BME2312 - Analog Electronics

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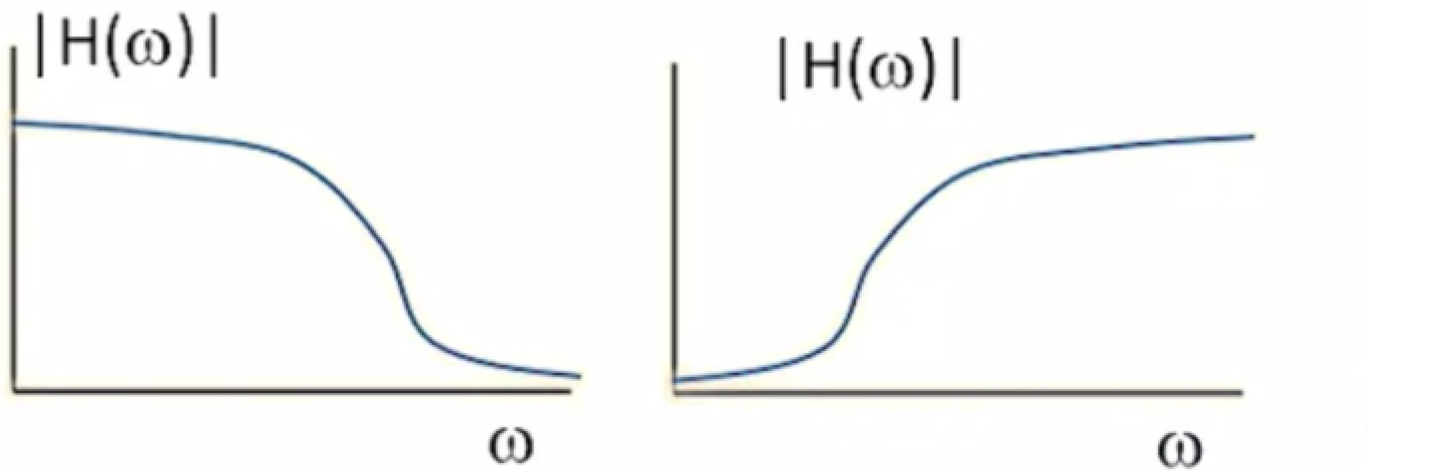
LECTURE 3

Passive Filters

Active Filters

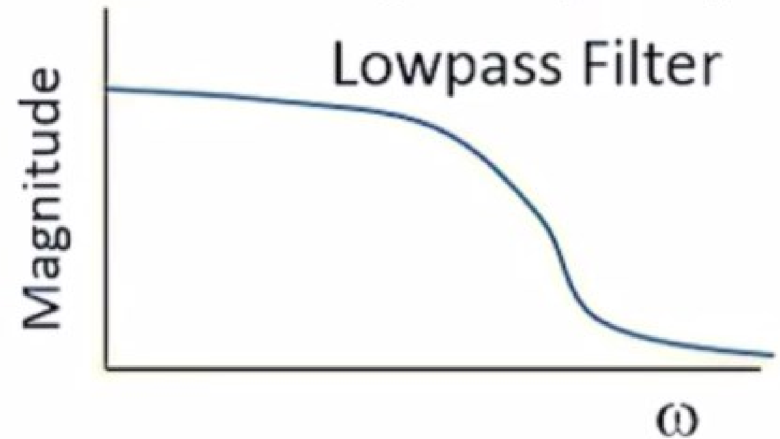
Lowpass and Highpass Filters

An **analog filter** is a circuit that has a specific shaped frequency response to attenuate (or filter) signals with specific frequency content

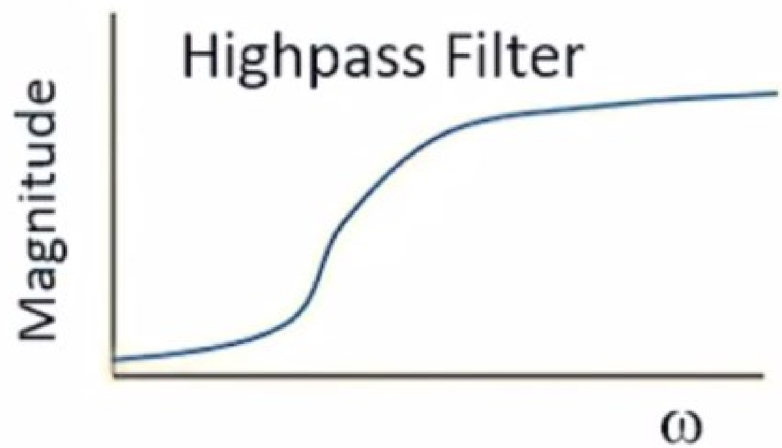


Lowpass and Highpass Filters

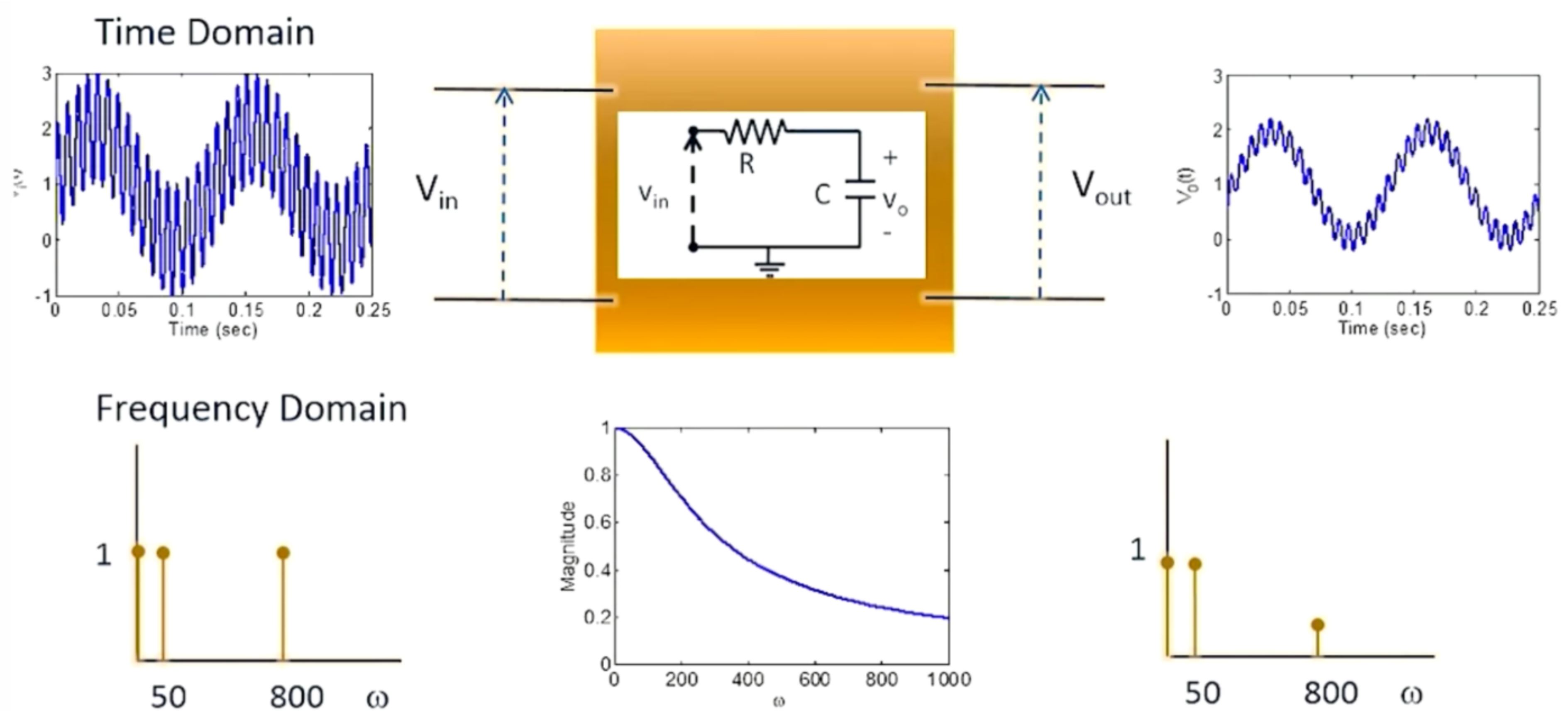
Lowpass Filter: Passes low frequency components and attenuates high frequency components



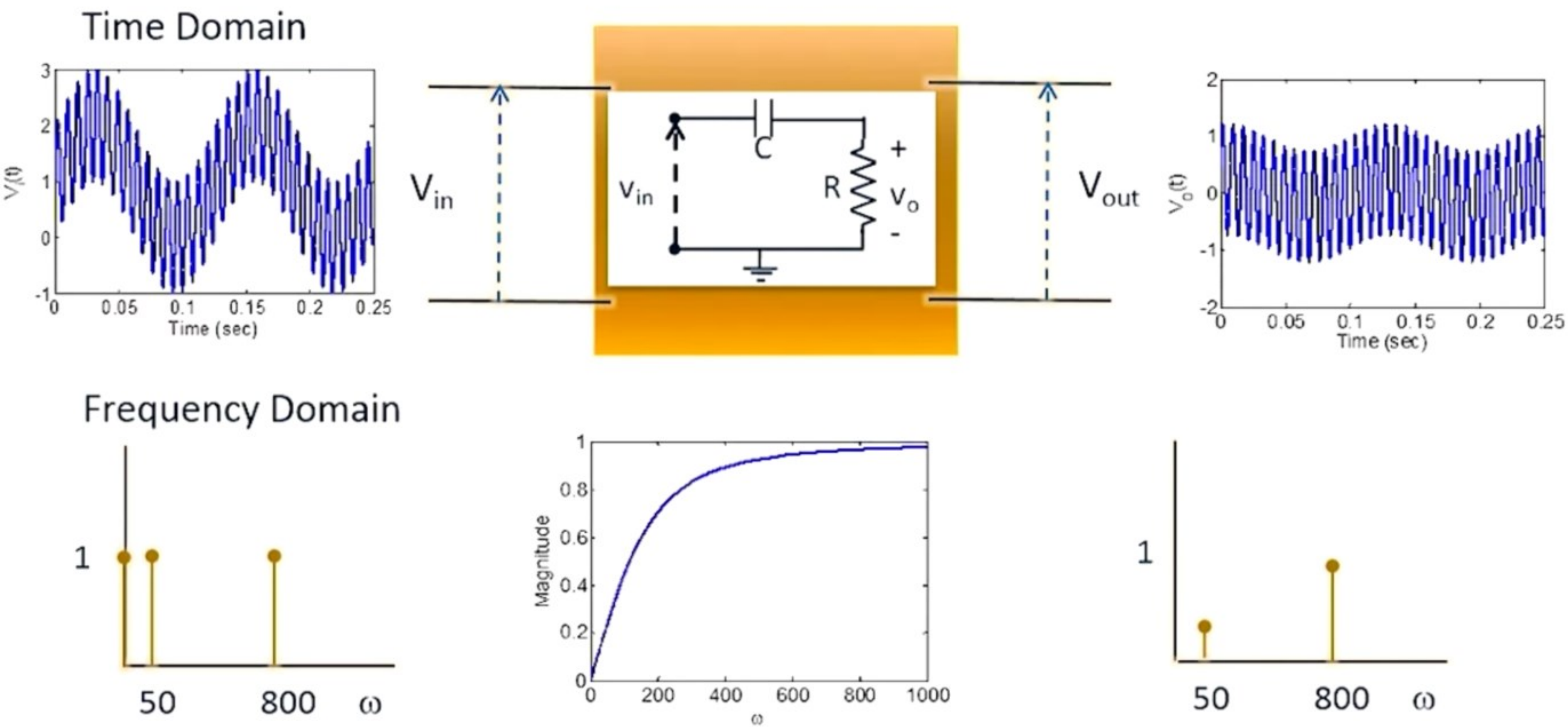
Highpass Filter: Passes high frequency components and attenuates low frequency components



