BME2301 - Circuit Theory

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Energy Storage Devices

Capacitors and Inductors

Objective of Lecture

- Describe
  - the construction of a capacitor
  - how charge is stored.
  - Introduce several types of capacitors
- The electrical properties of a capacitor
  - Relationship between charge, voltage, and capacitance; power; and energy
  - Equivalent capacitance when a set of capacitors are in series and in parallel
- Describe
  - The construction of an inductor
  - How energy is stored in an inductor
  - The electrical properties of an inductor
    - Relationship between voltage, current, and inductance; power; and energy
  - Equivalent inductance when a set of inductors are in series and in parallel

Capacitors

Energy Storage Devices

Capacitors

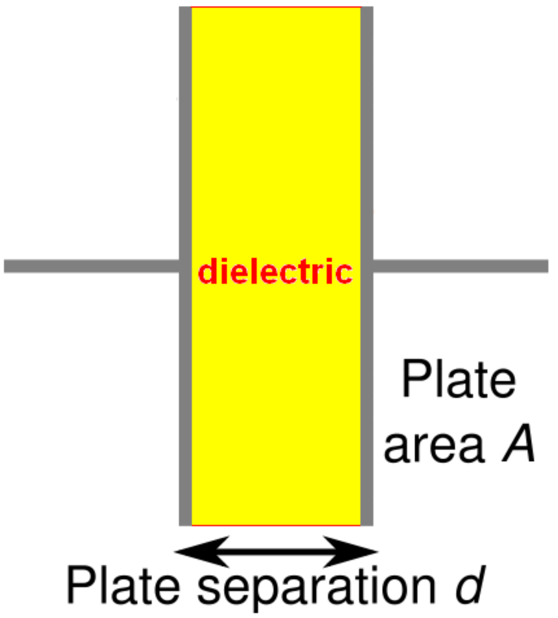
- Composed of two conductive plates separated by an insulator (or dielectric).
  - Commonly illustrated as two parallel metal plates separated by a distance, d.

$C = \varepsilon A/d$

where $\varepsilon = \varepsilon_r \varepsilon_o$

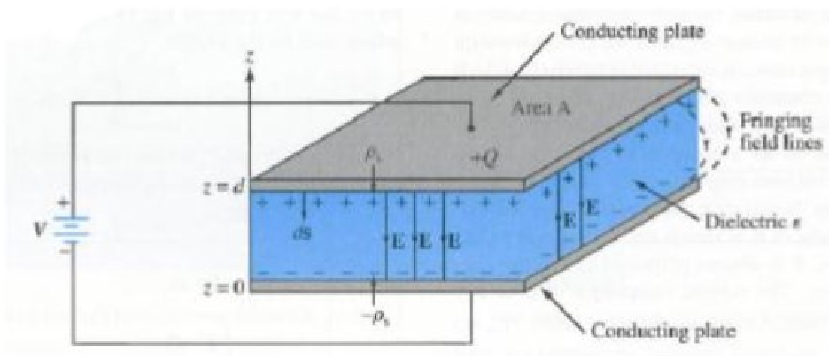
ε_r is the relative dielectric constant

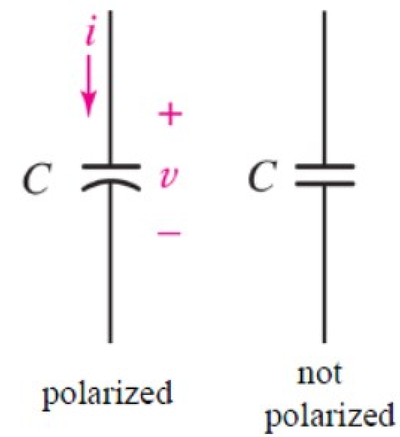
ε_o is the vacuum permittivity



Charging a Capacitor

A **capacitor** is a linear circuit element which stores energy in the **electric field** in the space between two conducting bodies occupied by a material with permittivity ε.





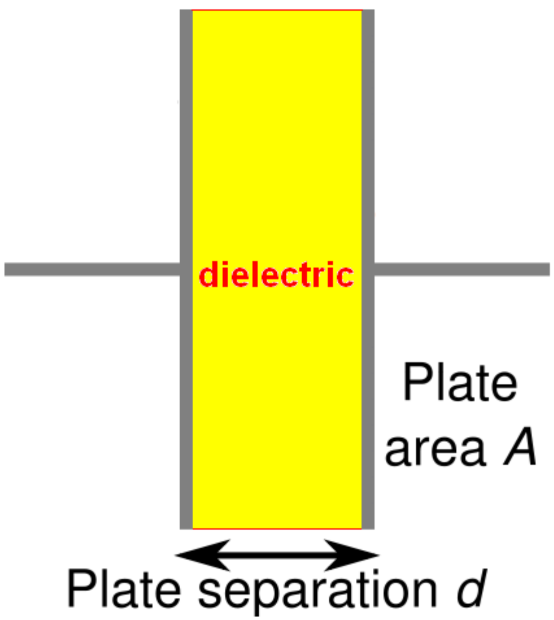
- *charged* by applying current (for a finite amount of time, from another source) to its terminals

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\cdot dt' + v(t_0)$$

- *discharged* when it provides current (for a finite amount of time, to a circuit) from its terminals

Effect of Dimensions

- Capacitance increases with



  – increasing surface area of the plates,
  – decreasing spacing between plates, and
  – increasing the relative dielectric constant of the insulator between the two plates.

Capacitor Current & Voltage

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\cdot d\tau + v(t_0)$$

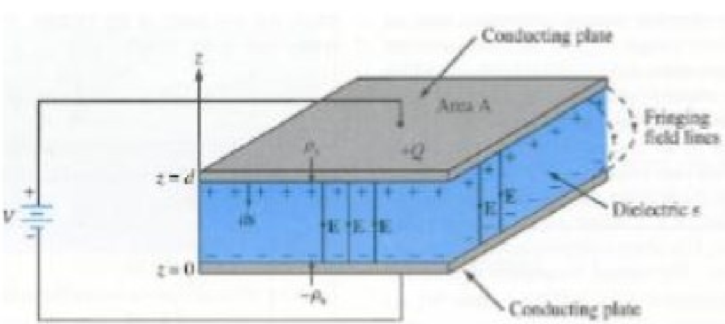
same equation

$$i(t) = C\frac{dv(t)}{dt}$$

$$q(t) = C \cdot v(t)$$

$$\downarrow$$

$$C = Q/V$$



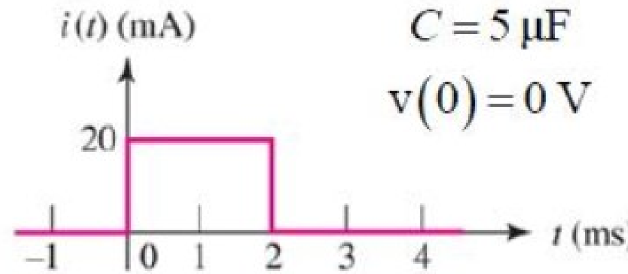
$$C = \varepsilon\frac{A}{d}$$

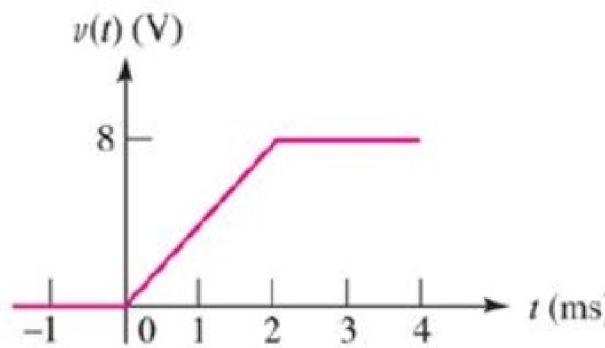
ε = permittivity

unit of capacitance = farad
1 F = 1 C / V

$C = 5\ \mu\text{F}$

$v(0) = 0\ \text{V}$



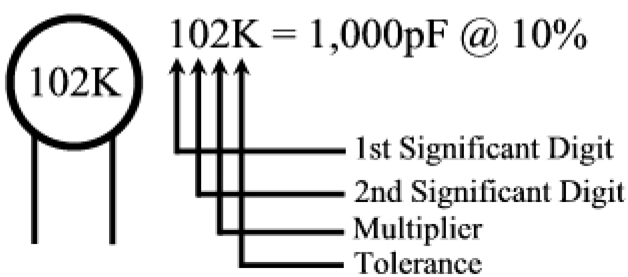


Types of Capacitors

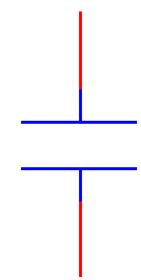
- Fixed Capacitors
  - Nonpolarized
    - May be connected into circuit with either terminal of capacitor connected to the high voltage side of the circuit.
      - Insulator: Paper, Mica, Ceramic, Polymer
  - Electrolytic
    - The negative terminal must always be at a lower voltage than the positive terminal
      - Plates or Electrodes: Aluminum, Tantalum

