BLM1612 Circuit Theory

RLC Circuits

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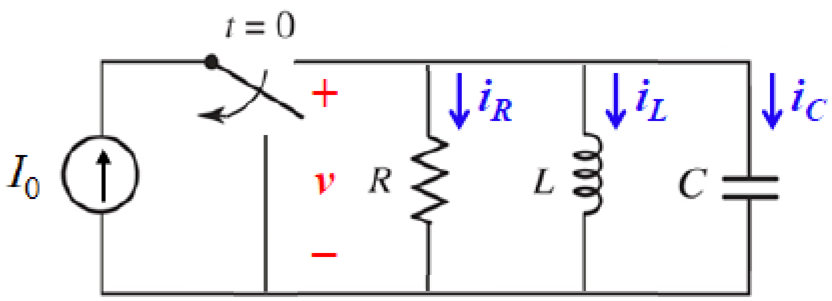
Parallel *RLC* Circuit



$$i_R(t) = v(t)/R$$

$$i_L(t) = \frac{1}{L}\int_{t_0}^{t} v(t')\cdot dt' + i_L(t_0)$$

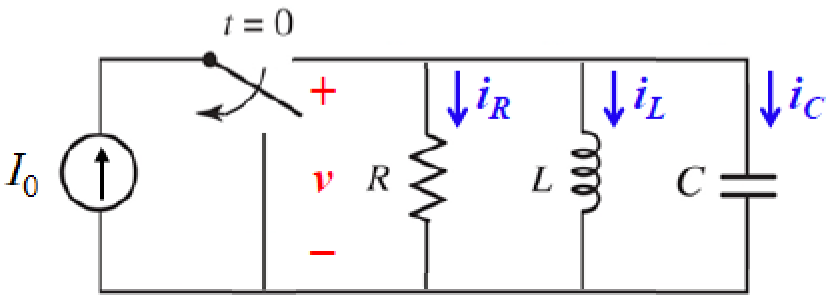
$$i_C(t) = C\cdot\frac{d}{dt}v(t)$$

KCL at the top node:

$$-I_0 + i_R(t) + i_L(t) + i_C(t) = 0$$

$$-I_0 + \frac{v(t)}{R} + \frac{1}{L}\int_{t_0}^{t} v(t')\cdot dt' + i_L(t_0) + C\cdot\frac{d}{dt}v(t) = 0$$

Parallel *RLC* Circuit



$$-I_0 + \frac{v(t)}{R} + C \cdot \frac{d}{dt} v(t) + \frac{1}{L} \int_{t_0}^{t} v(t') \cdot dt' + i_L(t_0) = 0$$

rearrange terms & divide by C...

$$\frac{d}{dt} v(t) + \frac{1}{RC} v(t) + \frac{1}{LC} \int_{t_0}^{t} v(t') \cdot dt' + \frac{i_L(t_0)}{C} = \frac{I_0}{C}$$

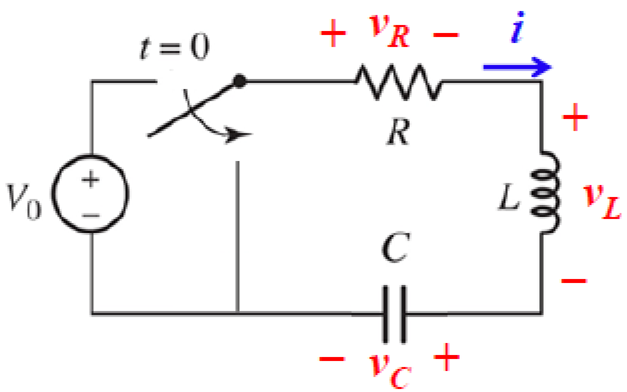
take d/dt of the entire equation... $$\frac{d^2}{dt^2} v(t) + \frac{1}{RC} \frac{d}{dt} v(t) + \frac{1}{LC} v(t) = 0$$

Damping coefficient

$$\frac{d^2}{dt^2} v(t) + 2\alpha \cdot \frac{d}{dt} v(t) + \omega_0{}^2 \cdot v(t) = 0 \quad \text{where} \quad \alpha = \frac{1}{2RC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

resonant frequency

Series *RLC* Circuit



$$v_R(t) = R \cdot i(t)$$

$$v_L(t) = L \cdot \frac{d}{dt} i(t)$$

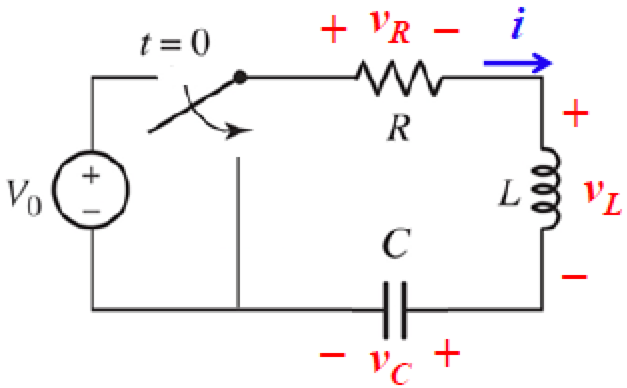
$$v_C(t) = \frac{1}{C}\int_{t_0}^{t} i(t') \cdot dt' + v_C(t_0)$$

KVL around the series loop:

$$-V_0 + v_R(t) + v_L(t) + v_C(t) = 0$$

$$-V_0 + R \cdot i(t) + L \cdot \frac{d}{dt} i(t) + \frac{1}{C}\int_{t_0}^{t} i(t') \cdot dt' + v_C(t_0) = 0$$

Series *RLC* Circuit



$$-V_0 + R \cdot i(t) + L \cdot \frac{d}{dt} i(t) + \frac{1}{C} \int_{t_0}^{t} i(t') \cdot dt' + v_C(t_0) = 0$$

rearrange terms & divide by L…

$$\frac{d}{dt} i(t) + \frac{R}{L} \cdot i(t) + \frac{1}{LC} \int_{t_0}^{t} i(t') \cdot dt' + \frac{v_C(t_0)}{L} = \frac{V_0}{L}$$

take d/dt of the entire equation… $$\frac{d^2}{dt^2} i(t) + \frac{R}{L} \frac{d}{dt} i(t) + \frac{1}{LC} i(t) = 0$$

$$\frac{d^2}{dt^2} i(t) + 2\alpha \cdot \frac{d}{dt} i(t) + \omega_0^{\ 2} \cdot i(t) = 0 \quad \text{where} \quad \alpha = \frac{R}{2L}, \ \ \omega_0 = \frac{1}{\sqrt{LC}}$$

Series & Parallel *RLC*: Solution

series: $$\frac{d^2}{dt^2}i(t)+2\alpha\cdot\frac{d}{dt}i(t)+\omega_0{}^2\cdot i(t)=0 \qquad \alpha=\frac{R}{2L},\ \omega_0=\frac{1}{\sqrt{LC}}$$

parallel: $$\frac{d^2}{dt^2}v(t)+2\alpha\cdot\frac{d}{dt}v(t)+\omega_0{}^2\cdot v(t)=0 \qquad \alpha=\frac{1}{2RC},\ \omega_0=\frac{1}{\sqrt{LC}}$$

