BME2301 - Circuit Theory

The Instructors:

Dr. Görkem SERBES (C317)

gserbes@yildiz.edu.tr

https://avesis.yildiz.edu.tr/gserbes/

Lab Assistants:

Nihat AKKAN

nakkan@yildiz.edu.tr

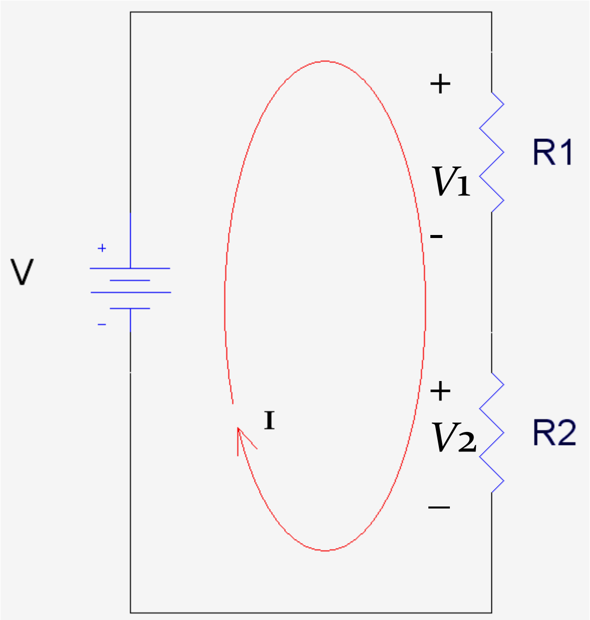
https://avesis.yildiz.edu.tr/nakkan

Objectives of the Lecture

- Explain mathematically how resistors in series are combined and their equivalent resistance.
- Explain mathematically how resistors in parallel are combined and their equivalent resistance.
- Rewrite the equations for conductances.
- Explain mathematically how a voltage that is applied to resistors in series is distributed among the resistors.
- Explain mathematically how a current that enters the a node shared by resistors in parallel is distributed among the resistors.
- Describe the equations that relate the resistances in a Wye (Y) and Delta (Δ) resistor network.
- Describe a bridge circuit in terms of wye and delta sub-circuits.

Voltage Division

- All resistors in series share the same current



  – From KVL and Ohm's Law :

$$0 = -V + V_1 + V_2$$
$$V = I \times R1 + I \times R2$$
$$V = I \times (R1 + R2) = I \times R_{eq}$$
$$R_{eq} = R1 + R2 = V/I \qquad I = V/R_{eq}$$

$$V_1 = I \times R1 = \frac{V}{R_{eq}} \times R1 = \frac{R1}{R1 + R2} \times V$$
$$V_2 = I \times R2 = \frac{V}{R_{eq}} \times R2 = \frac{R2}{R1 + R2} \times V$$

  – the source voltage V is divided among the resistors in direct proportion to their resistances;
    - the larger the resistance, the larger the voltage drop.
  – This is called the principle of voltage division, and the circuit is called a voltage divider.

Voltage Division

- In general, if a voltage divider has N resistors (R_1, R_2, . . . , R_N) in series with the source voltage V_{total}, the nth resistor (R_n) will have a voltage drop of

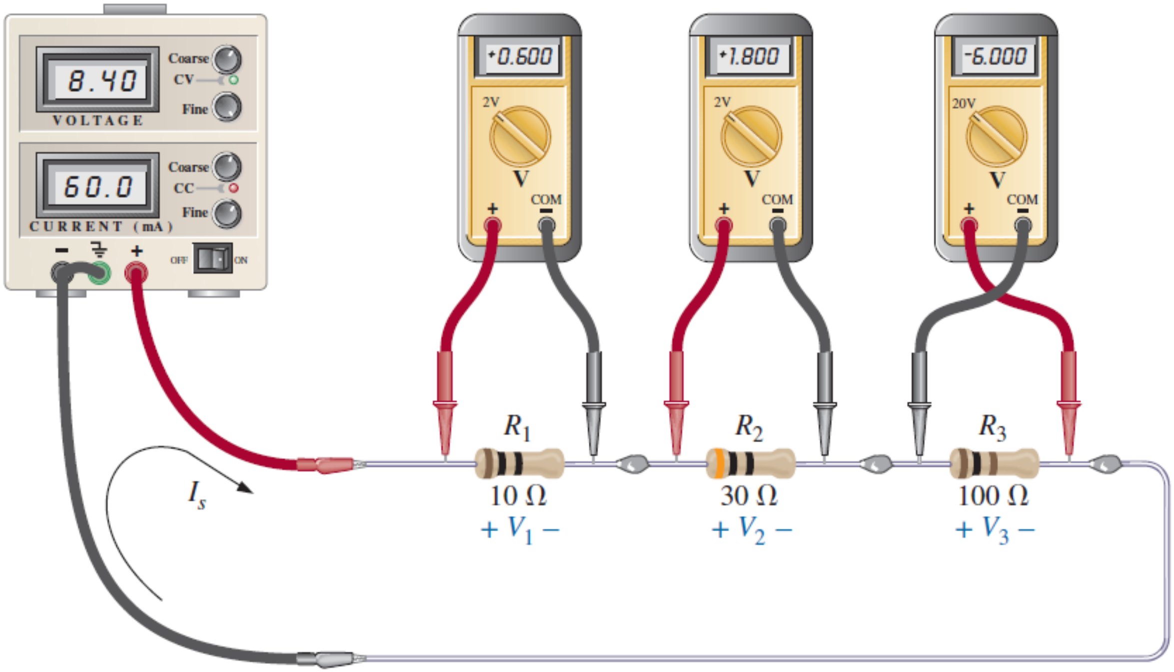
$$V_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} \times V_{total} = \left[\frac{R_n}{R_{eq}}\right] \times V_{total}$$

where V_{total} is the total of the voltages applied across the resistors and R_{eq} is equivalent series resistance.

- The percentage of the total voltage associated with a particular resistor is equal to the percentage that that resistor contributed to the equivalent resistance, R_{eq}.
  - The largest value resistor has the largest voltage.

Voltage Division

- Using voltmeters to measure the voltages across the resistors

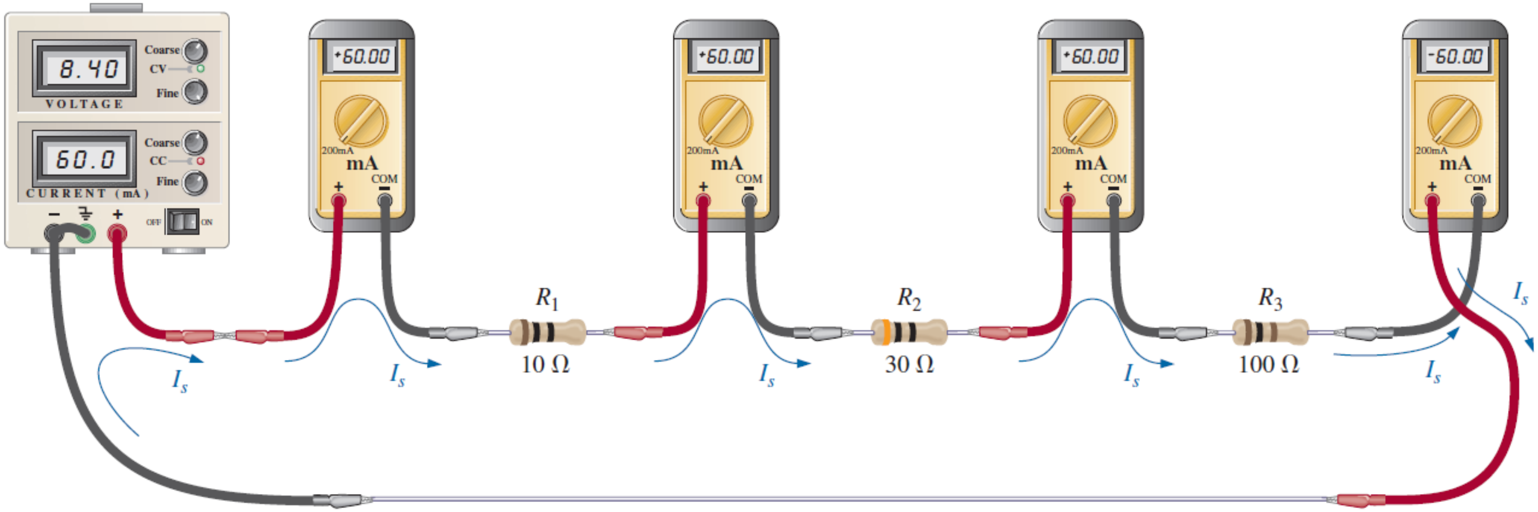


- The positive (normally red) lead of the voltmeter is connected to the point of higher potential (positive sign), with the negative (normally black) lead of the voltmeter connected to the point of lower potential (negative sign) for V_1 and V_2.

- The result is a positive reading on the display.
- If the leads were reversed, the magnitude would remain the same, but a negative sign would appear as shown for V_3.

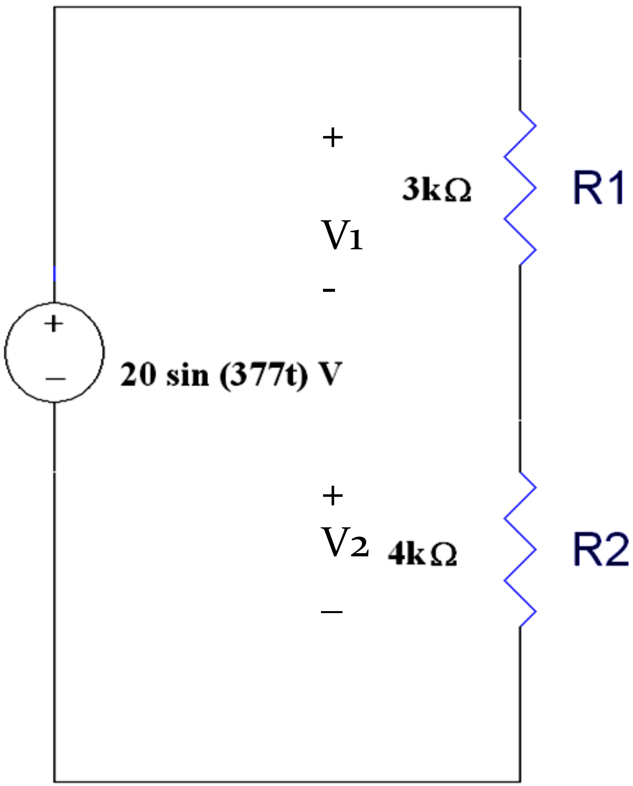
Voltage Division

- Measuring the current throughout the series circuit.



- If each ampermeter is to provide a positive reading, the connection must be made such that conventional current enters the positive terminal of the meter and leaves the negative terminal.
  - The ampermeter to the right of R_3 connected in the reverse manner, resulting in a negative sign for the current.

Example 01



- Find the V_1, the voltage across $R1$, and V_2, the voltage across $R2$

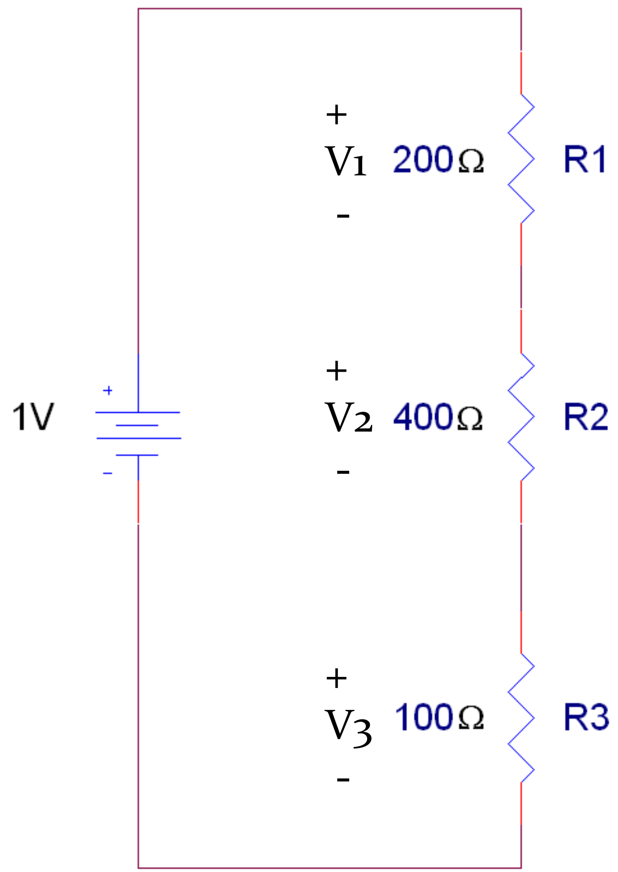
$$V_1 = [R_1/(R_1+R_2)]V_{total}$$
$$V_1 = [3k\Omega/(3k\Omega+4k\Omega)][20V\sin(377t)]$$
$$V_1 = 8.57V\sin(377t)$$

$$V_2 = [R_2/(R_1+R_2)]V_{total}$$
$$V_2 = [4k\Omega/(3k\Omega+4k\Omega)][20V\sin(377t)]$$
$$V_2 = 11.4V\sin(377t)$$

– Check: $V_1 + V_2$ should equal V_{total}
  - 8.57sin(377t) + 11.4sin(377t) = 20sin(377t) V

Example 02

- Find the voltages listed in the circuit below.