Types of Capacitors - Nonpolarized

- Difficult to make nonpolarized capacitors that store a large amount of charge or operate at high voltages.
  - Tolerance on capacitance values is very large
    - +50%/-25% is not unusual



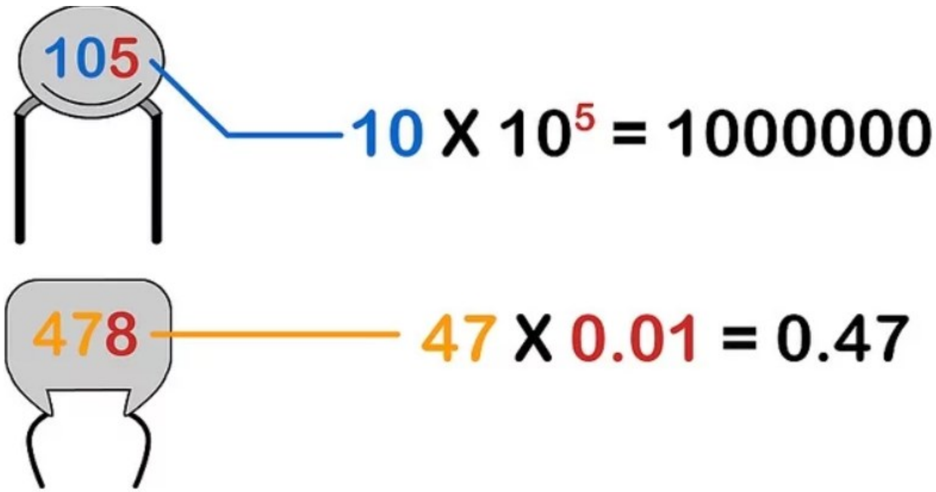
PSpice Symbol



http://www.marvac.com/fun/ceramic_capacitor_codes.aspx

Reading the Capacitor Codes

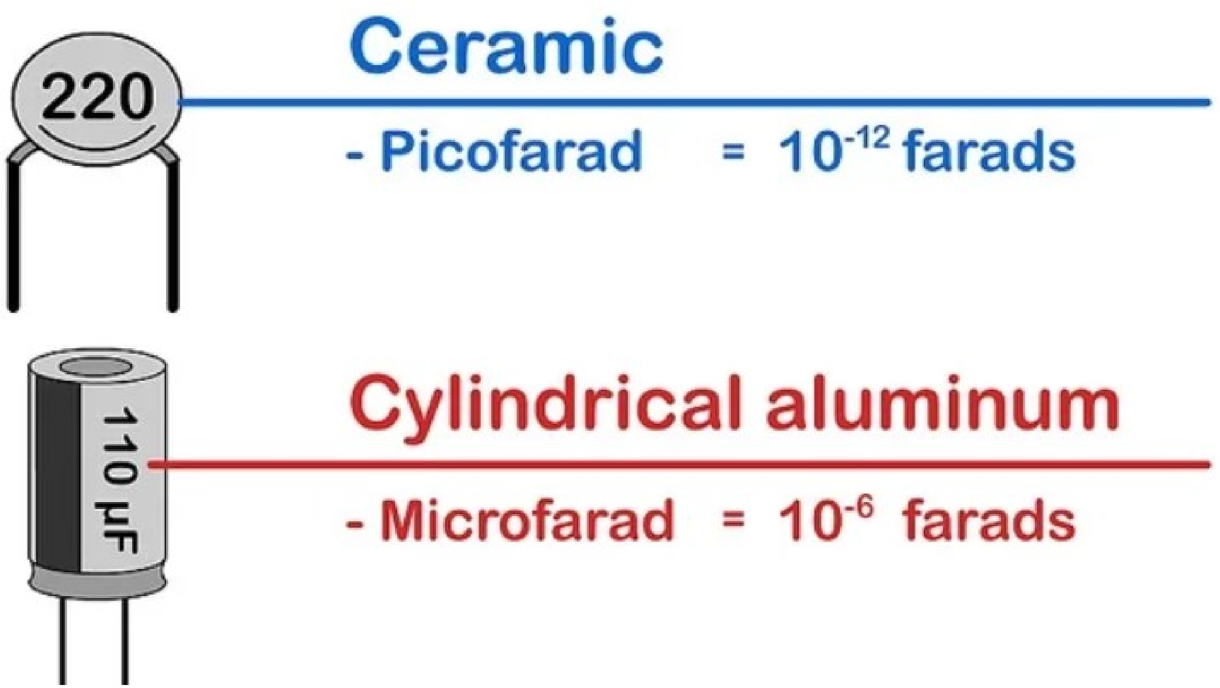
1. **Write down the first two digits of the capacitance.**
   – If your code starts with exactly two digits followed by a letter (e.g. 44M), the first two digits are the full capacitance code.



- If the third digit is 0 through 6, add that many zeroes to the end of the number. (For example, 453 → 45 x 10^3 → 45,000.)
- If the third digit is 8, multiply by 0.01. (e.g. 278 → 27 x 0.01 → 0.27)
- If the third digit is 9, multiply by 0.1. (e.g. 309 → 30 x 0.1 → 3.0)

Reading the Capacitor Codes

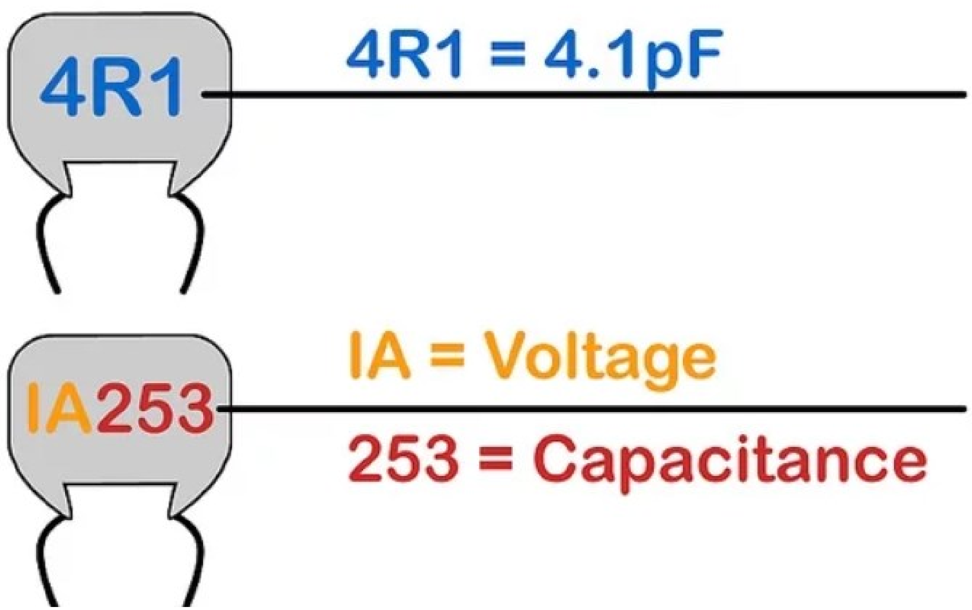
2. **Work out the capacitance units from context**.
   - The smallest capacitors (made from ceramic, film, or tantalum) use units of picofarads (pF), equal to 10-12 farads. Larger capacitors (the cylindrical aluminum electrolyte type or the double-layer type) use units of microfarads (uF or µF), equal to 10-6 farads.



Reading the Capacitor Codes

3. **Read codes that contain letters instead**. If your code includes a letter as one of the first two characters, there are three possibilities:.