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general differential equation for $v(t)$ or $i(t)$: $$\frac{d^2x}{dt^2}+2\alpha\frac{dx}{dt}+\omega_0{}^2x=0$$

solution to the general differential equation : $$x(t)=X_1e^{s_1t}+X_2e^{s_2t}+X_3$$

$$s_1=-\alpha+\sqrt{\alpha^2-\omega_0{}^2}$$
$$s_2=-\alpha-\sqrt{\alpha^2-\omega_0{}^2}$$

$$x(0^+)=X_1+X_2+X_3$$
$$\frac{dx}{dt}(0^+)=s_1X_1+s_2X_2$$

$$x(\infty)=X_3$$

RLC Solution: Over-damped

$$x(t) = X_1 e^{s_1 t} + X_2 e^{s_2 t} + X_3$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

The solution is said to be **overdamped** if $\alpha > \omega_0$

series: $\dfrac{R}{2L} > \dfrac{1}{\sqrt{LC}}$

parallel: $\dfrac{1}{2RC} > \dfrac{1}{\sqrt{LC}}$

…in which case the solution is

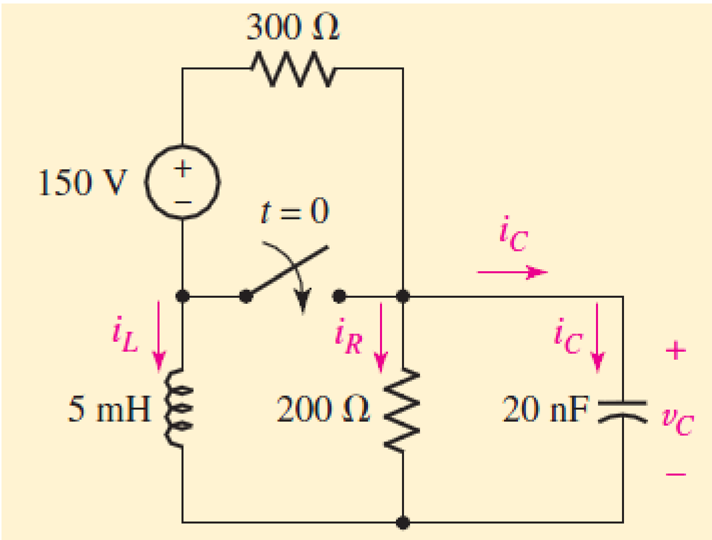
$$x(t) = X_1 e^{c_1 t} + X_2 e^{c_2 t} + X_3$$

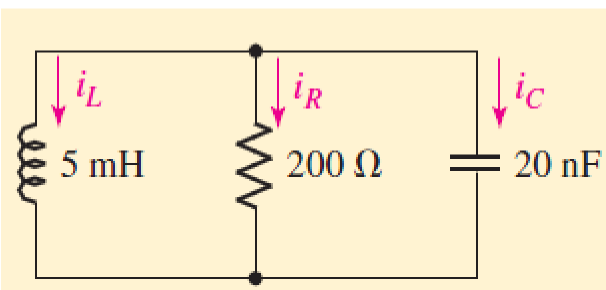
$$c_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$

$$c_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Example 9.2

Find an expression for $v_C(t)$ valid for $t > 0$ in the circuit.





$$\alpha = 1/2RC = 125\,000 \text{ s}^{-1}$$

$$\omega_0 = 1/\sqrt{LC} = 100\,000 \text{ rad/s.}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -50\,000 \text{ s}^{-1}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -200\,000 \text{ s}^{-1}$$

The circuit for $t > 0$, in which the 150 V source and the 300 resistor have been shorted out by the switch, and so are of no further relevance to v_C

Example 9.2 (Cont.)

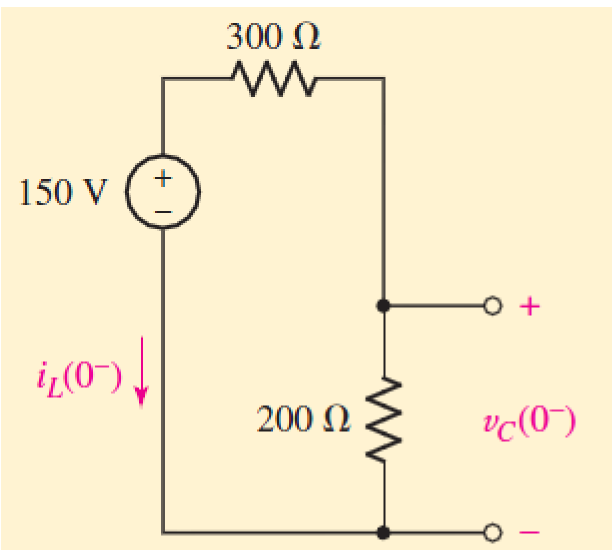
$\alpha > \omega_0$.

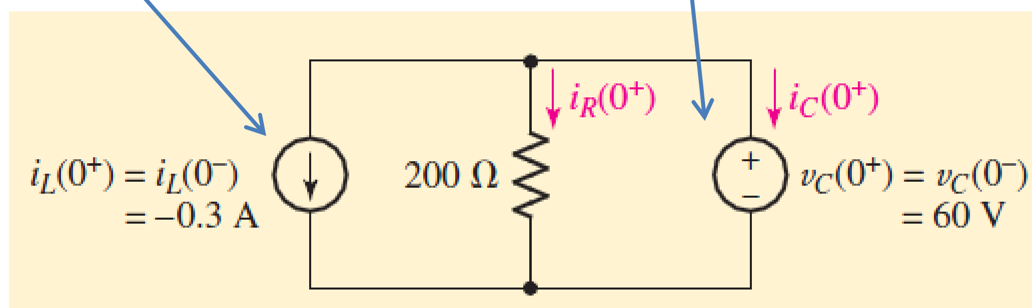
$v_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$$i_L(0^-) = -\frac{150}{200+300} = -300 \text{ mA}$$

$$v_C(0^-) = 150\frac{200}{200+300} = 60 \text{ V}$$

$v_C(0^+) = 60$ V.





The equivalent circuit at *t = 0*⁻

equivalent circuit at *t = 0*⁺, drawn using ideal sources to represent the initial inductor current and initial capacitor voltage

Example 9.2 (Cont.)

