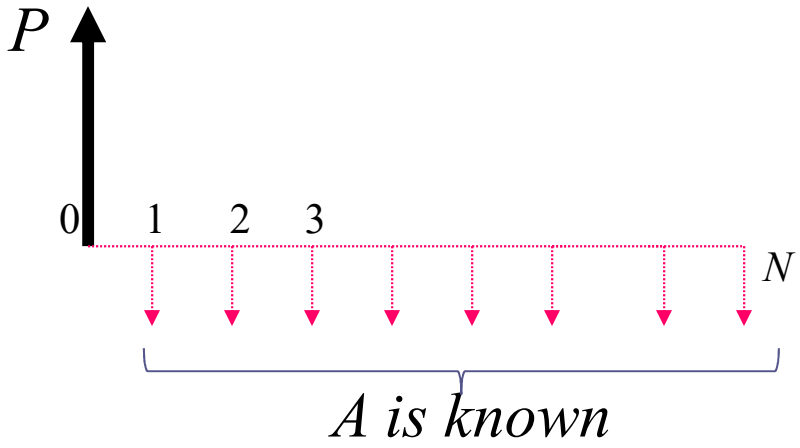
Yıldız Technical University
Civil Engineering Department
Construction Management Division

Engineering Economy- 2

Uniform-series Present Worth Factor



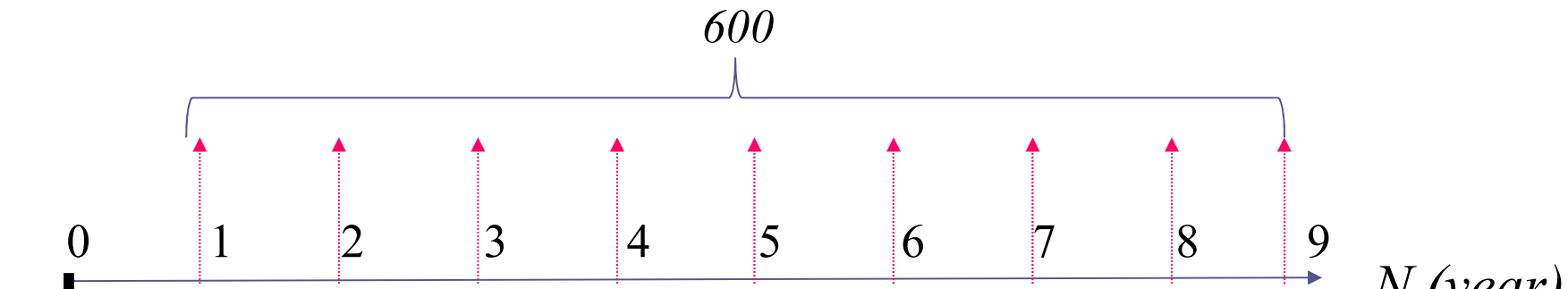
$$P = A\frac{(1+i)^N - 1}{i\,(1+i)^N}$$

$$P = A(P/A, i\%, N)$$

- **When A, i and N are known, calculation of P value:** The equivalent present worth P of a uniform series A end of period cash flows N at an interest rate i.

Example -1

- How much money should you be willing to pay now for a guaranteed $600 per year for 9 years starting next year, at a rate of return of 16% per year?



i= %16

P

P= A (P/A, %16, 9)

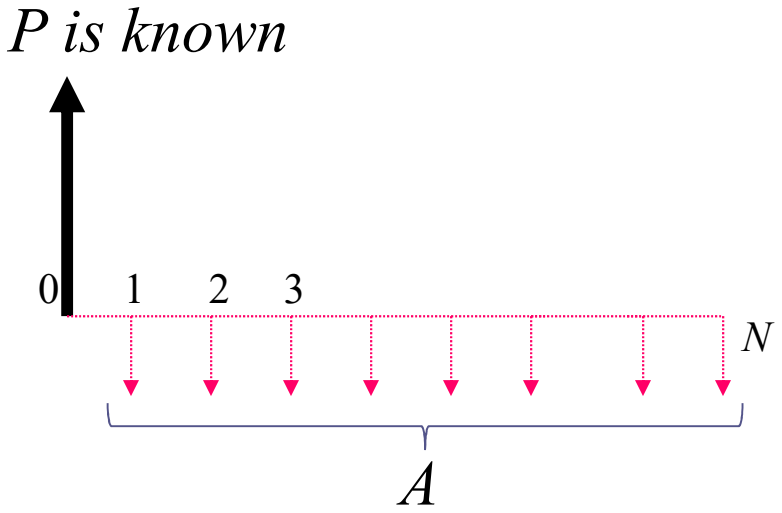
$P = 600^* \frac{(1+0{,}16)^9 - 1}{0{,}16\,(1+0{,}16)^9}$

$P = A \frac{(1+i)^N - 1}{i\,(1+i)^N}$

P=600*4,6065

P= 2763,9 TL

Capital Recovery Factor



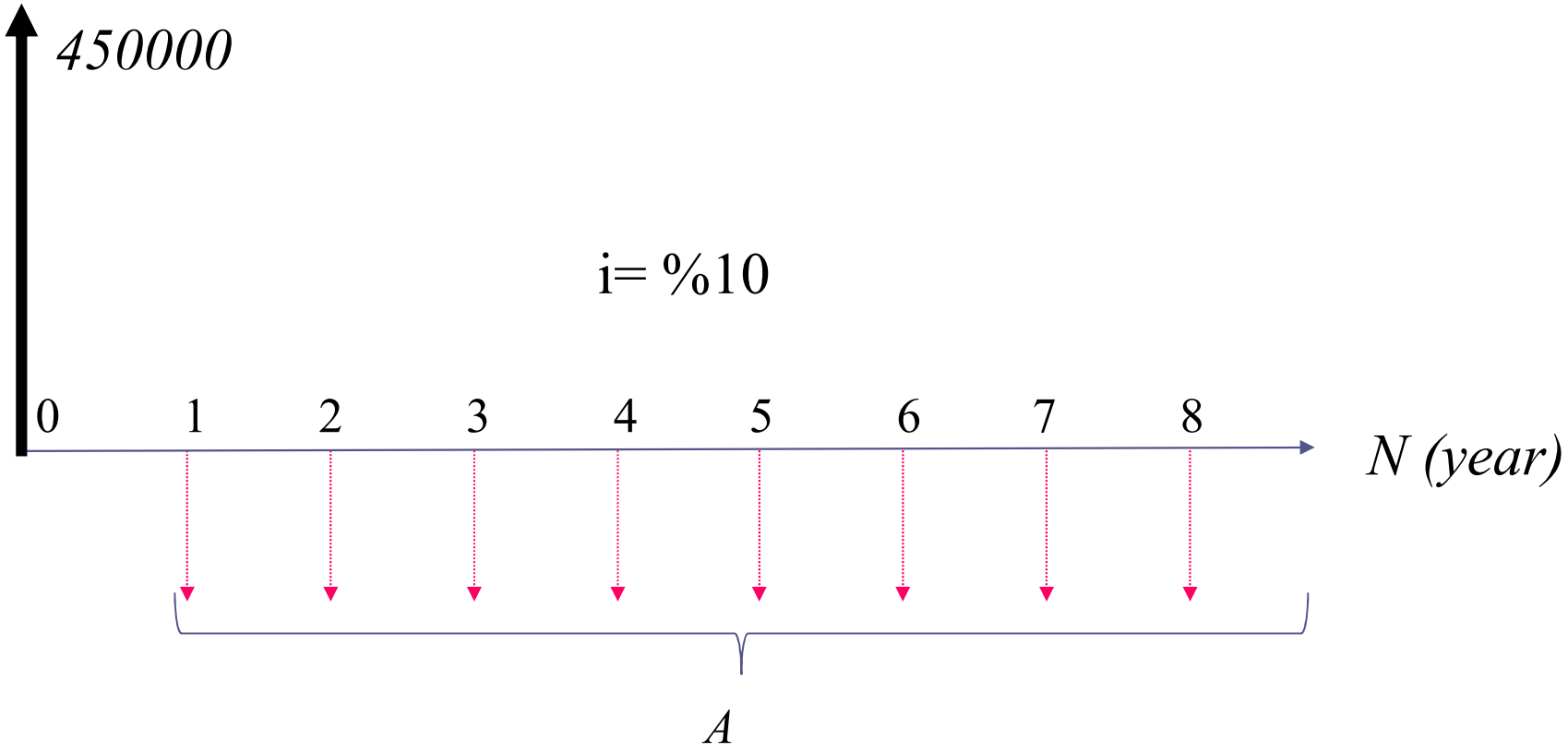
$$A = P\frac{i\,(1+i)^N}{(1+i)^N - 1}$$

$$A = P(A/P, i\%, N)$$

- **When P, i and N are known, calculation of value of A:** the present worth P is known and the equivalent uniform-series amount A is sought throughout a period N at an interest rate i.
- The payback of house and car credits are some of the examples of capital recovery factors.