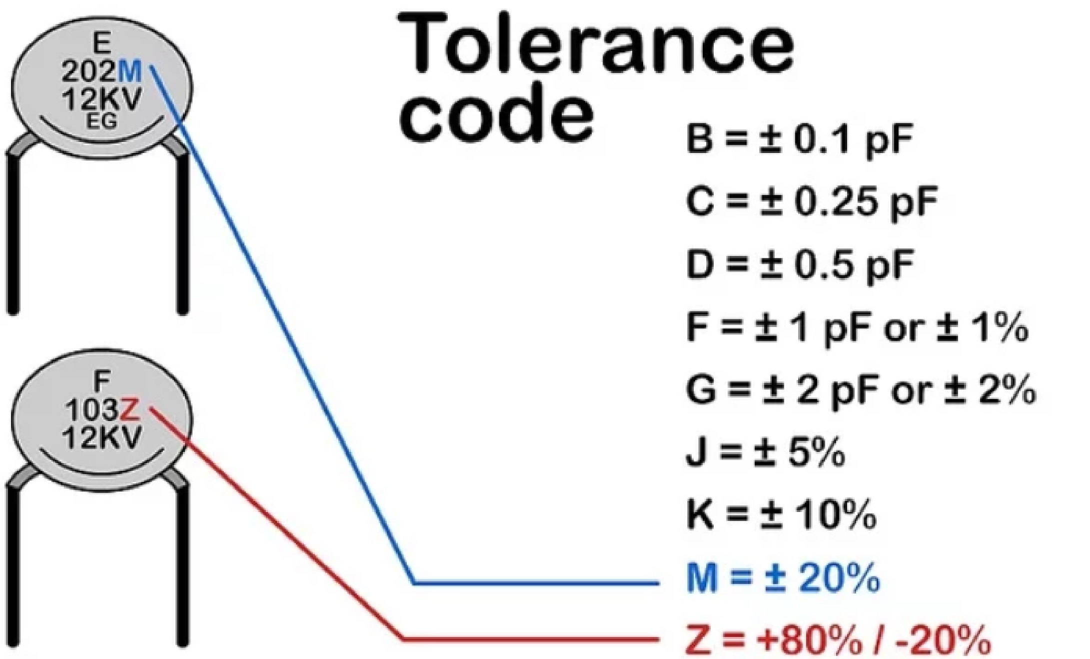
- If the letter is an R, replace it with a decimal point to get the capacitance in pF. For example, 4R1 means a capacitance of 4.1pF.
- If the letter is p, n, or u, this tells you the units (pico-, nano-, or microfarad). Replace this letter with a decimal point. For example, n61 means 0.61 nF, and 5u2 means 5.2 uF.
- A code like "1A253" is actually two codes. 1A tells you the voltage, and 253 tells you the capacitance as described above..



Reading the Capacitor Codes

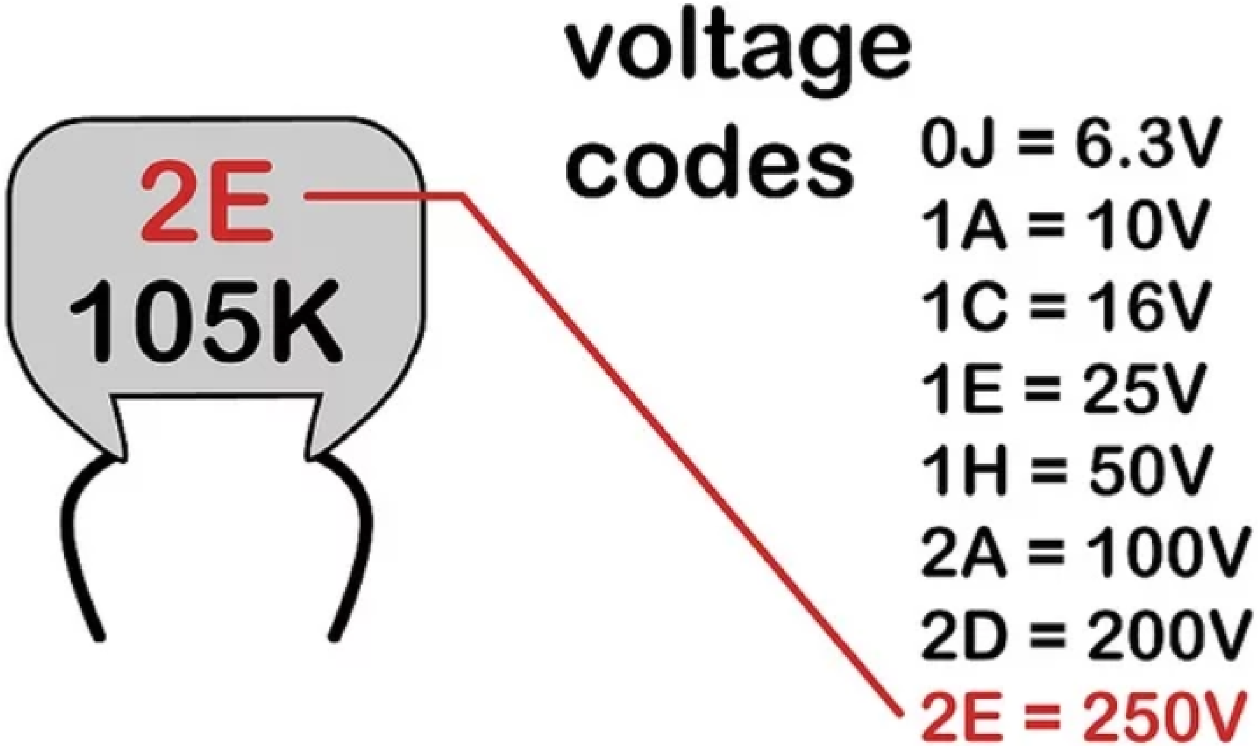
4. **Read the tolerance code on ceramic capacitors**. Ceramic capacitors, which are usually tiny "pancakes" with two pins, typically list the tolerance value as one letter immediately after the three-digit capacitance value.

- This letter represents the tolerance of the capacitor, meaning how close the actual value of the capacitor can be expected to be to the indicated value of the capacitor.



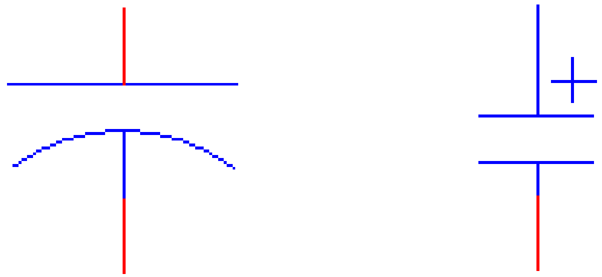
Reading the Capacitor Codes

5. **Interpret voltage codes**. You can look up the EIA voltage chart for a full list, but most capacitors use one of the following common codes for maximum voltage (values given for DC capacitors only).

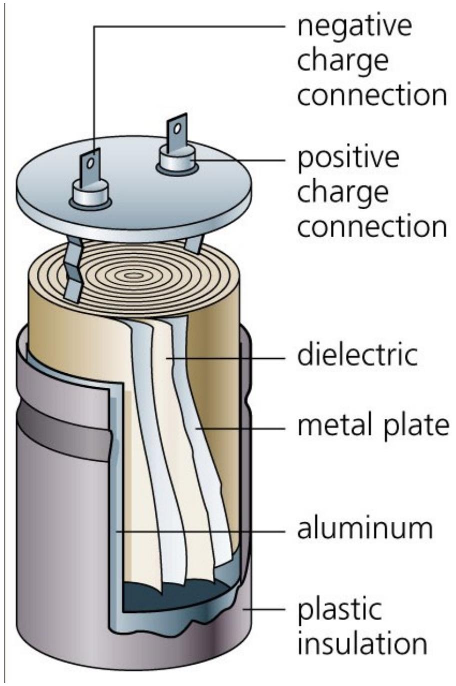


Types of Capacitors - Electrolytic

Pspice Symbols



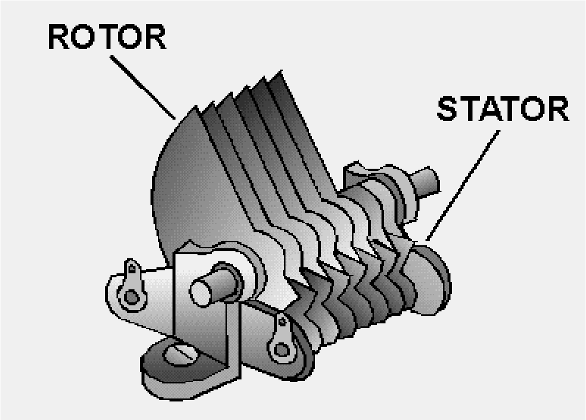
Fabrication



http://www.digitivity.com/articles/2008/11/choosing-the-right-capacitor.html

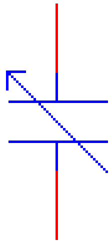
Types of Capacitors - Variable Capacitors

- Cross-sectional area is changed as one set of plates are rotated with respect to the other.