$v_C(t) = A_1 e^{-50,000t} + A_2 e^{-200,000t}$ equation for the capacitor voltage

$v_C(0) = 60$ V. we know the initial capacitor voltage

We have two unknowns, so we need an additional equation

$$\frac{dv_C}{dt} = -50,000 A_1 e^{-50,000t} - 200,000 A_2 e^{-200,000t}$$

$$i_C = C(dv_C/dt)$$

$$i_C(0^+) = -i_L(0^+) - i_R(0^+) = 0.3 - [v_C(0^+)/200] = 0$$

We have two unknowns and two equations

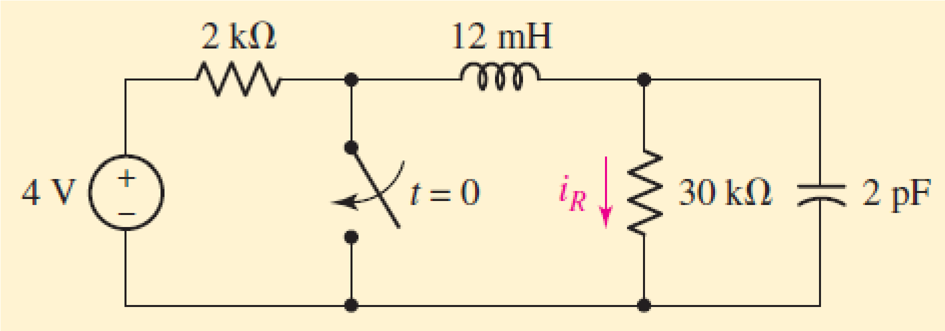
$v_C(0) = A_1 + A_2 = 60$ $\quad$ $i_C(0) = -20 \times 10^{-9}(50,000A_1 + 200,000A_2) = 0$

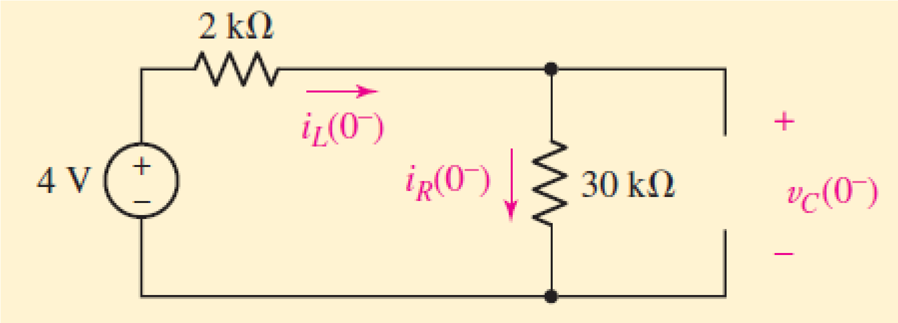
Solving, $A_1 = 80$ V and $A_2 = -20$ V, so that

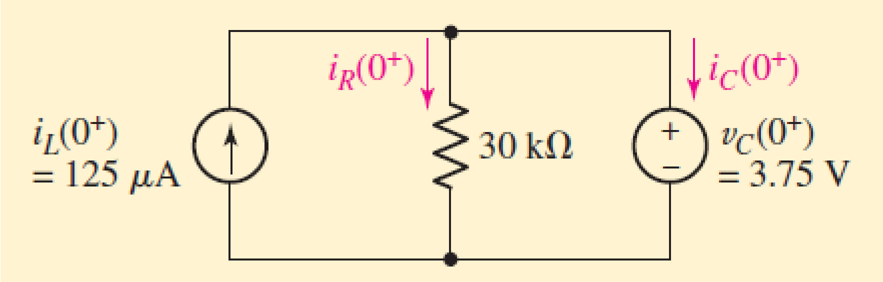
$$v_C(t) = 80e^{-50,000t} - 20e^{-200,000t} \text{ V}, \qquad t > 0$$

Example 9.3

The circuit below reduces to a simple parallel *RLC* circuit after $t = 0$. Determine an expression for the resistor current i_R valid for all time.







Equivalent circuit for $t = 0^-$.

Equivalent circuit for $t = 0^+$.

Example 9.3 (Cont.)

$R = 30\ \text{k}\Omega$ $\quad L = 12\ \text{mH}$ $\quad C = 2\ \text{pF}$

$\alpha = 8.333 \times 10^6\ \text{s}^{-1}$ $\quad \omega_0 = 6.455 \times 10^6\ \text{rad/s}$ $\quad \alpha > \omega_0$

The circuit is overdamped

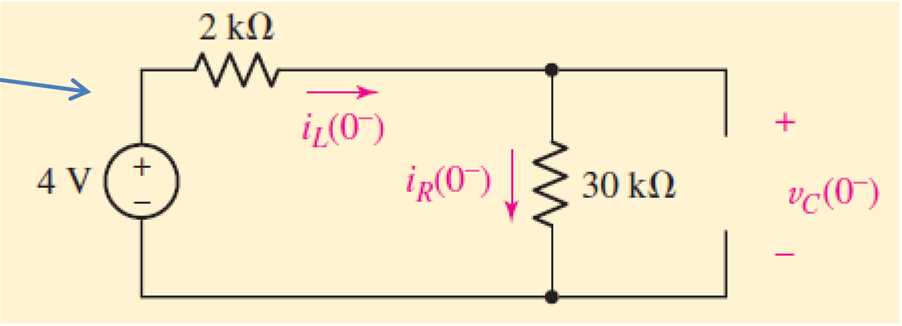
$$i_R(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, \qquad t > 0$$

$s_1 = -3.063 \times 10^6\ \text{s}^{-1}$ $\quad s_2 = -13.60 \times 10^6\ \text{s}^{-1}$

The circuit at t = 0⁻

$$i_L(0^-) = i_R(0^-) = 4/32 \times 10^3 = 125\ \mu\text{A}$$

$$v_C(0^-) = 4 \times 30/32 = 3.75\ \text{V}.$$

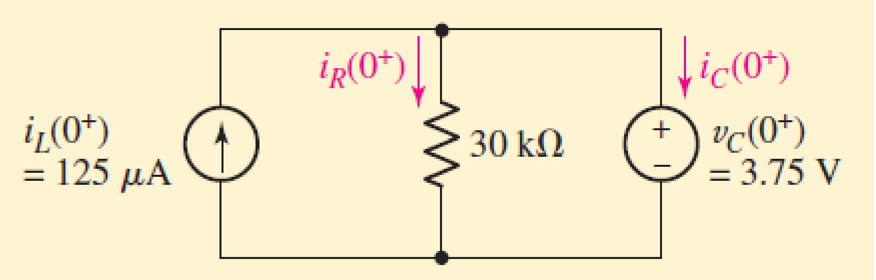


Equivalent circuit for *t = 0⁻*.

Example 9.3 (Cont.)

The circuit at t = 0^+

$i_L(0^+) = 125\ \mu\text{A}$ and $v_C(0^+) = 3.75\ \text{V}.$



Equivalent circuit for $t = 0^+$.

