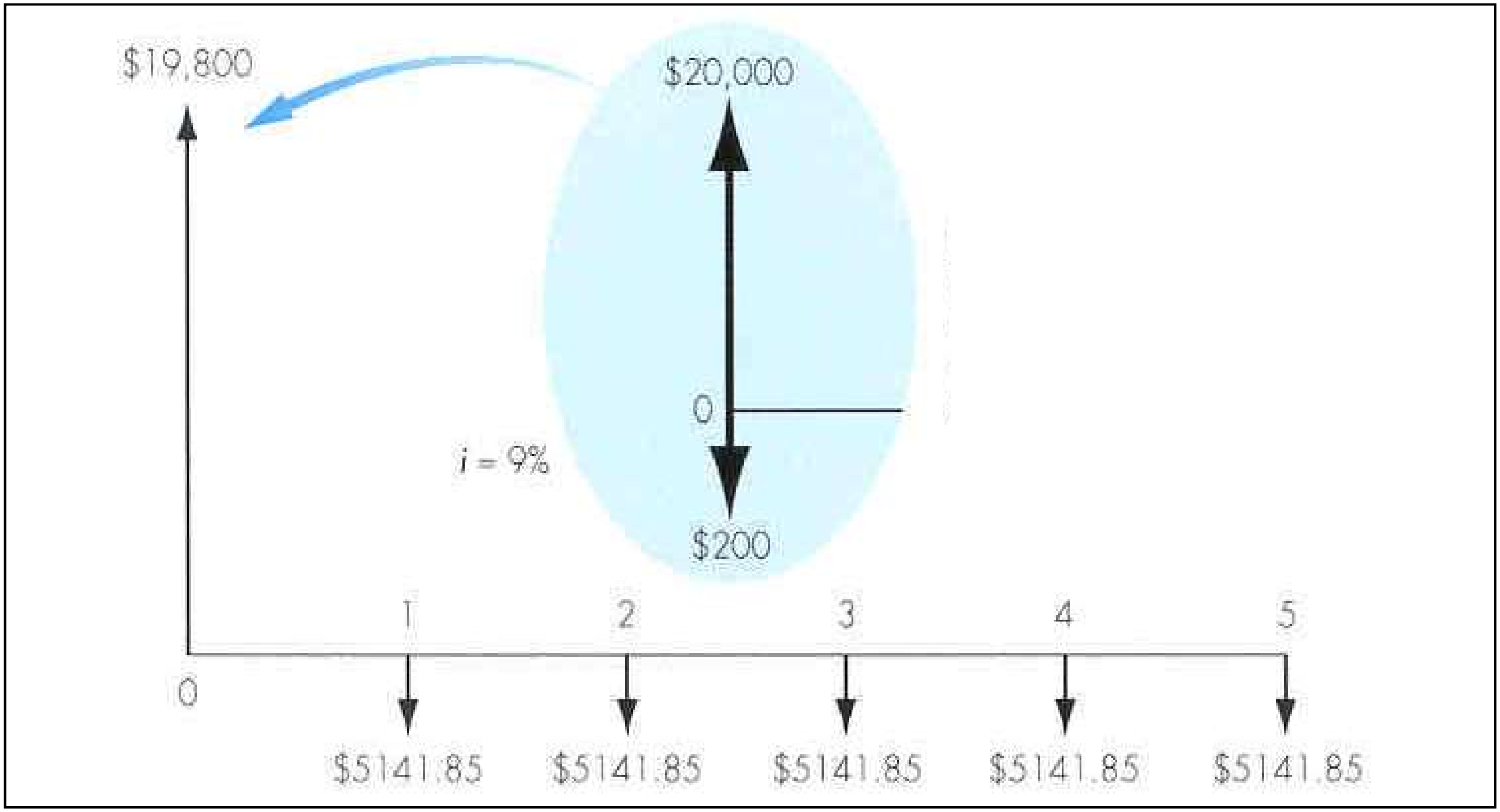
$F = 537824$ TL

# Cash Flow Diagrams

- The estimated inflows (revenues) and outflows (costs) of money are called cash flows.
- The cash flow is fundamental to every economy study.
- Without cash flow estimates over a stated time period, no engineering economy study can be conducted.
- They can be considered as free body diagrams.
- Every person or company has cash receipts-revenue and income (inflows); and cash disbursements-expenses, and costs (outflows).