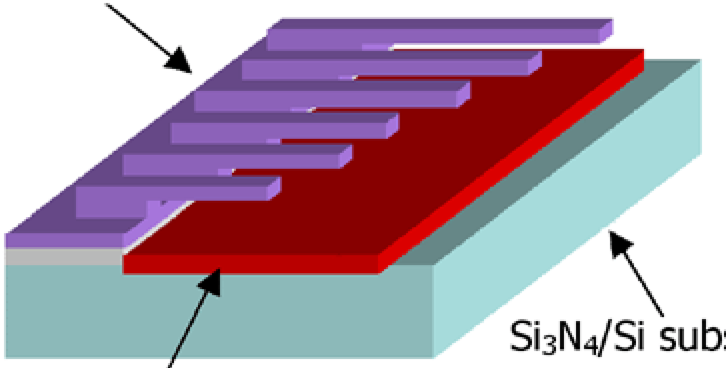
http://www.tpub.com/neets/book2/3f.htm

PSpice Symbol



Types of Capacitors - MEMS Capacitor

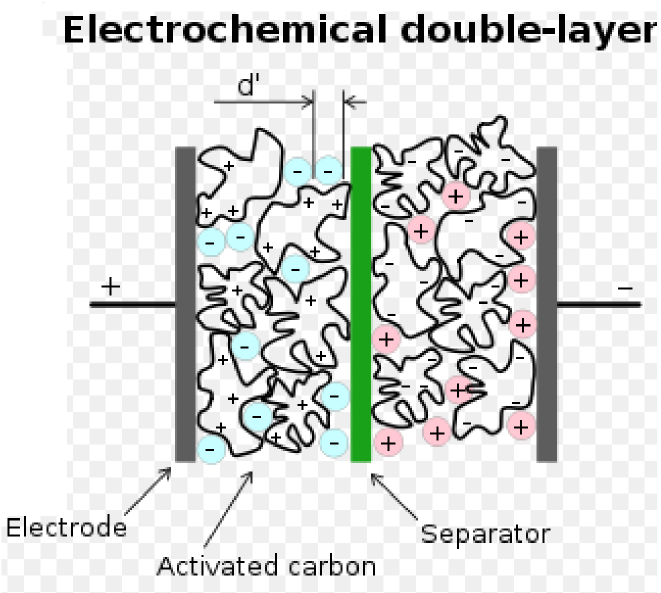
- MEMS (Microelectromechanical system)
  - Can be a variable capacitor by changing the distance between electrodes.
  - Use in sensing applications as well as in RF electronics.



http://www.silvaco.com/tech_lib_TCAD/simulationstandard/2005/aug/a3/a3.html

Types of Capacitors - Electric Double Layer Capacitor

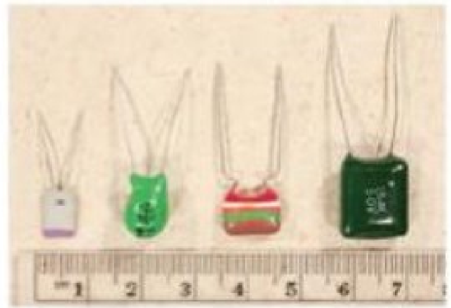
- Also known as a supercapacitor or ultracapacitor
  – Used in high voltage/high current applications.
    - Energy storage for alternate energy systems.



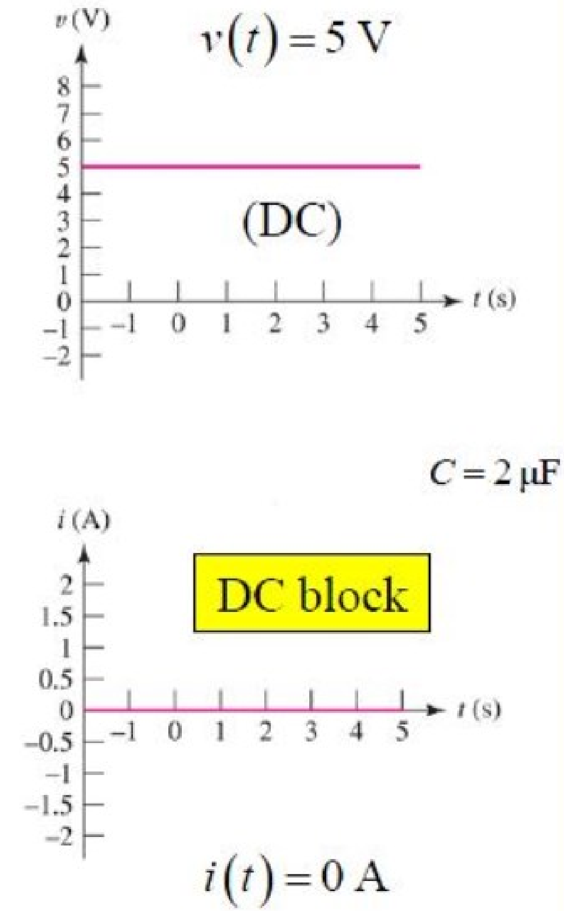
http://en.wikipedia.org/wiki/File:Supercapacitor_diagram.svg

Electrical Properties of a Capacitor

- Acts like an **open circuit** at steady state when connected to a **D.C. voltage or current** source.

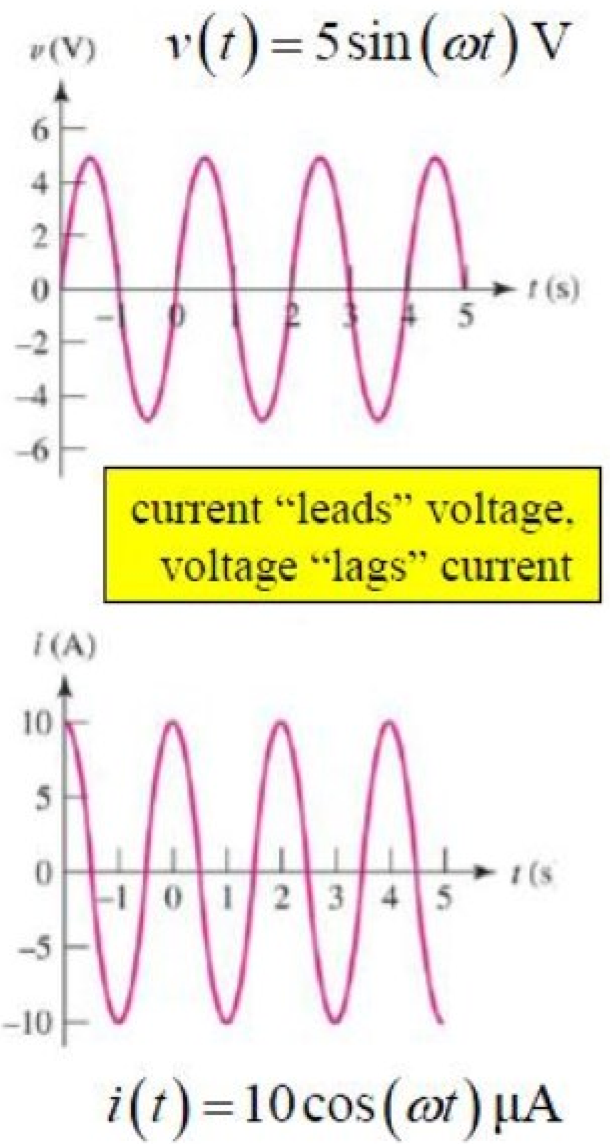


$$i = C\frac{dv}{dt}$$



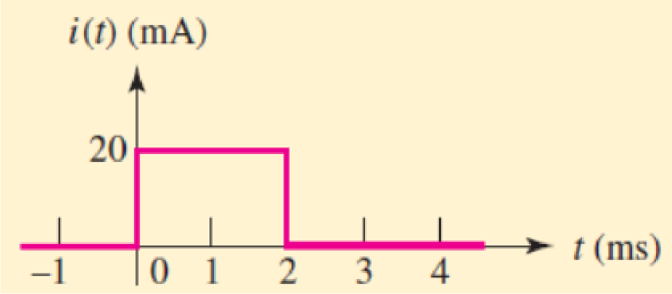
Electrical Properties of a Capacitor

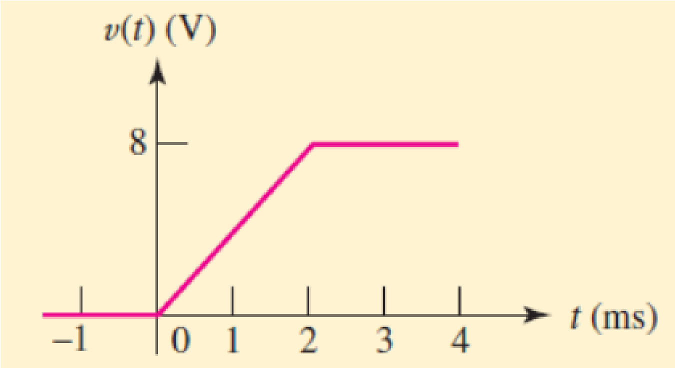
- Voltage on a capacitor must be continuous
  - There are no abrupt changes to the voltage
- An ideal capacitor does not dissipate energy, it takes power when storing energy and returns it when discharging.



Example 1

- Find the capacitor voltage that is associated with the current shown graphically in below Figure. The value of the capacitance is 5 μF.