We can calculate i_R , $v_R = v_C$

$i_R(0^+) = 3.75/30 \times 10^3 = 125\ \mu\text{A}.$

$i_R(0) = A_1 + A_2 = 125 \times 10^{-6}$ → first equation

to find the second initial condition

$$i_C = C\frac{dv_C}{dt} = (2 \times 10^{-12})\underbrace{(30 \times 10^3)(A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t})}_{dv_R/dt,\ v_R = v_C}$$

By KCL,

$$i_C(0^+) = i_L(0^+) - i_R(0^+) = 0$$

Thus,

$-(2 \times 10^{-12})(30 \times 10^3)(3.063 \times 10^6 A_1 + 13.60 \times 10^6 A_2) = 0$ → second equation

$$i_R = \begin{cases} 125\ \mu\text{A} & t < 0 \\ 161.3e^{-3.063\times10^6 t} - 36.34e^{-13.6\times10^6 t}\ \mu\text{A} & t > 0 \end{cases}$$

RLC Solution: Critically Damped

$$x(t) = X_1 e^{s_1 t} + X_2 e^{s_2 t} + X_3 \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\,2}}$$

The solution is said to be **critically damped** if $\alpha = \omega_0$

series: $\dfrac{R}{2L} = \dfrac{1}{\sqrt{LC}}$

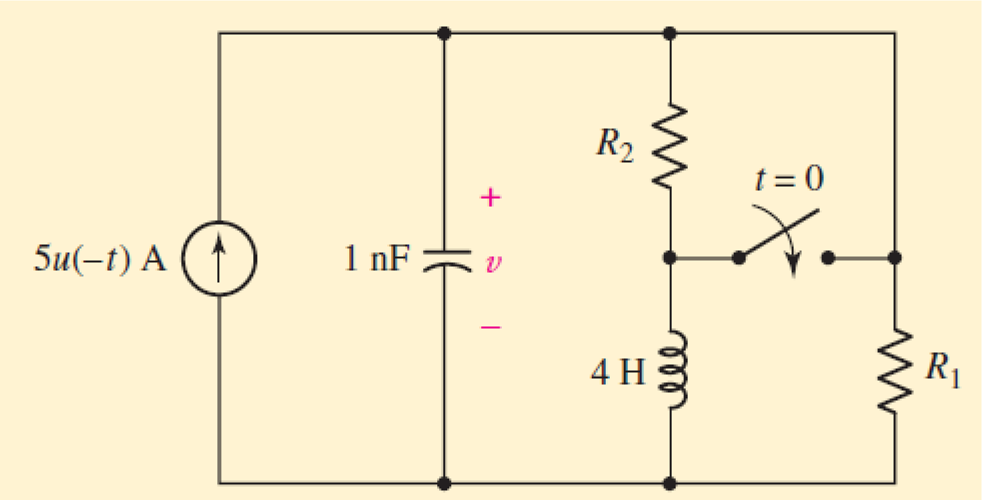
parallel: $\dfrac{1}{2RC} = \dfrac{1}{\sqrt{LC}}$

…in which case the solution is

$$x(t) = e^{-\alpha t}\left(X_1 t + X_2\right) + X_3$$

Example 9.5

Select a value for R_1 such that the circuit will be characterized by a critically damped response for $t > 0$, and a value for R_2 such that $v(0) = 2$ V.



We note that at $t = 0^-$, the current source is on, and the inductor can be treated as a short circuit. Thus, $v(0^-)$ appears across R_2, and is given by

$$v(0^-) = 5R_2$$

and a value of 400 mΩ should be selected for R_2 to obtain $v(0) = 2$ V.

Example 9.5 (Cont.)

After the switch is thrown, the current source has turned itself off and R_2 is shorted. We are left with a parallel *RLC* circuit comprised of R_1, a 4 H inductor, and a 1 nF capacitor.

We may now calculate (for $t > 0$)

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10^{-9} R_1}$$

and

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times 10^{-9}}} = 15{,}810 \text{ rad/s}$$

Therefore, to establish a critically damped response in the circuit for $t > 0$, we need to set $R_1 = 31.63\ \text{k}\Omega$. (*Note: since we have rounded to four significant figures, the pedantic can rightly argue that this is still not* ***exactly*** *a critically damped response—a difficult situation to create.*)

RLC Solution: Under-damped

$$x(t) = X_1 e^{s_1 t} + X_2 e^{s_2 t} + X_3 \qquad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^{\,2}}$$

The solution is said to be **underdamped** if $\alpha < \omega_0$

series: $\dfrac{R}{2L} < \dfrac{1}{\sqrt{LC}}$

parallel: $\dfrac{1}{2RC} < \dfrac{1}{\sqrt{LC}}$

…in which case the solution is

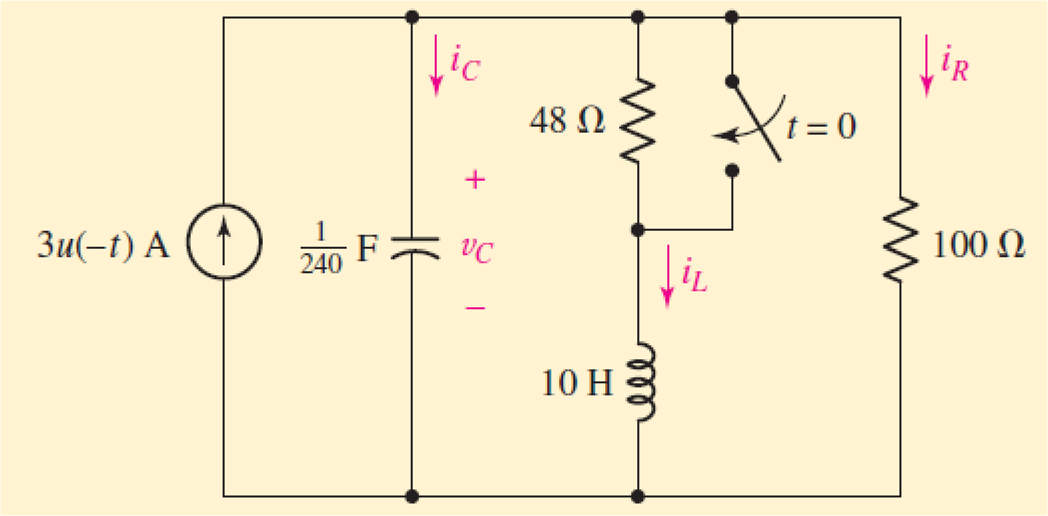
$$x(t) = e^{-\alpha t}\left[X_1 \cos(\omega_d t) + X_2 \sin(\omega_d t)\right] + X_3$$

$$\omega_d = \sqrt{\omega_0^{\,2} - \alpha^2}$$

natural resonant frequency

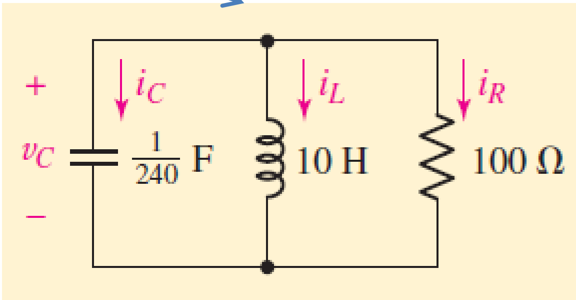
Example 9.6

Determine $i_L(t)$ for the circuit, and plot the waveform.