# Cash Flow Diagram

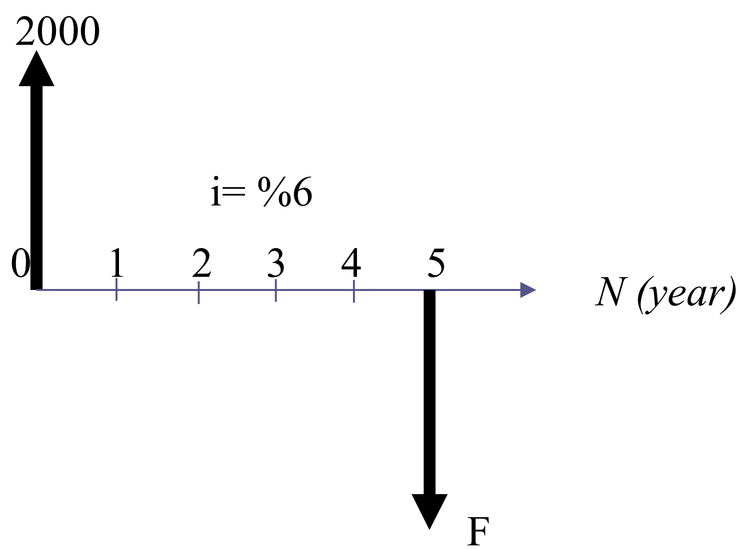
An example of cash flow diagram is shown. The inflows and outflows of the company are illustrated as arrows. The direction of the arrows indicate the direction of the money. The time period is shown on the horizontal line. Finally, the interest rate valid for this cash flow diagram is stated.



# Example 2

- If 2000 TL is borrowed at the present time for 5 year period at 6% interest rate compounded yearly. Show these transactions on the cash flow diagram.

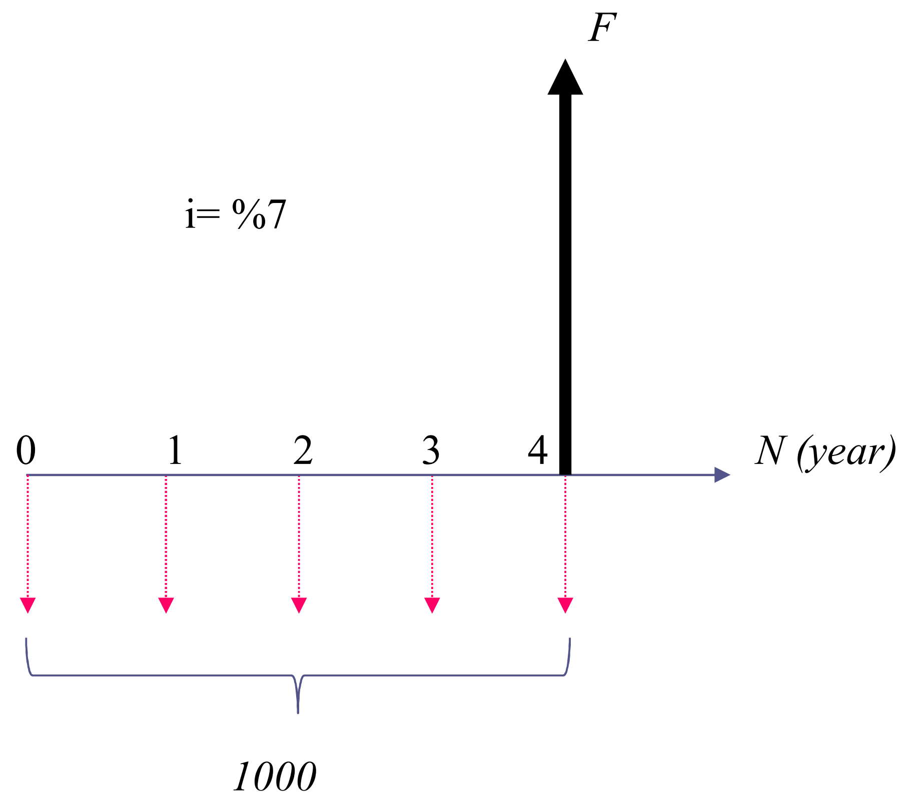
# Example 2



# Example 3

- 1000 TL is saved for 5 times from starting of present time at %7 interest rate per year. The saved total amount is withdrawn with the last saving. Draw the cash flow diagram of these transactions.