Example - 2

- Company A borrows 450 000 TL for buying laboratory equipment at a rate of return of %10 per year for 8 years starting next year. What should the company pay annually throughout the credit period?



A= P (A/P, %10, 8)

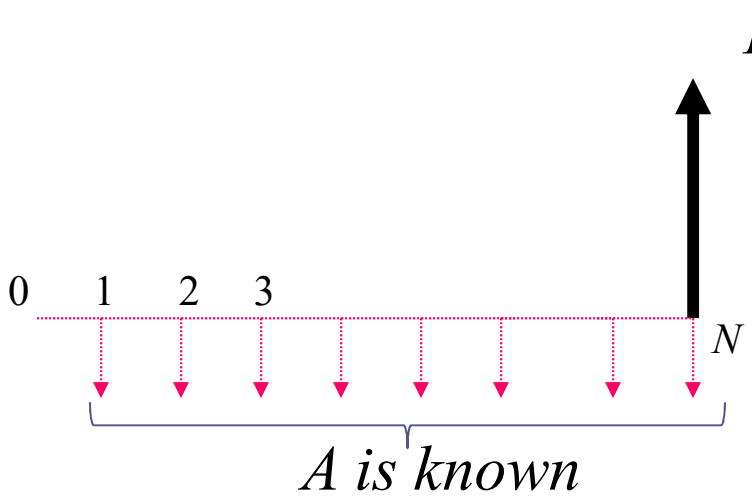
A= 450000* $\frac{0{,}10\ (1+0{,}10)^8}{(1+0{,}10)^8-1}$

A=450000*0,1874

A= 84330 TL

$$A = P\,\frac{i\,(1+i)^N}{(1+i)^N-1}$$

Uniform Series Compound Amount



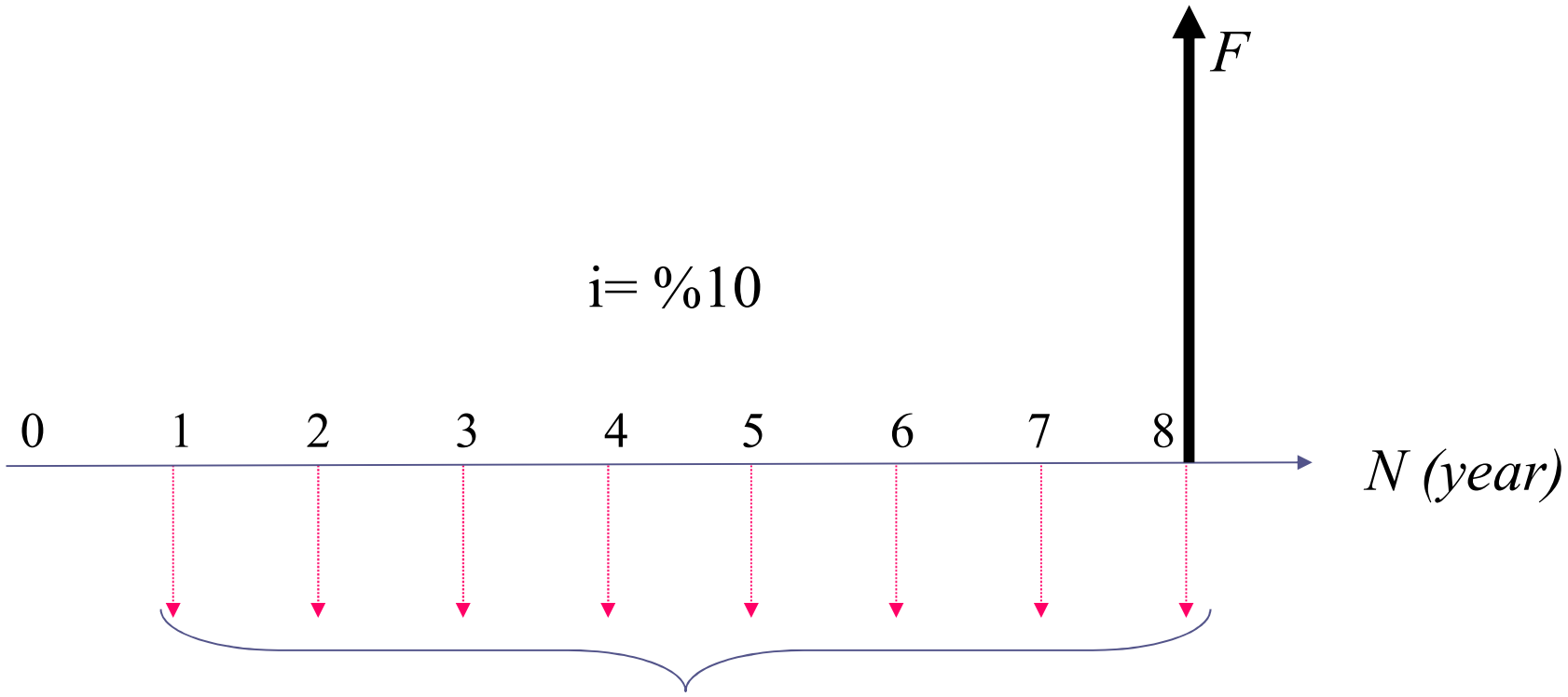
$$F = A\frac{(1+i)^N - 1}{i}$$

$$F = A(F/A, i\%, N)$$

- **When A, i and N are known, calculation of F value:** The equivalent future worth F of a uniform series A end of period cash flows N at an interest rate i.

Example - 3

At the end of each year, 4500 TL is deposited throughout 8 years at an interest rate of %10 per year. What is the amount of the money which can be withdrawn at the end of 8^{th} year?



F= A (F/A, %10, 8)