$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)$$

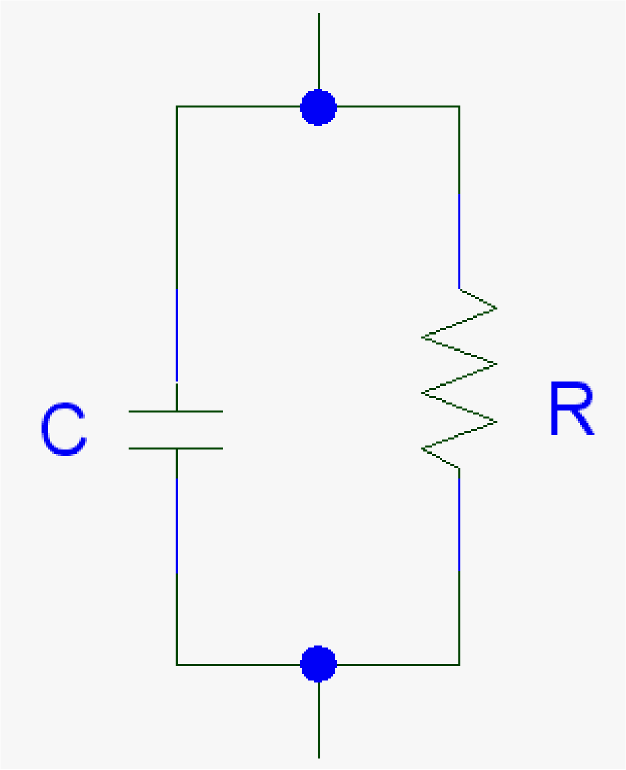
$$v(t) = \frac{1}{5 \times 10^{-6}}\int_{0}^{t} 20 \times 10^{-3}\,dt' + v(0)$$

Since $v(0) = 0$,

$$v(t) = 4000t \qquad 0 \le t \le 2 \text{ ms}$$

Properties of a Real Capacitor

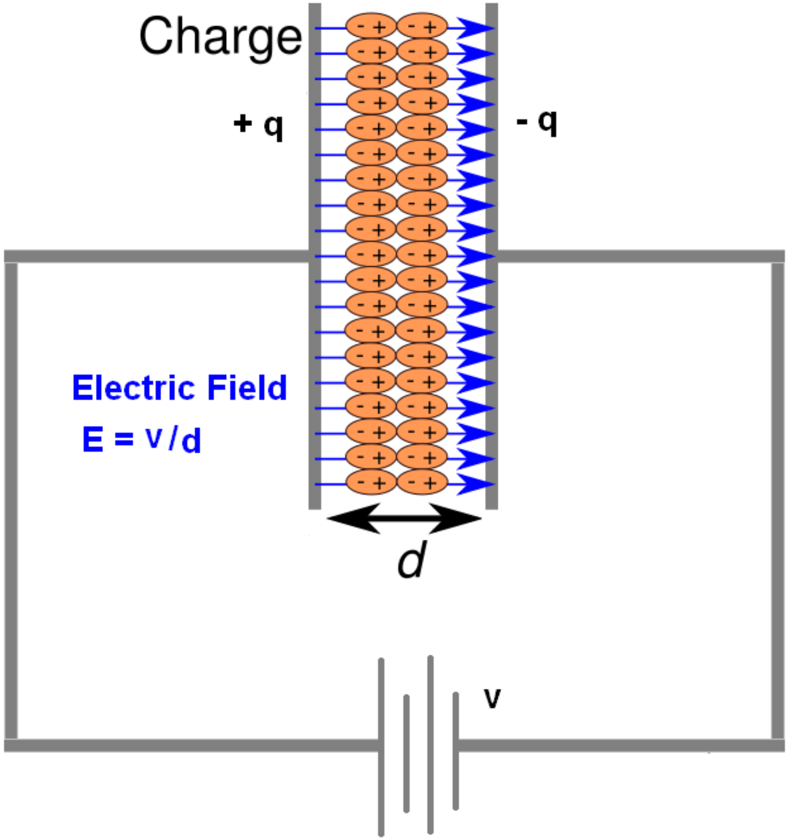
- A real capacitor does dissipate energy due to leakage of charge through its insulator.



  - This is modeled by putting a resistor in parallel with an ideal capacitor.

Energy Storage

- Charge is stored on the plates of the capacitor.



Equation:

$$Q = CV$$

Units:

Coulomb = Farad·Voltage

C = F V

Adding Charge to Capacitor

- The ability to add charge to a capacitor depends on:
  - the amount of charge already on the plates of the capacitor

  and

  - the force (voltage) driving the charge towards the plates (i.e., current)

Charging a Capacitor

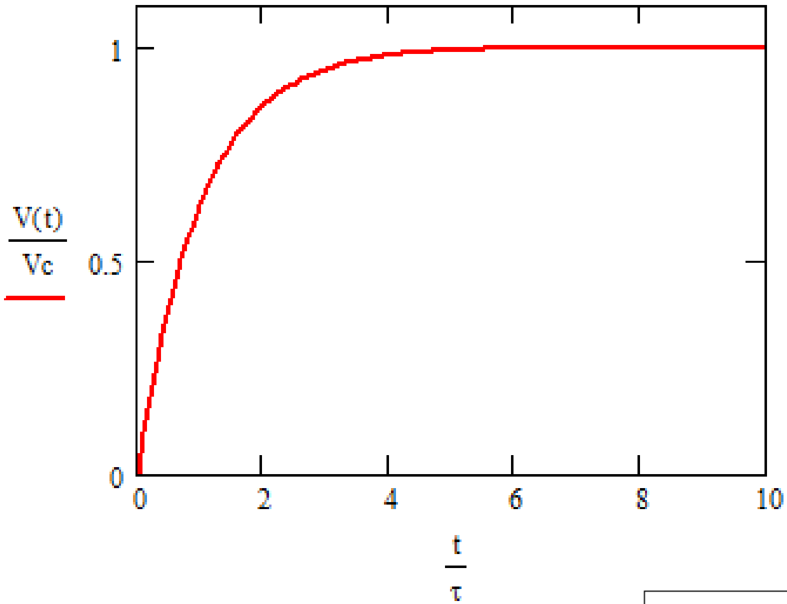
- At first, it is easy to store charge in the capacitor.
- As more charge is stored on the plates of the capacitor, it becomes increasingly difficult to place additional charge on the plates.
  - Coulombic repulsion from the charge already on the plates creates an opposing force to limit the addition of more charge on the plates.
    - Voltage across a capacitor increases rapidly as charge is moved onto the plates when the initial amount of charge on the capacitor is small.
    - Voltage across the capacitor increases more slowly as it becomes difficult to add extra charge to the plates.

Discharging a Capacitor

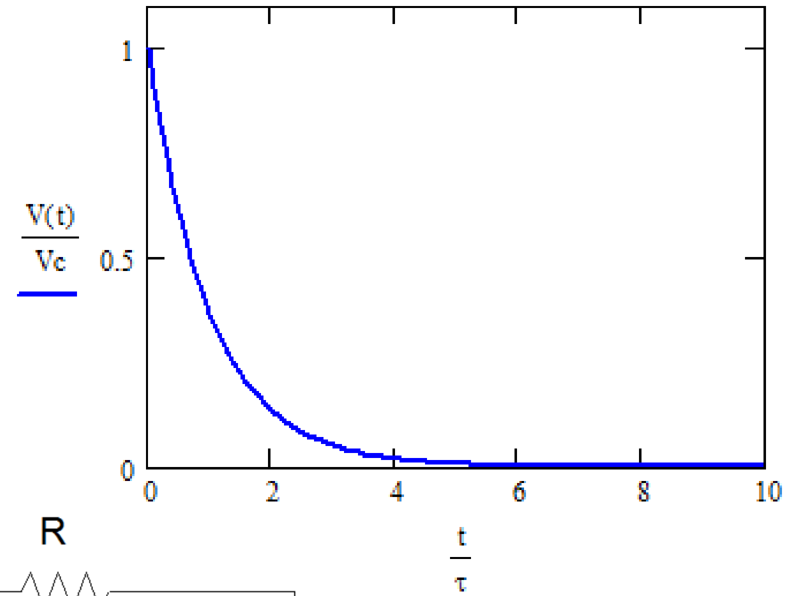
- At first, it is easy to remove charge in the capacitor.
  - Coulombic repulsion from the charge already on the plates creates a force that pushes some of the charge out of the capacitor once the force (voltage) that placed the charge in the capacitor is removed (or decreased).
- As more charge is removed from the plates of the capacitor, it becomes increasingly difficult to get rid of the small amount of charge remaining on the plates.
  - Coulombic repulsion decreases as the charge spreads out on the plates. As the amount of charge decreases, the force needed to drive the charge off of the plates decreases.
    - Voltage across a capacitor decreases rapidly as charge is removed from the plates when the initial amount of charge on the capacitor is small.
    - Voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.