Lowpass Passive Filter Example

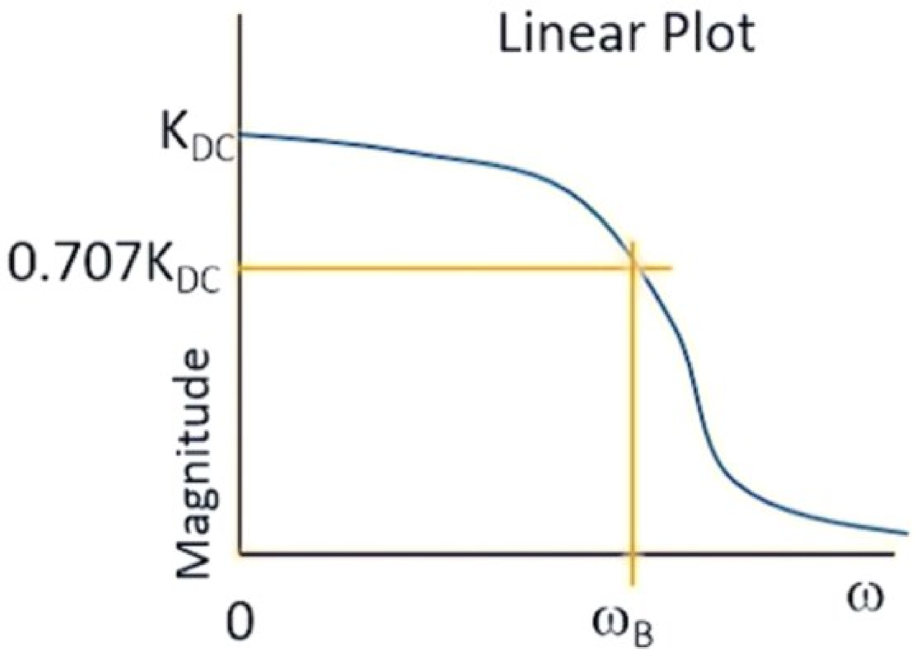


Highpass Passive Filter Example



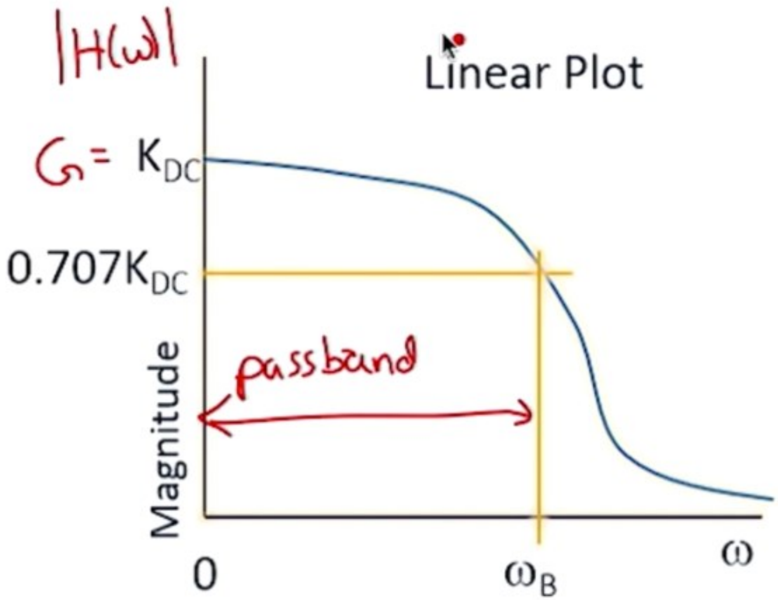
Lowpass Filter Properties

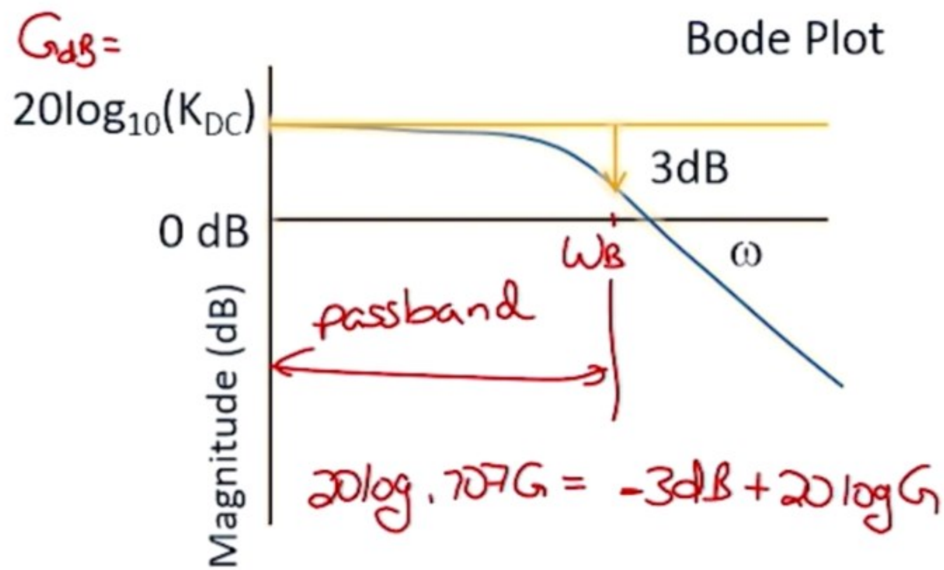
Bandwidth, ω_B Passband Region Passband Gain G



Lowpass Filter Properties

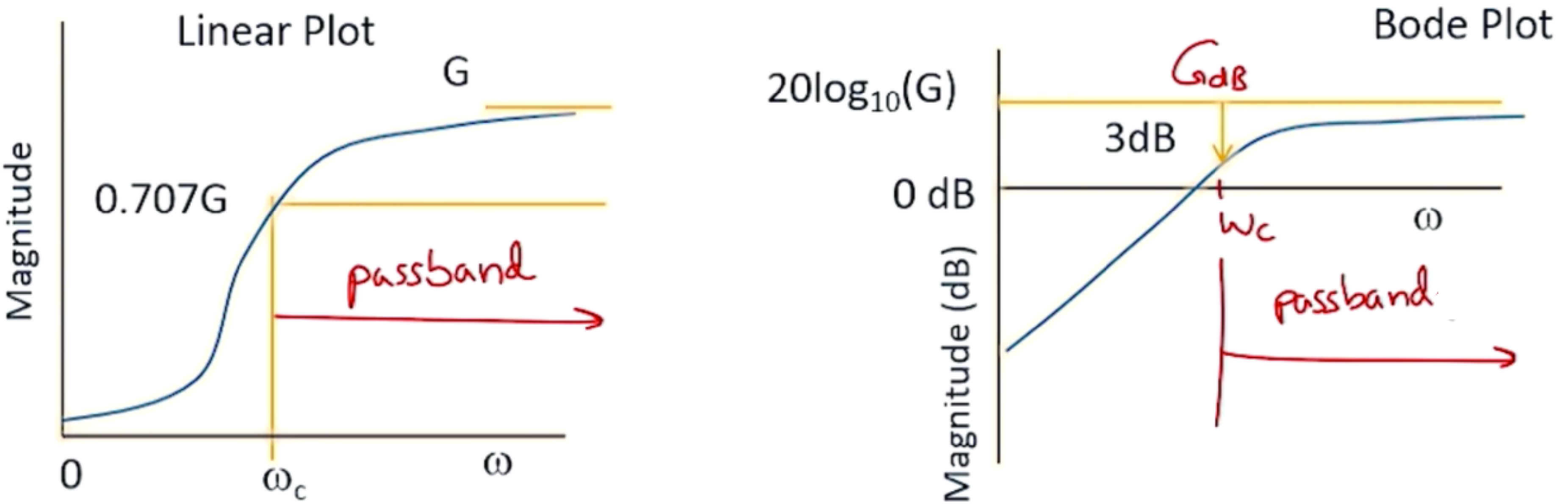
Bandwidth, ω_B Passband Region Passband Gain G $= K_{DC} = |H(0)|$





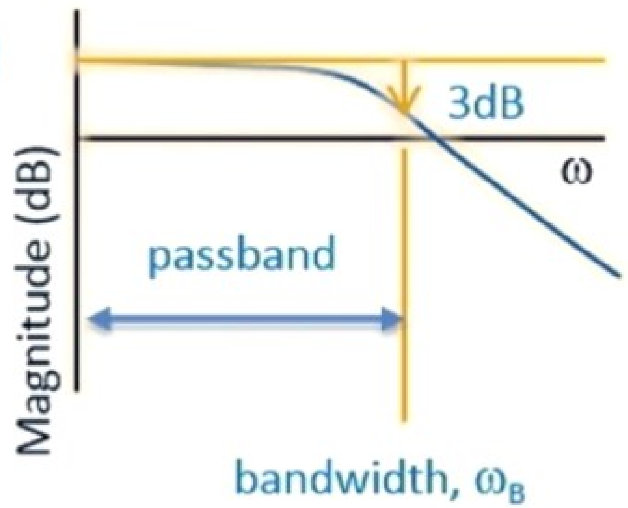
Highpass Filter Properties

Corner Frequency, ω_c Passband Region Passband Gain G

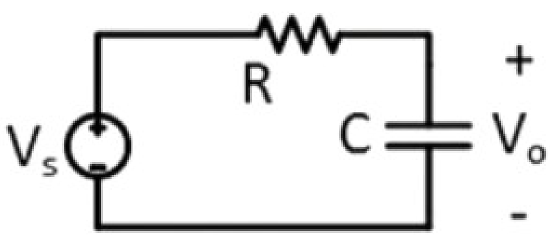


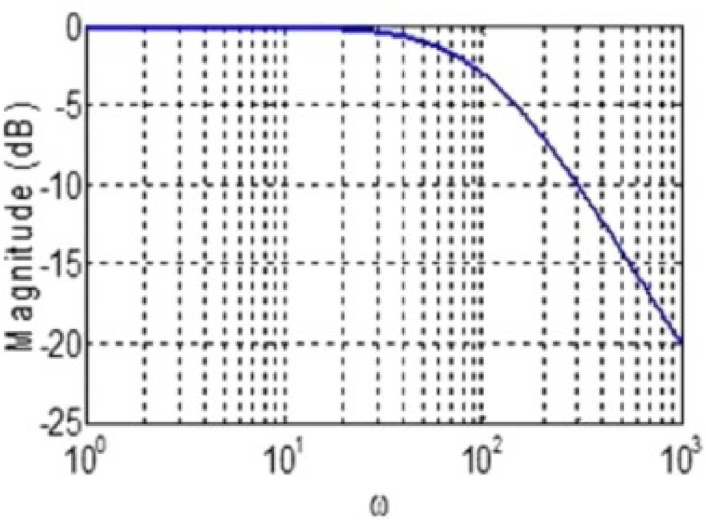
Lowpass RC Filter Design

Filters



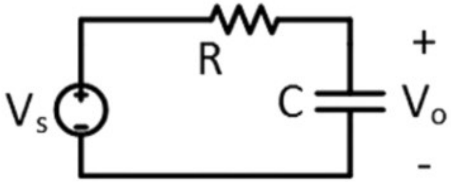
RC, RLC Bode Plots

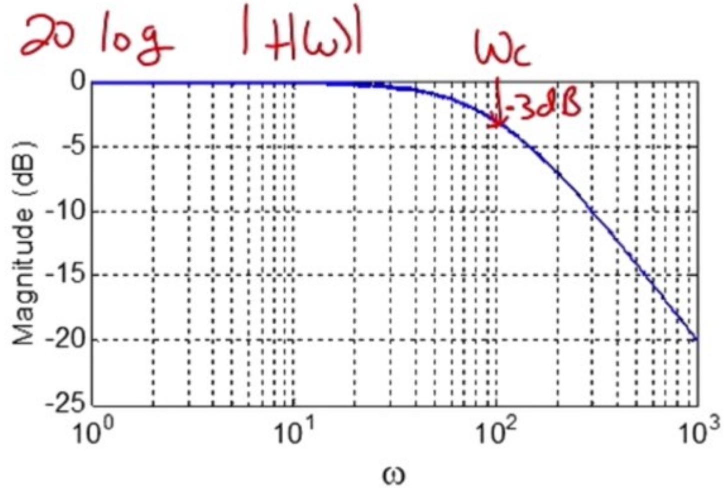


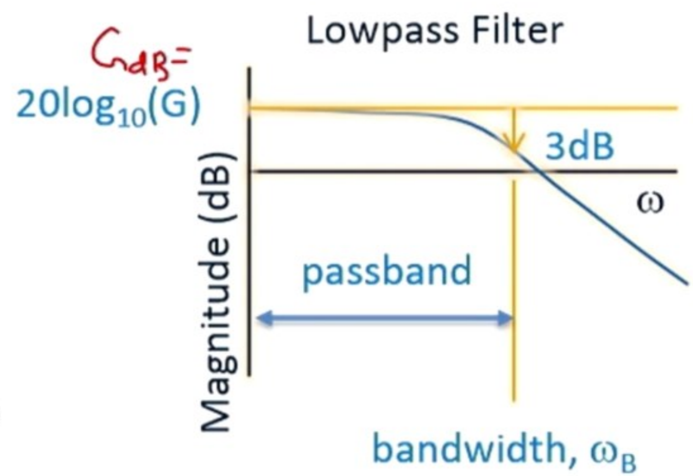


Lowpass RC Filter Design

Lowpass RC Filter







Low Frequency:
Magnitude: 0dB

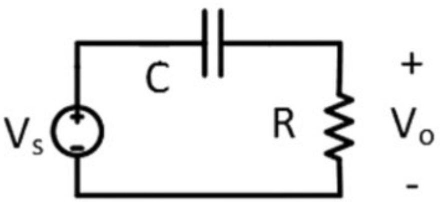
Corner Frequency:

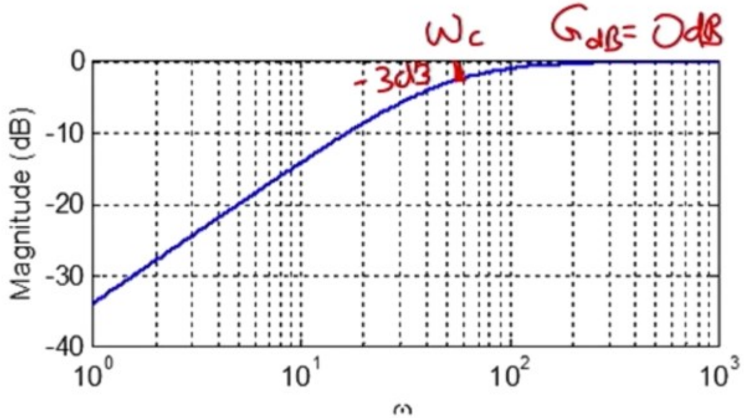
$$\omega_C = \frac{1}{RC}\,\text{rad/sec}$$

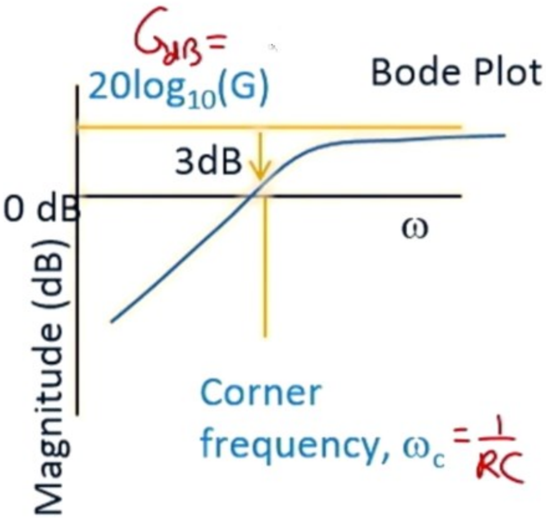
$$\omega_B = \omega_c = \frac{1}{RC}$$

$$G_{dB} = 0\,dB$$

Highpass RC Filter Design







High Frequency:
Magnitude: 0dB

Corner Frequency:

$$\omega_c = \frac{1}{RC}\ \text{rad/sec}$$

$\omega_c = \frac{1}{RC}$

passband gain $G_{dB} = 0dB$

Example 1

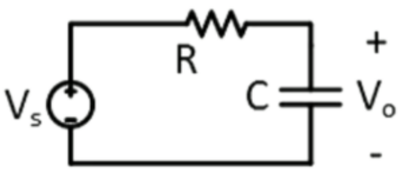
Design an RC filter that attenuates frequencies above 200 Hz.

Solution 1

f

Design an RC filter that attenuates frequencies above 200 Hz.

$\omega_B = 200(2\pi) = 1256$ rad/sec



$$\omega_C = \frac{1}{RC} \text{ rad/sec} = \omega_B = 1256$$

let $C = 1\,\mu f$

$$R = \frac{1}{\omega_B C} = 796\,\Omega$$

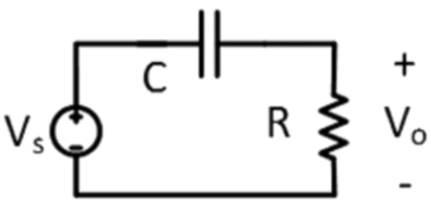
Example 2

Design an RC filter that attenuates frequencies below 50 Hz.

Solution 2

Design an RC filter that attenuates frequencies below 50 Hz.

$W_c = 50(2\pi) = 314$ rad/sec

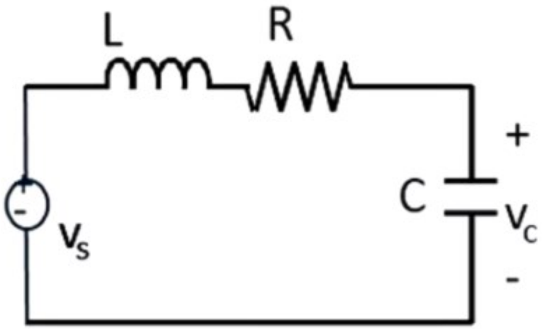


$R = \frac{1}{W_c C}$ let $C = 1\mu f$

$\Rightarrow R = 3184\,\Omega$

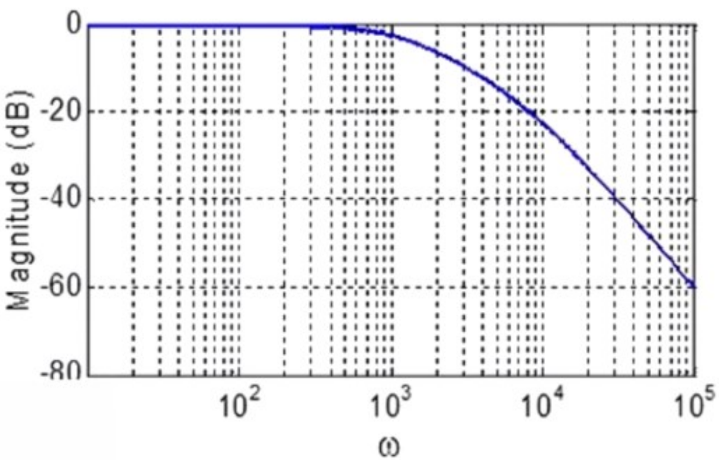
$$\omega_C = \frac{1}{RC} \text{rad/sec}$$