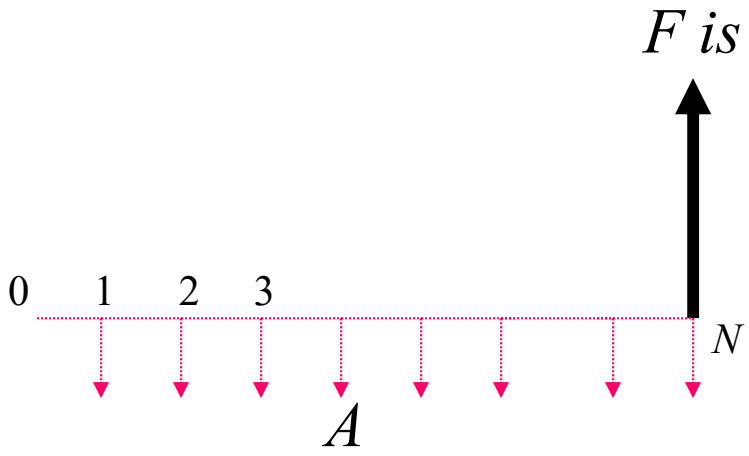
F= 4500*$\frac{(1+0,10)^8-1}{0,10}$

F=4500*11,4359

F= 51461,55 TL

$$F = A\frac{(1+i)^N-1}{i}$$

Sinking Fund Factor



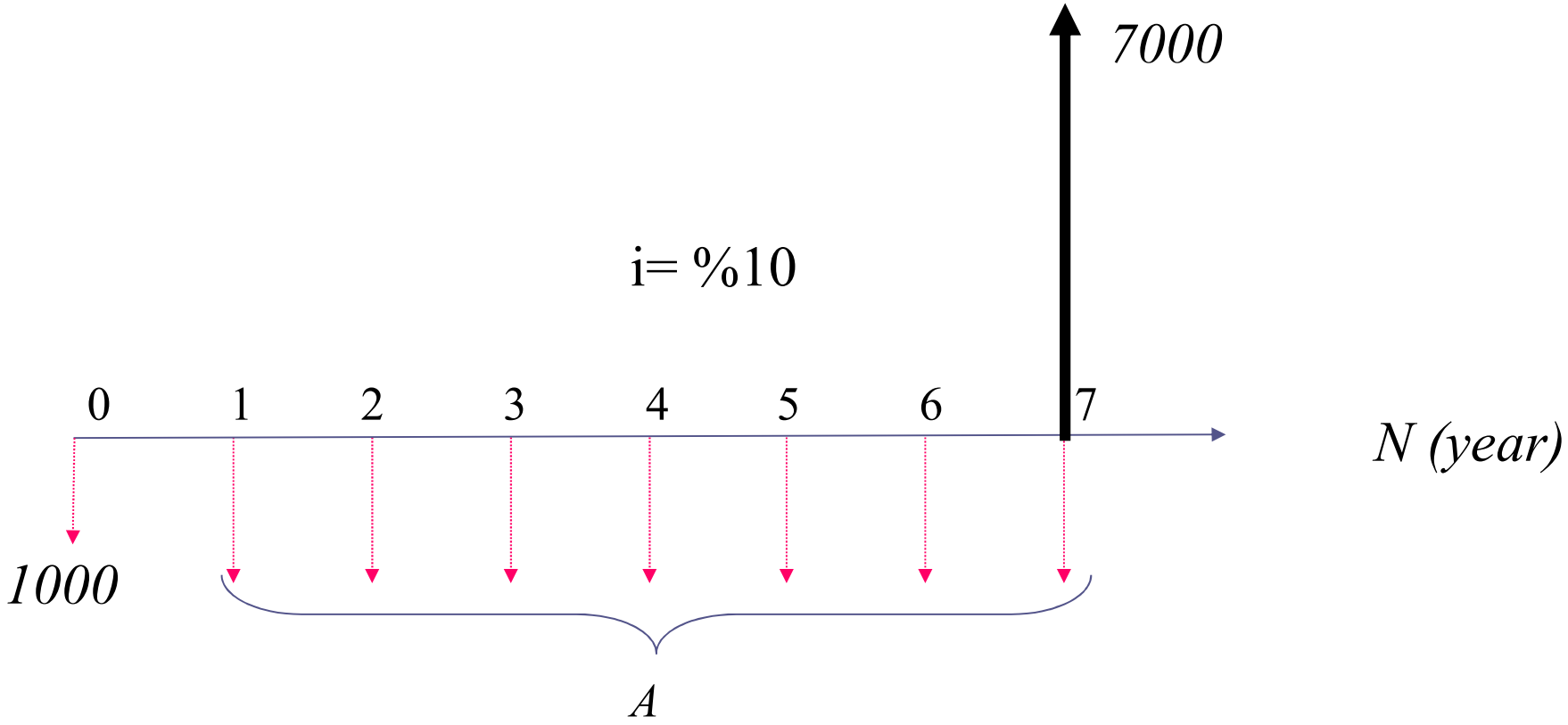
$$A = F\frac{i}{(1+i)^N - 1}$$

$$A = F(A/F, i\%, N)$$

- **When F, i and N are known, calculation of A value:** This function determines the A value for n years, given F in year n, at a given interest rate.
- These calculations are performed for determining the periodically deposited amount required to replace the fixed assets.

Example - 4

- At the present time, a father proposes to give 1000 TL to his son who wants to collect 7000 TL at the end of 7^{th} year. He deposits this money to a bank. On the other hand, the son is planning to deposit equal amount of money earned by working at a part time job at the end of each year. If the interest rate is %10 per year, how much money should he earn?



A= F(A/F,10%,7)-P (A/P, %10, 7)

A= $7000*\frac{0,10}{(1+0,10)^7-1}$ - $1000*\frac{0,10\ (1+0,10)^7}{(1+0,10)^7-1}$

A=7000*0,1054-1000*0,2054

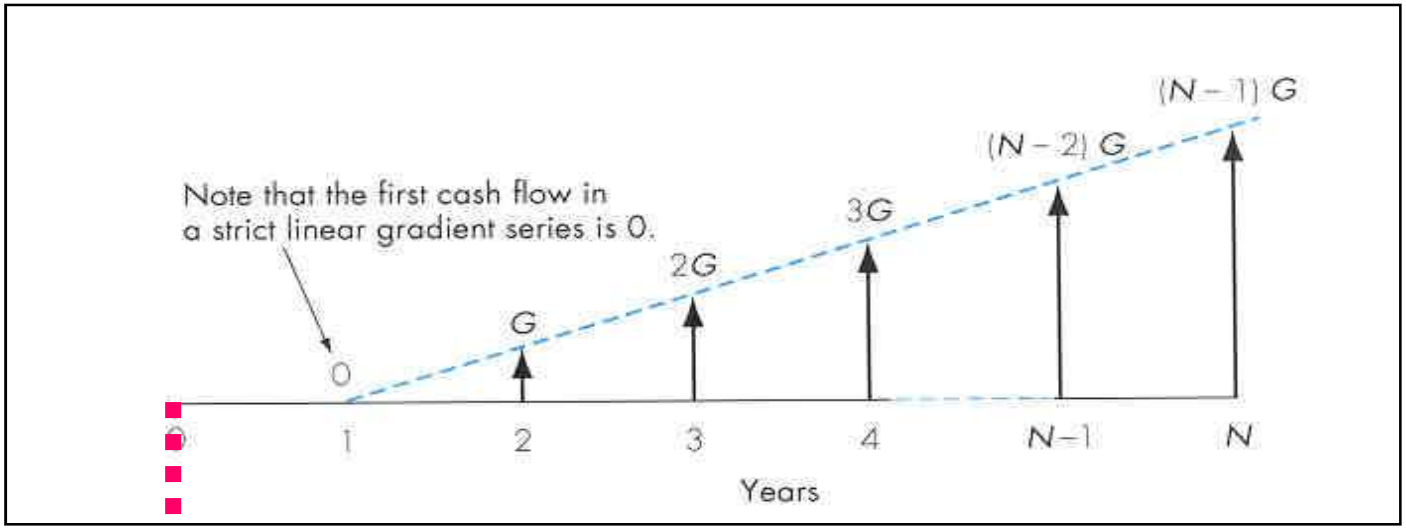
A= 532,4 TL

$$A = P\frac{i\ (1+i)^N}{(1+i)^N-1}$$

$$A = F\frac{i}{(1+i)^N-1}$$

Arithmetic Gradient Factors

An arithmetic gradient is a cash flow series that either increases or decreases by a constant amount. The amount of the increase or decrease is the gradient.