$$R_{eq} = 200\Omega + 400\Omega + 100\Omega$$
$$R_{eq} = 700\Omega$$

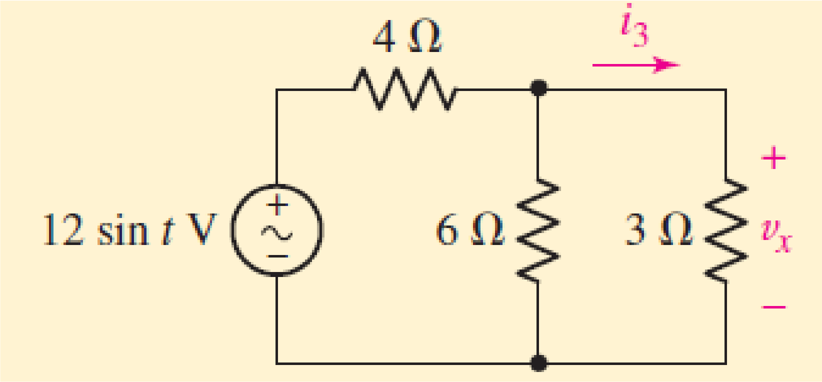
$$V_1 = [200\Omega / 700\Omega](1V)$$
$$V_1 = 0.286V$$

$$V_2 = [400\Omega / 700\Omega](1V)$$
$$V_2 = 0.571V$$

$$V_3 = [100\Omega / 700\Omega](1V)$$
$$V_3 = 0.143V$$

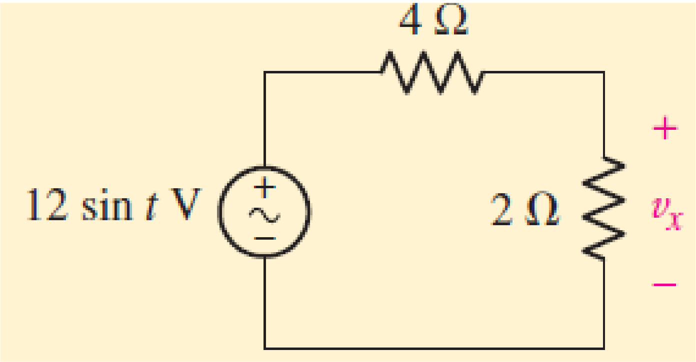
– Check: $V_1 + V_2 + V_3 = 1\text{V}$

Example 03



- Determine v_x in this circuit:

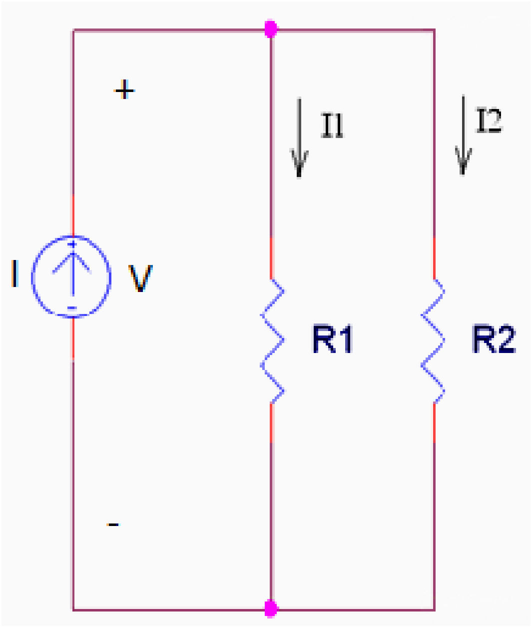
  $6\ \Omega \parallel 3\ \Omega = 2\ \Omega$



$$v_x = (12 \sin t)\frac{2}{4+2} = 4 \sin t$$

Symbol for Parallel Resistors

- To make writing equations simpler, we use a symbol to indicate that a certain set of resistors are in parallel.



  – Here, we would write

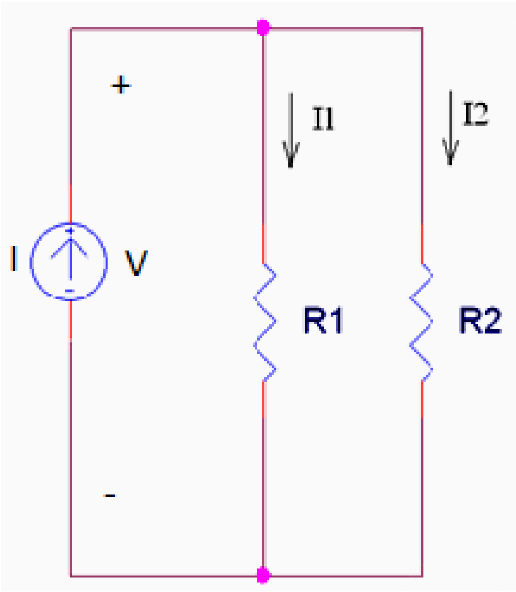
$$R1 \| R2$$

  to show that R1 is in parallel with R2.

  – This also means that we should use the equation for equivalent resistance if this symbol is included in a mathematical equation.

Current Division

- All resistors in parallel share the same voltage



  - From KCL and Ohm's Law :

$$0 = -I + I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} = V \times \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$I = \frac{V}{R_{eq}} = \frac{V}{R_1 \| R_2} \qquad V = I \times R_{eq}$$

$$R_{eq} = R_1 \| R_2 = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} = \frac{I \times R_{eq}}{R_1} = \frac{R_1 \| R_2}{R_1} \times I = \frac{R_2}{R_1 + R_2} \times I$$

$$I_2 = \frac{V}{R_2} = \frac{I \times R_{eq}}{R_2} = \frac{R_1 \| R_2}{R_2} \times I = \frac{R_1}{R_1 + R_2} \times I$$

  - The total current *I* is shared by the resistors in inverse proportion to their resistances
    - the smaller the resistance, the larger the current flow.
  - This is called the principle of current division, and the circuit is called a current divider.

Current Division

- In general, if a current divider has N resistors (R_1, R_2, . . . , R_N) in parallel with the source current I_{total}, the nth resistor (R_n) will have a current flow

$$I_n = \frac{1/R_n}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}} \times I_{total} = \left[\frac{R_{eq}}{R_n}\right] \times I_{total}$$

 where I_{total} is the total of the currents applied to the resistors and R_{eq} is equivalent parallel resistance.

- The percentage of the total current associated with a particular resistor is equal to the percentage that that resistor contributed to the equivalent resistance, R_{eq}.
  - The smallest value resistor has the largest current

Current Division

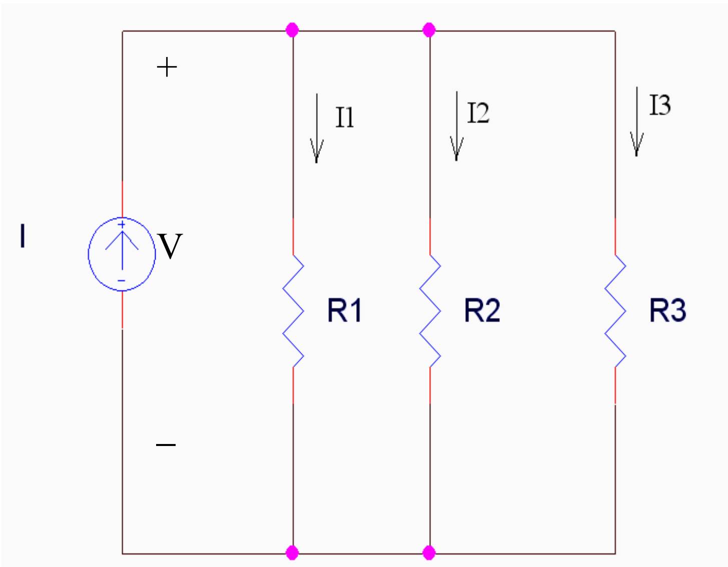
- If a current divider circuit with N resistors (having conductances $G_1, G_2, \ldots, G_N$) in parallel with the source current I_{total}, the nth resistor (with conductance G_n) will have a current flow

$$I_n = \frac{G_n}{G_1 + G_2 + \cdots + G_N} \times I_{total} = \left[\frac{G_n}{G_{eq}}\right] \times I_{total}$$

  where I_{total} is the total of the currents applied to the resistors and G_{eq} is equivalent parallel conductance.
- The percentage of the total current associated with a particular resistor is equal to the percentage that that resistor contributed to the equivalent conductance, G_{eq}.
  - The largest conductance value resistor has the largest current

Current Division

- For three resistors parallel circuit, current in branches:



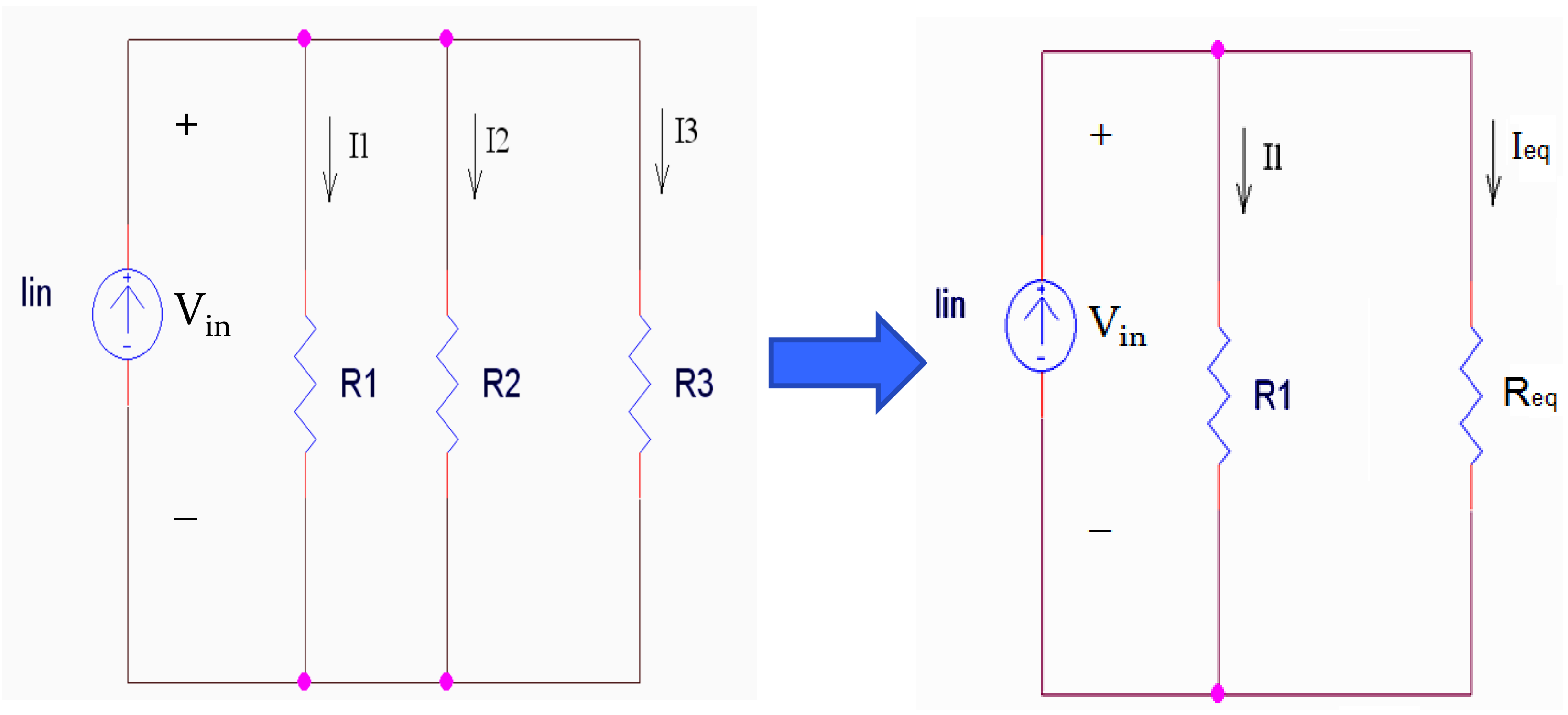
$$I_1 = \frac{R_2 \| R_3}{R_1 + R_2 \| R_3} I_{in}$$

$$I_2 = \frac{R_1 \| R_3}{R_2 + R_1 \| R_3} I_{in}$$

$$I_3 = \frac{R_1 \| R_2}{R_3 + R_1 \| R_2} I_{in}$$

- Alternatively, you can reduce the number of resistors in parallel from 3 to 2 using an equivalent resistor.
- If you want to solve for current I_1, then find an equivalent resistor for R_2 in parallel with R_3.