RLC Lowpass Passive Filters

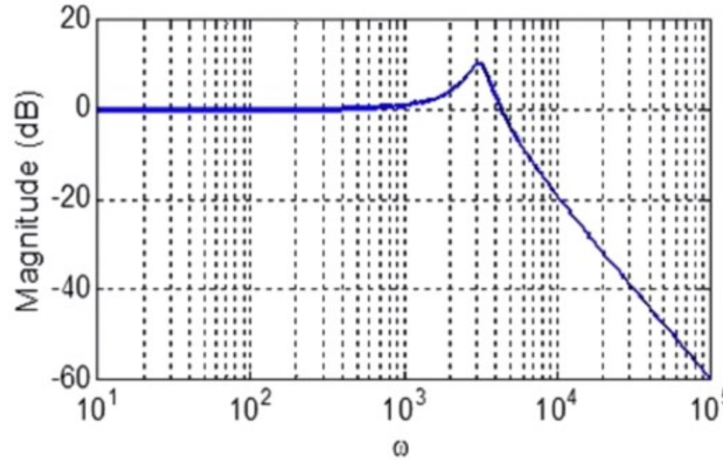


Corner Frequency: $\omega_C = \dfrac{1}{\sqrt{LC}}$

overdamped

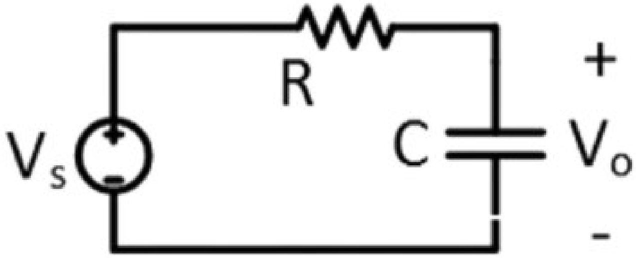


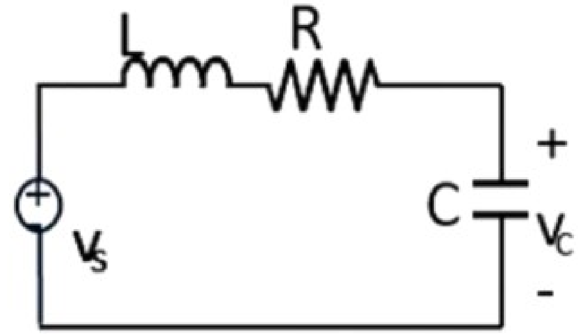
underdamped



Key Concepts

Lowpass Circuits

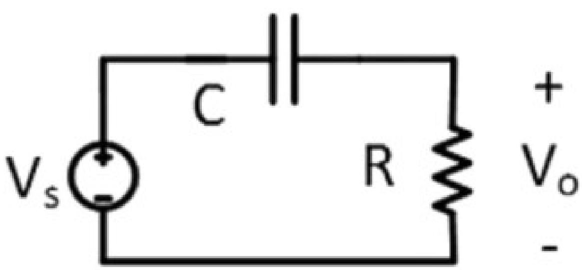


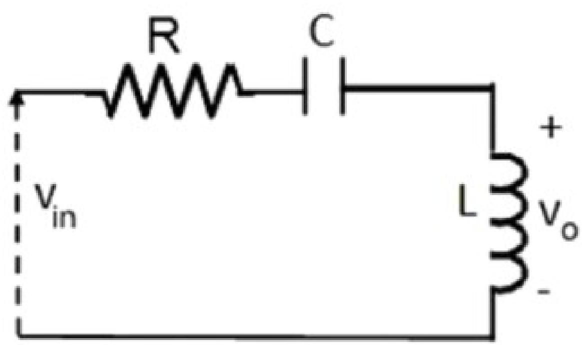


$$\omega_B = \frac{1}{RC} \text{ rad/sec}$$

Passband gain = 1

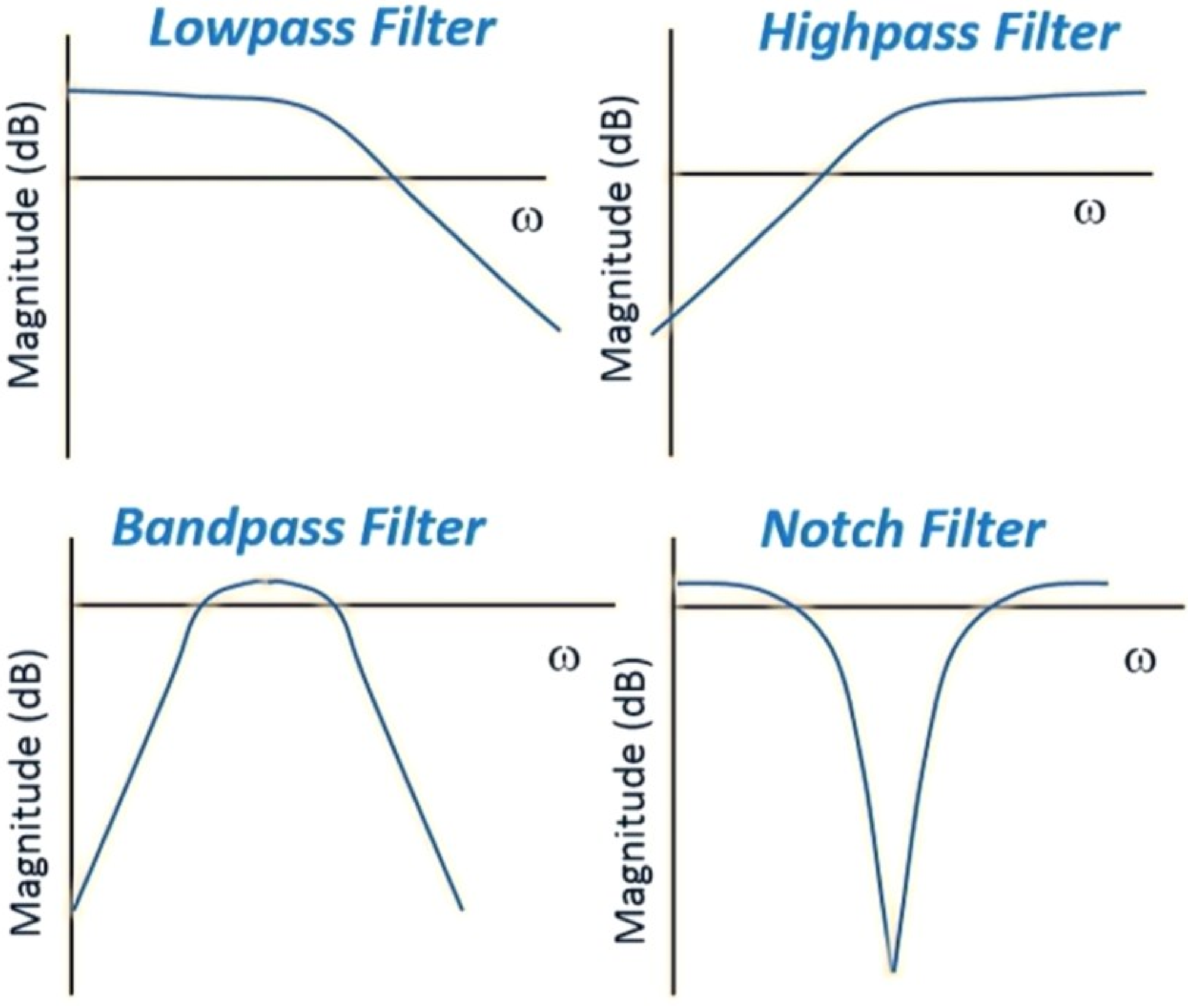
Highpass Circuits



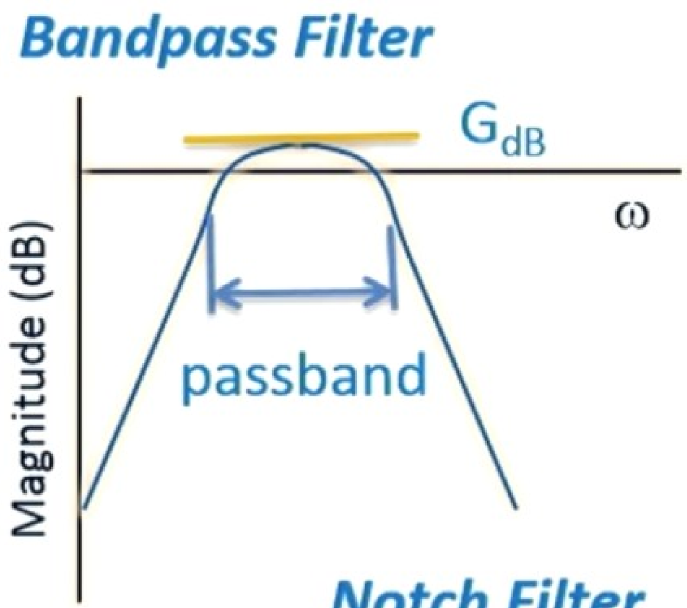


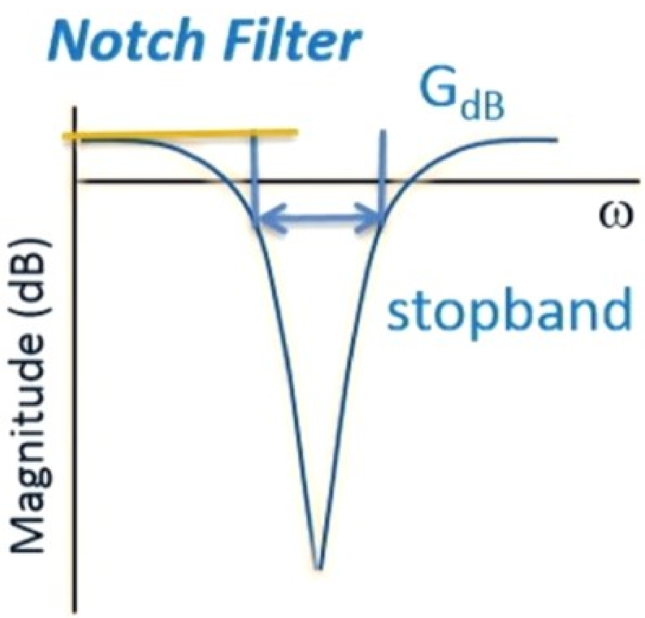
$$\omega_c = \frac{1}{RC} \text{ rad/sec}$$