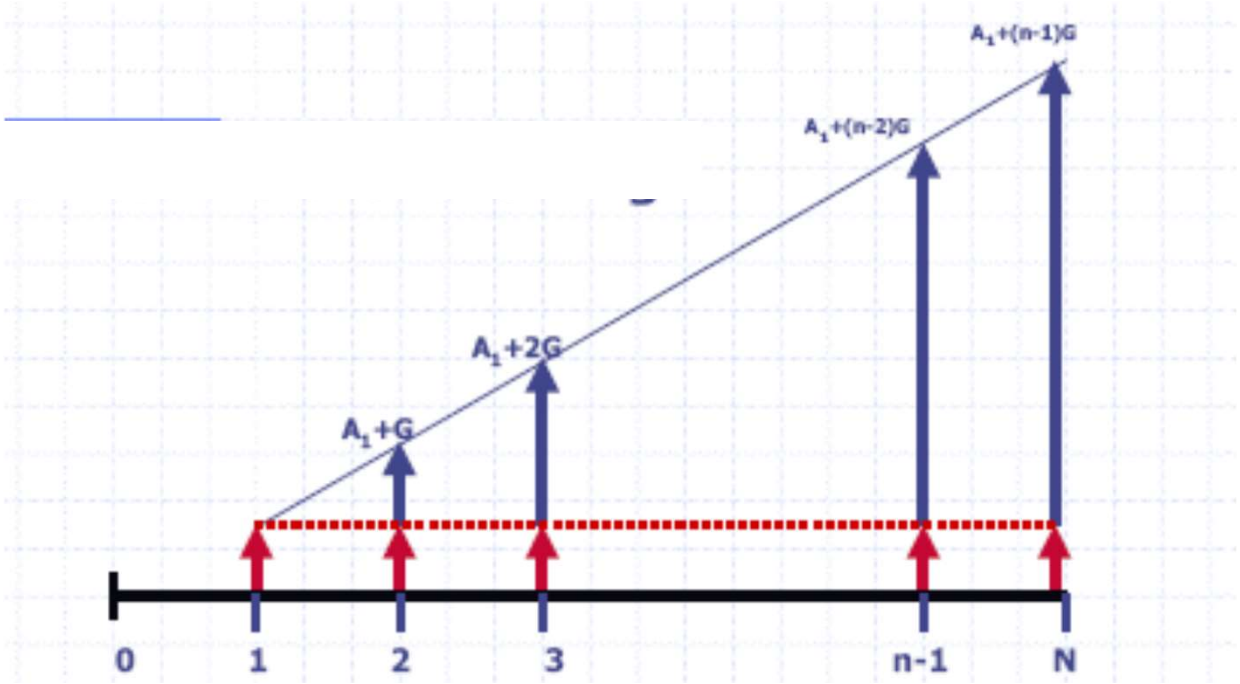


Gradient Present Worth

$$P = G\frac{(1+i)^N - iN - 1}{i^2 * (1+i)^N}$$

$$P = G(P/G, i, N)$$

Gradient uniform series

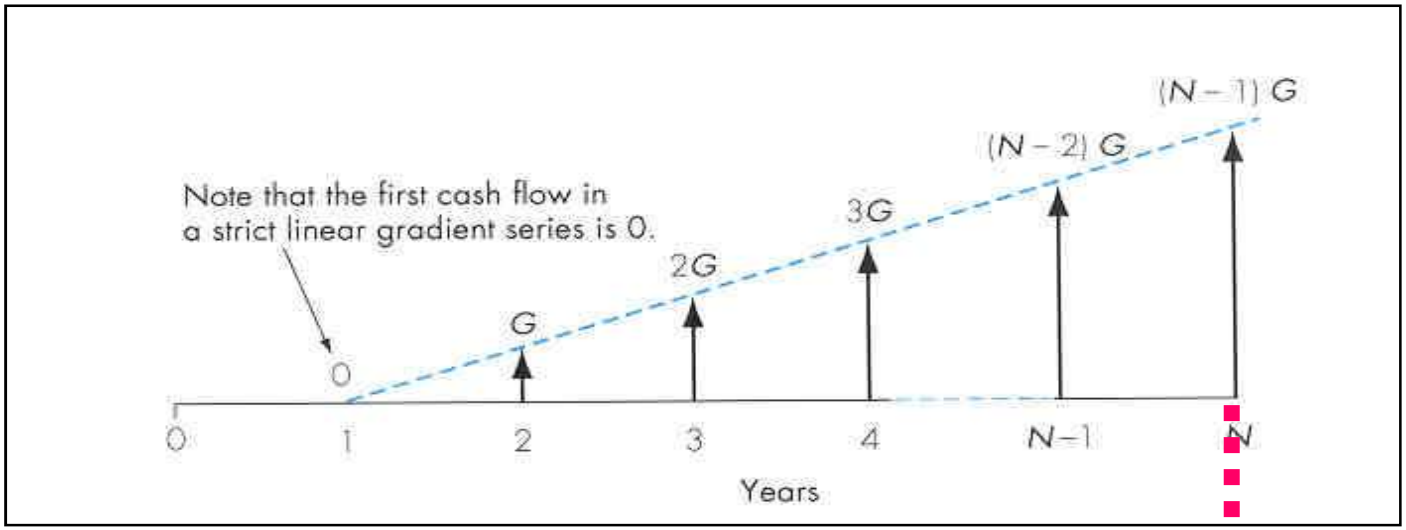


Gradient Uniform Series

$$A = A_1 + G\left[\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right]$$

$$A = A_1 + G(A/G, i\%, N)$$

Gradient uniform series



Gradient
Future Worth

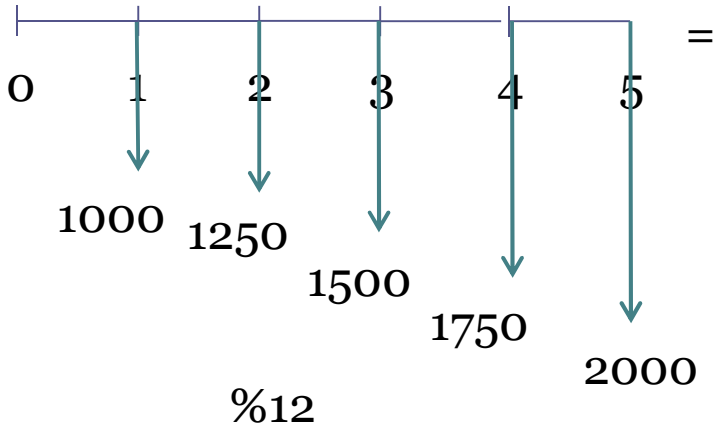
$$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i} - N\right]$$

$$F = G(F/G, i, N)$$

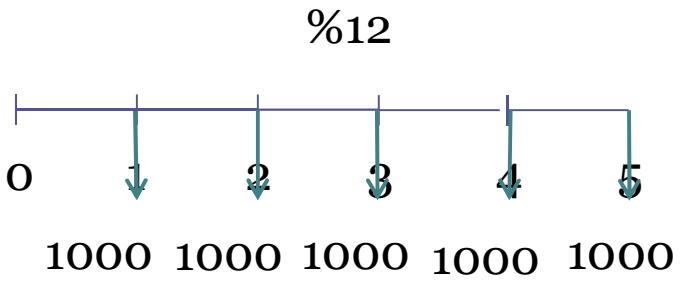
F

Example - 5

- A textile company wants to buy a manufacturing machine whose economic life is years. The engineers predict that the operating and maintenance expenses of this machine will be $1000. These expenses are expected to increase $250 each year uniformly and it is assumed that these expenses occurred at the end of the year. If the company wants to deposit an amount of money at %12 interest rate to compensate these expenses. Then how much money should they deposit?