Current Division



$$\text{where } R_{eq} = R_2 \| R_3 = \frac{R_2 R_3}{R_2 + R_3} \quad \text{and } I_1 = \frac{R_{eq}}{R_1 + R_{eq}} I_{in}$$

Current Division

The current associated with one resistor R_1 in parallel with one other resistor is:

$$I_1 = \left[\frac{R_2}{R_1 + R_2} \right] I_{total}$$

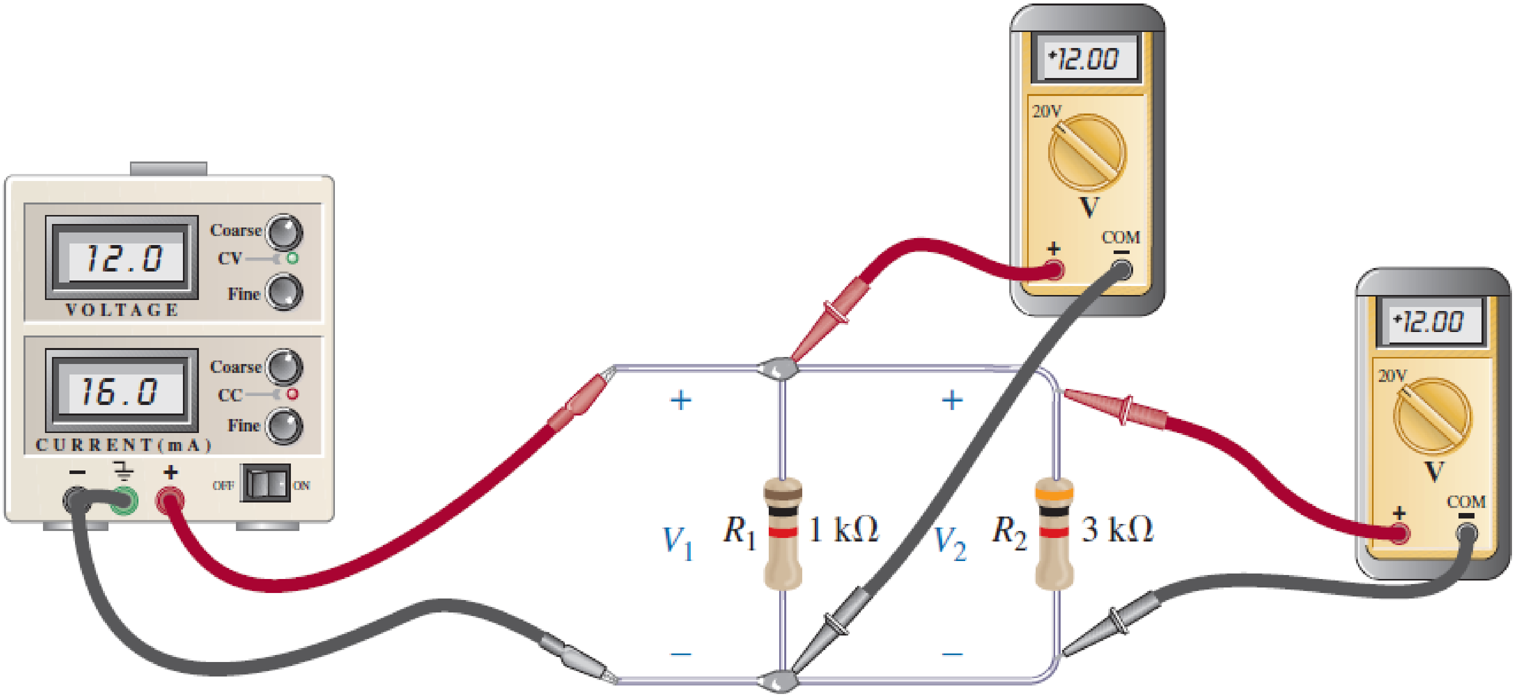
The current associated with one resistor R_m in parallel with two or more resistors is:

$$I_m = \left[\frac{R_{eq}}{R_m} \right] I_{total}$$

where I_{total} is the total of the currents entering the node shared by the resistors in parallel.

Resistors in Parallel

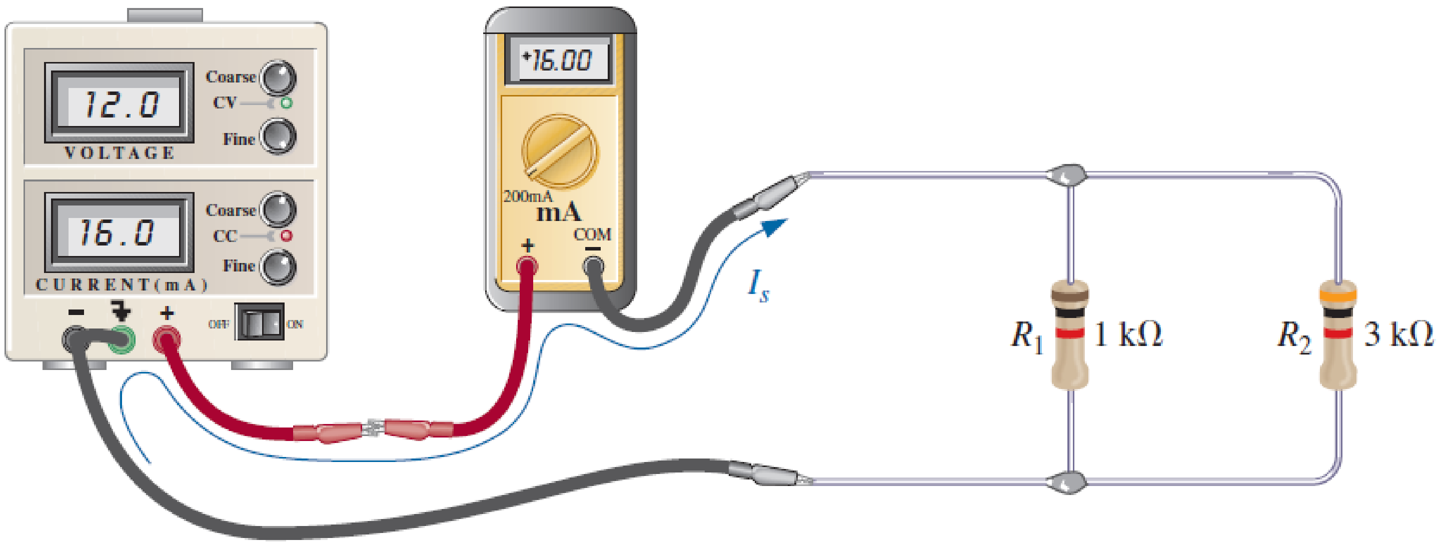
- Measuring the voltages of a parallel dc network



  – Note that the positive or red lead of each voltmeter is connected to the high (positive) side of the voltage across each resistor to obtain a positive reading.

Resistors in Parallel

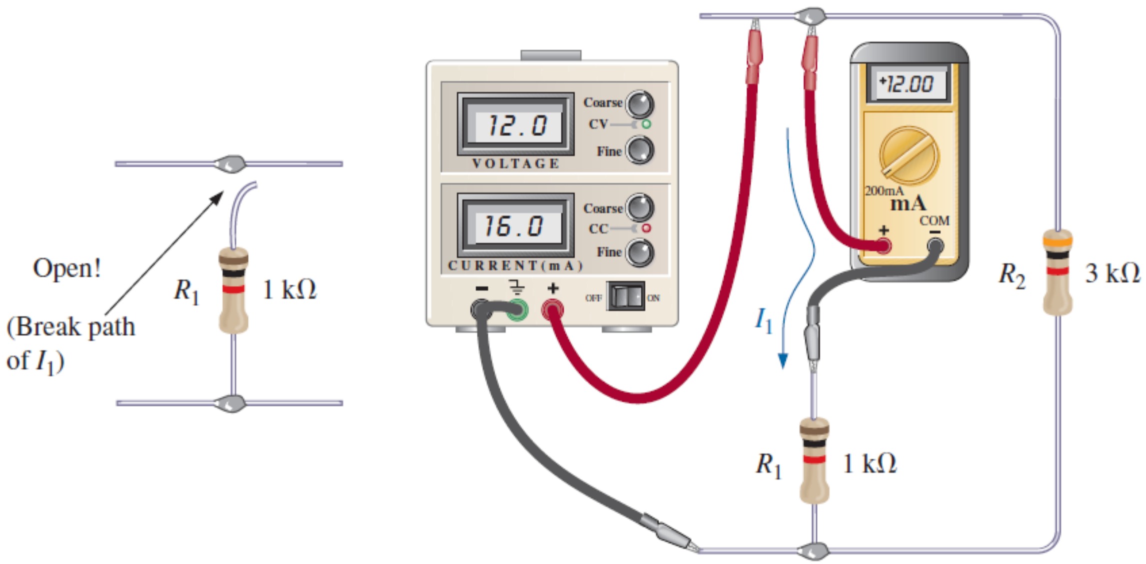
- Measuring the source current of a parallel network



  – The red or positive lead of the meter is connected so that the source current enters that lead and leaves the negative or black lead to ensure a positive reading.

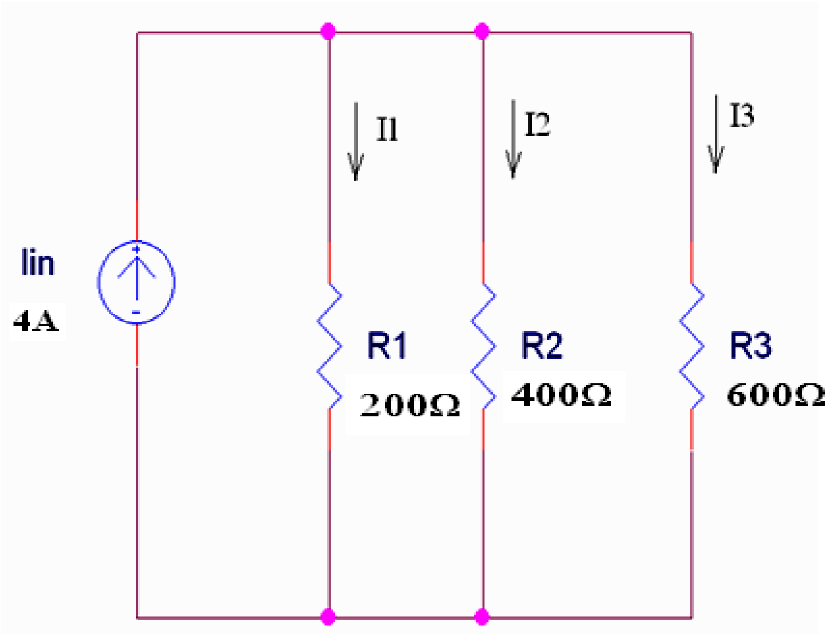
Resistors in Parallel

- Measuring the current through resistor R_1



  – resistor R_1 must be disconnected from the upper connection point to establish an open circuit.
    - The ampermeter is then inserted between the resulting terminals so that the current enters the positive or red terminal

Example 04



- Find currents I_1, I_2, and I_3 in the circuit

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2}}$$

$$= \frac{1}{\frac{1}{200} + \frac{1}{400} + \frac{1}{600}} = 109\ \Omega$$

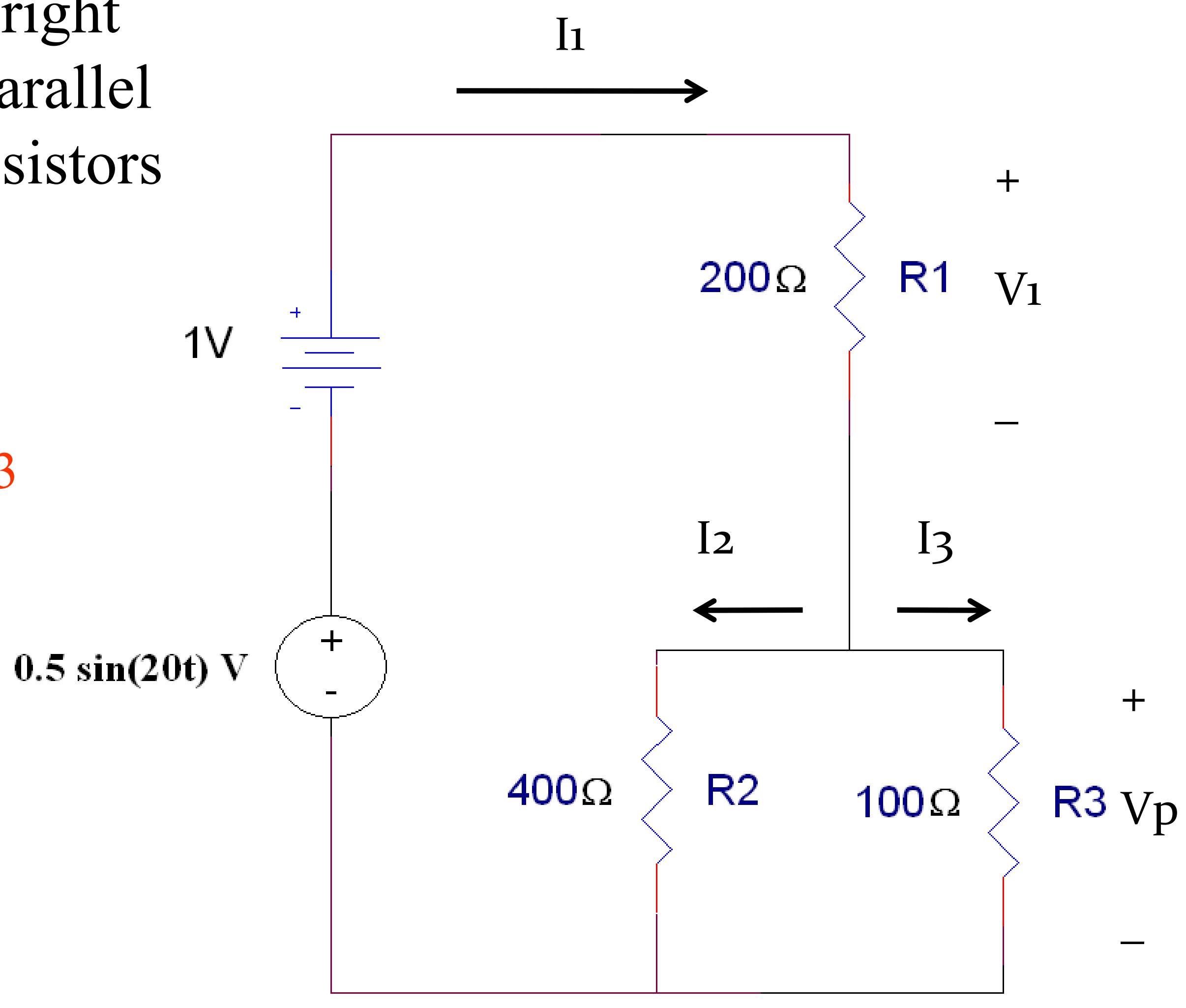
$$I_1 = \frac{R_{eq}}{R_1} \times I_{in} = \frac{109}{200} \times 4 = 2.18\ \text{A}$$

$$I_1 = \frac{R_{eq}}{R_2} \times I_{in} = \frac{109}{400} \times 4 = 1.09\ \text{A}$$

$$I_1 = \frac{R_{eq}}{R_3} \times I_{in} = \frac{109}{600} \times 4 = 0.727\ \text{A}$$