Bandpass and Notch Filters

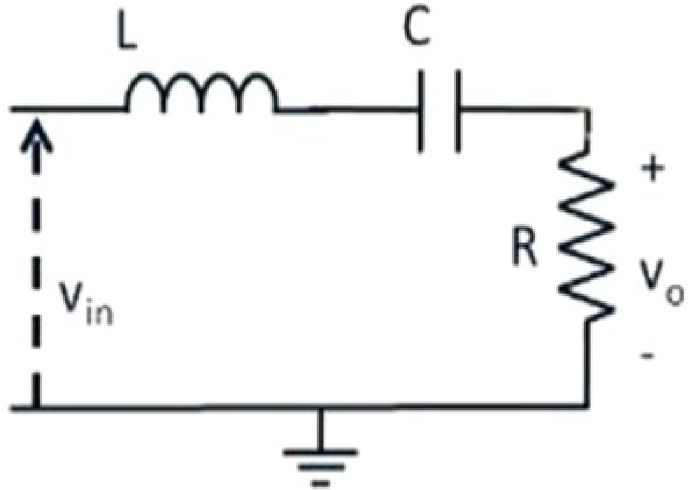


Filter Characteristics

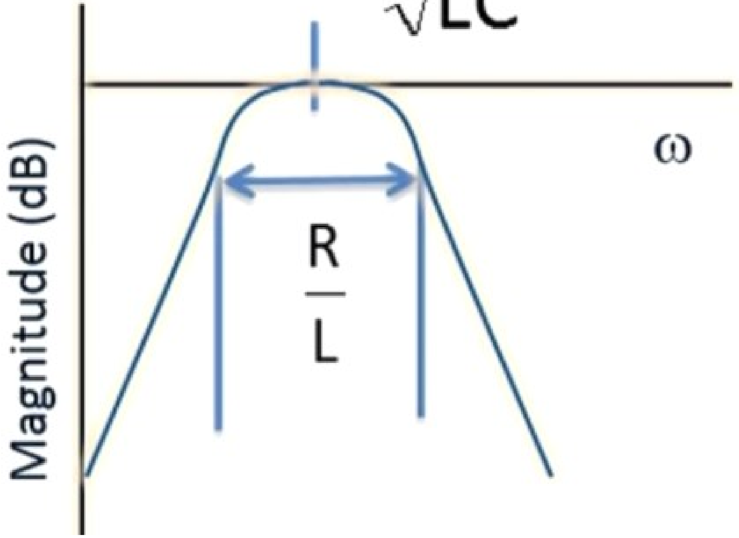




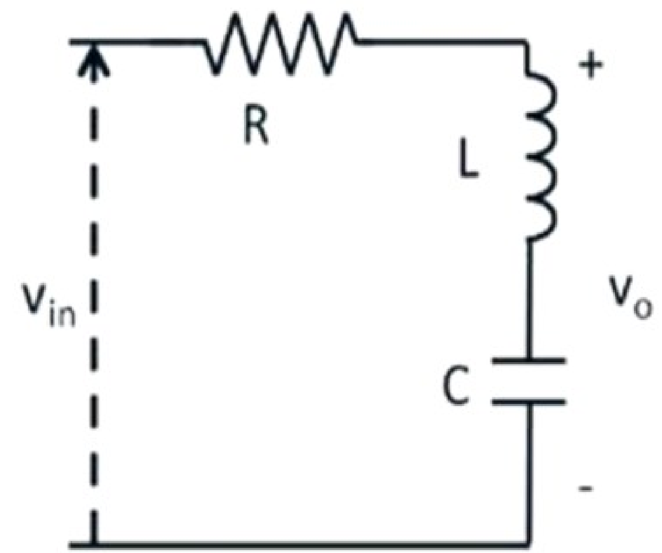
RLC Bandpass Filter

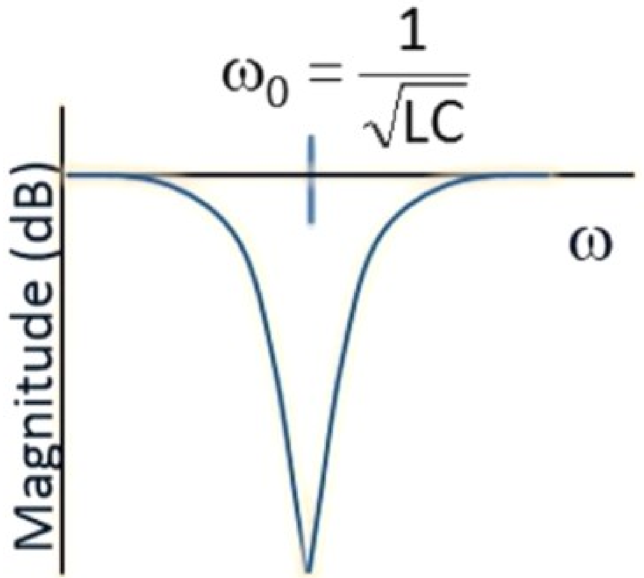


$$\omega_0 = \frac{1}{\sqrt{LC}}$$



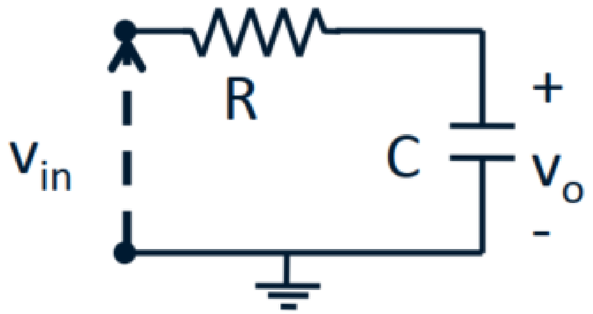
RLC Notch Filter





Limitations of Passive Filters

- Depletes power



- No isolation

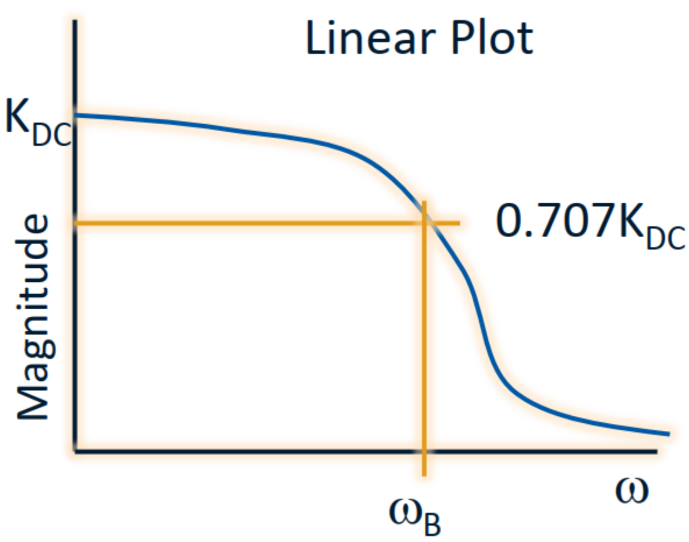
Advantages of Active Filters

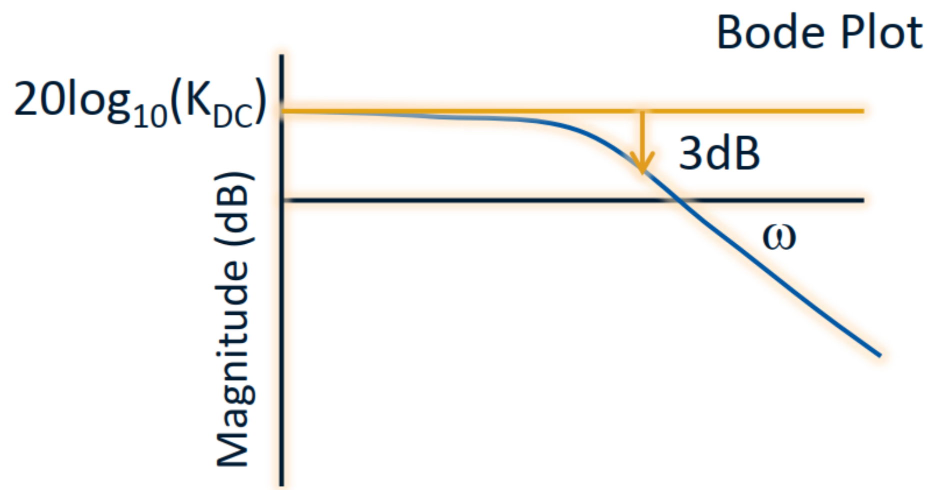
- **Less cost**: Inexpensive opamps and absence of costly inductors (especially at lower frequencies)
- **Gain and frequency adjustment flexibility**: Opamp provides gain (adjustable) -> input signal not attenuated as in case of passive filters
- **No loading problem**: Excellent isolation between stages due to high input impedance (opamps again) and low output impedance. The output can drive other circuitry without loading the source or load
- **Size**: Small in size (due to absence of bulky 'L')
- **Non-floating terminals**: Active filters generally have single ended input and output which do not float with respect to the system power supply

First Order Lowpass Active Filters

- Lowpass filters pass low frequency components and attenuate high frequency components

Transfer Function H(ω)

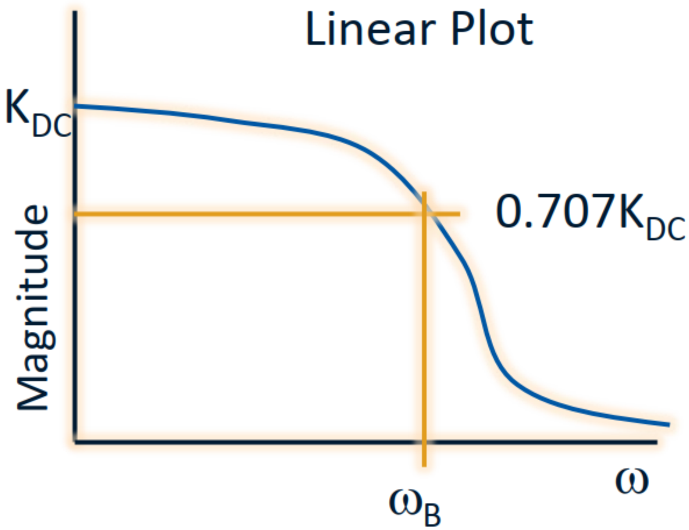




First Order Lowpass Active Filters

- Lowpass filters pass low frequency components and attenuate high frequency components

Transfer Function $H(\omega)$

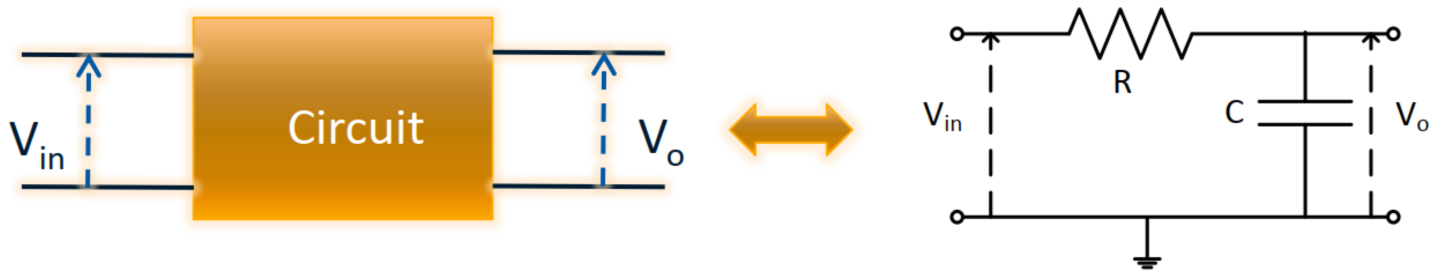


$$H(\omega) = K_{DC} \frac{1}{\tau j\omega + 1}$$

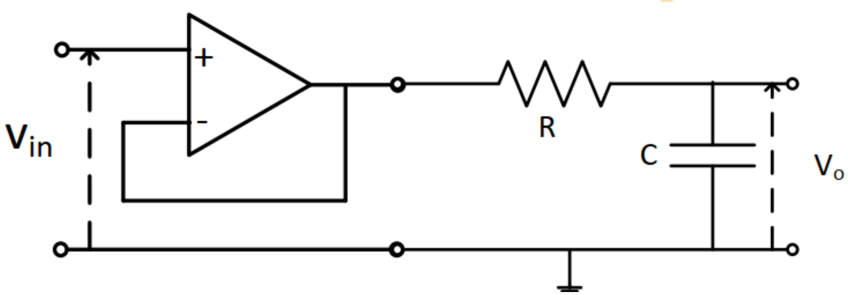
Bandwidth, $\omega_B = 1/\tau$

DC Gain = $H(0) = K_{DC}$

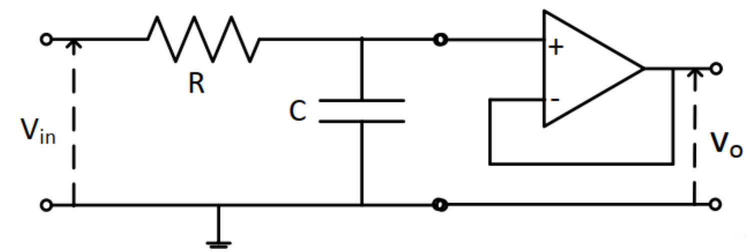
First Order Lowpass Active Filters



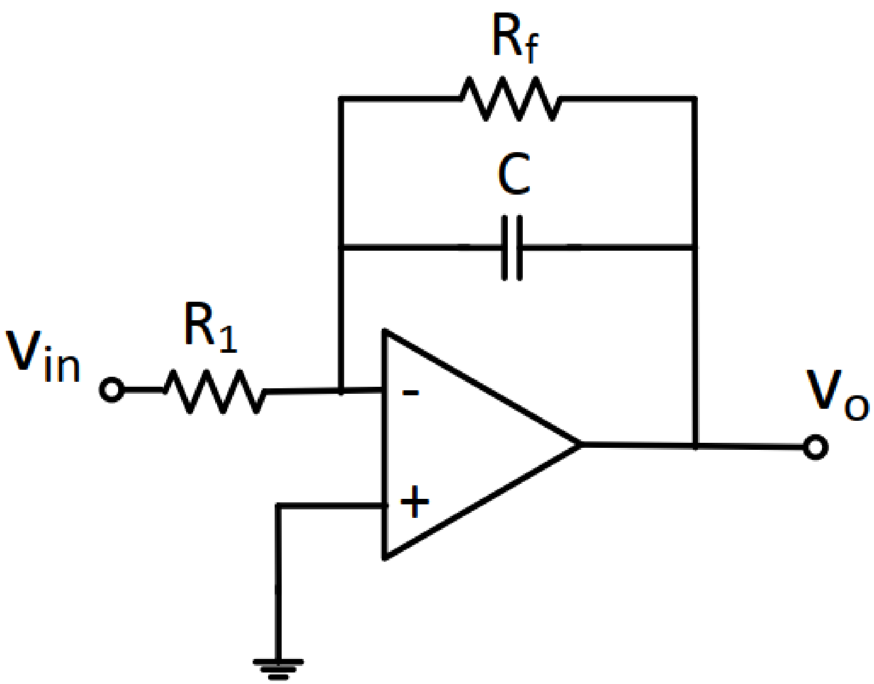
Isolation at the input:



Isolation in the output:



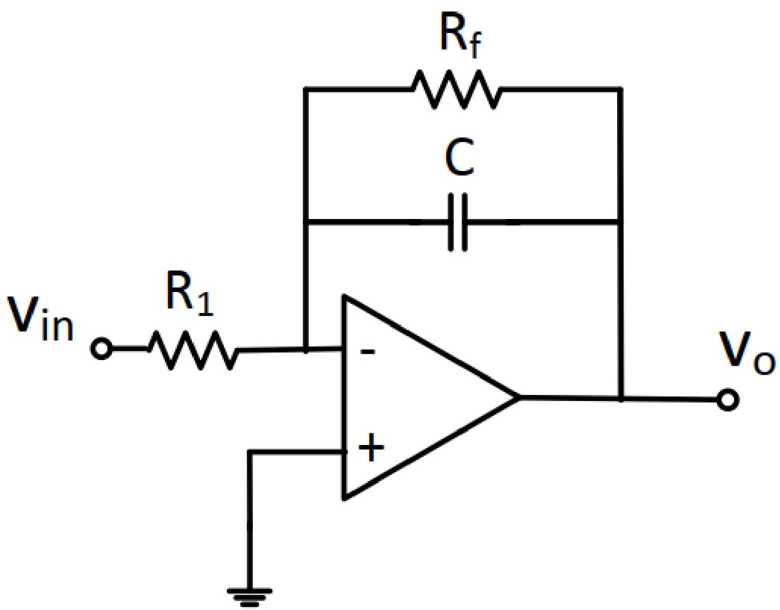
First Order Lowpass Active Filters

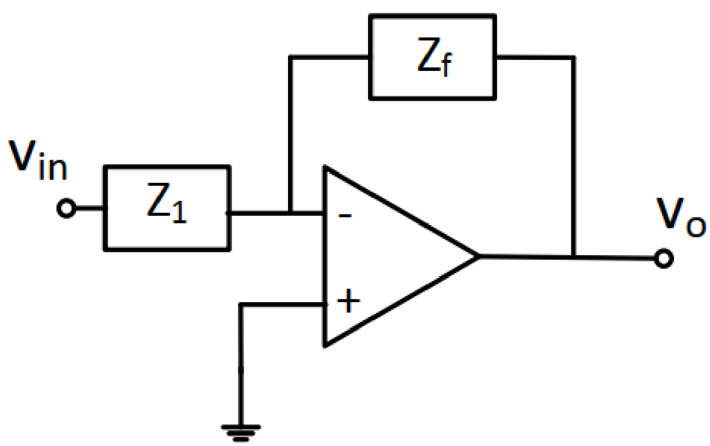


$$V_o = -\frac{R_f}{R_1}\frac{1}{R_f C j\omega + 1} V_{in}$$

First Order Lowpass Active Filters

Derivation: Lowpass Filter





First Order Lowpass Active Filters

Frequency Characteristics of LP Filter

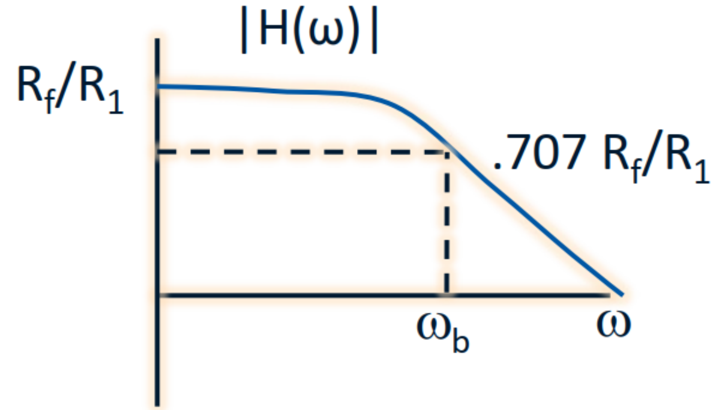
$$H(\omega) = -\frac{R_f}{R_1}\frac{1}{(R_f C j\omega + 1)}$$