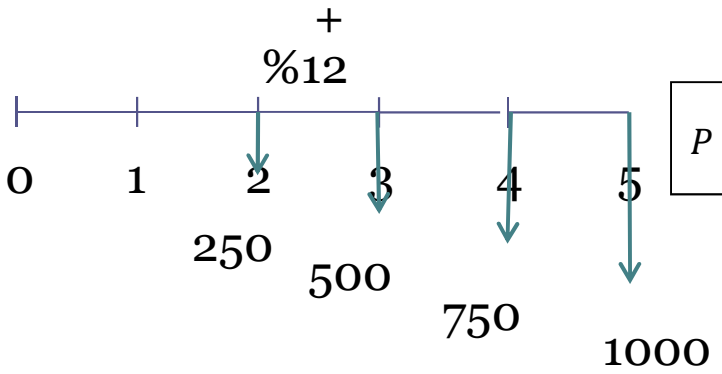


=



$$P = A\frac{(1+i)^N - 1}{i\,(1+i)^N}$$

+



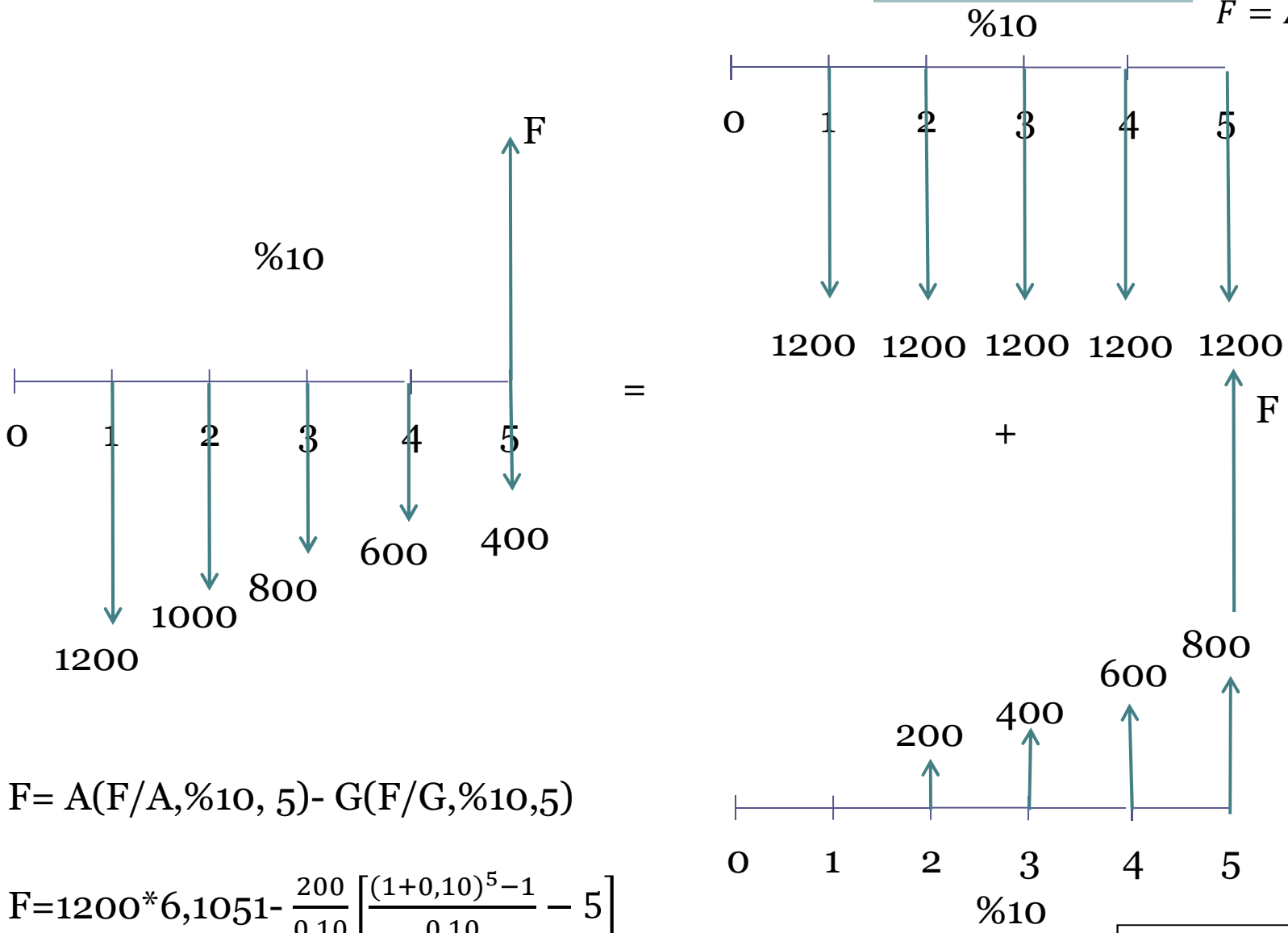
$$P = G\frac{(1+i)^N - iN - 1}{i^2 * (1+i)^N}$$

P = A(P/A, 12%, 5) + G (P/G, 12%, 5)
P= 1000*3,6048+250*6,3970
P= $5204,05

Example - 6

- A company wants to deposit money to a bank at %10 interest rate. The money deposited at the end of first year is $1200, and it will decrease by $200 per year for 4 years. What is the amount of money will the company earn at the end of 5^{th} year?

$$F = A\frac{(1+i)^N - 1}{i}$$



F= A(F/A,%10, 5)- G(F/G,%10,5)

$$F=1200*6{,}1051-\frac{200}{0{,}10}\left[\frac{(1+0{,}10)^5-1}{0{,}10}-5\right]$$

F=$5115,92

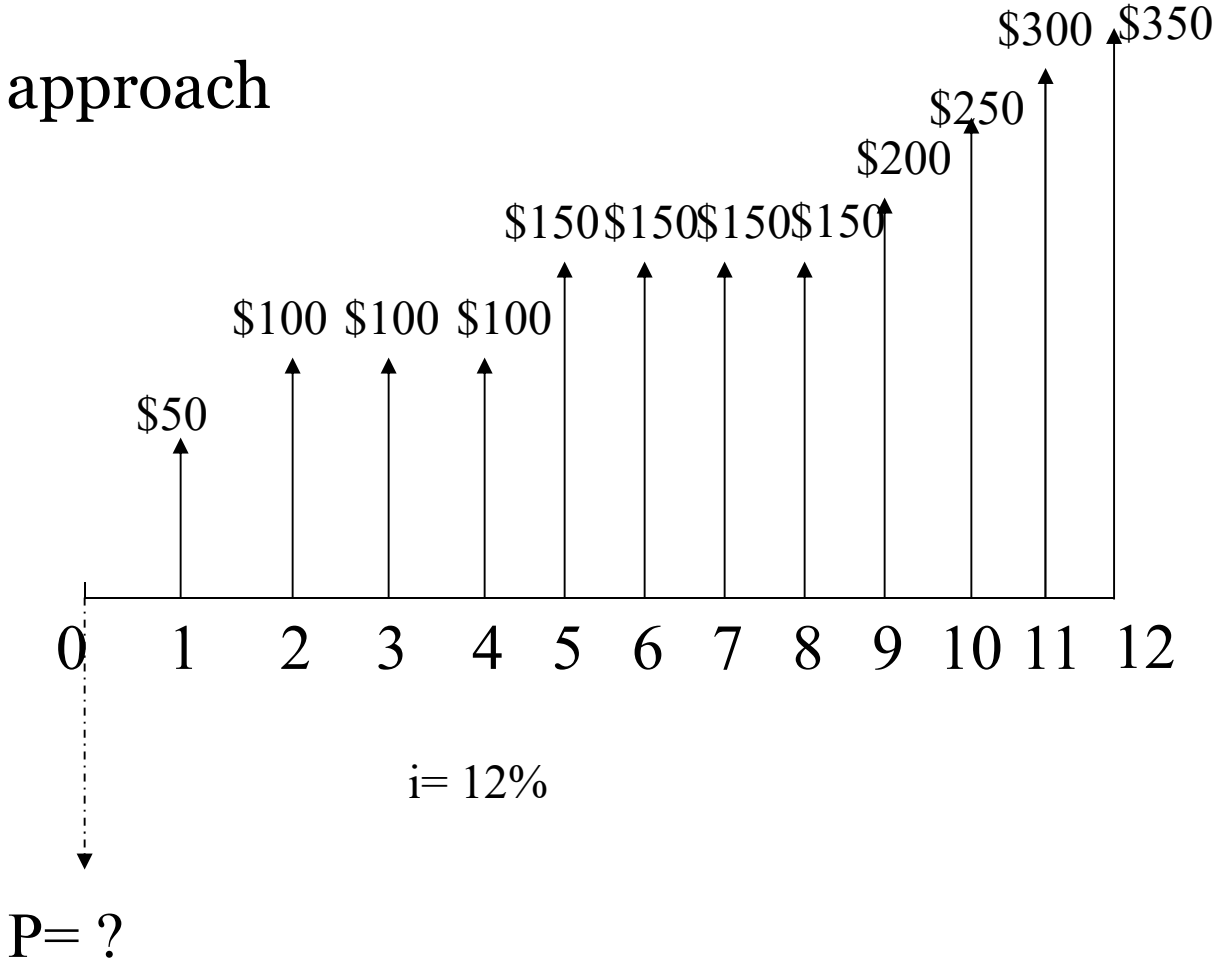
$$F = \frac{G}{i}\left[\frac{(1+i)^N - 1}{i} - N\right]$$