Example 9.6 (Cont.)

At $t = 0$, both the 3 A source and the 48 Ω resistor are removed.



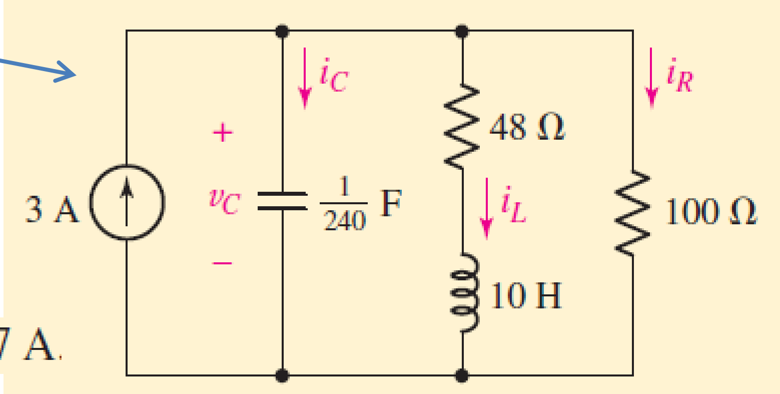
$\alpha = 1.2\ \text{s}^{-1}$ $\omega_0 = 4.899$ rad/s.

Since $\alpha < \omega_0$, the circuit is *underdamped*,

$$i_L(t) = e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t) \quad [28]$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 4.750 \text{ rad/s.}$$

the circuit as it exists at $t = 0^-$



$v_C(0^-) = 97.30$ V and $i_L(0^-) = 2.027$ A.

$v_C(0^+) = 97.30$ V $i_L(0^+) = 2.027$ A

Substituting $i_L(0) = 2.027$ into Eq. [28] yields $B_1 = 2.027$ A.

Example 9.6 (Cont.)

for the other constant

$$\frac{di_L}{dt} = e^{-\alpha t}(-B_1\omega_d \sin\omega_d t + B_2\omega_d \cos\omega_d t) - \alpha e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$$

we know that

$v_L(t) = L(di_L/dt)$

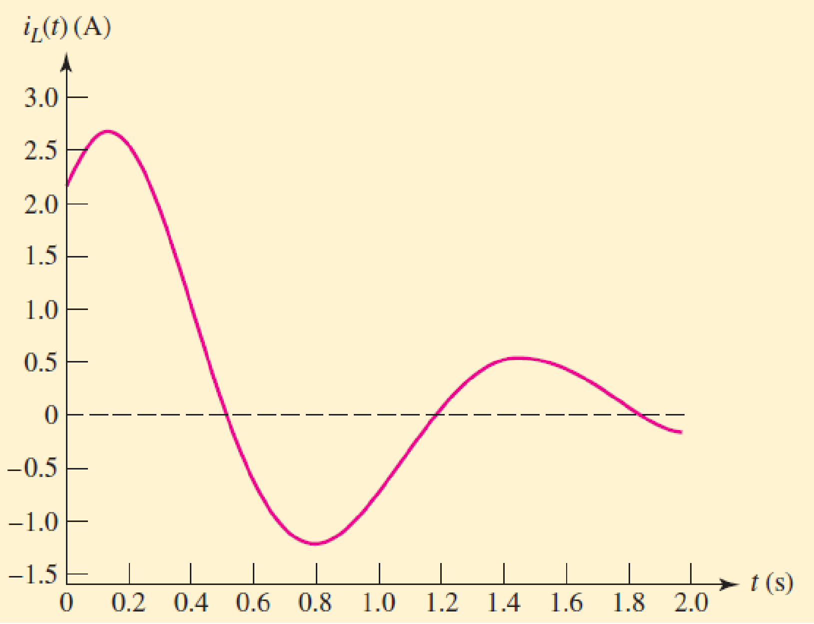
$v_L(0^+) = v_C(0^+) = 97.3 \text{ V}$

if we multiply Eq. [29] by *L = 10 H* and setting *t = 0*

$$v_L(0) = 10(B_2\omega_d) - 10\alpha B_1 = 97.3$$

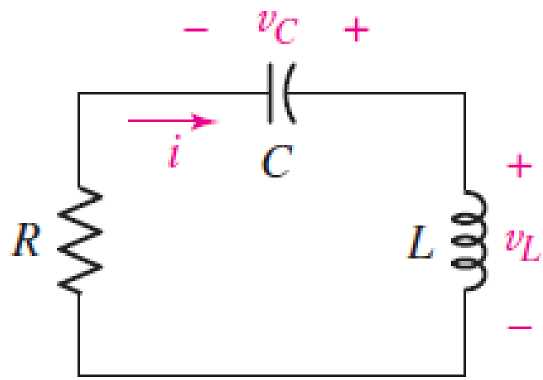
Solving, $B_2 = 2.561$ A, so that

$$i_L = e^{-1.2t}(2.027 \cos 4.75t + 2.561 \sin 4.75t)$$



Example 9.7

Given the series RLC circuit in which L = 1 H, R = 2 kΩ, C = 1/401 μF, $i(0)$ = 2 mA, and $v_C(0)$ = 2 V, find and sketch $i(t)$, $t > 0$.



Example 9.7 (Cont.)

We find that $\alpha = R/2L = 1000\ \text{s}^{-1}$ and $\omega_0 = 1/\sqrt{LC} = 20{,}025$ rad/s. This indicates an *underdamped* response; we therefore calculate the value of ω_d and obtain 20,000 rad/s. Except for the evaluation of the two arbitrary constants, the response is now known:

$$i(t) = e^{-1000t}(B_1 \cos 20{,}000t + B_2 \sin 20{,}000t)$$

Since we know that $i(0) = 2$ mA, we may substitute this value into our equation for $i(t)$ to obtain

$$B_1 = 0.002\ \text{A}$$

and thus

$$i(t) = e^{-1000t}(0.002 \cos 20{,}000t + B_2 \sin 20{,}000t) \qquad \text{A}$$

Example 9.7 (Cont.)