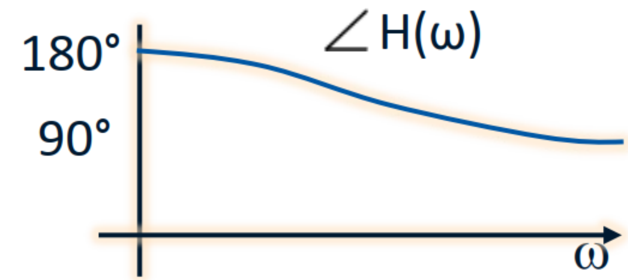
$$|H(\omega)| = \frac{R_f}{R_1}\frac{1}{\sqrt{(R_f C_f \omega)^2 + 1}}$$

$$\angle H(\omega) = 180 - \arctan(R_f C_f \omega)$$

$$\text{DC Gain} = -\frac{R_f}{R_1}$$

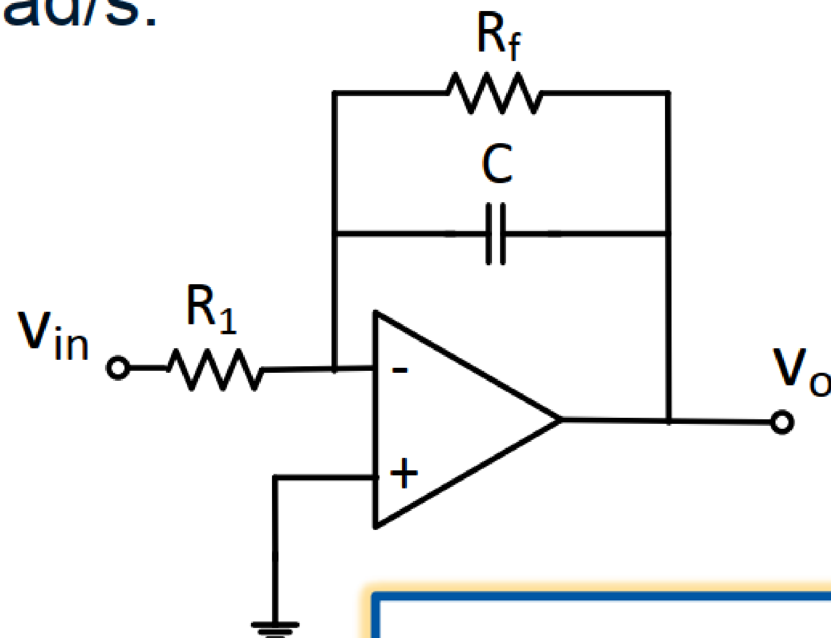
$$\text{Bandwidth, } \omega_b = \frac{1}{R_f C_f}$$





Example 3

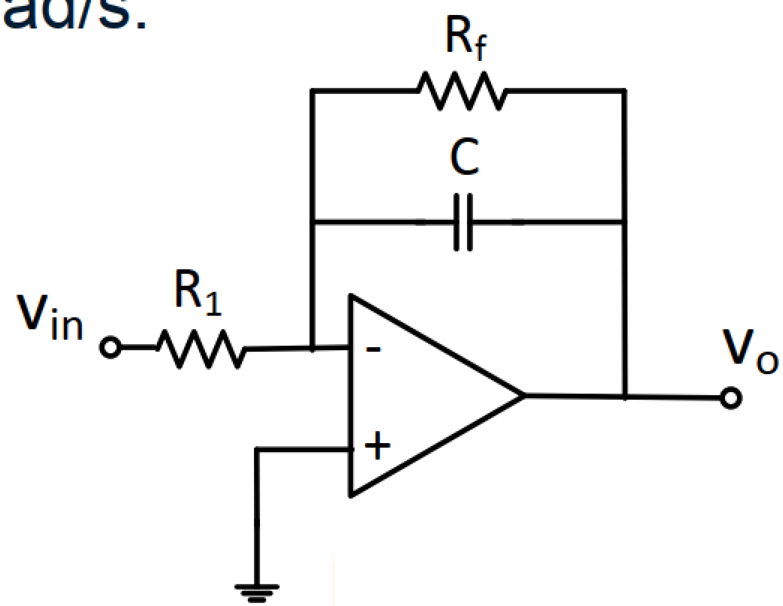
Design an inverting lowpass filter to have a DC gain of -2 and a bandwidth of 500 rad/s:



$$H(\omega) = -\frac{R_f}{R_1}\frac{1}{R_f C j\omega + 1}$$

Solution 3

Design an inverting lowpass filter to have a DC gain of -2 and a bandwidth of 500 rad/s:



$$\text{DC gain} = H(0) = \frac{-R_f}{R_1} = -2$$

$$\omega_B = \frac{1}{R_f C} = 500$$

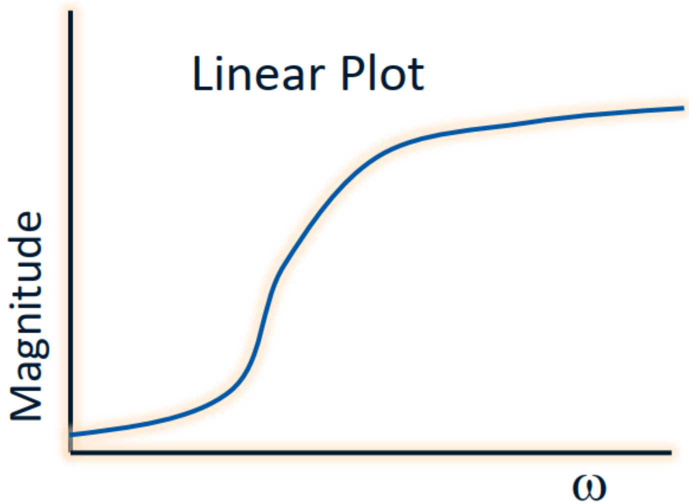
pick $R_1 = 1000\,\Omega$

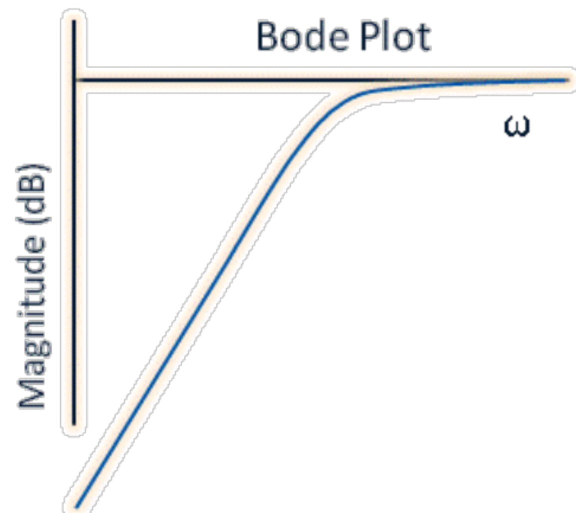
$R_f = 2000\,\Omega$

$C = 1\,\mu f$

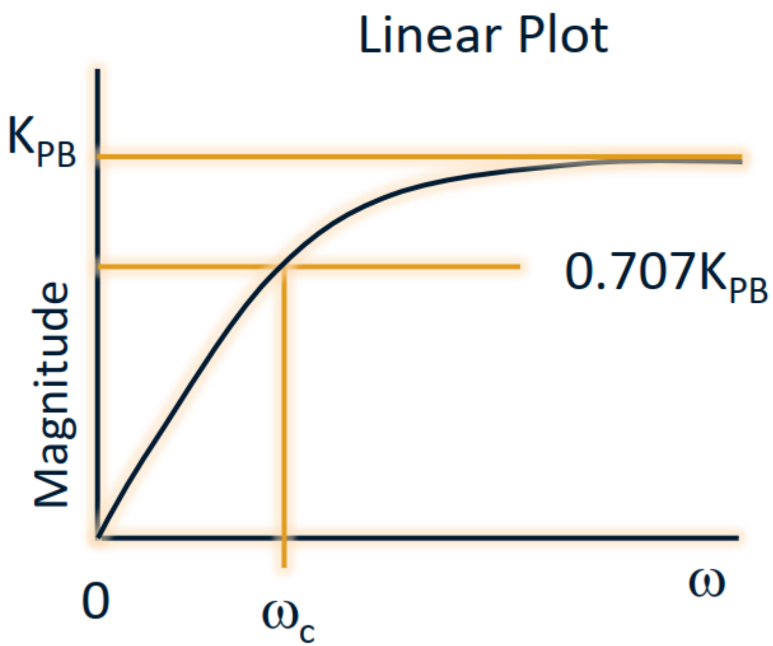
First Order Highpass Active Filters

- Passes high frequency components and attenuates low frequency components





First Order Highpass Active Filters

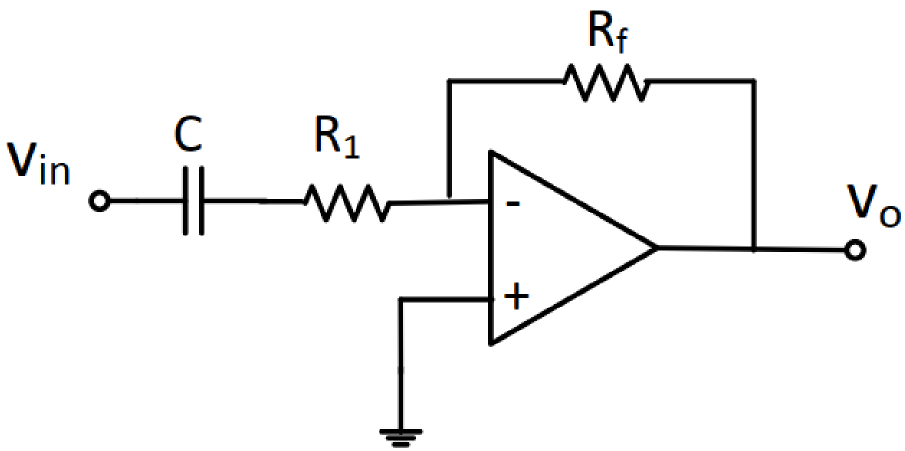


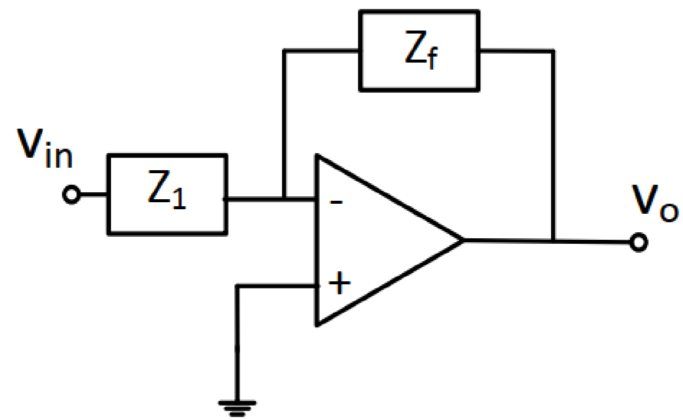
$$H(\omega) = \frac{Kj\omega}{\tau j\omega + 1}$$