Composite Cash Flows

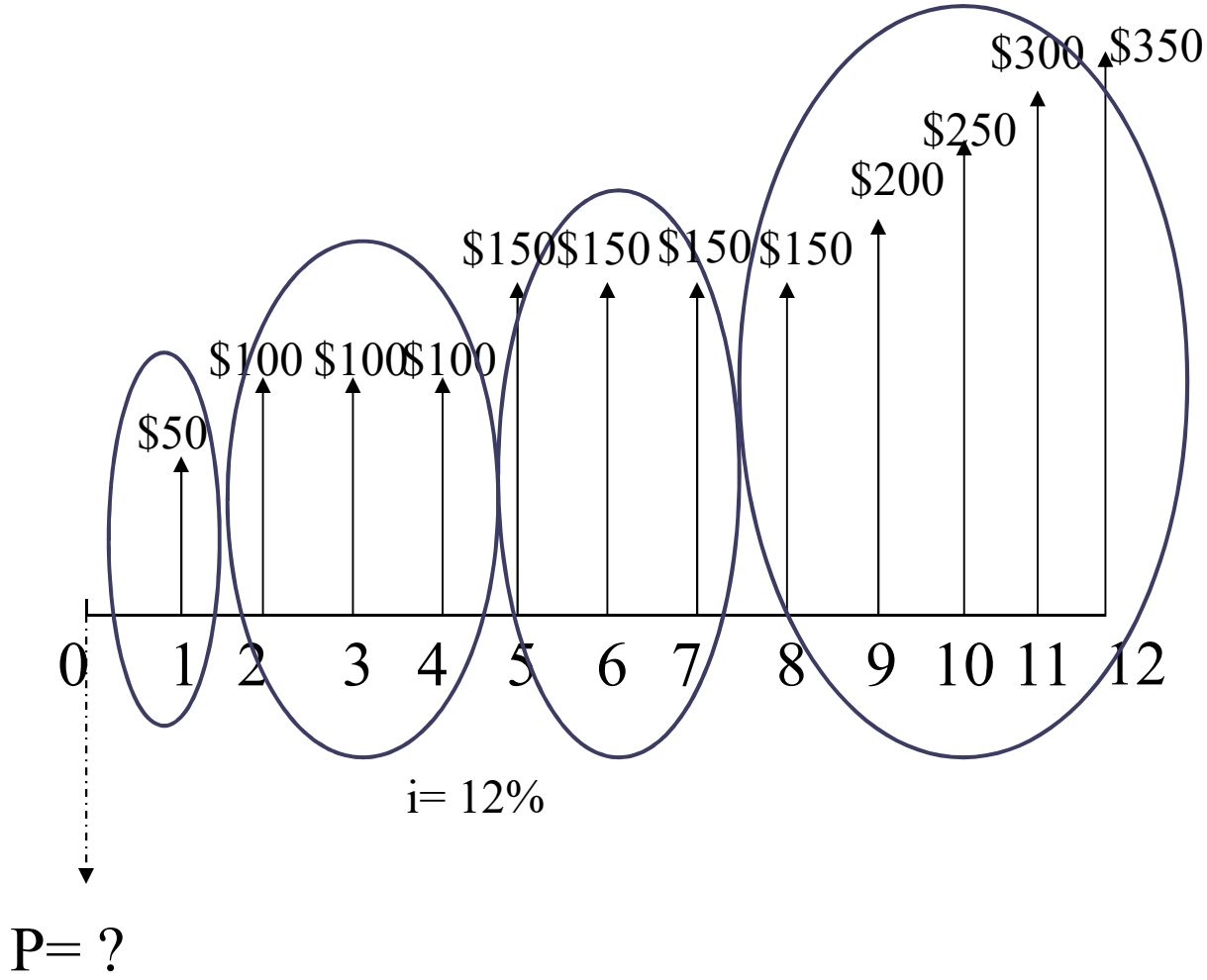
- Carrying all inflows and outflows one by one