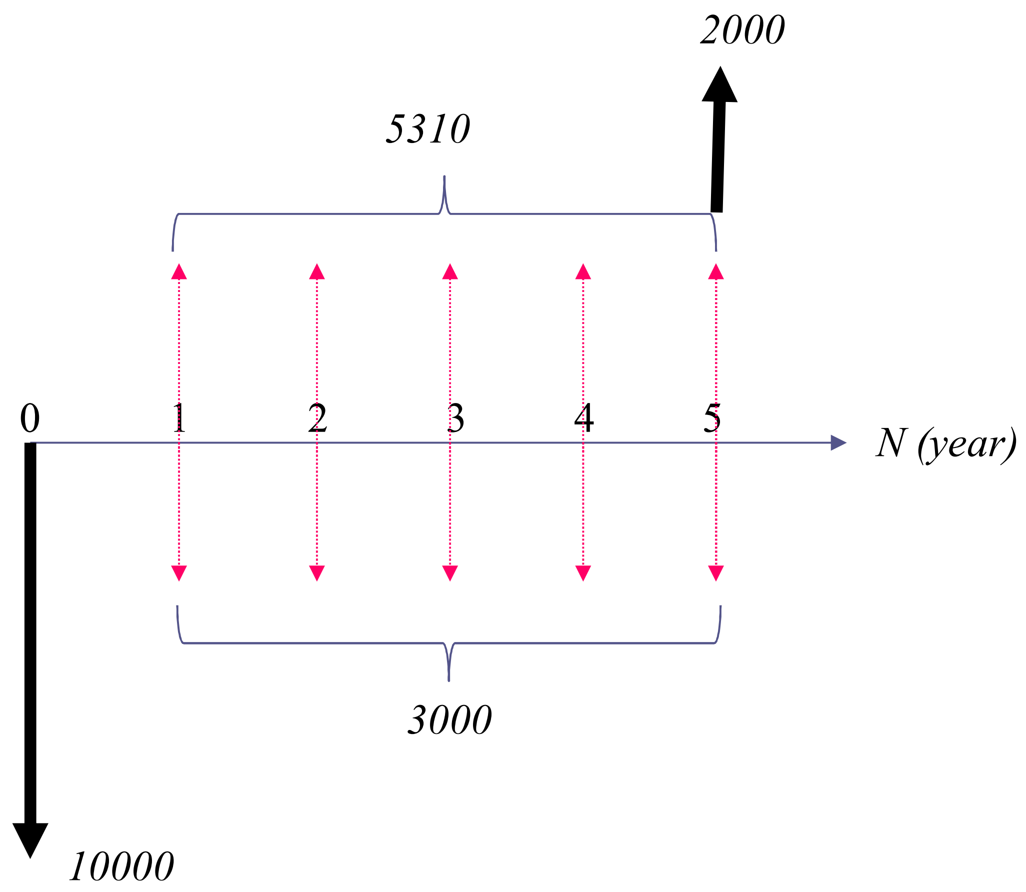
# Example 3



# Example 4

- A company invests 10000 TL at the present time. The incomes of this invest are predicted as 5310 TL per year for 5 years, and the salvage value of this investment is 2000 TL at the end of $5^{th}$ year. The maintenance and operation costs of this investment is calculated as 3000 TL per year. According to these data, please draw the cash flow diagram of this investment.

# Example 4



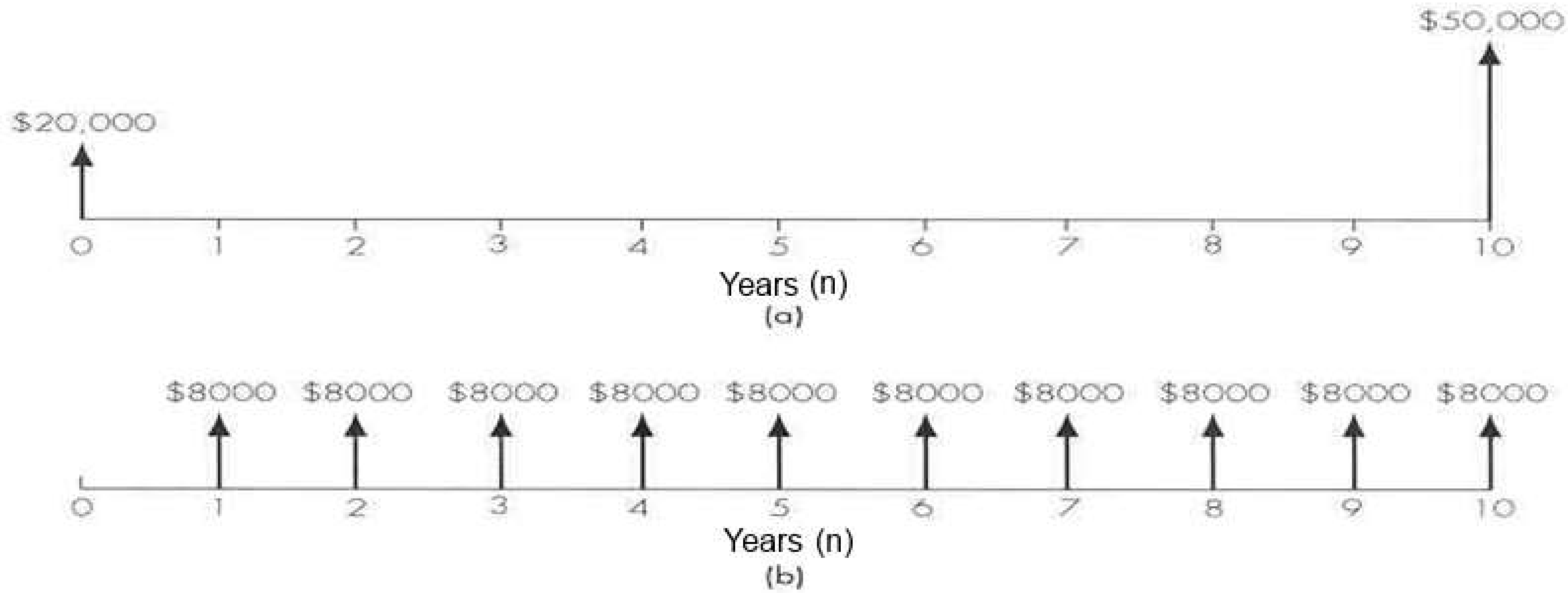
# Economic Equivalence

- Economic equivalence, means that different sums of money at different times would be equal in economic value.
- For example, if the interest rate is 6% per year, $100 today (present time) is equivalent to $106 one year from today.
- Each payment has different economic equivalences for different times, in other words by selecting different times, the economic value of the payment can change. If the present time is selected, then the present worth is calculated, on the other hand the future time is selected, future worth is calculated.