Corner Frequency, $\omega_c = 1/\tau$

Passband Gain= $K_{PB} = K/\tau$

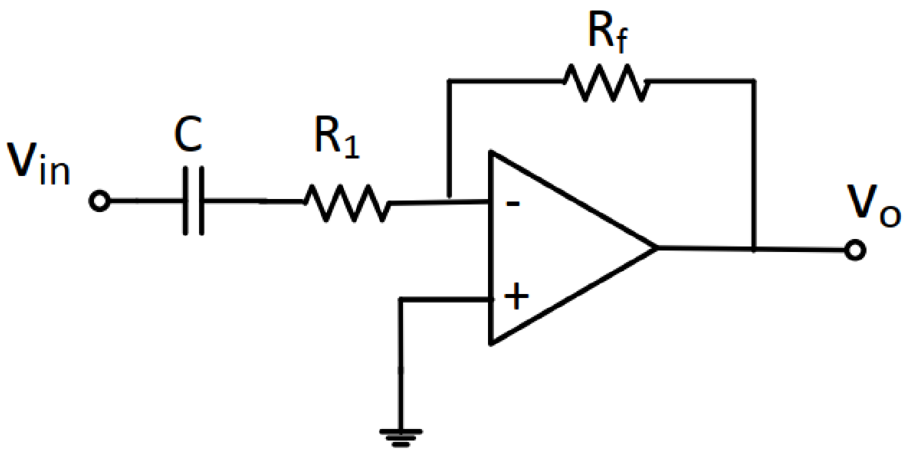
First Order Highpass Active Filters

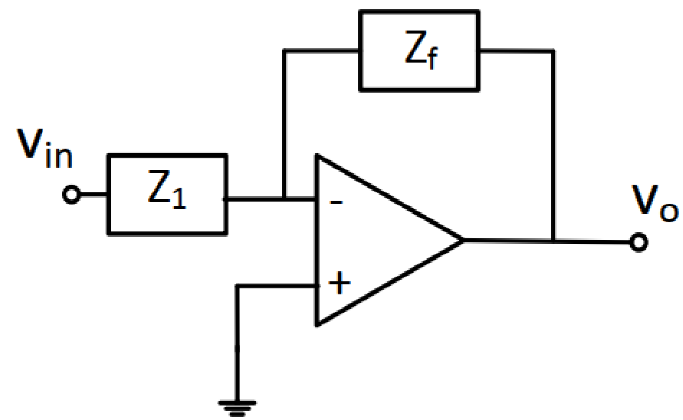




$$\boxed{V_o = \frac{-R_f C j\omega}{(R_1 C j\omega + 1)} V_{in}}$$

First Order Highpass Active Filters





$$\boxed{V_o = \frac{-R_f C j\omega}{(R_1 C j\omega + 1)} V_{in}}$$

$$H(\omega) = \frac{-Z_f}{Z_1}$$

$$Z_f = R_f$$

$$Z_1 = Z_C + R_1 = \frac{1}{j\omega C}$$

passband gain

$$H(\infty) = \frac{-R_f}{R_1}$$

$$\omega_c = \frac{1}{R_1 C}$$

First Order Highpass Active Filters

Frequency Characteristics of HP Filter

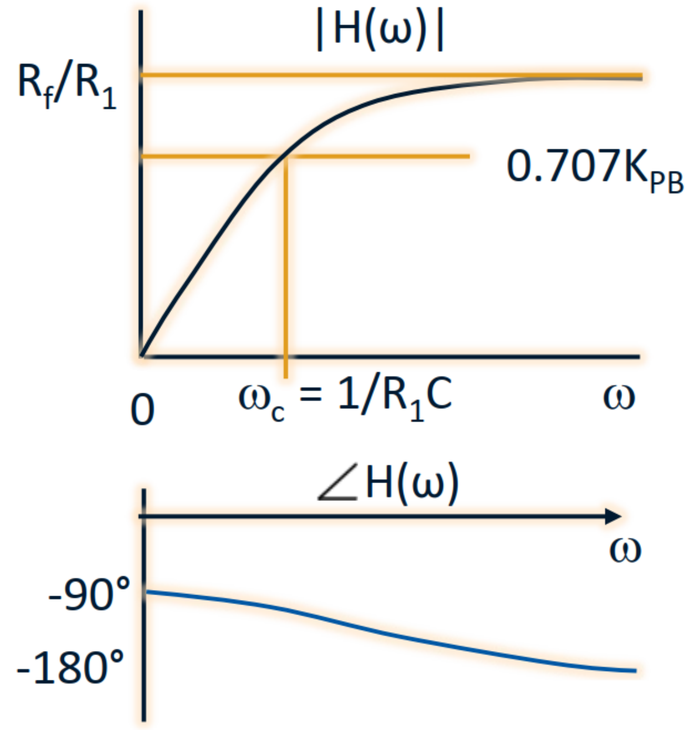
$$H(\omega) = \frac{-R_f C j\omega}{(R_1 C j\omega + 1)}$$

$$|H(\omega)| = \frac{R_f C\omega}{\sqrt{(R_1 C\omega)^2 + 1}}$$

$$\angle H(\omega) = -90^\circ - \arctan(R_1 C\omega)$$

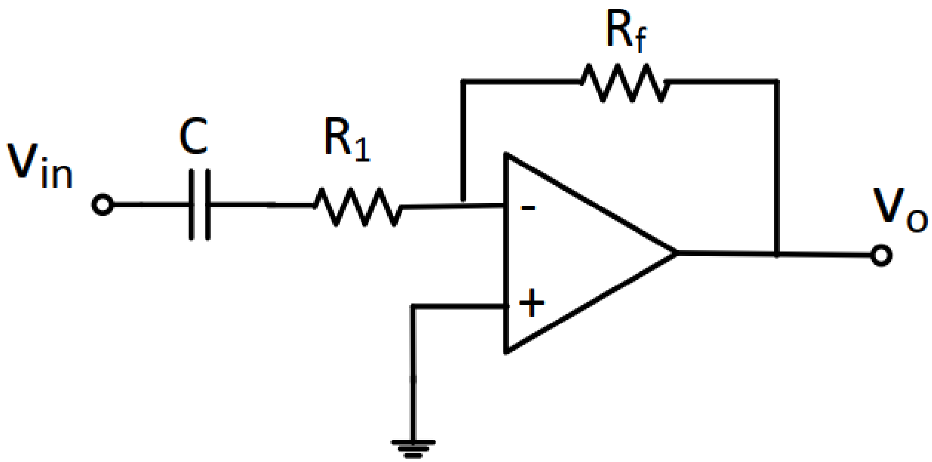
$$\text{Passband Gain } (\omega \to \infty) = -\frac{R_f}{R_1}$$

$$\text{Corner Freq., } \omega_c = \frac{1}{R_1 C}$$



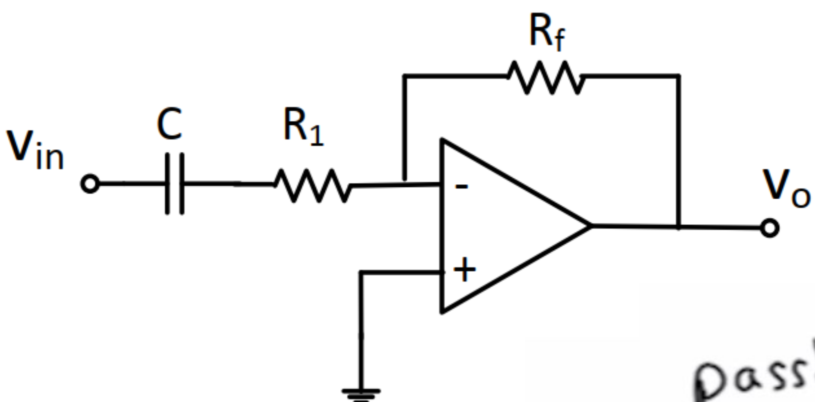
Example 4

Design a highpass filter to have a passband gain of 2 and a corner frequency of 1k rad/s:



Solution 4

Design a highpass filter to have a passband gain of 2 and a corner frequency of 1k rad/s:



passband gain $H(\infty) = \frac{-R_f}{R_1} = -2$

$$H(\omega) = \frac{-R_f C j\omega}{R_1 C j\omega + 1}$$

$\omega_c = \frac{1}{R_1 C} = 1000$

let $R_1 = 1000\,\Omega$

$\Rightarrow R_f = 2000\,\Omega \quad C = 1\mu f$

if f_c in Hz $\quad \omega = 2\pi f$

$2\pi f_c = \frac{1}{R_1 C}$

Cascaded First Order Filters

Transfer Functions in Hertz f

Lowpass

$$H(\omega) = K_{DC}\frac{1}{\tau j\omega + 1}$$

Bandwidth, $\omega_B = 1/\tau$
DC Gain = H(0) = K_{DC}

$\omega = 2\pi f$
$\omega_B = 1/\tau = \omega_0$
$\omega_0 = 2\pi f_0$

$$H(f) = K_{DC}\frac{1}{\frac{j2\pi f}{2\pi f_0}+1}$$

$$\boxed{H(f) = K_{DC}\frac{1}{\frac{jf}{f_0}+1}}$$

Highpass

$\omega = 2\pi f$
$\omega_c = 1/\tau = \omega_0$
$\omega_0 = 2\pi f_0$
$K_{PB} = K/\tau$

$$H(\omega) = \frac{Kj\omega}{\tau j\omega + 1}$$

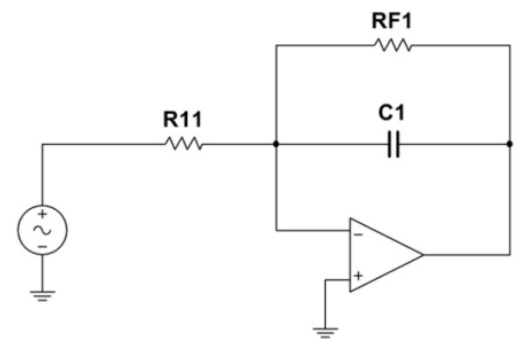
Corner Frequency, $\omega_c = 1/\tau$
Passband Gain= $K_{PB} = K/\tau$

$$H(f) = K\frac{j2\pi f}{\frac{j2\pi f}{2\pi f_0}+1}$$

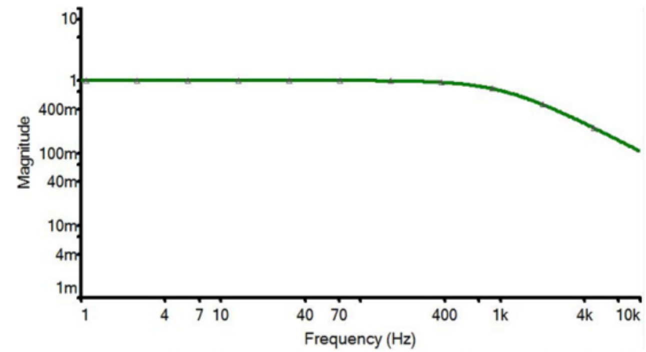
$$= 2\pi f_0 K\frac{\frac{j2\pi f}{2\pi f_0}}{\frac{jf}{f_0}+1}$$

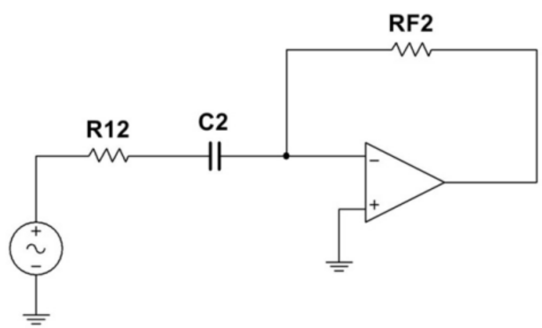
$$\boxed{H(f) = K_{PB}\frac{jf/f_0}{jf/f_0+1}}$$