or

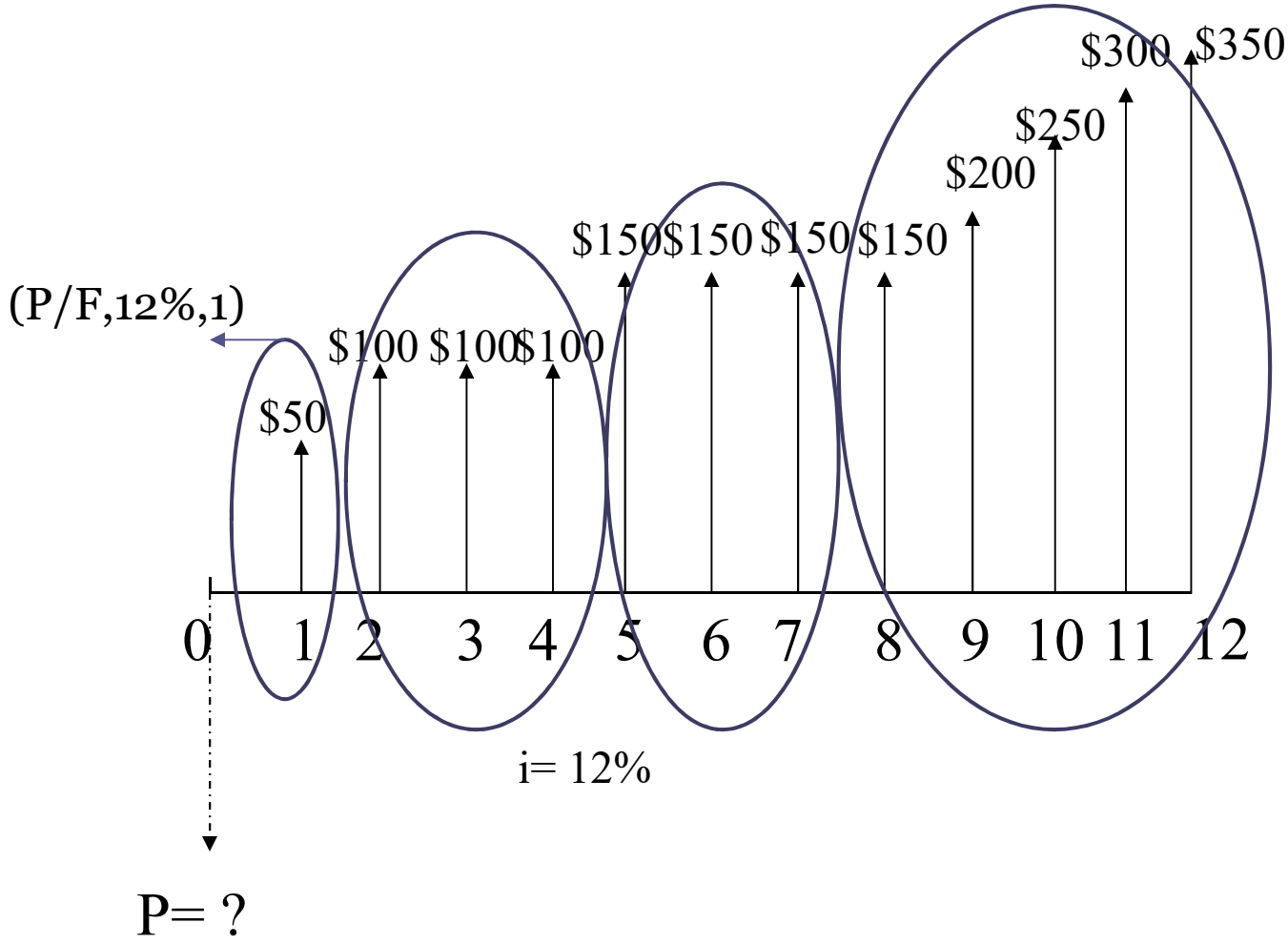
- Grouping approach



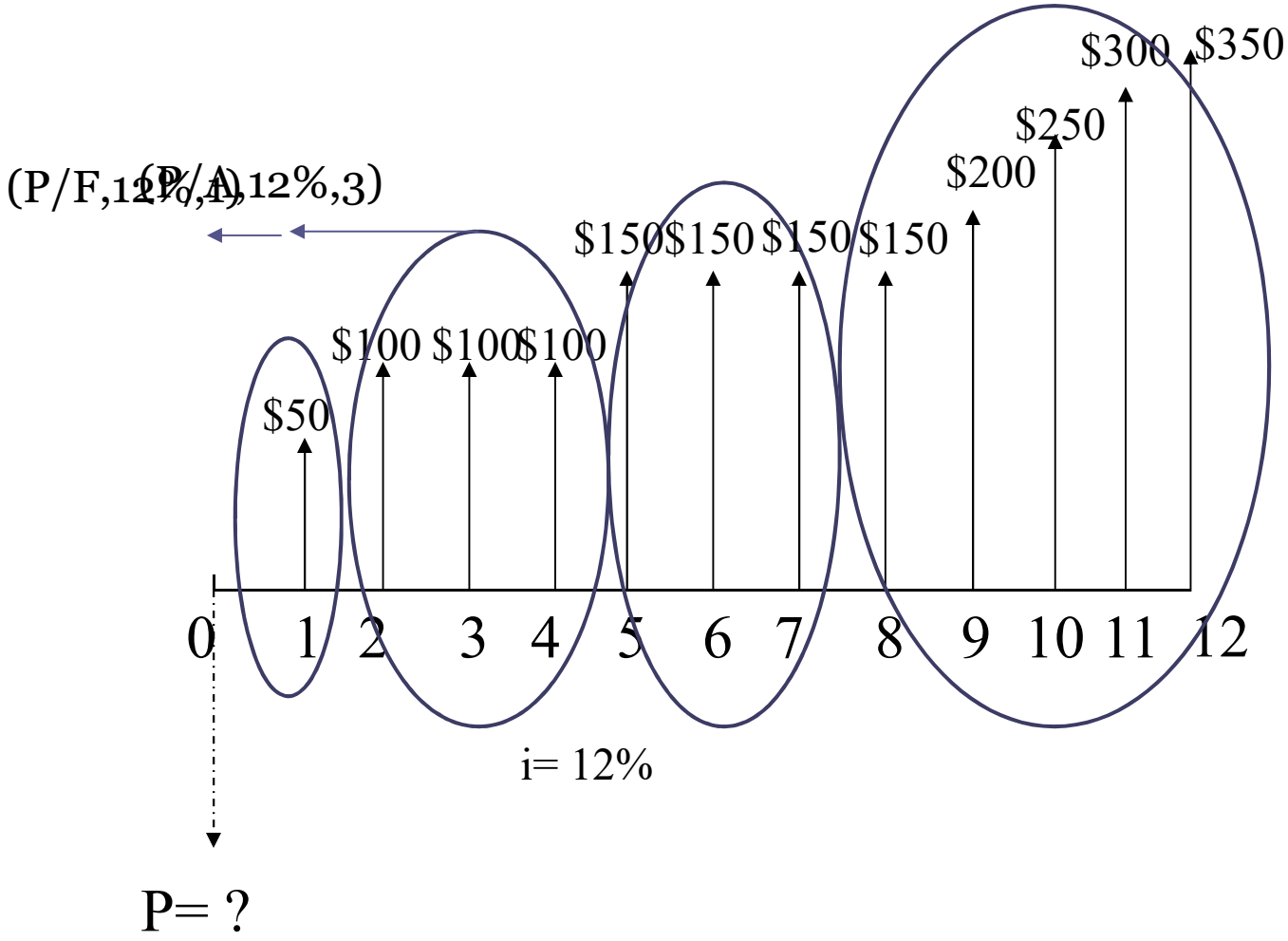
Composite Cash Flows



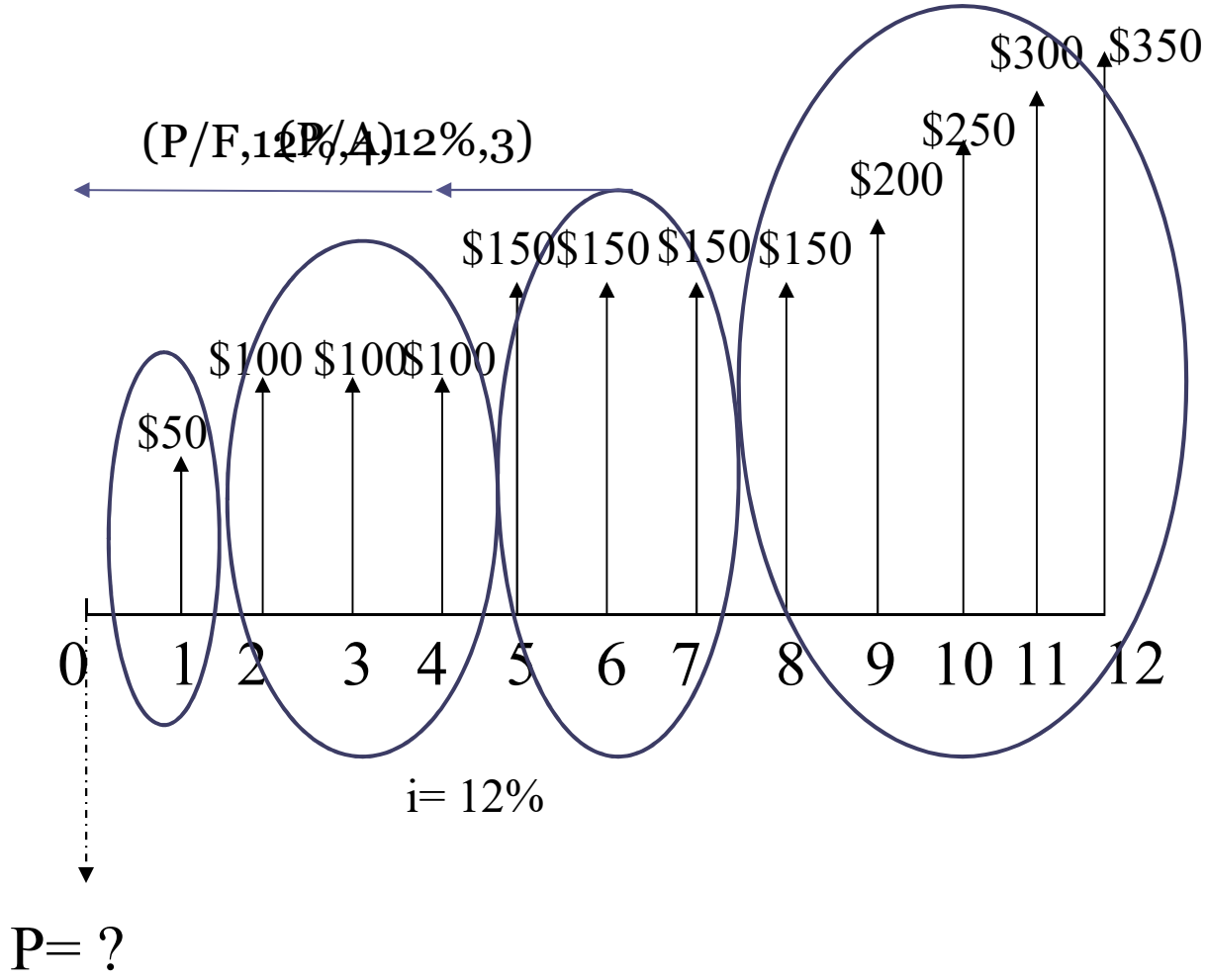
Composite Cash Flows



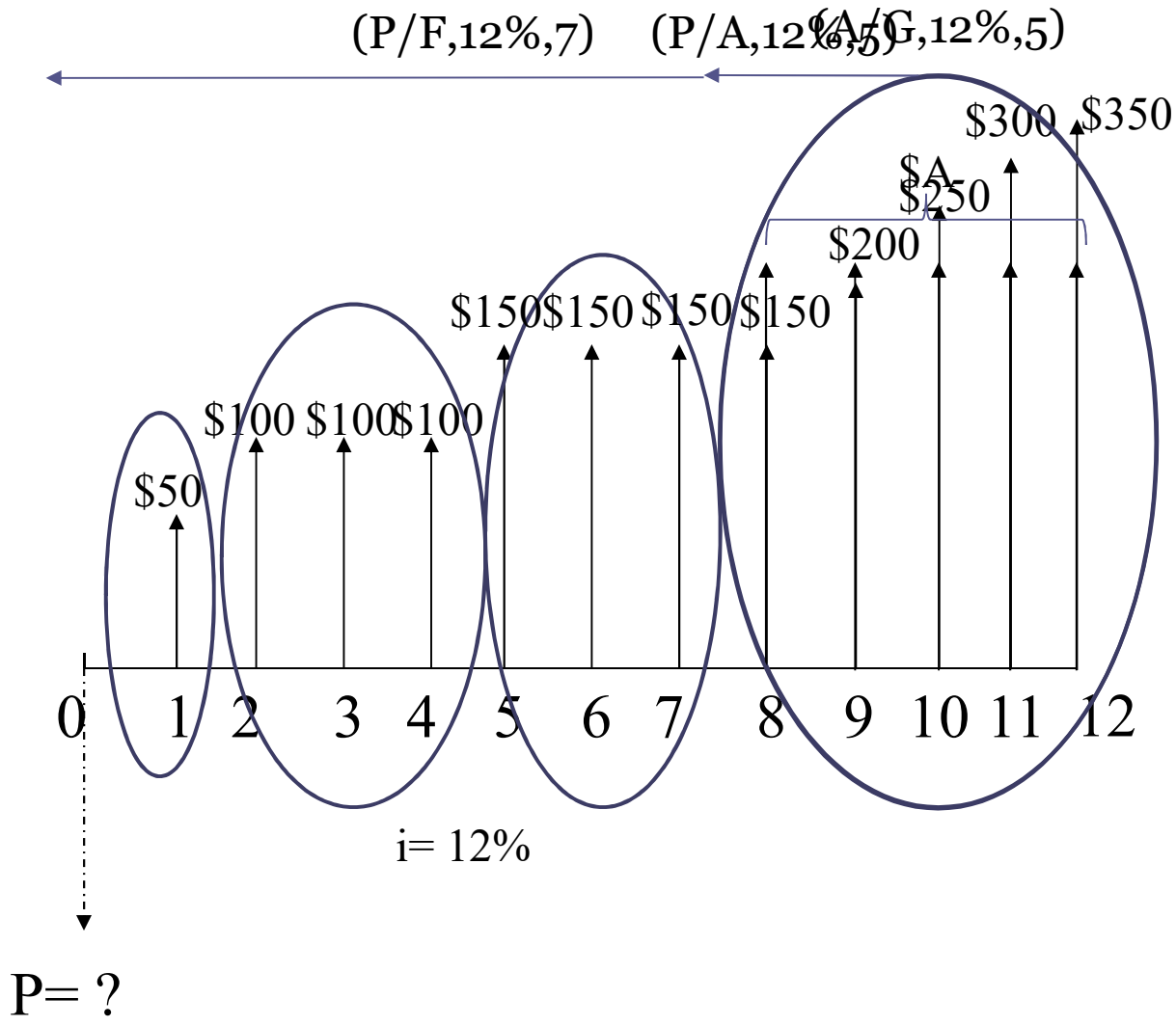
Composite Cash Flows



Composite Cash Flows



Composite Cash Flows



P= F (P/F,%12,1)+ A_1 (P/A, %12, 3)* (P/F,%12,1)+ A_2 (P/A, %12, 3)* (P/F,%12,4)+(A_3+G(A/G,%12,5))*(P/A,12%,5)*(P/F,12%,7)

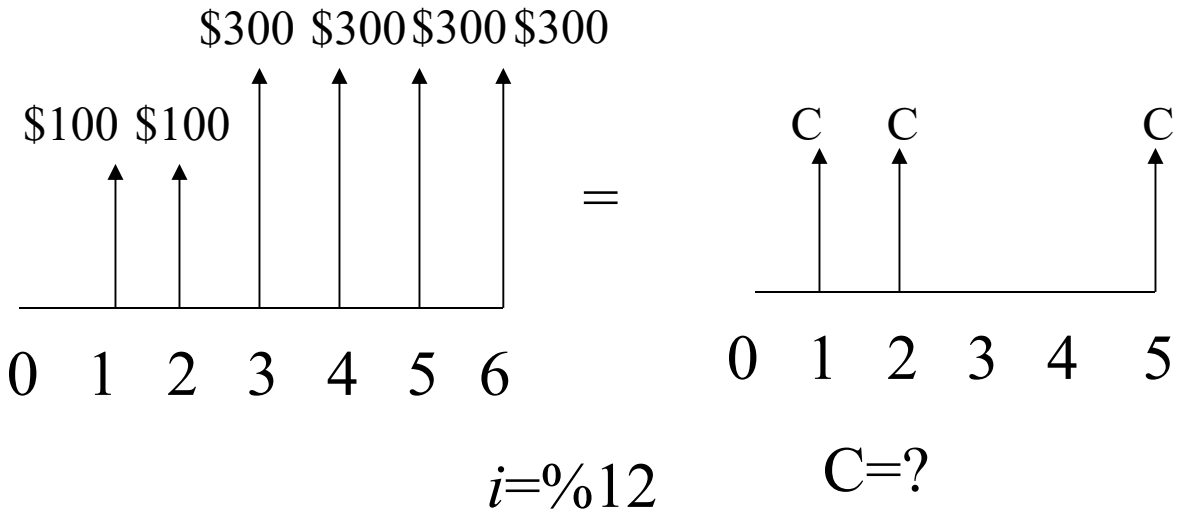
P= 50 (P/F,%12,1)+ 100 (P/A, %12, 3)* (P/F,%12,1)+ 150 (P/A, %12, 3)* (P/F,%12,4)+(150+50(A/G,%12,5))*(P/A,12%,5)*(P/F,12%,7)