# Economic Equivalence

- In comparing of different cash flows in terms of economic equivalence, the economic equivalences of each inflows and outflows should be calculated for a specific time.
- However, this specific time does not affect the economic equivalence of two cash flows, in other words, by changing this specific time, the equivalence of two cash flows does not change.
- In control of economic equivalence, all transactions in the cash flow could be required to be converted into one transaction at the targeted time.
- Equivalence depends on selected interest rate.
- Two cash flows which seem different due to the different transactions can be economically equivalent due to the interest rate.

# Example




As it can be seen, these two cash flow diagrams have different inflows and outflows at different time periods, however at %32,04 interest rate per year, these two cash flows are economically equivalent to each other.

# Example 5

- If a company owes 8000 TL at 10% compound interest rate per year for 4 year period. According to the following payback plans, which payback plan is the best alternative.
  - 2000 TL and the interests are paid annually.
  - The interests are paid annually, and principal is repaid at the end of the payment period.
  - All payments are made at the end of the payment period.

# 2000 TL and the interest are paid annually

| End of year | Total owed at the beginning of the year | Interest owed for the year | Total owed at the end of the year | Payment from principal | Total payment at the end of the year |
|---|---|---|---|---|---|
| 1 | 8000 | 800 | 8800 | 2000 | 2800 |
| 2 | 6000 | 600 | 6600 | 2000 | 2600 |
| 3 | 4000 | 400 | 4400 | 2000 | 2400 |
| 4 | 2000 | 200 | 2200 | 2000 | 2200 |
| Total | | 2000 | | | 10000 |

# The interest is paid annually, and principal repaid at the end of the period

| End of year | Total owed at the beginning of the year | Interest owed for the year | Total owed at the end of the year | Payment from principal | Total payment at the end of the year |
|---|---|---|---|---|---|
| 1 | 8000 | 800 | 8800 | 0 | 800 |
| 2 | 8000 | 800 | 8800 | 0 | 800 |
| 3 | 8000 | 800 | 8800 | 0 | 800 |
| 4 | 8000 | 800 | 8800 | 8000 | 8800 |
| Total | | 3200 | | | 11200 |

# All payment is made at the end of the period

| End of year | Total owed at the beginning of the year | Interest owed for the year | Total owed at the end of the year | Payment from principal | Total payment at the end of the year |
|---|---|---|---|---|---|
| 1 | 8000 | 800 | 8800 | 0 | 0 |
| 2 | 8800 | 880 | 9680 | 0 | 0 |
| 3 | 9680 | 968 | 10648 | 0 | 0 |
| 4 | 10648 | 1064,8 | 11712,8 | 8000 | 11712,8 |
| Total | | 3712,8 | | | 11712,8 |