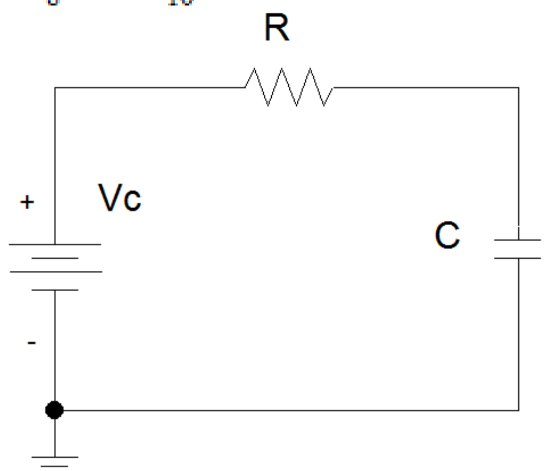
Capacitor Voltage vs. Time

d.c. voltage, Vc, is applied at t = 0s



d.c. voltage, Vc, is removed at t = 0s





Time constant, τ

- The rate at which charge can be added to or removed from the plates of a capacitor as a function of time can be fit to an exponential function.

Charging

$$V(t) = V_c\left(1 - e^{-t/\tau}\right)$$

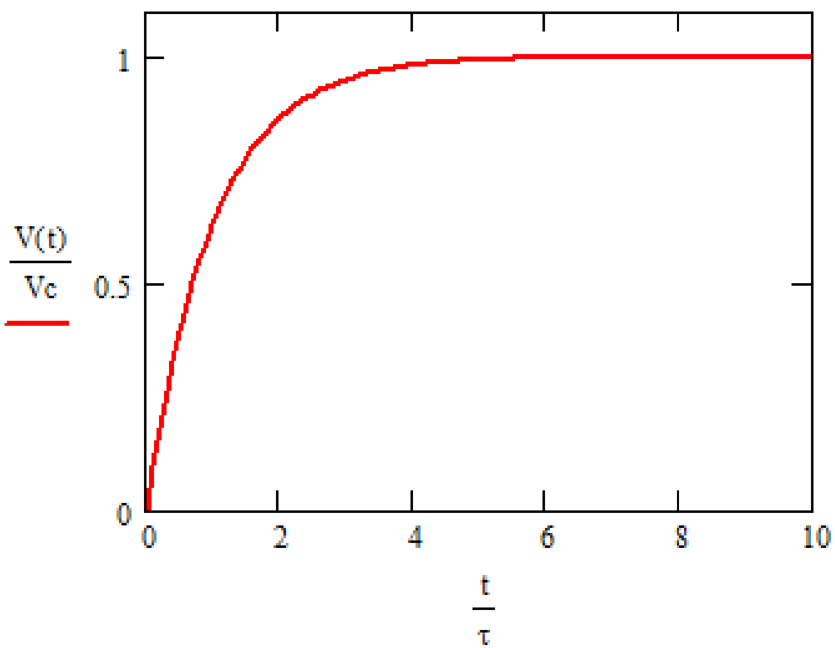
Discharging

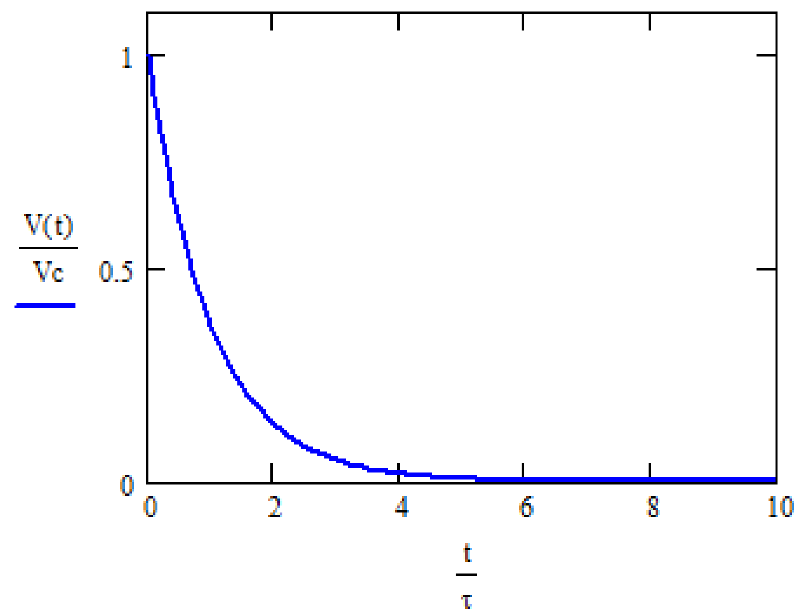
$$V(t) = V_c e^{-t/\tau}$$

$$\tau = RC$$

Transition to steady state

- We approximate that the exponential function reaches its final value when the charging or discharging time is equal to 5τ.





Current-Voltage Relationships

$$i_C = \frac{dq}{dt}$$

$$q = Cv_C$$

$$i_C = C\frac{dv_C}{dt}$$

$$v_C = \frac{1}{C}\int_{t_o}^{t_1} i_C dt$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(\tau)\cdot d\tau + v(t_0)$$

Initial Condition

Power and Energy

$$p = i \cdot v$$

$$i = C\frac{dv}{dt}$$

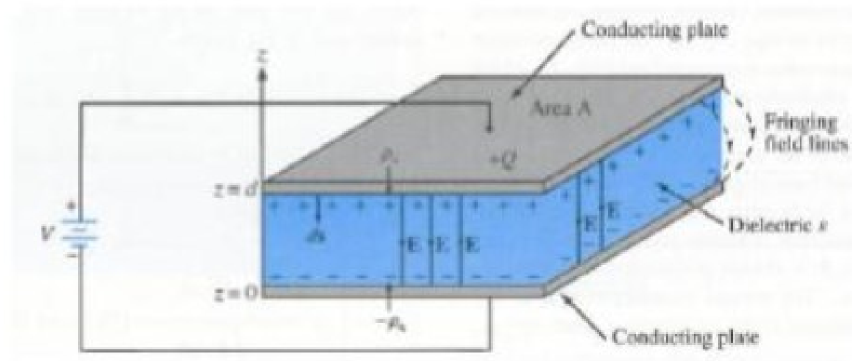
$$w = \int_0^t p(\tau)\cdot d\tau$$

$$= \int_{v(0)}^{v(t)} C\frac{dv}{d\tau}\cdot v \cdot d\tau$$

$$= C\cdot\int_0^t v\cdot dv$$

$$= C\cdot\frac{1}{2}v^2\Big|_0^t$$

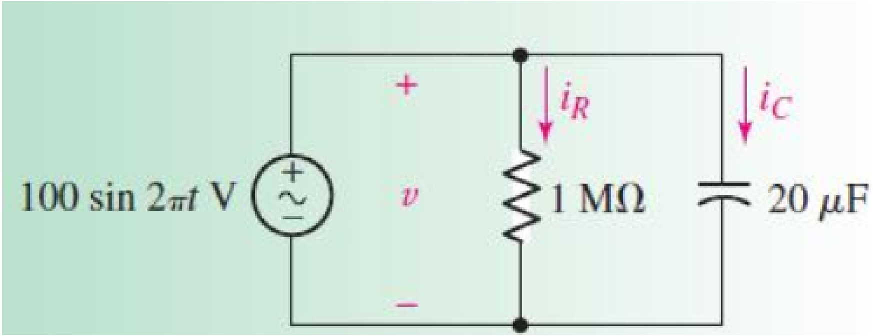
$$= C\cdot\frac{1}{2}\left[v(t)^2 - v(0)^2\right] \rightarrow \boxed{w = \frac{1}{2}C\cdot v^2}$$



electrical energy stored in a capacitor with voltage v across its plates:

Example 2

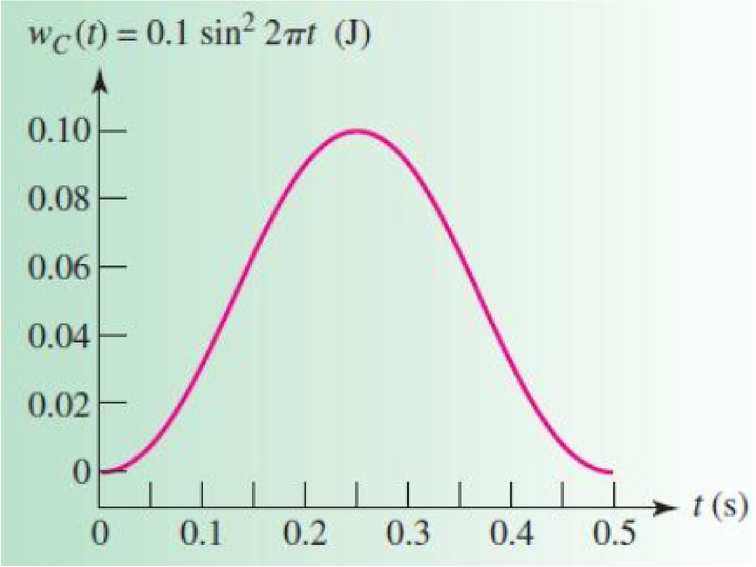
Find the maximum energy stored in the capacitor and the energy dissipated in the resistor over the interval 0 < t < 0.5 s.