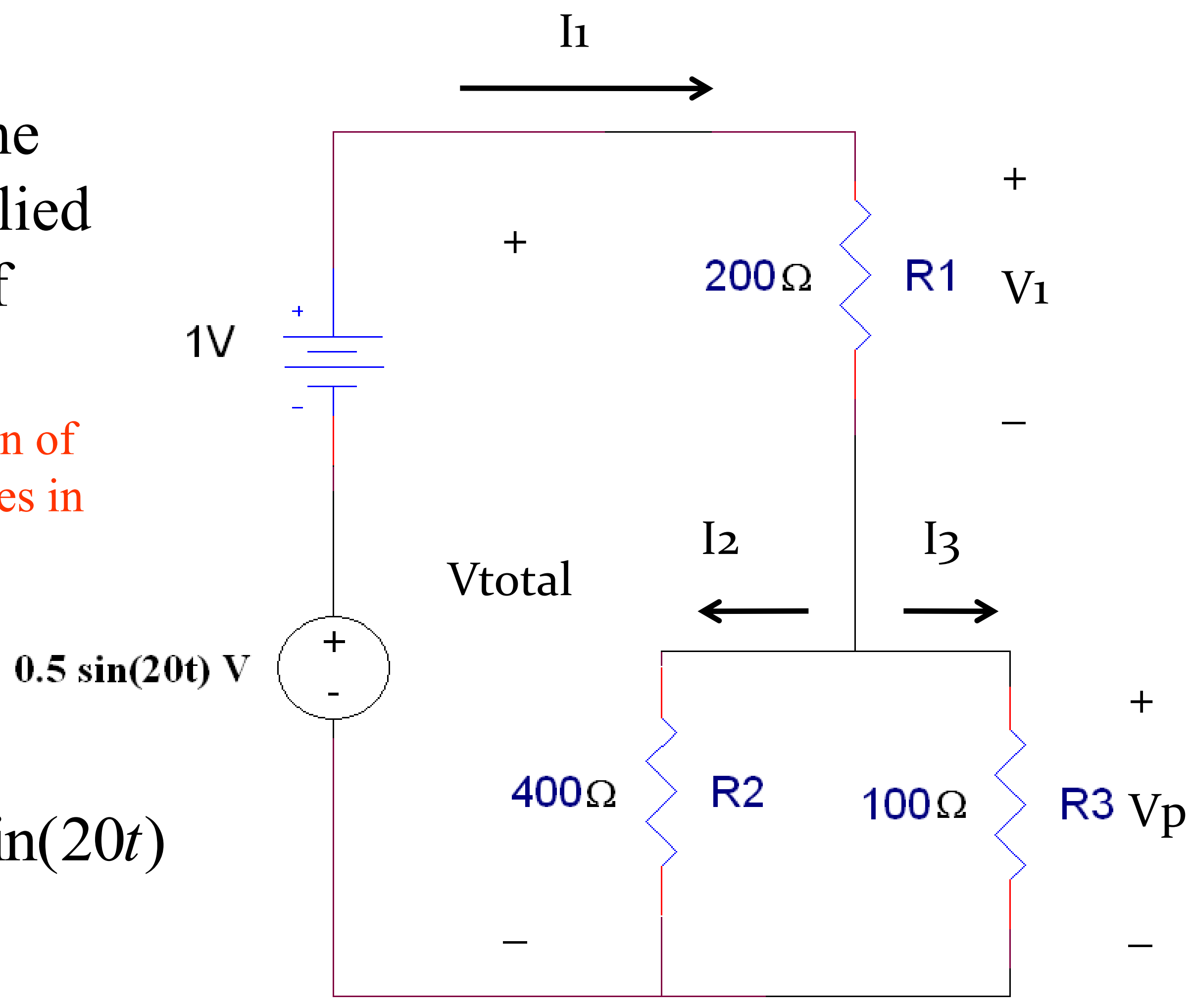
Example 05…

- The circuit to the right has a series and parallel combination of resistors plus two voltage sources.
  - Find V1 and Vp
  - Find I1, I2, and I3



...Example 05...

- First, calculate the total voltage applied to the network of resistors.
  - This is the addition of two voltage sources in series.



$$V_{total} = 1V + 0.5V\sin(20t)$$

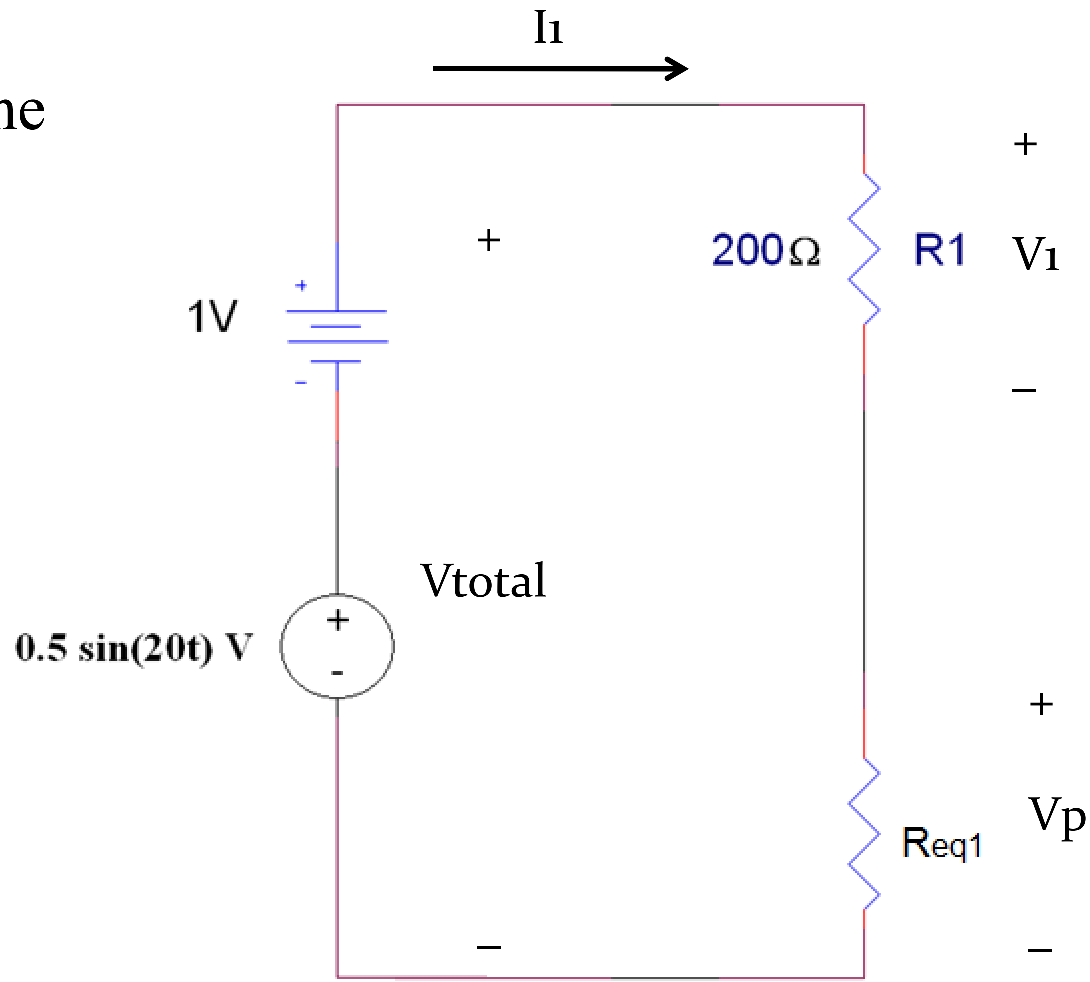
…Example 05…

- Second, calculate the equivalent resistor that can be used to replace the parallel combination of R2 and R3.

$$R_{eq1} = \frac{R_2 R_3}{R_2 + R_3}$$

$$R_{eq1} = \frac{400\Omega(100\Omega)}{400\Omega + 100\Omega}$$

$$R_{eq1} = 80\Omega$$



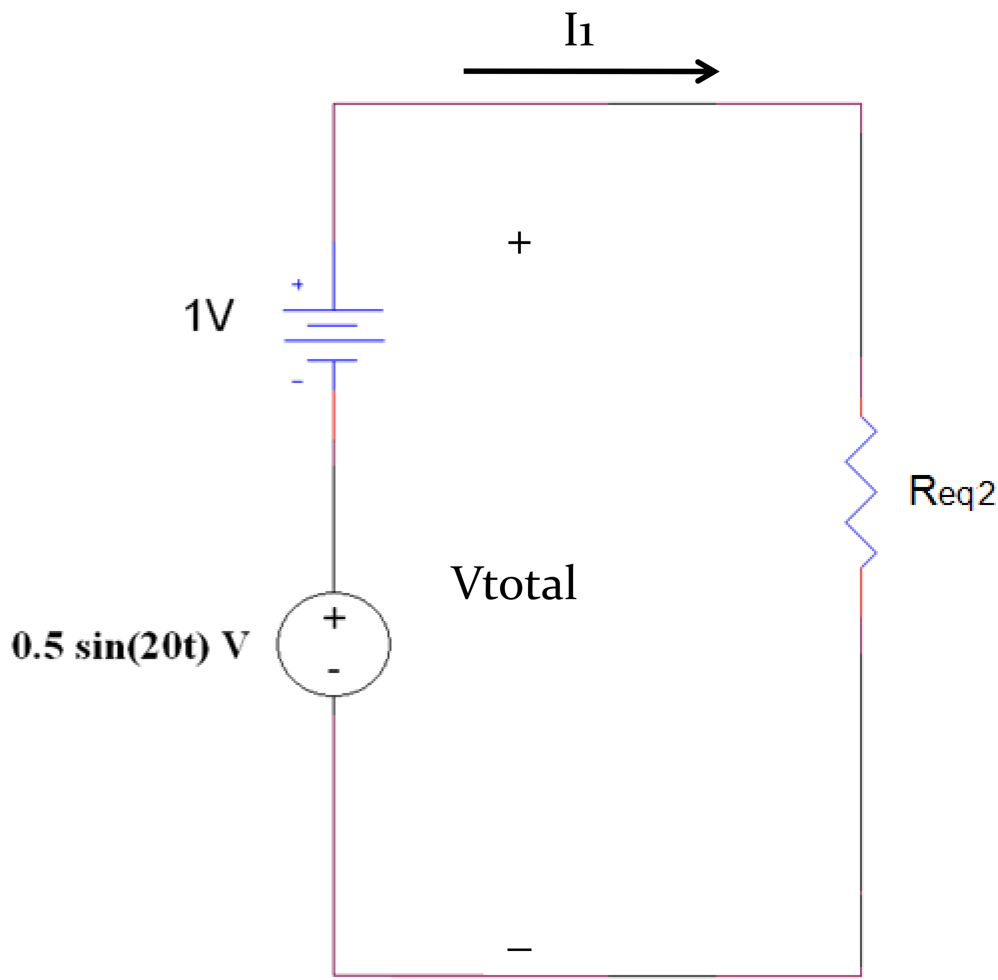
…Example 05…

- To calculate the value for I1, replace the series combination of R1 and Req1 with another equivalent resistor.