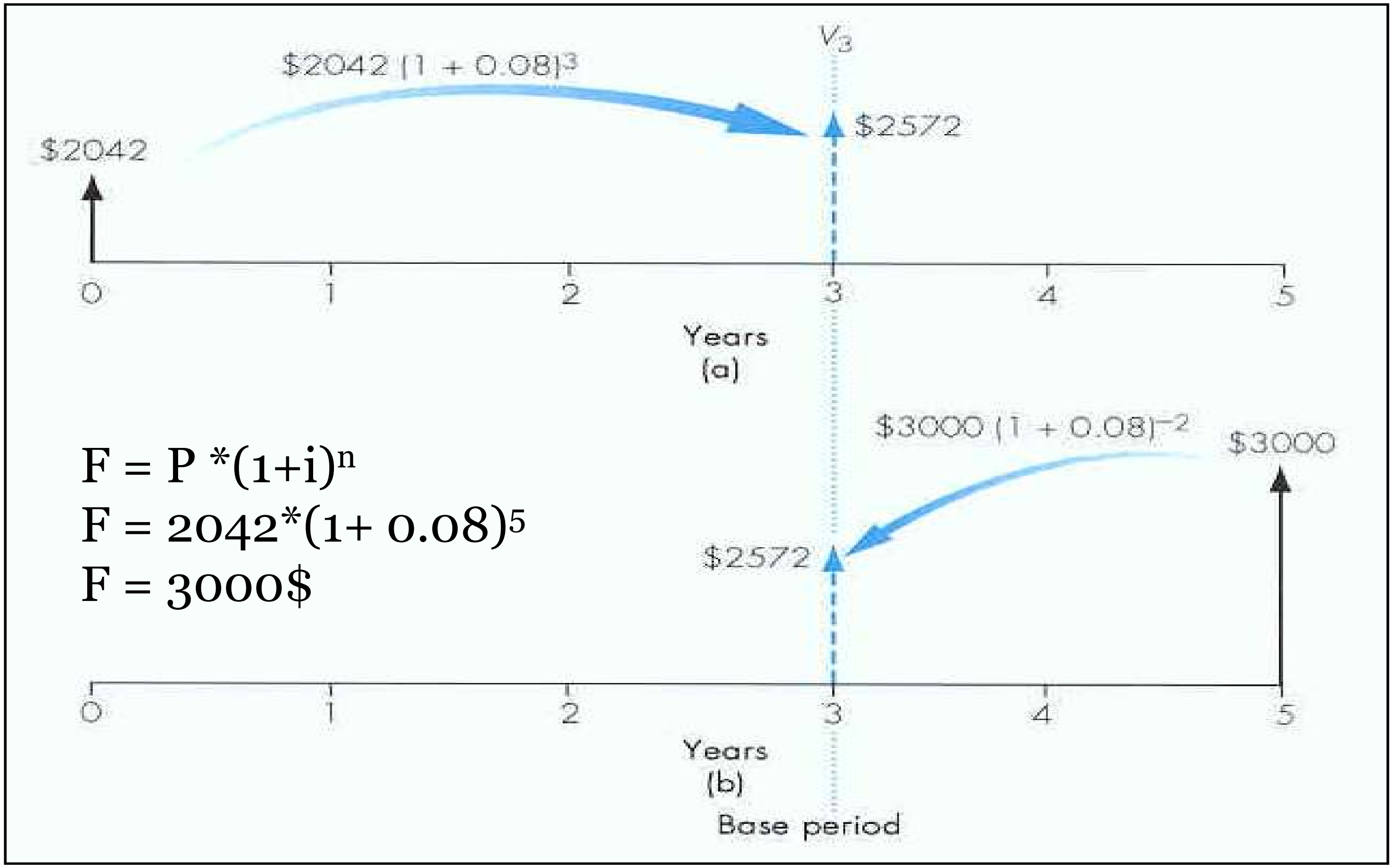
# Economic Equivalence

- As you can see all these three alternatives have different cash flows and the total amount of payments made at these alternatives are different.
- Actually, the present worth of all these alternatives is equivalent to 8000 TL, in other words they have same attractiveness. This shows that it is impossible to reach a correct conclusion by just considering the cash flows at this stage. In order to reach a conclusion, these cash flows are required to be rearranged in order to compare with each other and the equivalent values of all cash flows at a specific time should be calculated.

# Example 6

- If you deposit $2042 today in a savings account that pays 8% interest annually, how much would you have at the end of 5 years?