$$w_C = \frac{1}{2} C \cdot v^2$$

$$w_C = \frac{1}{2}(20\ \mu\text{F}) \cdot \{100 \sin(2\pi t)\ \text{V}\}^2$$

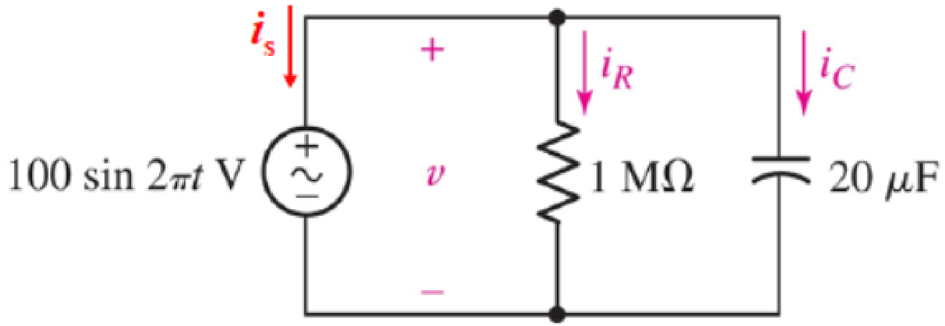
$$= 100 \sin^2(2\pi t)\ \text{mJ}$$



… Example 2 …

$$i_C = C \cdot \frac{dv}{dt} = (20\,\mu) \cdot \frac{d}{dt} 100 \sin(2\pi t)$$

$$= (2\,\text{m}) \cdot 2\pi \cdot \cos(2\pi t) = 4\pi \cdot \cos(2\pi t)\ \text{mA}$$



$$i_R = \frac{v}{R} = \frac{100 \sin(2\pi t)\ \text{V}}{10^6\ \Omega} = 0.1 \sin(2\pi t)\ \text{mA}$$

$$i_s + i_R + i_C = 0$$

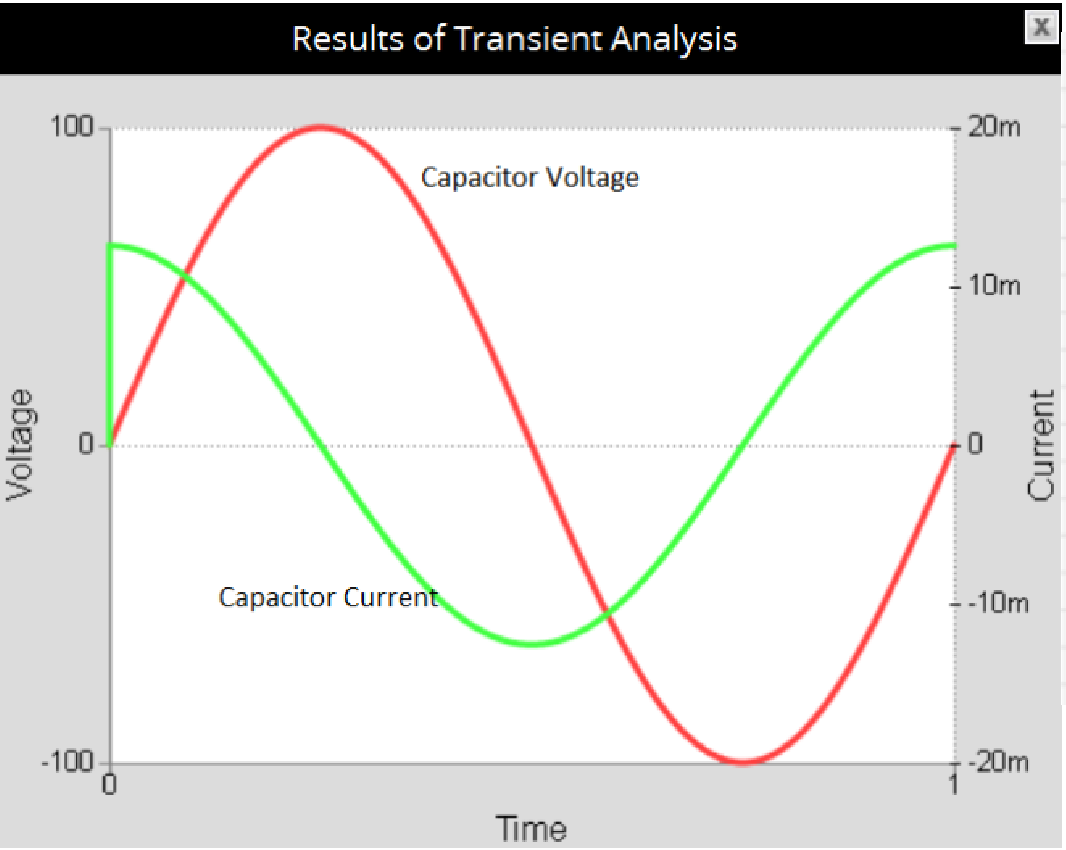
$$i_s = -0.1 \sin(2\pi t) - 4\pi \cdot \cos(2\pi t)\ \text{mA}$$

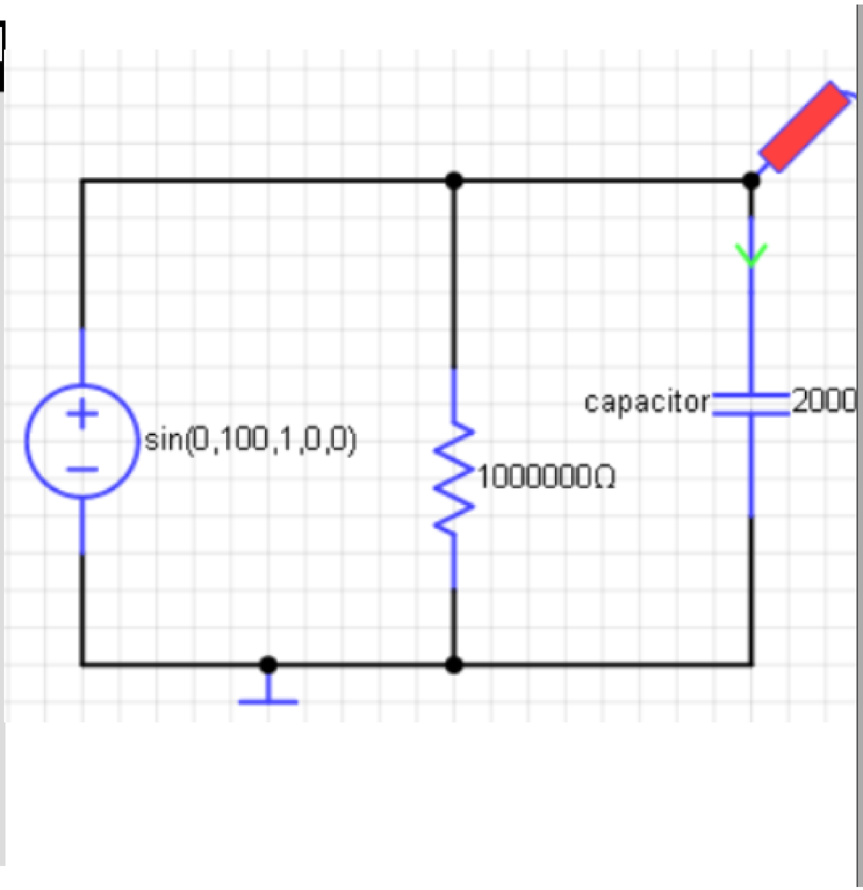
$$p_R = i_R^2 R = (10^{-8})(10^6) \sin^2 2\pi t$$