Cascaded First Order Filters

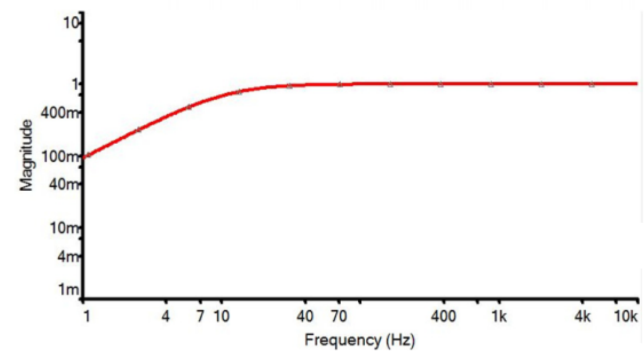


$$H_{LP}(f) = K_{DC}\frac{1}{\frac{jf}{f_0}+1}$$



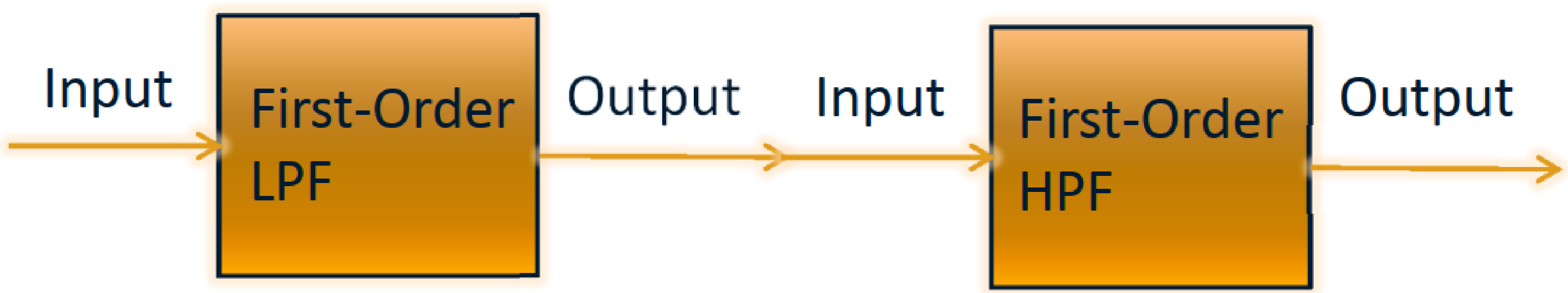


$$H_{HP}(f) = K_{PB}\frac{\frac{jf}{f_0}}{\frac{jf}{f_0}+1}$$



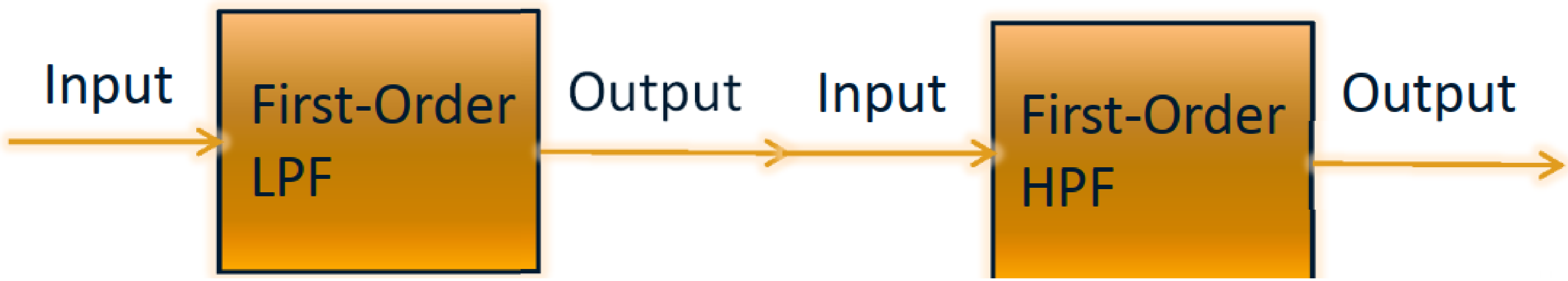
Cascaded First Order Filters

Cascaded Filter



Cascaded First Order Filters

Cascaded Filter

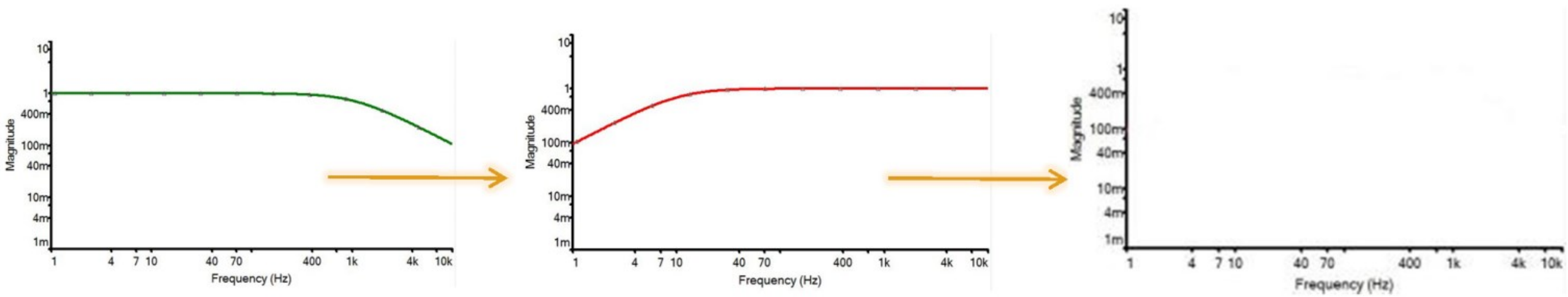


V_{IN} V_{OUT1} V_{IN2} V_{OUT}

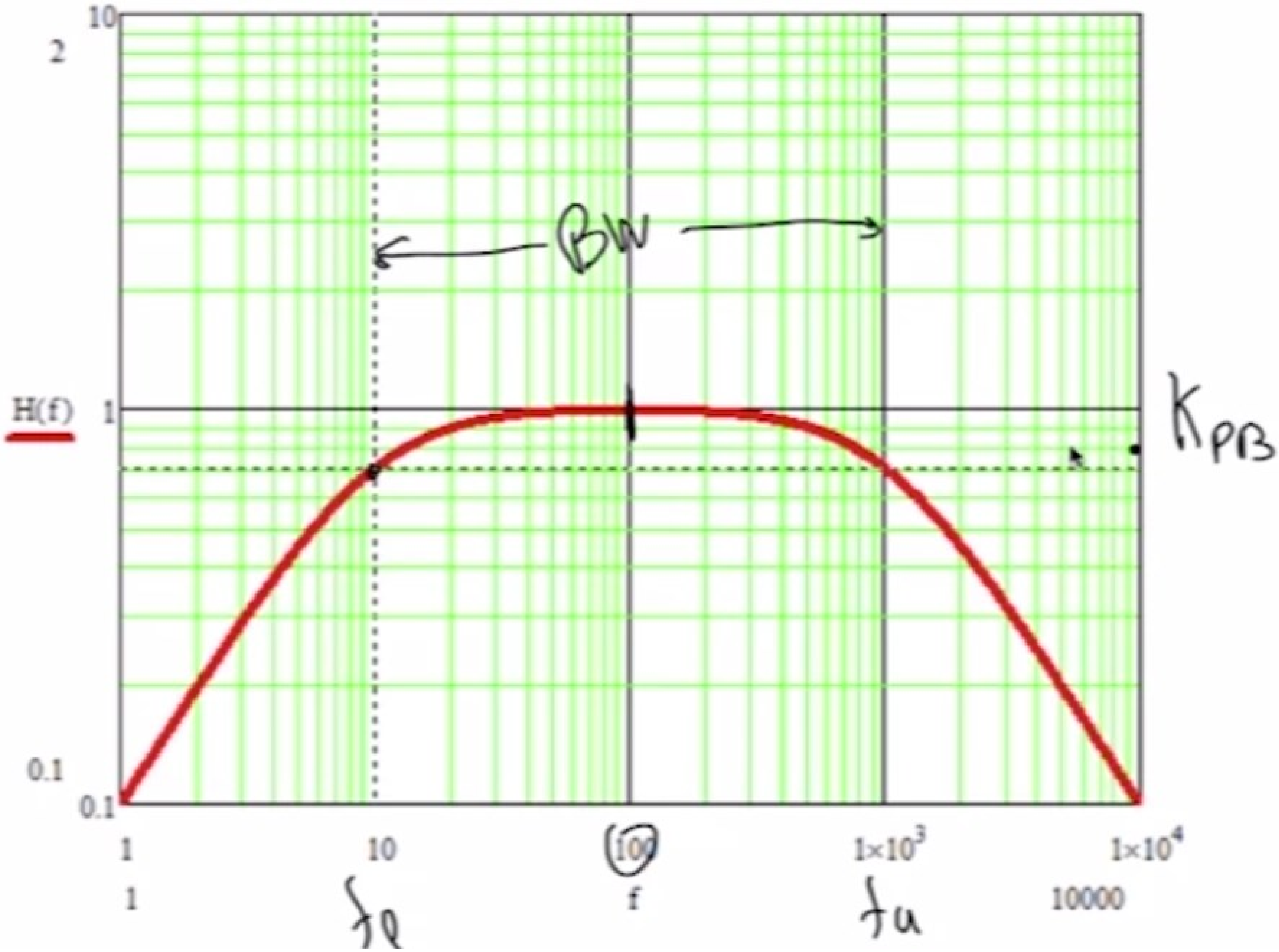
$$H_C = \frac{V_{OUT}}{V_{IN}} = \underbrace{\frac{V_{OUT1}}{V_{IN}}}_{H_{LP}} \cdot \underbrace{\frac{V_{OUT}}{V_{IN2}}}_{H_{HP}} = H_{LP} H_{HP} .$$

Cascaded First Order Filters

Cascaded Filter



Bandpass Filter Characteristics



$$BW = f_u - f_l$$

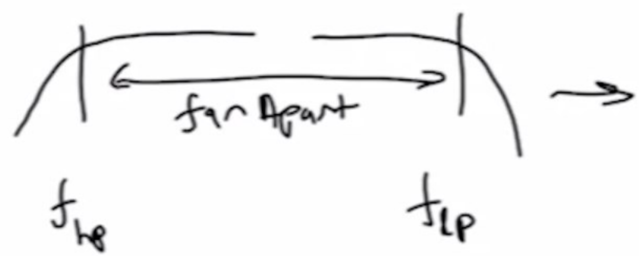
$$f_0 = \sqrt{f_u f_l}$$

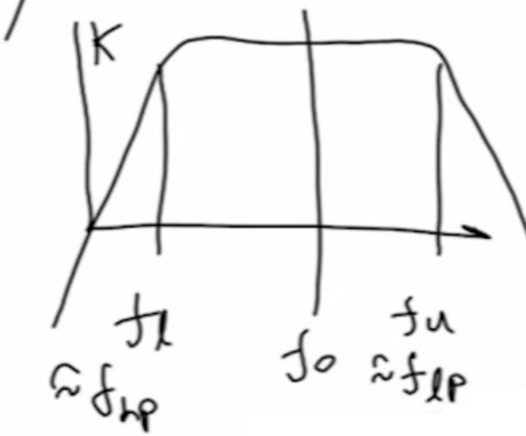
$$Q = \frac{f_0}{BW}$$

Cascaded Filter Transfer Function

$$H_{BP}(f) = H_{LP}(f)H_{HP}(f) = \left(K_{DC}\frac{1}{\frac{jf}{f_{lp}}+1}\right)\left(K_{PB}\frac{\frac{jf}{f_{hp}}}{\frac{jf}{f_{hp}}+1}\right)$$

$$K = K_{DC}K_{PB}\left(\frac{f_{lp}}{f_{lp}+f_{hp}}\right)$$





$$f_0 = \sqrt{f_{lp}f_{hp}}$$

$$Q = \frac{\sqrt{f_{lp}f_{hp}}}{f_{lp}+f_{hp}}$$

$Q < \frac{1}{2}$ $\boxed{Q = \frac{1}{2} \quad f_{lp} = f_{hp}}$

$$BW = f_{lp} + f_{hp}$$