# “Equivalent cash flows are equivalent at any common point in time”




$F = P * (1+i)^n$

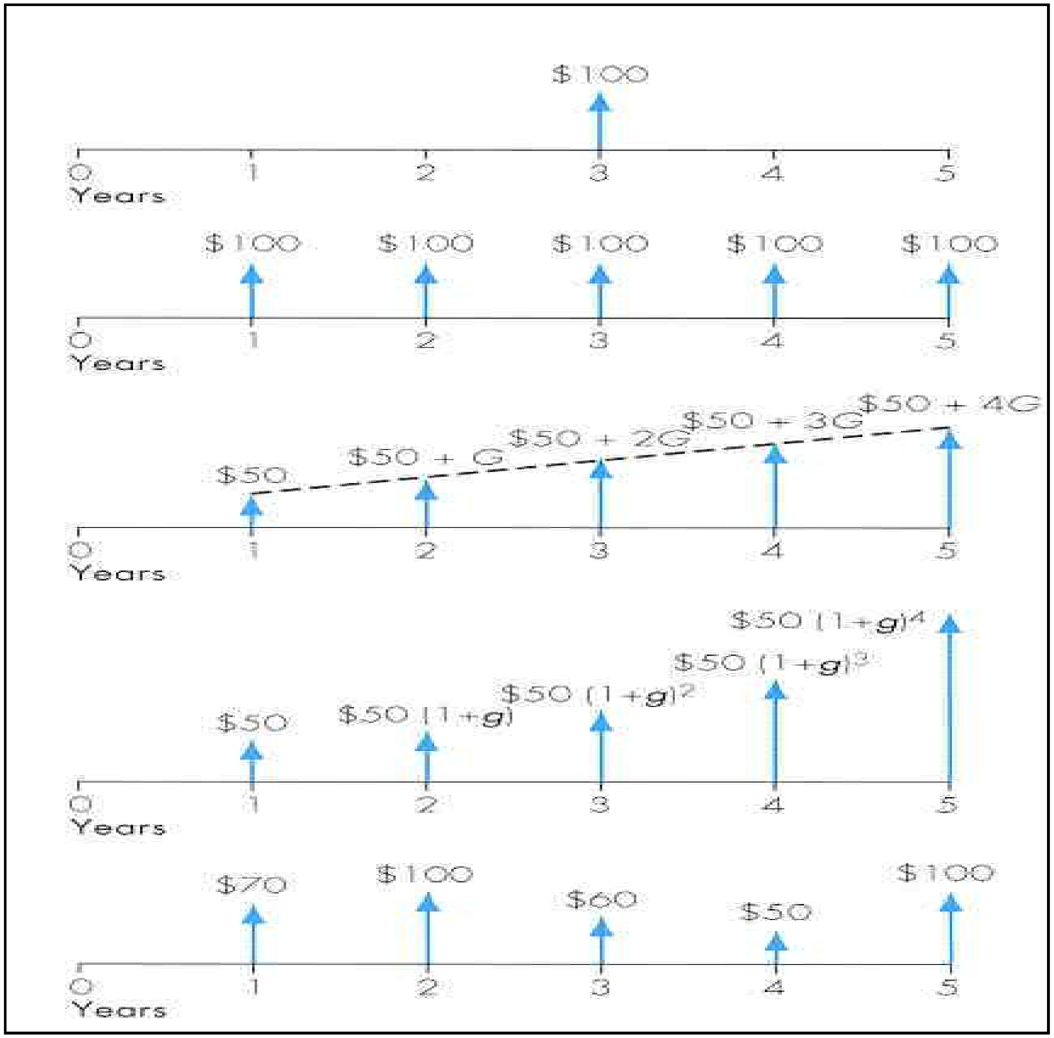
$F = 2042 * (1+ 0.08)^5$

$F = 3000\$$

P value which satisfies the equivalence of these two cash flows is calculated as shown. Since these two cash flows are equivalent, they are equivalent at any common point in time. This is also valid for third year.

# Types of Cash Flows

- Single payment
- Uniform payment series
- Linear gradient series
- Geometrical gradient series
- Irregular payment series



# Single Payment

- Single Payment, compound interest, future value
- Given:

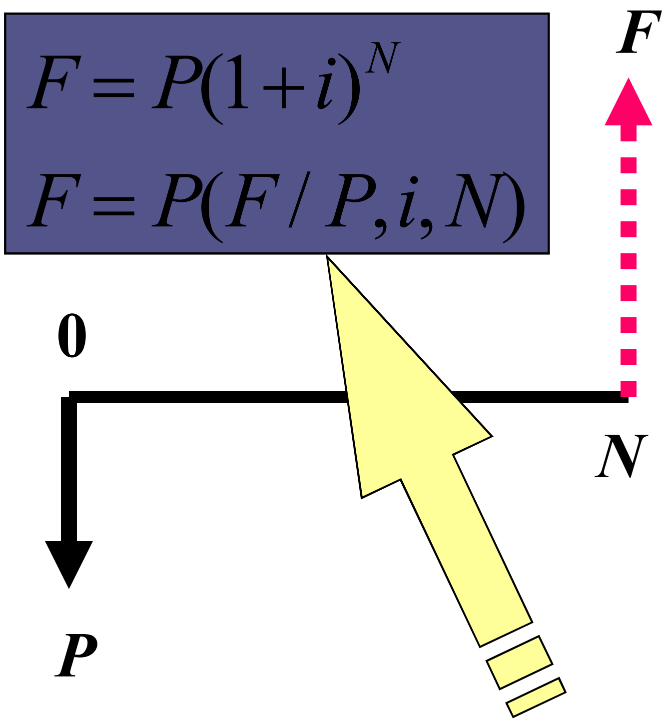
$$i = 10\%$$
$$N = 8 \text{ years}$$
$$P = \$2,000$$

- Demand:

$$F = \$2,000(1+0.10)^8$$
$$= \$2,000(F/P,10\%,8)$$
$$= \$4,287.18$$

- *Compound factor*

$$F = P(1+i)^N$$
$$F = P(F/P,i,N)$$




The data obtained from interest tables can be used instead of formulas!!!

$$F = P \times (1+i)^n$$

- **P** : Present value; value or amount of money at a time designated as the present or time 0. (TL, ...)
- **F** : future value; value or amount of money at some future time. (TL, ...)
- **n** : number of interest periods; years, months, days
- **i** : interest rate or rate of return per time period; percent per year, percent per month; percent per day