The remaining initial condition must be applied to the derivative; thus,

$$\frac{di}{dt} = e^{-1000t}(-40 \sin 20{,}000t + 20{,}000 B_2 \cos 20{,}000t$$
$$- 2 \cos 20{,}000t - 1000 B_2 \sin 20{,}000t)$$

and

$$\left.\frac{di}{dt}\right|_{t=0} = 20{,}000 B_2 - 2 = \frac{v_L(0)}{L}$$
$$= \frac{v_C(0) - Ri(0)}{L}$$
$$= \frac{2 - 2000(0.002)}{1} = -2 \text{ A/s}$$

so that

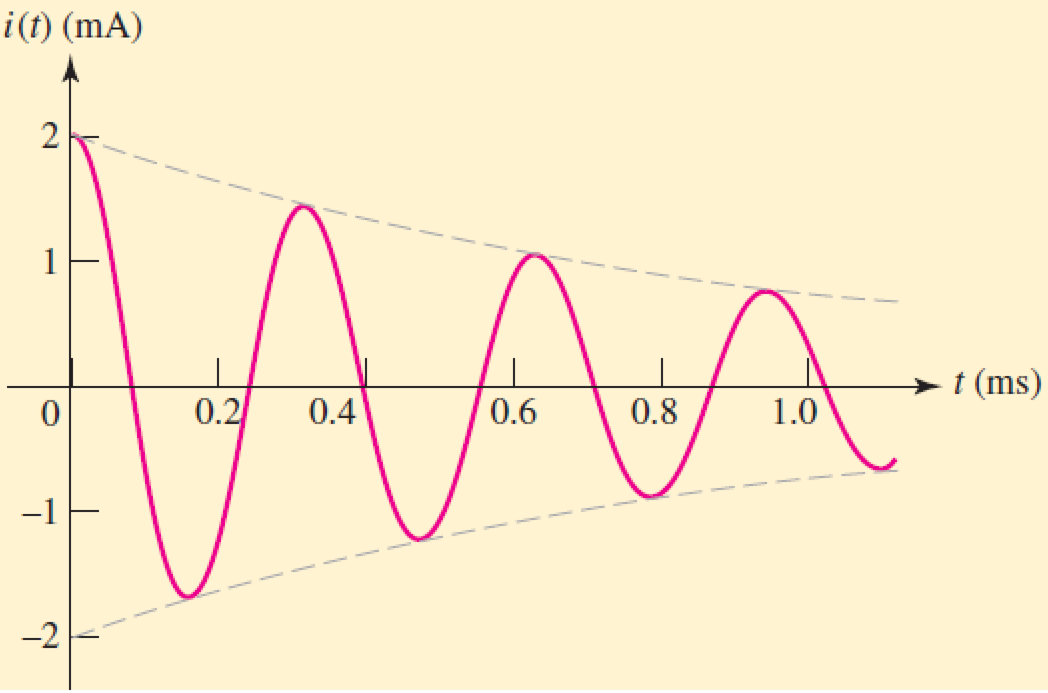
$$B_2 = 0$$

Example 9.7 (Cont.)

The desired response is therefore

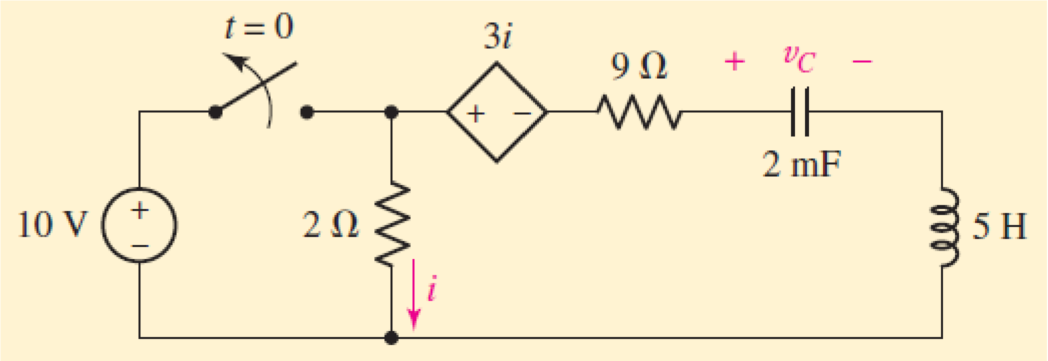
$$i(t) = 2e^{-1000t} \cos 20{,}000t \qquad \text{mA}$$

A good sketch may be made by first drawing in the two portions of the exponential *envelope*, $2e^{-1000t}$ and $-2e^{-1000t}$ mA, as shown by the broken lines in Fig. 9.23. The location of the quarter-cycle points of the sinusoidal wave at $20{,}000t = 0, \pi/2, \pi$, etc., or $t = 0.07854k$ ms, $k = 0, 1, 2, \ldots$, by light marks on the time axis then permits the oscillatory curve to be sketched in quickly.



Example 9.8

Find an expression for $v_C(t)$ in the circuit, valid for $t > 0$.

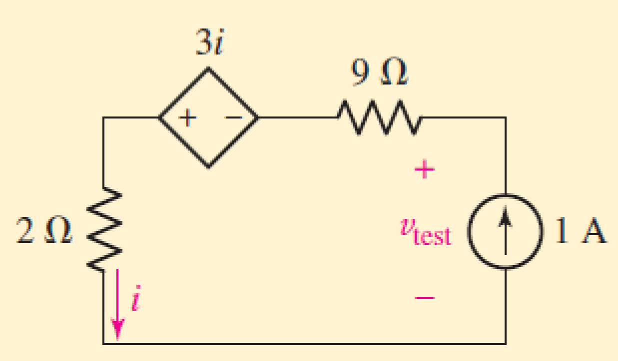


Example 9.8 (Cont.)

Thévenin equivalent resistance

$$v_{\text{test}} = 11i - 3i = 8i = 8(1) = 8 \text{ V}$$

$$R_{\text{eq}} = 8\ \Omega,$$



$\alpha = R/2L = 0.8\ \text{s}^{-1}$ and $\omega_0 = 1/\sqrt{LC} = 10$ rad/s, we expect an underdamped response

$$v_C(t) = e^{-0.8t}(B_1 \cos 9.968t + B_2 \sin 9.968t)$$

In considering the circuit at $t = 0^-$, we note that $i_L(0^-) = 0$ due to the presence of the capacitor. By Ohm's law, $i(0^-) = 5$ A, so

$$v_C(0^+) = v_C(0^-) = 10 - 3i = 10 - 15 = -5 \text{ V}$$

This last condition substituted into Eq. [32] yields $B_1 = -5$ V. Taking the derivative of Eq. [32] and evaluating at $t = 0$ yield