$$R_{eq2} = R_1 + R_{eq1}$$

$$R_{eq2} = 200\Omega + 80\Omega$$

$$R_{eq2} = 280\Omega$$



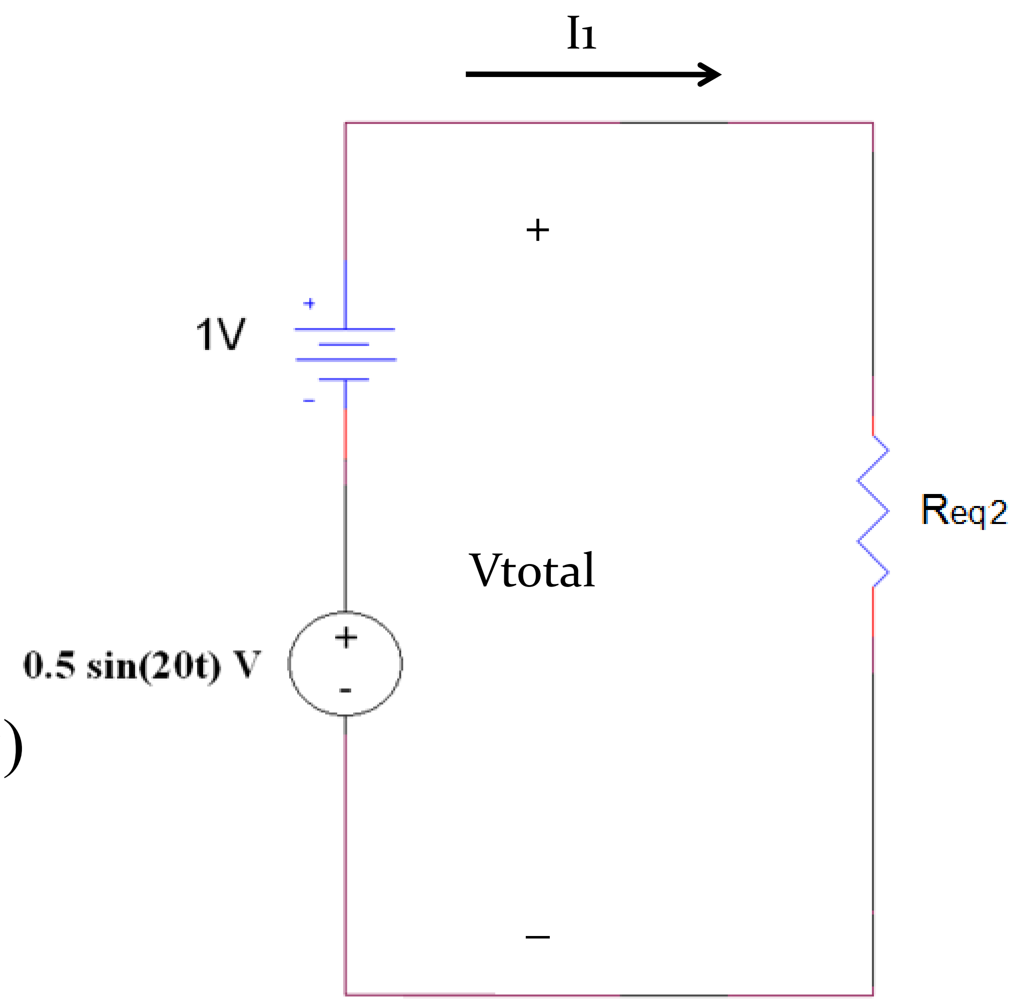
…Example 05…

$$I_1 = \frac{V_{total}}{R_{eq2}}$$

$$I_1 = \frac{1V + 0.5V\sin(20t)}{280\Omega}$$

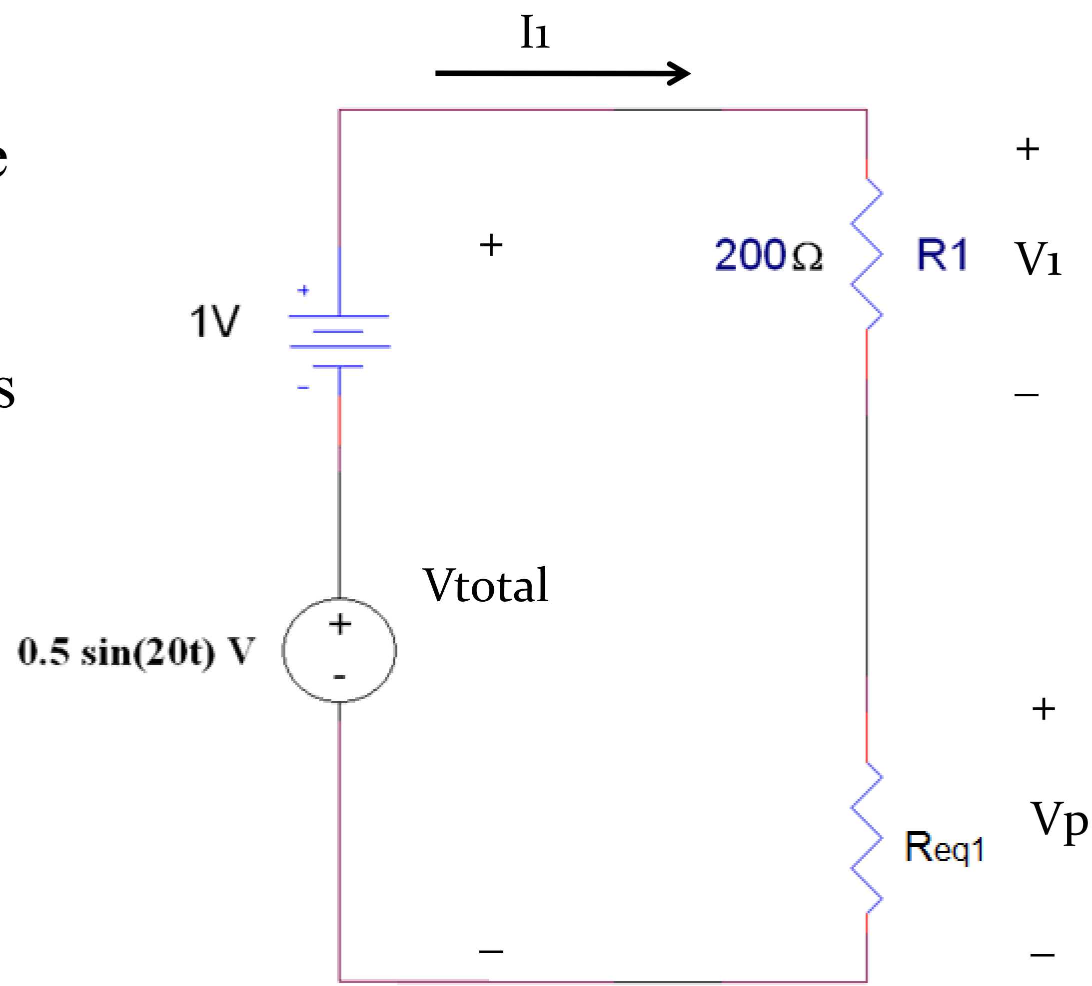
$$I_1 = \frac{1V}{280\Omega} + \frac{0.5V\sin(20t)}{280\Omega}$$

$$I_1 = 3.57mA + 1.79mA\sin(20t)$$



…Example 05…

- To calculate V1, use one of the previous simplified circuits where R1 is in series with Req1.



$$V_1 = \frac{R_1}{R_1 + R_{eq}} V_{total}$$

or

$$V_1 = R_1 I_1$$

$$V_1 = 0.714V + 0.357V \sin(20t)$$

…Example 05…

To calculate Vp:

$$V_p = \frac{R_{eq1}}{R_1 + R_{eq1}} V_{total}$$

or

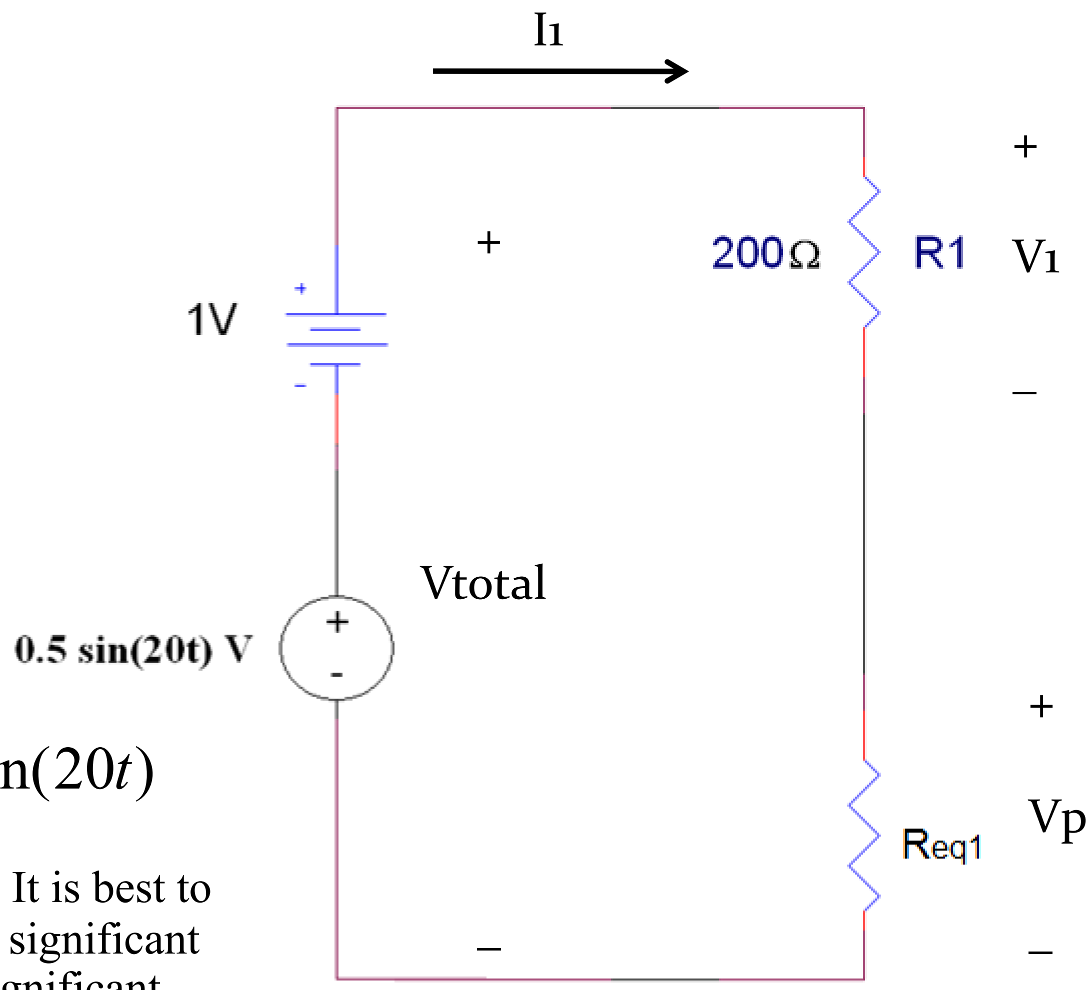
$$V_p = R_{eq1} I_1$$

or

$$V_p = V_{total} - V_1$$

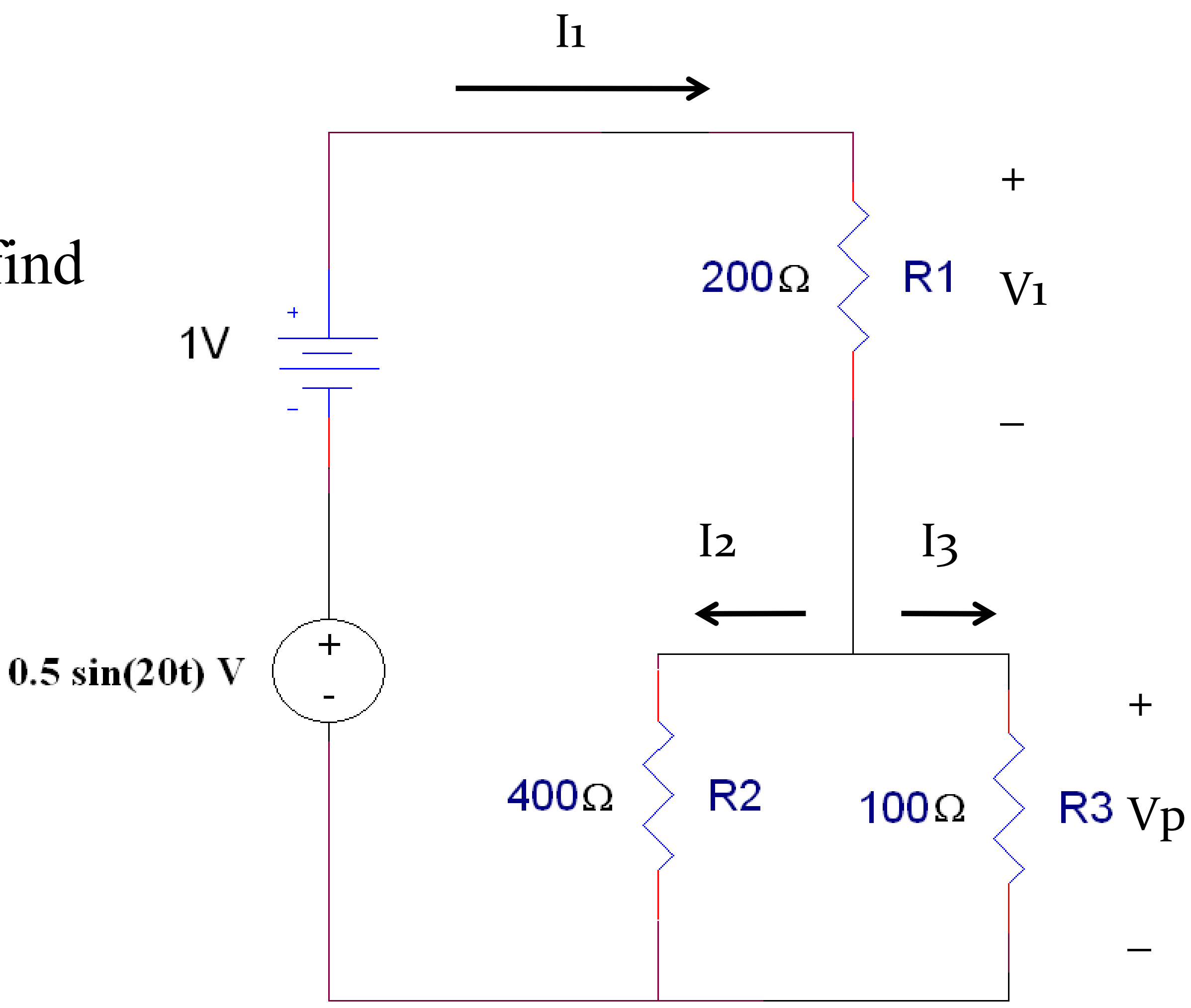
$$V_p = 0.287V + 0.143V \sin(20t)$$

Note: rounding errors can occur. It is best to carry the calculations out to 5 or 6 significant figures and then reduce this to 3 significant figures when writing the final answer.