# Interest table for 1% interest rate

| 1% | TABLE 4 Discrete Cash Flow: Compound Interest Factors | | | | | | | 1% |
|---|---|---|---|---|---|---|---|---|
| | Single Payments | | Uniform Series Payments | | | | Arithmetic Gradients | |
| *n* | F/P Compound Amount | P/F Present Worth | A/F Sinking Fund | F/A Compound Amount | A/P Capital Recovery | P/A Present Worth | P/G Gradient Present Worth | A/G Gradient Uniform Series |
| 1 | 1.0100 | 0.9901 | 1.00000 | 1.0000 | 1.01000 | 0.9901 | | |
| 2 | 1.0201 | 0.9803 | 0.49751 | 2.0100 | 0.50751 | 1.9704 | 0.9803 | 0.4975 |
| 3 | 1.0303 | 0.9706 | 0.33002 | 3.0301 | 0.34002 | 2.9410 | 2.9215 | 0.9934 |
| 4 | 1.0406 | 0.9610 | 0.24628 | 4.0604 | 0.25628 | 3.9020 | 5.8044 | 1.4876 |
| 5 | 1.0510 | 0.9515 | 0.19604 | 5.1010 | 0.20604 | 4.8534 | 9.6103 | 1.9801 |
| 6 | 1.0615 | 0.9420 | 0.16255 | 6.1520 | 0.17255 | 5.7955 | 14.3205 | 2.4710 |
| 7 | 1.0721 | 0.9327 | 0.13863 | 7.2135 | 0.14863 | 6.7282 | 19.9168 | 2.9602 |
| 8 | 1.0829 | 0.9235 | 0.12069 | 8.2857 | 0.13069 | 7.6517 | 26.3812 | 3.4478 |
| 9 | 1.0937 | 0.9143 | 0.10674 | 9.3685 | 0.11674 | 8.5660 | 33.6959 | 3.9337 |
| 10 | 1.1046 | 0.9053 | 0.09558 | 10.4622 | 0.10558 | 9.4713 | 41.8435 | 4.4179 |
| 11 | 1.1157 | 0.8963 | 0.08645 | 11.5668 | 0.09645 | 10.3676 | 50.8067 | 4.9005 |
| 12 | 1.1268 | 0.8874 | 0.07885 | 12.6825 | 0.08885 | 11.2551 | 60.5687 | 5.3815 |
| 13 | 1.1381 | 0.8787 | 0.07241 | 13.8093 | 0.08241 | 12.1337 | 71.1126 | 5.8607 |
| 14 | 1.1495 | 0.8700 | 0.06690 | 14.9474 | 0.07690 | 13.0037 | 82.4221 | 6.3384 |
| 15 | 1.1610 | 0.8613 | 0.06212 | 16.0969 | 0.07212 | 13.8651 | 94.4810 | 6.8143 |
| 16 | 1.1726 | 0.8528 | 0.05794 | 17.2579 | 0.06794 | 14.7179 | 107.2734 | 7.2886 |
| 17 | 1.1843 | 0.8444 | 0.05426 | 18.4304 | 0.06426 | 15.5623 | 120.7834 | 7.7613 |
| 18 | 1.1961 | 0.8360 | 0.05098 | 19.6147 | 0.06098 | 16.3983 | 134.9957 | 8.2323 |
| 19 | 1.2081 | 0.8277 | 0.04805 | 20.8109 | 0.05805 | 17.2260 | 149.8950 | 8.7017 |
| 20 | 1.2202 | 0.8195 | 0.04542 | 22.0190 | 0.05542 | 18.0456 | 165.4664 | 9.1694 |

# Single payment

- Single Payment, compound interest, <u>future value</u>
- Given:

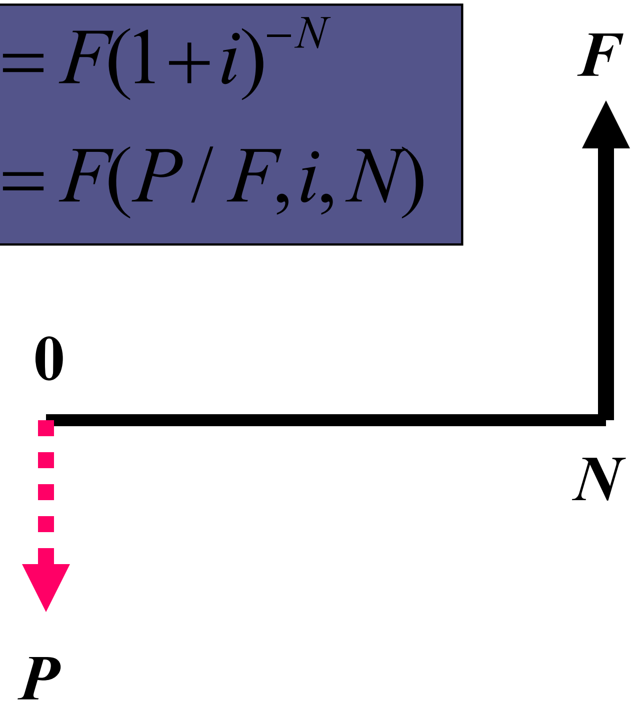
$$i = 12\%$$
$$N = 5 \text{ years}$$
$$F = \$1,000$$

- Demand:

$$P = \$1,000(1+0.12)^{-5}$$
$$= \$1,000(P/F,12\%,5)$$
$$= \$567.40$$

- *Discount factor*

$$P = F(1+i)^{-N}$$
$$P = F(P/F, i, N)$$



# Single Payment

- Example: What is the value of 3680 $ at the end of 8 year at %12 compound interest rate per year?