$$\left.\frac{dv_C}{dt}\right|_{t=0} = -0.8B_1 + 9.968B_2 = 4 + 9.968B_2$$

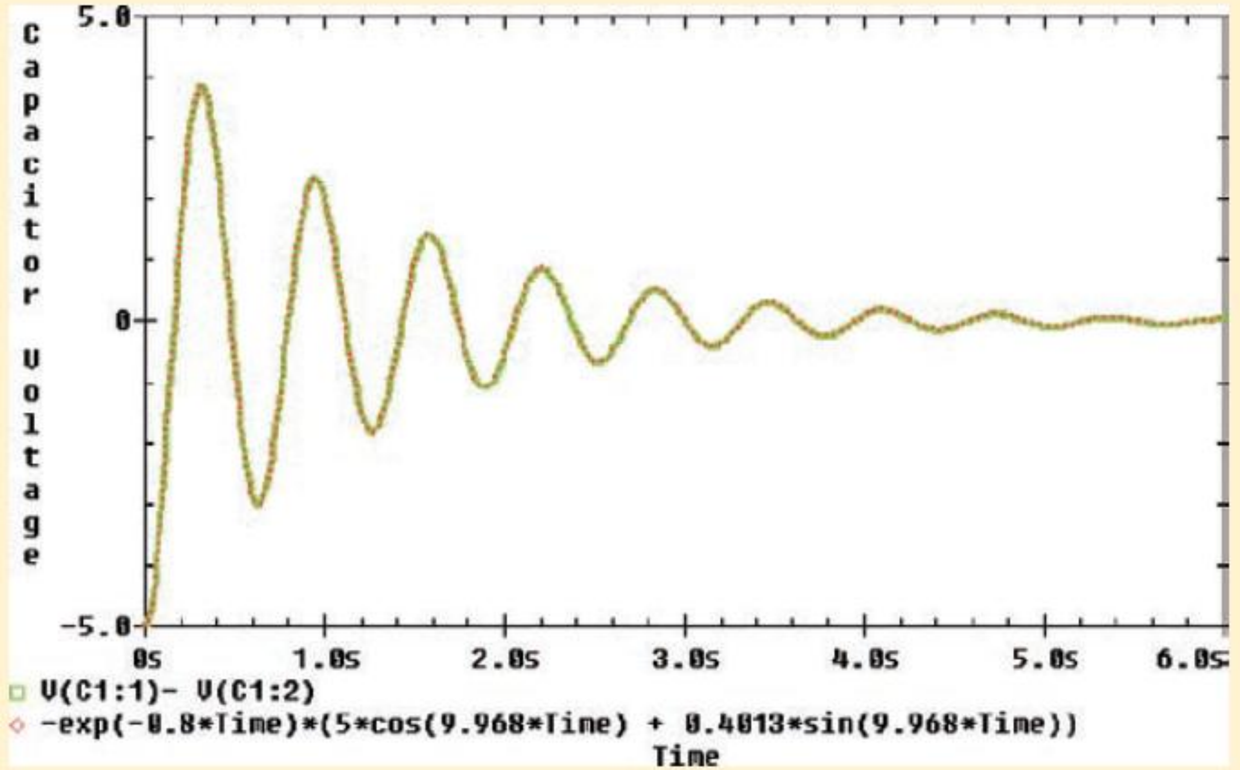
Example 9.8 (Cont.)

We see from figure that

$$i = -C\frac{dv_C}{dt}$$

Thus, making use of the fact that $i(0^+) = i_L(0^-) = 0$ in Eq. [33] yields $B_2 = -0.4013$ V, and we may write

$$v_C(t) = -e^{-0.8t}(5\cos 9.968t + 0.4013\sin 9.968t) \qquad \text{V} \qquad t > 0$$



Summary of Relevant Equations for Source-Free RLC Circuits

Summary of Relevant Equations for Source-Free *RLC* Circuits

Type	Condition	Criteria	α	ω_0	Response
Parallel Series	Overdamped	$\alpha > \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$A_1 e^{s_1 t} + A_2 e^{s_2 t}$, where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$
Parallel Series	Critically damped	$\alpha = \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(A_1 t + A_2)$
Parallel Series	Underdamped	$\alpha < \omega_0$	$\frac{1}{2RC}$ $\frac{R}{2L}$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$, where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

THE COMPLETE RESPONSE OF THE *RLC* CIRCUIT

$$v(t) = v_f(t) + v_n(t)$$

$$v_f(t) = V_f \qquad v_n(t) = Ae^{s_1 t} + Be^{s_2 t}$$

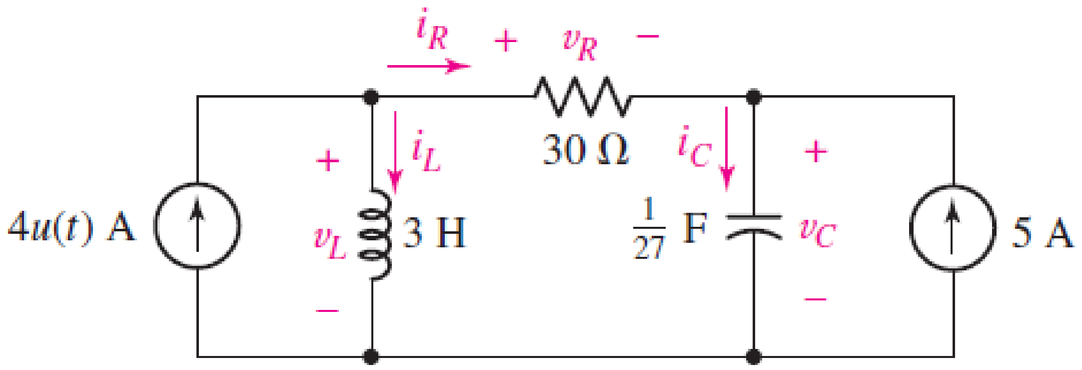
We now consider those *RLC* circuits in which dc sources are switched into the network and produce forced responses that do not necessarily vanish as time becomes infinite.

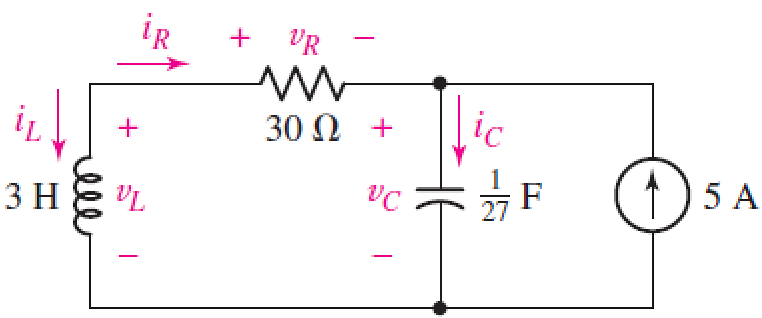
The general solution is obtained by the same procedure that was followed for *RL* and *RC* circuits. The basic steps are (not necessarily in this order) as follows:

1. Determine the initial conditions.
2. Obtain a numerical value for the forced response.
3. Write the appropriate form of the natural response with the necessary number of arbitrary constants.
4. Add the forced response and natural response to form the complete response.
5. Evaluate the response and its derivative at $t = 0$, and employ the initial conditions to solve for the values of the unknown constants.

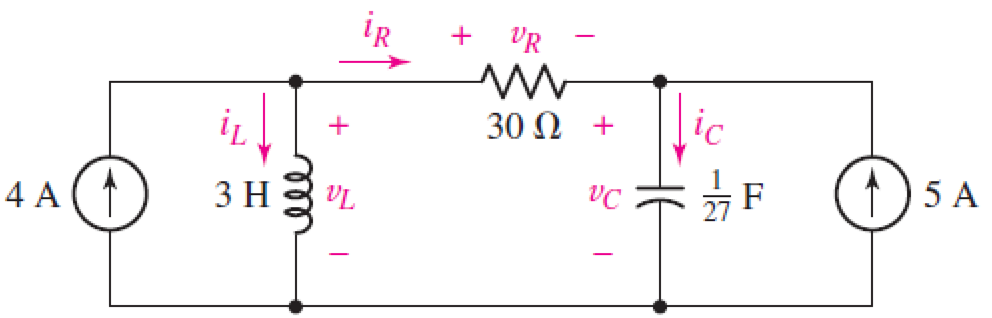
Example 9.9

There are three passive elements in the, and a voltage and a current are defined for each. Find the values of these six quantities at both t = 0^- and t = 0^+.