…Example 05…

- Finally, use the original circuit to find I2 and I3.



$$I_2 = \frac{R_3}{R_2 + R_3} I_1$$

or

$$I_2 = \frac{R_{eq1}}{R_2} I_1$$

$$I_2 = 0.714mA + 0.357mA\sin(20t)$$

...Example 05

- Lastly, the calculation for I3.

$$I_3 = \frac{R_2}{R_2 + R_3} I_1$$

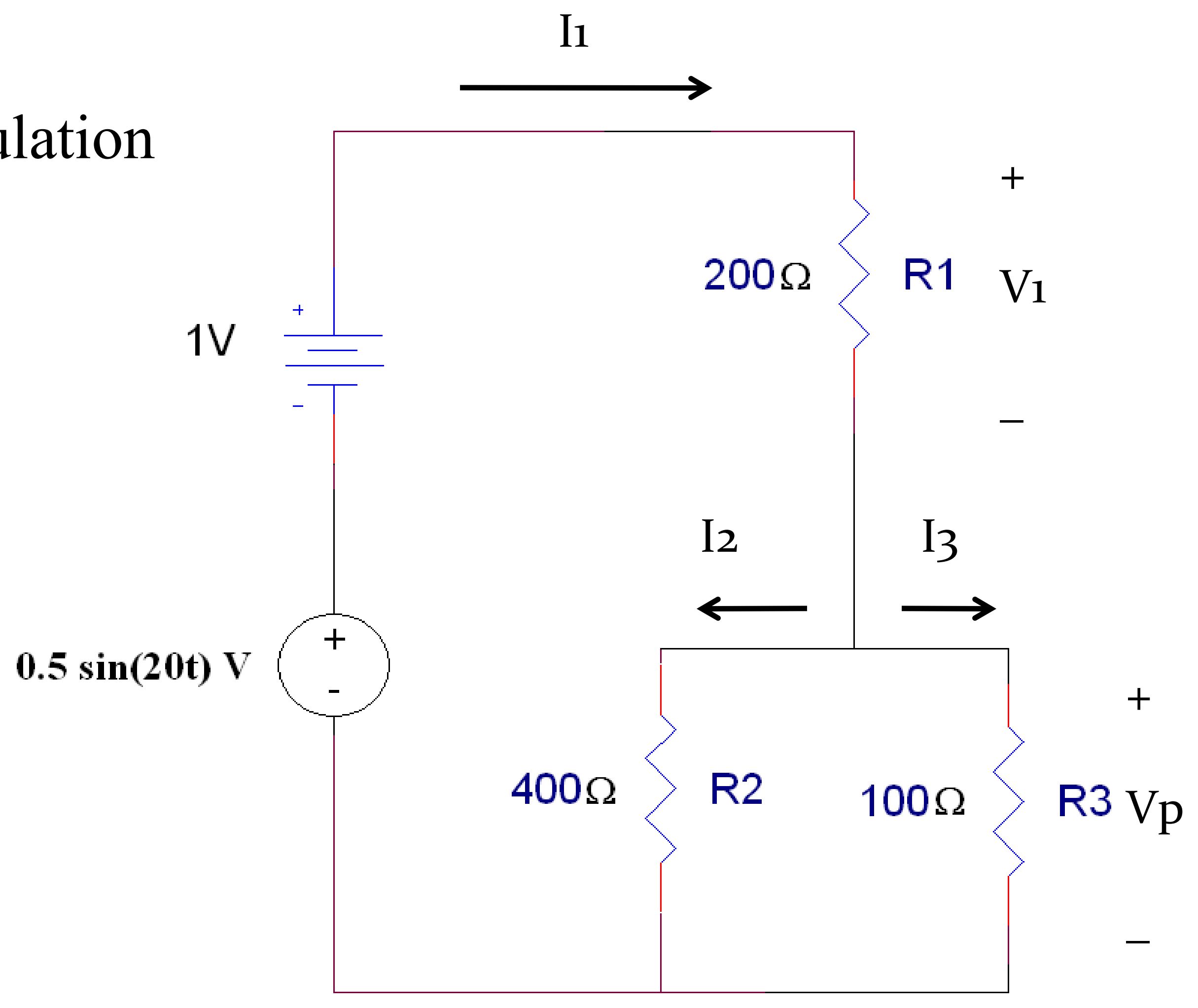
or

$$I_3 = \frac{R_{eq1}}{R_3} I_1$$

or

$$I_3 = I_1 - I_2$$

$$I_3 = 2.86mA + 1.43mA \sin(20t)$$



Summary

- The equations used to calculate the voltage across a specific resistor R_n in a set of resistors in series are:

$$V_n = \left[\frac{R_n}{R_{eq}}\right] V_{total}$$

$$V_n = \left[\frac{G_{eq}}{G_n}\right] V_{total}$$

- The equations used to calculate the current flowing through a specific resistor R_m in a set of resistors in parallel are:

$$I_m = \frac{R_{eq}}{R_m} \mathrm{I}_{\mathrm{total}}$$

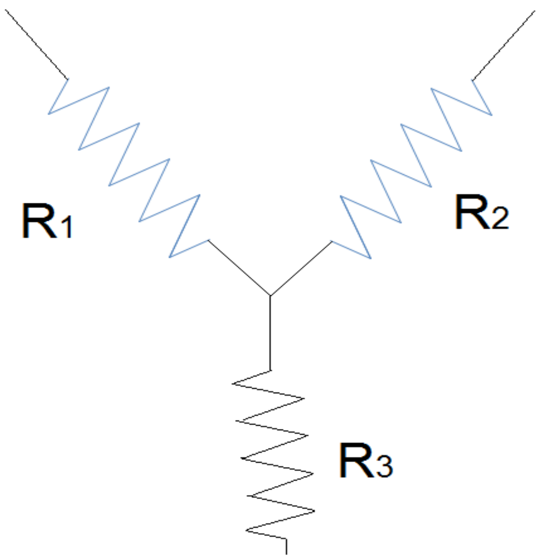
$$I_m = \frac{G_m}{G_{eq}} \mathrm{I}_{\mathrm{total}}$$

Summary Table

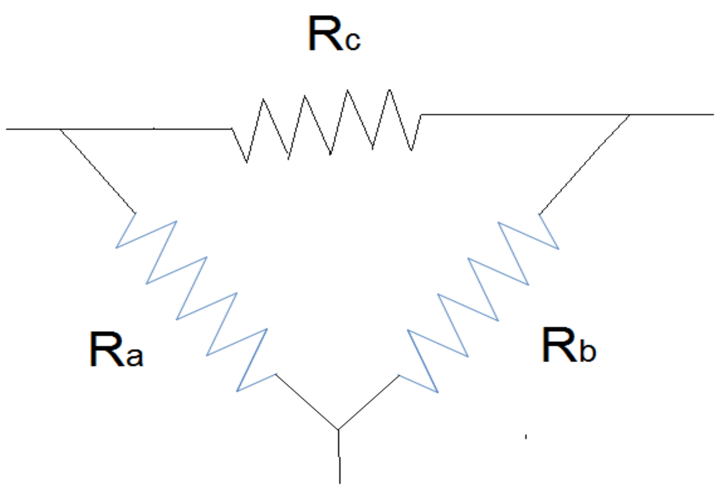
Series and Parallel Circuits		
Series	**Duality**	**Parallel**
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \rightleftarrows G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \rightleftarrows G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements	$R \rightleftarrows G$	$G_T = G_1 + G_2$
$R_T = R_1 + R_2$		and $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{R_1 R_2}{R_1 + R_2}$
I the same through series elements	$I \rightleftarrows V$	V the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \rightleftarrows I$	$I_T = I_1 + I_2 + I_3$
Largest V across largest R	$V \rightleftarrows I$ and $R \rightleftarrows G$	Greatest I through largest G (smallest R)
$V_x = \dfrac{R_x E}{R_T}$	$E, V \rightleftarrows I$ and $R \rightleftarrows G$	$I_x = \dfrac{G_x I_T}{G_T} = \dfrac{R_T I_T}{R_x}$
		with $I_1 = \dfrac{R_2 I_T}{R_1 + R_2}$ and $I_2 = \dfrac{R_1 I_T}{R_1 + R_2}$
$P = EI_T$	$E \rightleftarrows I$ and $I \rightleftarrows E$	$P = I_T E$
$P = I^2 R$	$I \rightleftarrows V$ and $R \rightleftarrows G$	$P = V^2 G = V^2/R$
$P = V^2/R$	$V \rightleftarrows I$ and $R \rightleftarrows G$	$P = I^2/G = I^2 R$

Wye and Delta Networks (3 Terminals)

- 3 terminal arrangements – commonly used in power systems



Wye (Y)



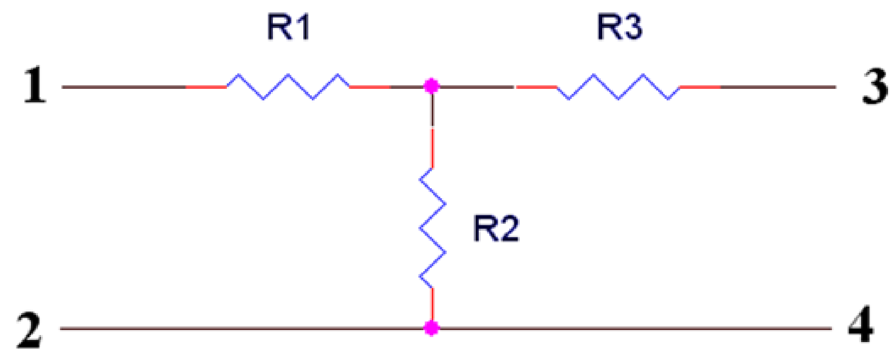
Delta (Δ)

Wye and Delta Networks

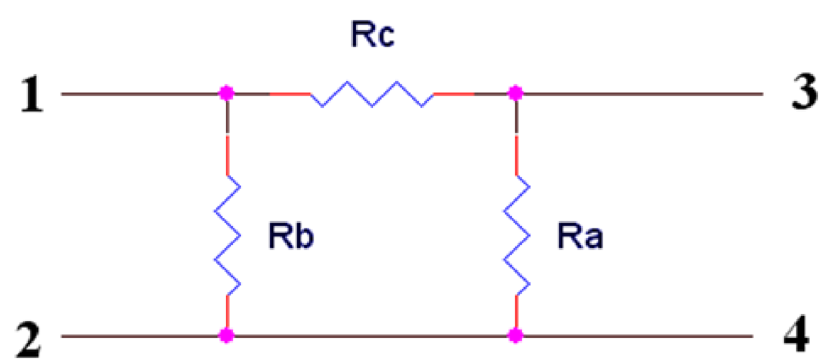
- Sometimes when you are simplifying a resistor network, you get stuck.
- Some resistor networks cannot be simplified using the usual series and parallel combinations. This situation can often be handled by trying the Delta (Δ) −Wye (Y) transformation.
- The names *Delta* and *Wye* come from the shape of the schematics, which resemble letters. The transformation allows you to replace three resistors in a Δ configuration by three resistors Y configuration, and the other way around.

T and Π (4 Terminals)

- Drawn as a 4 terminal arrangement of components.



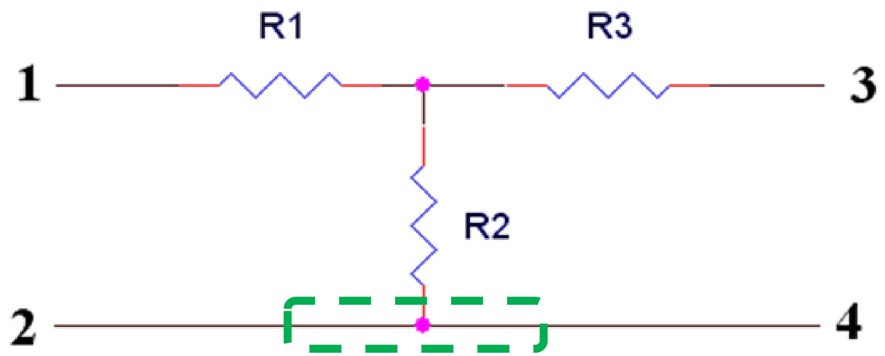
T



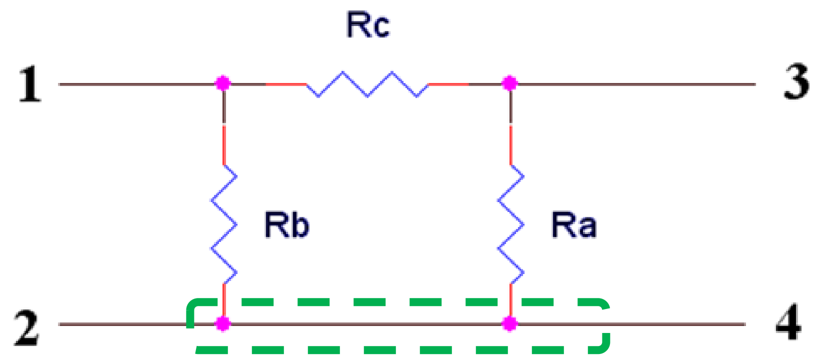
Π

T and Π

- 2 of the terminals are connecting at one node. The node is a distributed node in the case of the Π network.