  - P = 3680 $
  - i = %12
  - n = 8 year

$$F = P \times (1+i)^n$$

$$= 3680 \times (1 + 0.12)^8$$

$$= 3680 \times 2.476$$

$$= \mathbf{9112\ \$}$$

# Single Payment

- **Example: The stock is bought at $10 and it is sold at $20 after 5 years. Then what is the interest rate of this operation?**

- Solution:
  - Trial and error method (time consuming and ineffective method)
  - Interest tables (difficult for interest rates which are not integer)
  - Financial functions of calculators and excel

$$F=P(1+i)^N$$
$$20 = 10(1+i)^5$$
$$i=\%14.87$$

# Single Payment

- **Example:** XYZ company buys 100 stocks at \$60/stock price. The plan of this company is selling these stocks when its value increases to \$120 \$/stock. If it is predicted that the value of stocks will increase at %20 per year, how many years should the company wait for selling the stocks?

$F=P(1+i)^N = P(F/P, i,N)$

$12{,}000 = 6{,}000\ (1+0.20)^N$

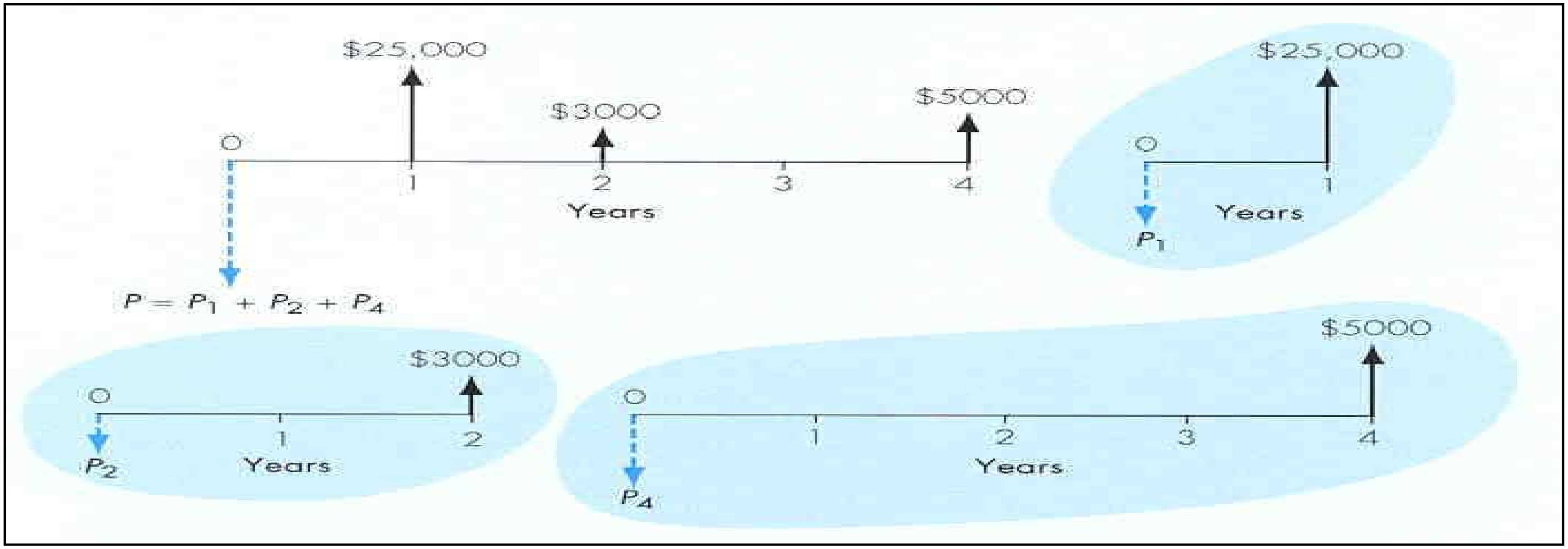
$\log 2 = N \cdot \log 1.2$

$N$=3.80 or approximately 4 year

# Irregular payment series

- **Example:** The payments for 4 year are stated below, how much money should be deposited to the bank (interest ratio is %10 per year)?
  - Year 1: Computer and software for customer services $25,000
  - Year 2: Upgrade the existing system $3000
  - Year 3:  No payment
  - Year 4: Upgrade software $5,000

# Irregular Payment Series




$P_1 = 25000*(P/F, 10\%,1) \rightarrow P_1 = 25000*0,9091 = 22727,5$

$P_2 = 3000*(P/F, 10\%,2) \rightarrow P_2 = 3000*0,8264 = 2479,2$

$P = F / (1+i)^n$ $\quad P_3 = 5000*(P/F, 10\%,4) \rightarrow P_3 = 5000* 0,6830 = 3415$

$P = \$28621,7$

These can be calculated by using the formula or interest table of 10% interest rate