The circuit for $t = 0^-$.



The circuit for $t > 0$

Example 9.9 (Cont.)

Our object is to find the value of each current and voltage at both $t = 0^-$ and $t = 0^+$. Once these quantities are known, the initial values of the derivatives may be found easily.

1. $\boldsymbol{t = 0^-}$ At $t = 0^-$, only the right-hand current source is active as depicted in Fig. 9.28*b*. The circuit is assumed to have been in this state forever, so all currents and voltages are constant. Thus, a dc current through the inductor requires zero voltage across it:

$$v_L(0^-) = 0$$

and a dc voltage across the capacitor ($-v_R$) requires zero current through it:

$$i_C(0^-) = 0$$

We next apply Kirchhoff's current law to the right-hand node to obtain

$$i_R(0^-) = -5 \text{ A}$$

which also yields

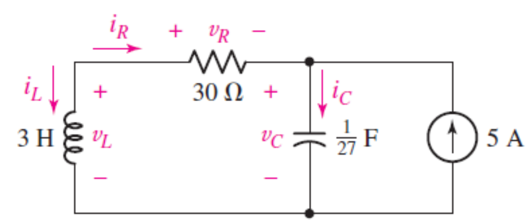
$$v_R(0^-) = -150 \text{ V}$$

We may now use Kirchhoff's voltage law around the left-hand mesh, finding

$$v_C(0^-) = 150 \text{ V}$$

while KCL enables us to find the inductor current,

$$i_L(0^-) = 5 \text{ A}$$



The circuit for $t = 0^-$.

Example 9.9 (Cont.)

2. $\boldsymbol{t = 0^+}$ During the interval from $t = 0^-$ to $t = 0^+$, the left-hand current source becomes active and many of the voltage and current values at $t = 0^-$ will change abruptly. The corresponding circuit is shown in Fig. 9.28c. However, we should *begin by focusing our attention on those quantities which cannot change, namely, the inductor current and the capacitor voltage*. Both of these must remain constant during the switching interval. Thus,

$$i_L(0^+) = 5 \text{ A} \qquad \text{and} \qquad v_C(0^+) = 150 \text{ V}$$

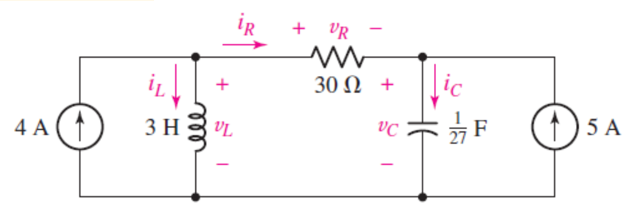
Since two currents are now known at the left node, we next obtain

$$i_R(0^+) = -1 \text{ A} \qquad \text{and} \qquad v_R(0^+) = -30 \text{ V}$$

so that

$$i_C(0^+) = 4 \text{ A} \qquad \text{and} \qquad v_L(0^+) = 120 \text{ V}$$

and we have our six initial values at $t = 0^-$ and six more at $t = 0^+$. Among these last six values, only the capacitor voltage and the inductor current are unchanged from the $t = 0^-$ values.



The circuit for $t > 0$

Example 9.10

Complete the determination of the initial conditions in the circuit, by finding values at $t = 0^+$ for the first derivatives of the three voltage and three current variables defined on the circuit diagram.

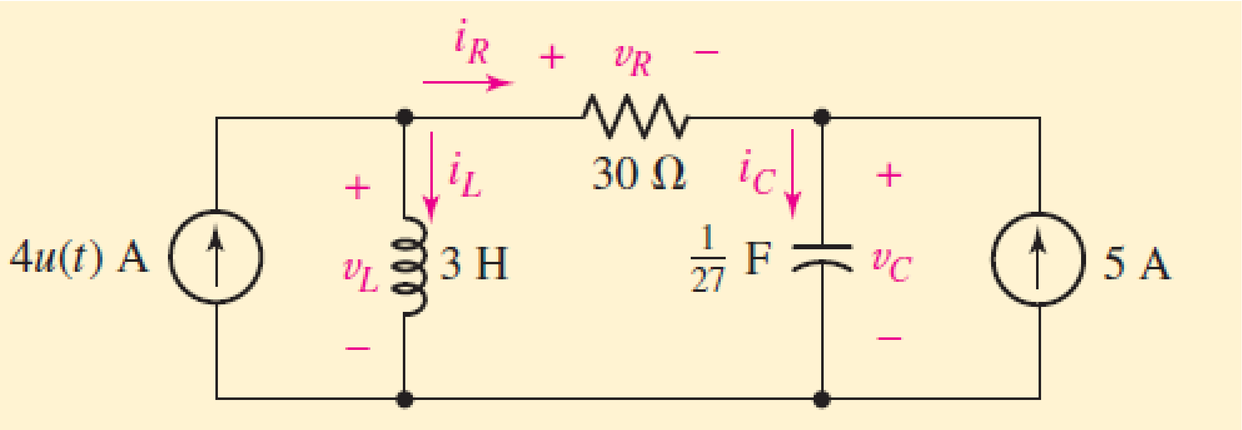


Figure 9.31

Example 9.10 (Cont.)

We begin with the two energy storage elements. For the inductor,

$$v_L = L\frac{di_L}{dt}$$

and, specifically,

$$v_L(0^+) = L\left.\frac{di_L}{dt}\right|_{t=0^+}$$

Thus,

$$\left.\frac{di_L}{dt}\right|_{t=0^+} = \frac{v_L(0^+)}{L} = \frac{120}{3} = 40 \text{ A/s}$$

Similarly,

$$\left.\frac{dv_C}{dt}\right|_{t=0^+} = \frac{i_C(0^+)}{C} = \frac{4}{1/27} = 108 \text{ V/s}$$

Example 9.10 (Cont.)

The other four derivatives may be determined by realizing that KCL and KVL are both satisfied by the derivatives also. For example, at the left-hand node in Fig. 9.31,

$$4 - i_L - i_R = 0 \qquad t > 0$$

and thus,

$$0 - \frac{di_L}{dt} - \frac{di_R}{dt} = 0 \qquad t > 0$$

and therefore,

$$\left.\frac{di_R}{dt}\right|_{t=0^+} = -40 \text{ A/s}$$

The three remaining initial values of the derivatives are found to be

$$\left.\frac{dv_R}{dt}\right|_{t=0^+} = -1200 \text{ V/s}$$

$$\left.\frac{dv_L}{dt}\right|_{t=0^+} = -1092 \text{ V/s}$$

and

$$\left.\frac{di_C}{dt}\right|_{t=0^+} = -40 \text{ A/s}$$