T



Π

Wye and Delta Networks

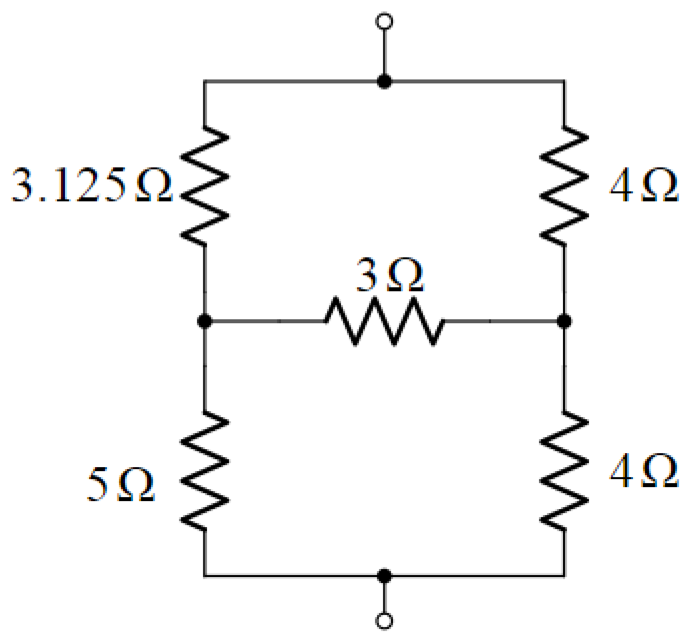
To transform a Delta into a Wye	To transform a Wye into a Delta
$R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c}$	$R_a = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$
$R_2 = \dfrac{R_a R_c}{R_a + R_b + R_c}$	$R_b = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$
$R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c}$	$R_c = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$

If $R_1 = R_2 = R_3 = R$, then Ra = Rb =Rc = 3R

If $R_a = R_b = R_c = R'$, then $R_1 = R_2 = R_3 = R'/3$

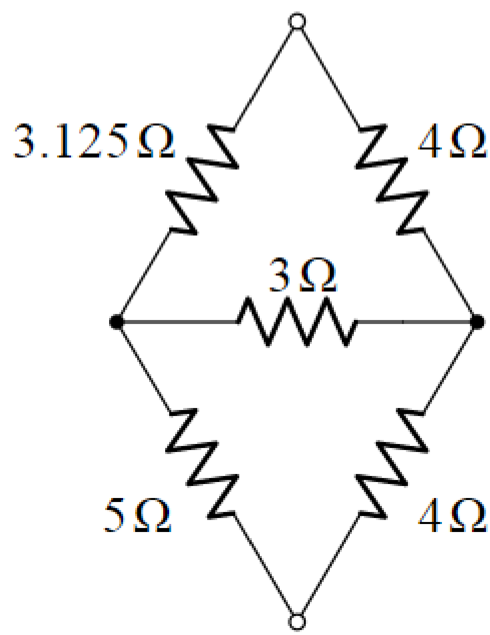
Example

- We want to find the equivalent resistance between the top and bottom terminals



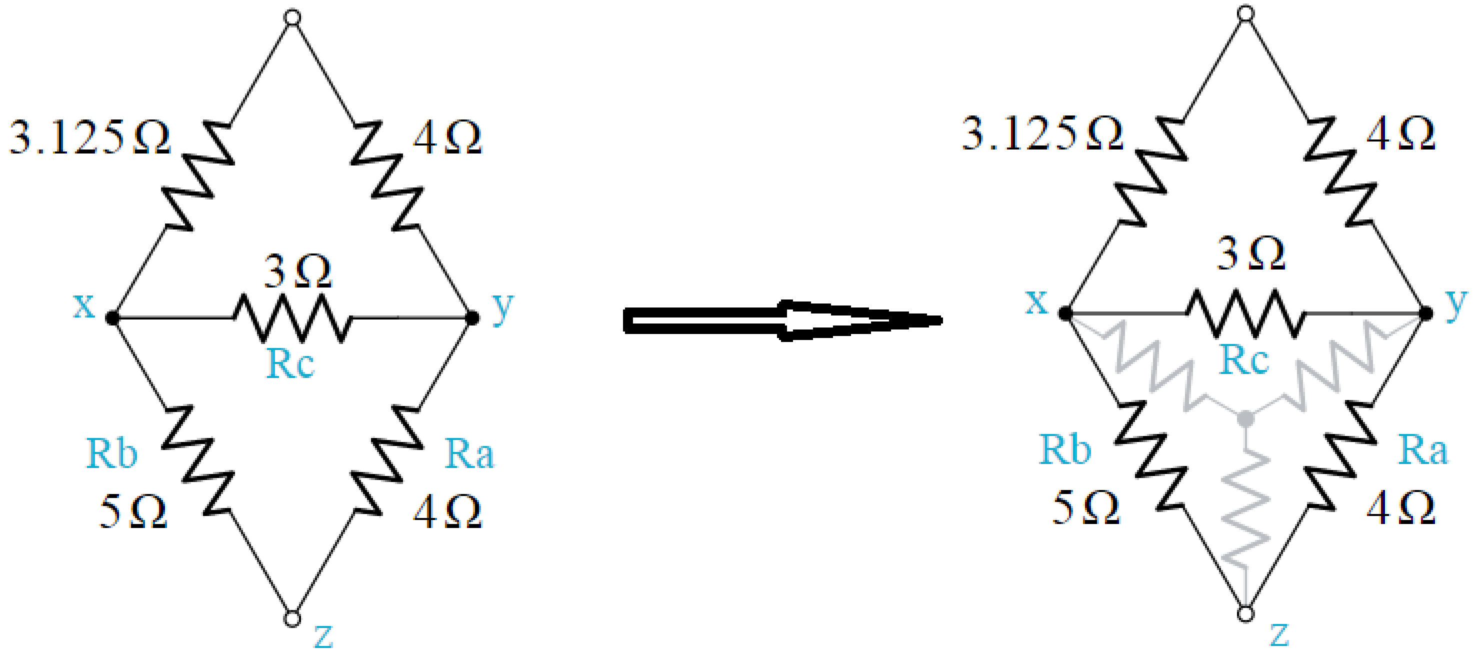
Example (cont.)

- First, let's redraw the schematic to emphasize we have two Δ connections stacked one on the other.



Example (cont.)

- Now select one of the Δ's to convert to a Y. We will perform a Δ→Y transformation and see if it breaks the logjam, opening up other opportunities for simplification.

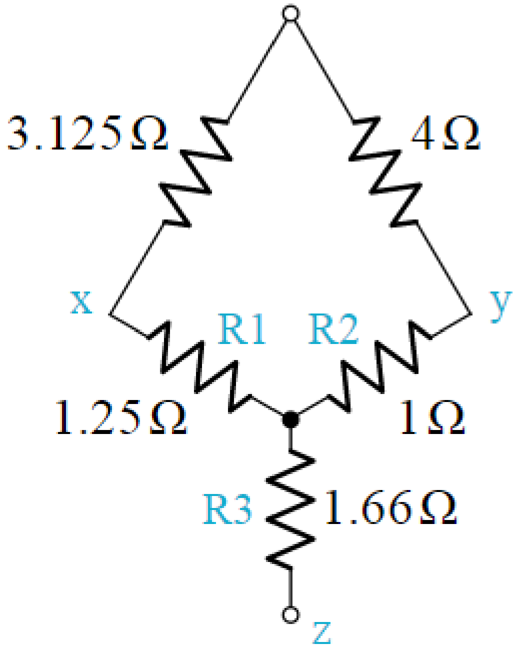


Example (cont.)

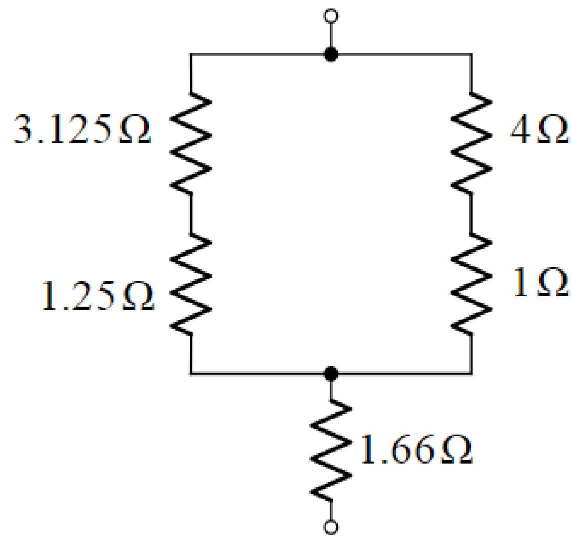
$$R1 = \frac{Rb\,Rc}{Ra + Rb + Rc} = \frac{5 \cdot 3}{4 + 5 + 3} = \frac{15}{12} = 1.25\,\Omega$$

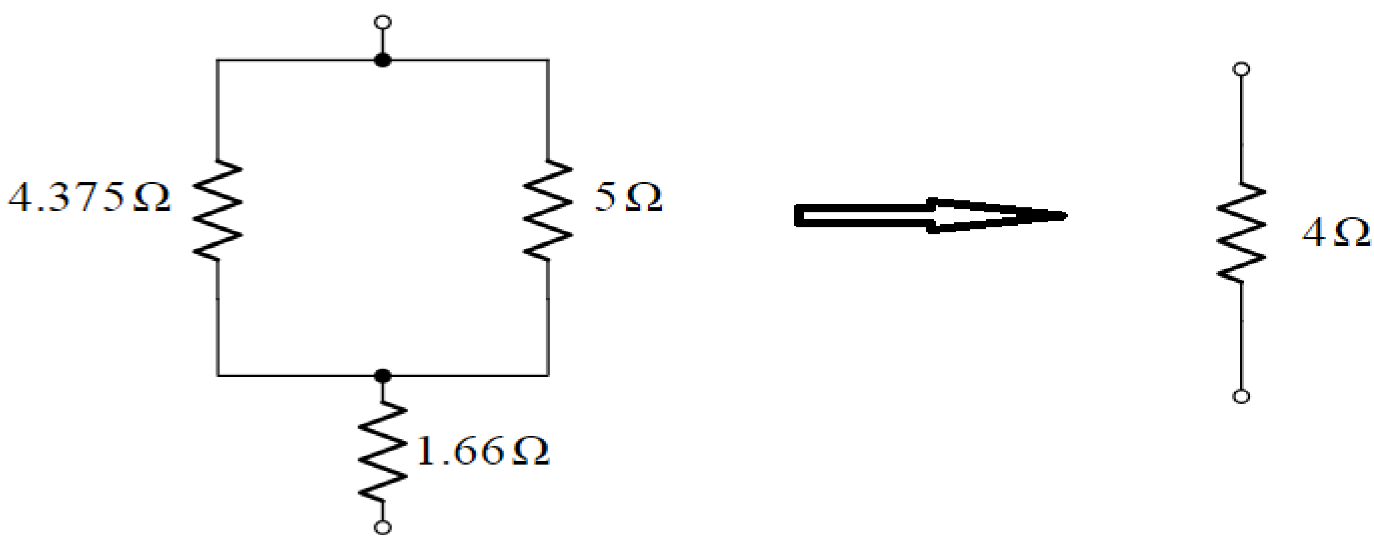
$$R2 = \frac{Ra\,Rc}{Ra + Rb + Rc} = \frac{4 \cdot 3}{4 + 5 + 3} = \frac{12}{12} = 1\,\Omega$$

$$R3 = \frac{Ra\,Rb}{Ra + Rb + Rc} = \frac{4 \cdot 5}{4 + 5 + 3} = \frac{20}{12} = 1.66\,\Omega$$

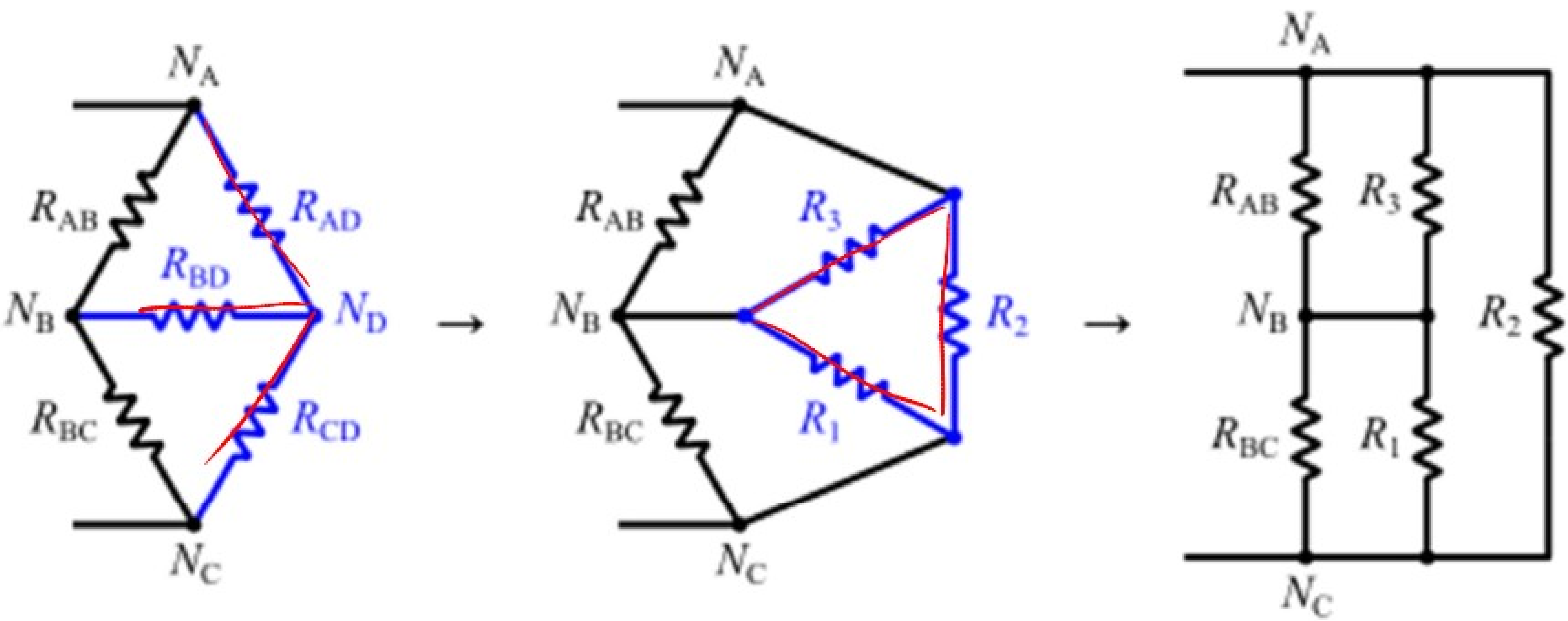


Example (cont.)





Example (cont.) – A different approach



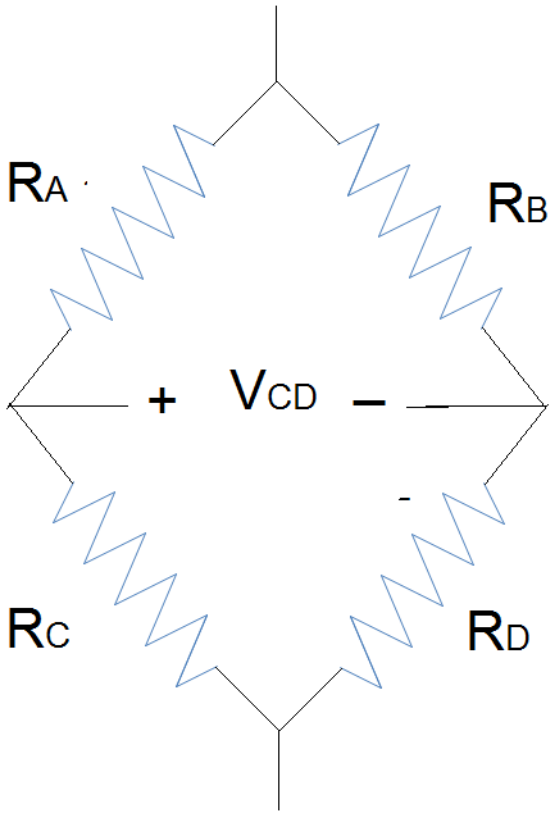
Uses

- Distribution of 3 phase power
- Distribution of power in stators and windings in motors/generators.
  - Wye windings provide better torque at low rpm and delta windings generates better torque at high rpm.