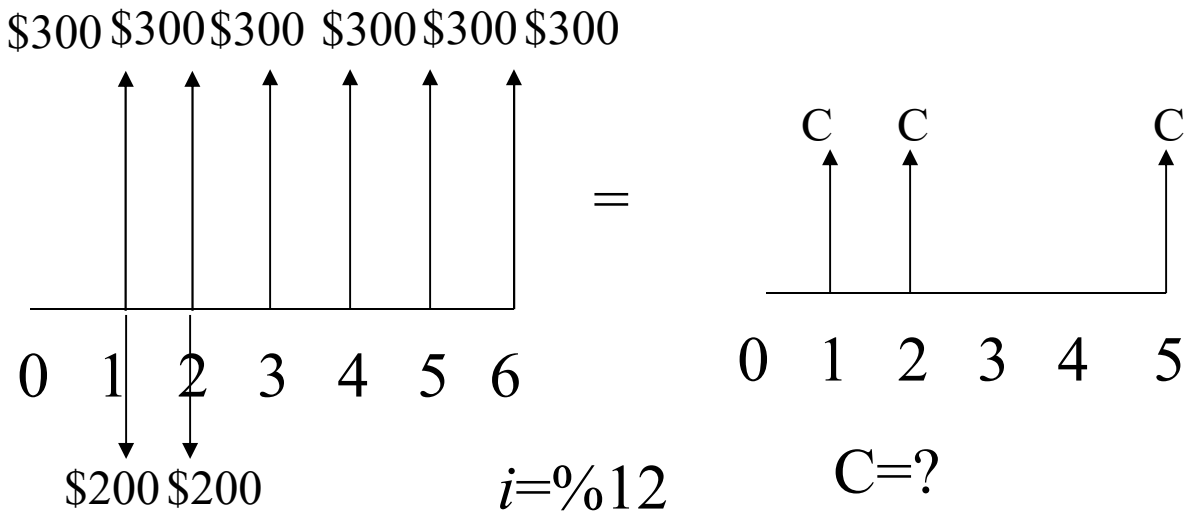
P= 50*0,8929+100*2,4018*0,8929+150*2,4018*0,6355+ (150+50*1,7746)*3,6048*0,4523

P= $ 877,29

Composite Cash Flows

Example 2: In order to satisfy the equivalent of these two cash flows, what should be the value of C?





$P_1 = A_1 (P/A, \%12, 6) - A_2 (P/A, \%12, 2)$

$P_1 = 300 * 4{,}1114 - 200 * 1{,}6901$

$P_1 = \$895.4$

$P_2 = A (P/A, \%12, 2) + F (P/F, \%12, 5)$

$P_2 = C * 1{,}6901 + C * 0{,}5674$

$P_2 = 2{,}2575C$

$P_1 = P_2$

$C = \$396.73$