Bridge Circuits

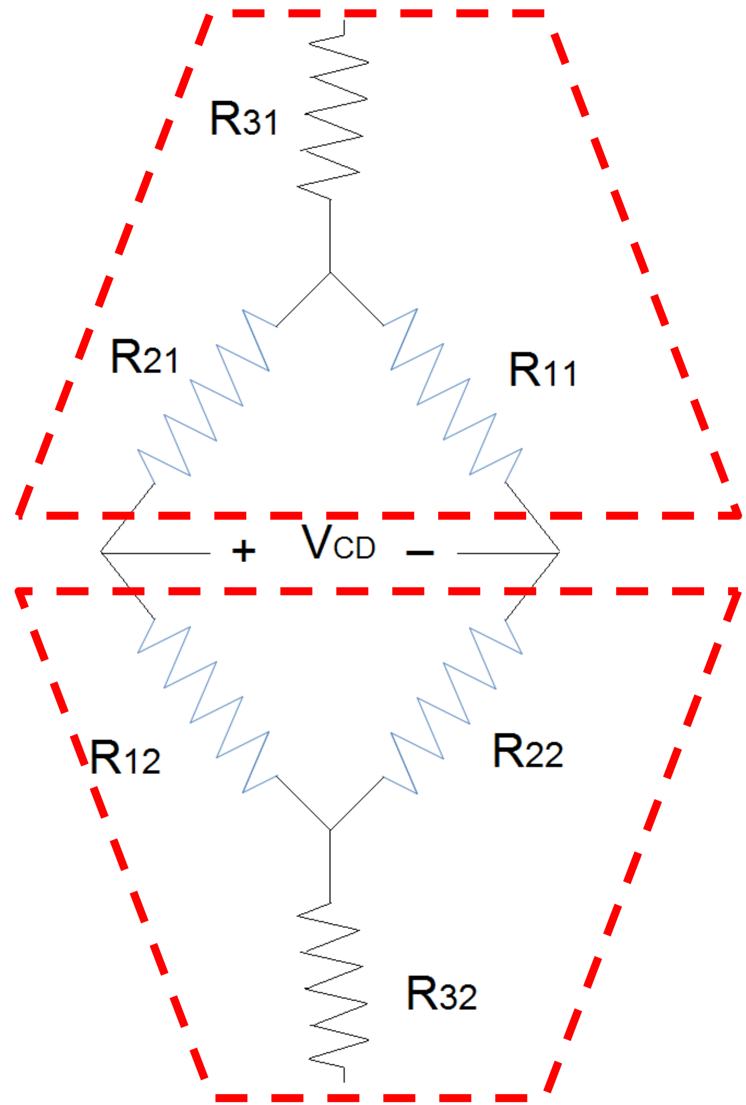
Measurement of the voltage V_{CD} is used in sensing and full-wave rectifier circuits.

- If $R_A = R_B = R_C = R_D$, $V_{CD} = 0V$
- In sensing circuits, the resistance of one resistor (usually R_D) is proportional to some parameter – temperature, pressure, light, etc. , then V_{CD} becomes a function of that same parameter.



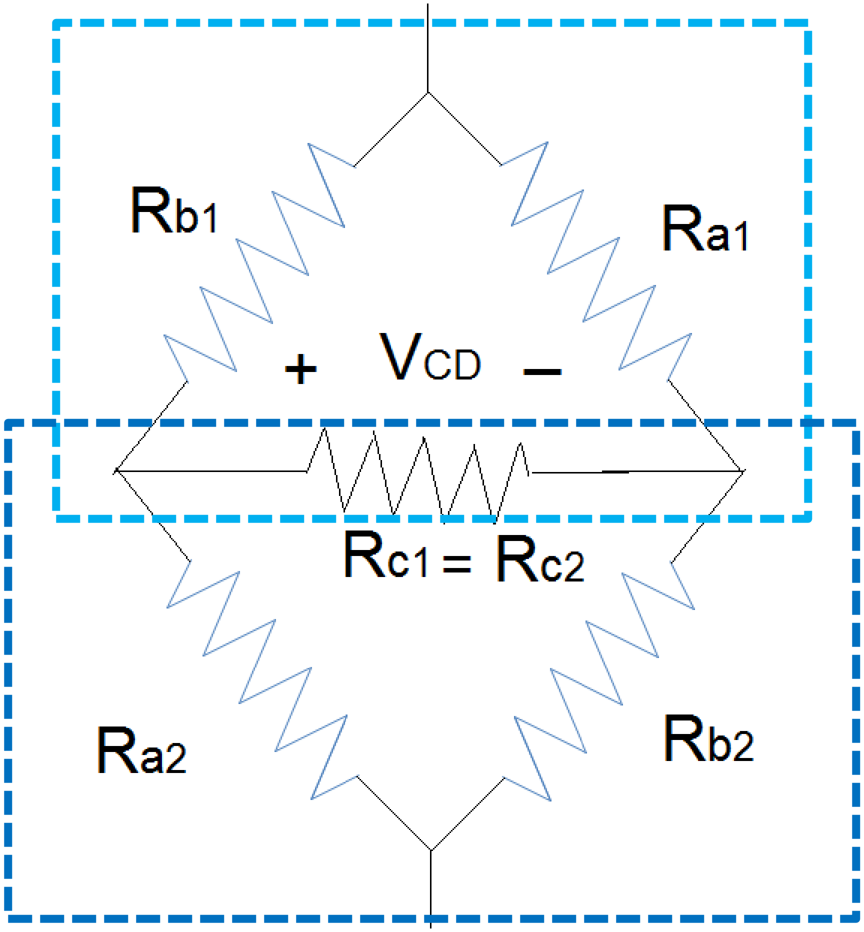
Bridge Circuits

- Back-to-back Wye networks



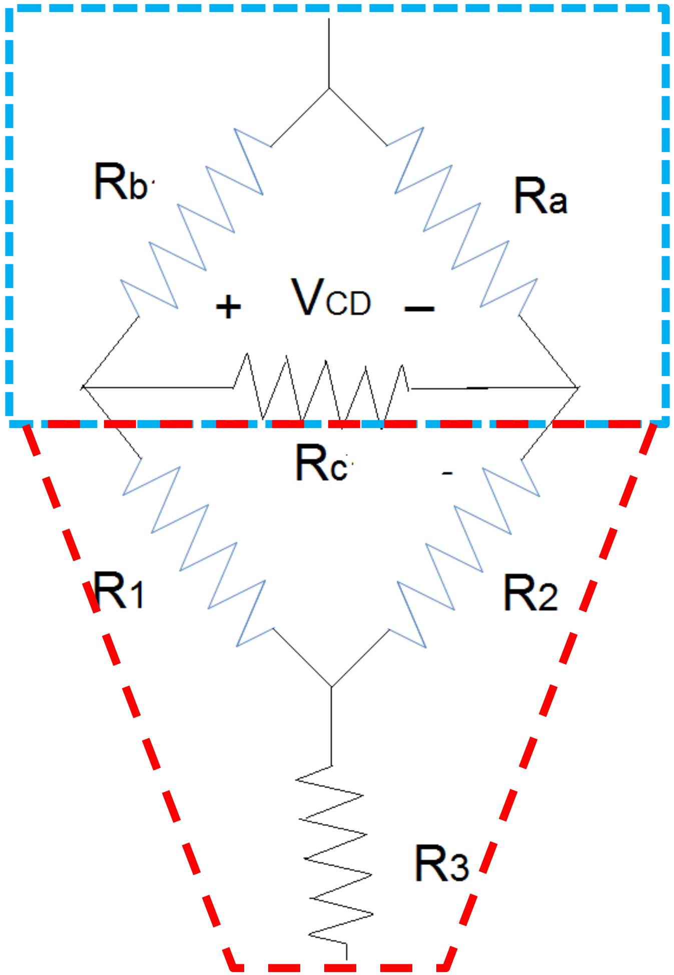
Bridge Circuit

- Or two Delta networks where Rc1 = Rc2 = ∞Ω.



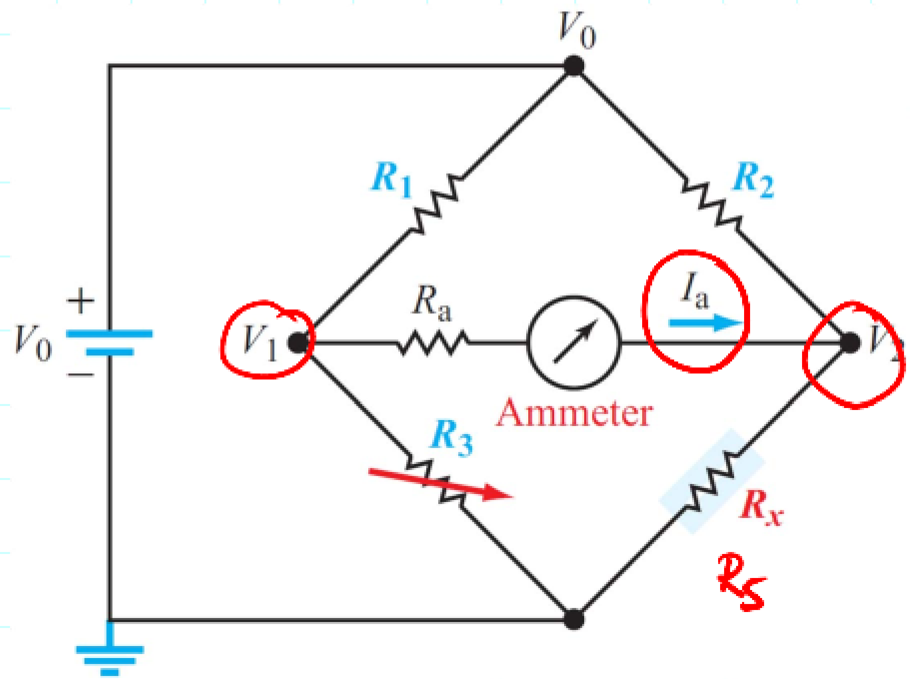
Bridge Circuits

- Alternatively, the bridge circuit can be constructed from one Delta and one Wye network where $R_c = \infty\Omega$.



Wheatstone Bridge Circuit

- Measurement instrument based on differential measurement
- Balanced Condition:
  $$I_a = 0$$
- Determine unknown resistance based on "balanced" condition
  $$V_1 = V_2$$



Wheatstone Bridge Circuit

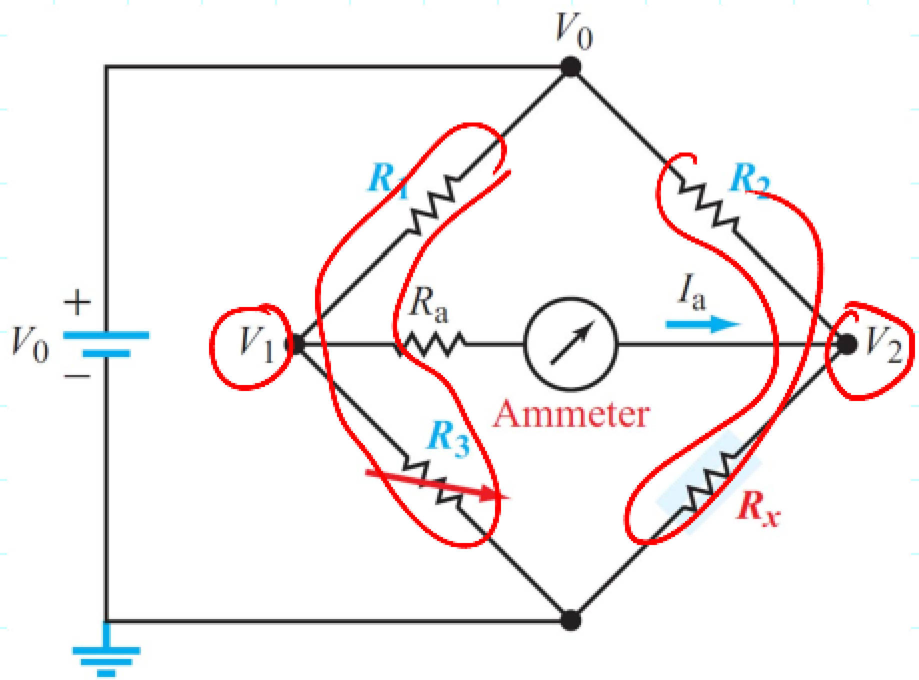
- Measurement instrument based on differential measurement
- Balanced Condition:

$$I_a = 0$$

- Determine unknown resistance based on "balanced" condition

$$V_1 = V_2$$

$$\frac{R_3 V_0}{R_1 + R_3} = \frac{R_x V_0}{R_2 + R_x}$$



$$\boxed{R_x = \left(\frac{R_2}{R_1}\right) R_3}$$

For balanced condition

Summary

- There is a conversion between the resistances used in wye and delta resistor networks.
- Bridge circuits can be considered to be a combination of wye-wye, delta-delta, or delta-wye circuits.
  - Voltage across a bridge can be related to the change in the resistance of one resistor if the resistance of the other three resistors is constant.