so that the energy dissipated in the resistor between 0 and 0.5 s is

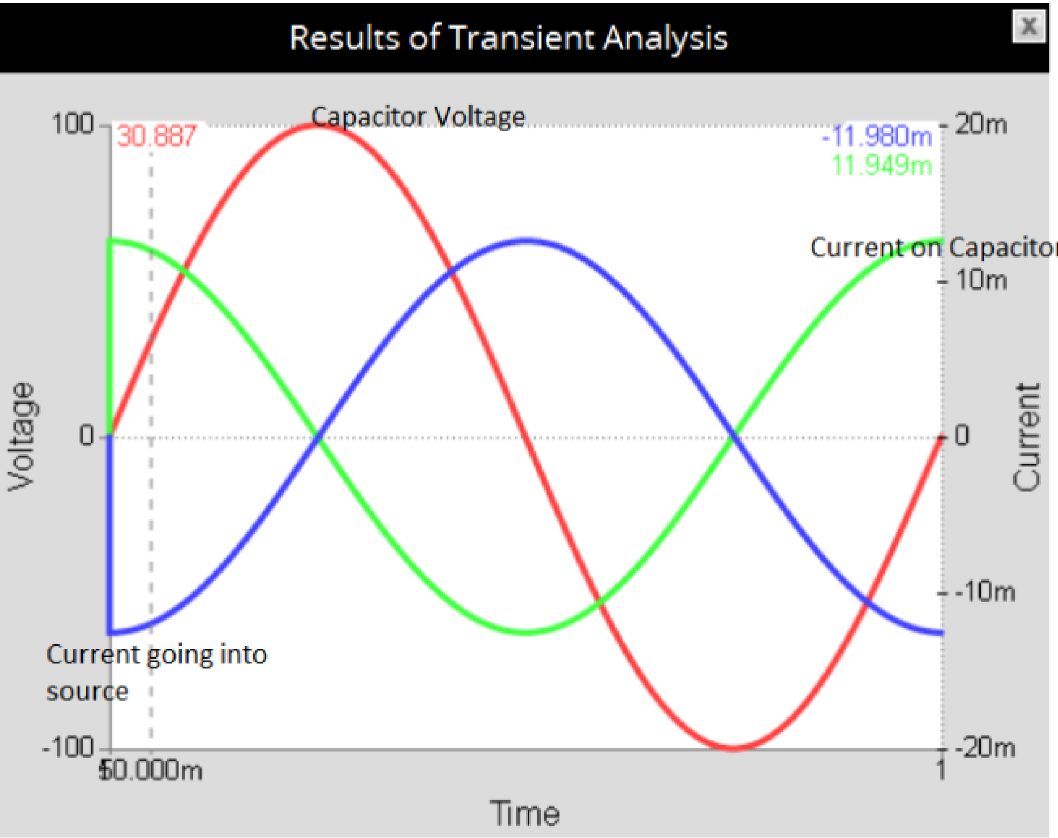
$$w_R = \int_0^{0.5} p_R\, dt = \int_0^{0.5} 10^{-2} \sin^2 2\pi t\, dt \qquad \text{J}$$

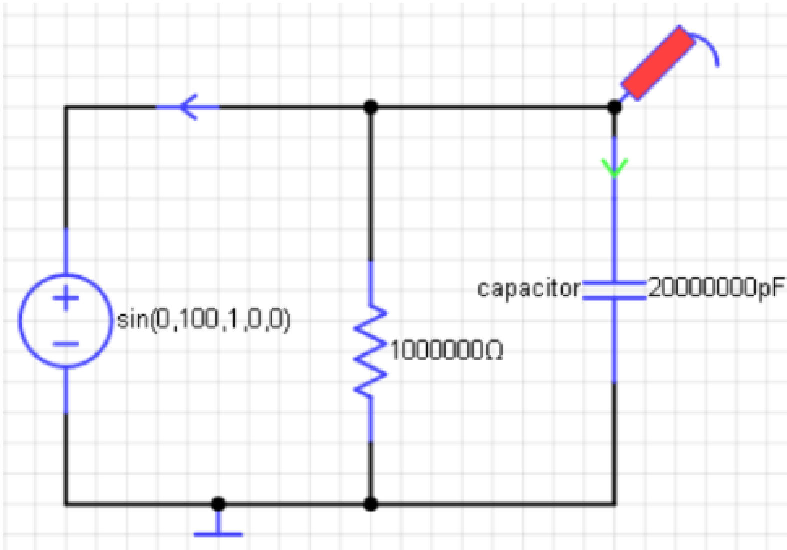
… Example 2 …

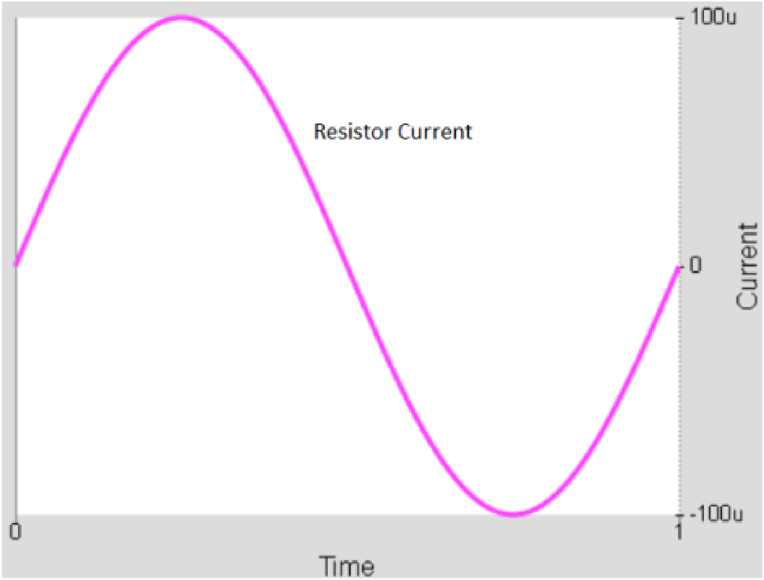

Results of Transient Analysis
100
0
-100
20m
10m
0
-10m
-20m
0
1
Time
Voltage
Current
Capacitor Voltage
Capacitor Current


sin(0,100,1,0,0)
1000000Ω
capacitor
2000

… Example 2

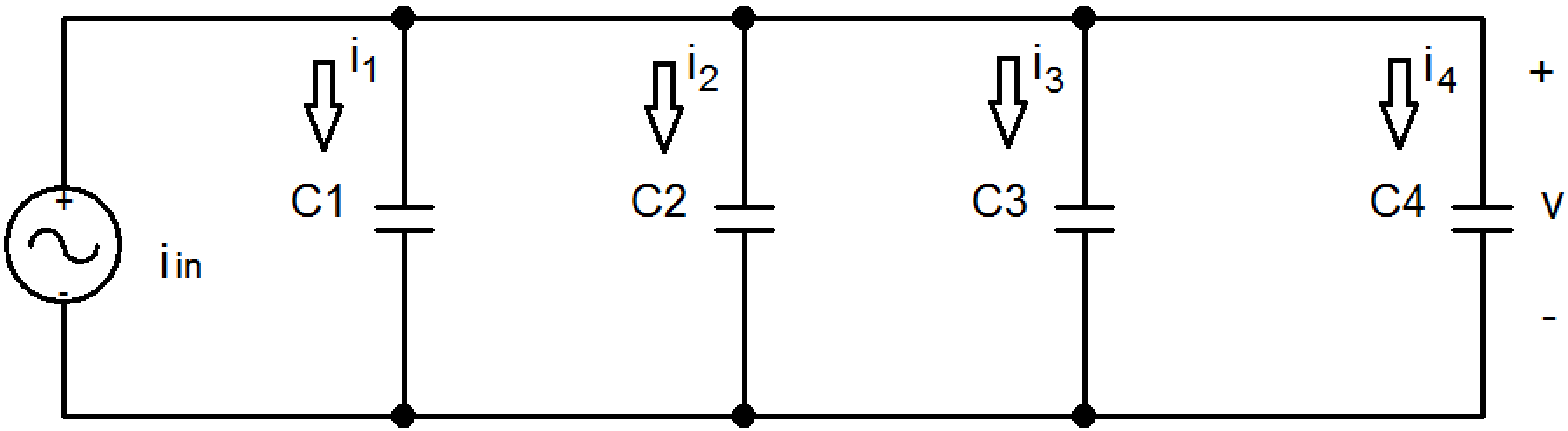

Results of Transient Analysis
Capacitor Voltage
100
30.887
-11.980m
11.949m
20m
Current on Capacitor
10m
Voltage
0
Current
-10m
Current going into source
-100
50.000m
1
-20m
Time


sin(0,100,1,0,0)
1000000Ω
capacitor
20000000pF


100u
Resistor Current
0
Current
-100u
0
1
Time

Equivalent Capacitance

- Capacitors in parallel



C_{eq} for Capacitors in Parallel

$$i_{in} = i_1 + i_2 + i_3 + i_4$$

$$i_1 = C_1 \frac{dv}{dt} \qquad i_2 = C_2 \frac{dv}{dt}$$

$$i_3 = C_3 \frac{dv}{dt} \qquad i \quad i_4 = C_4 \frac{dv}{dt}$$

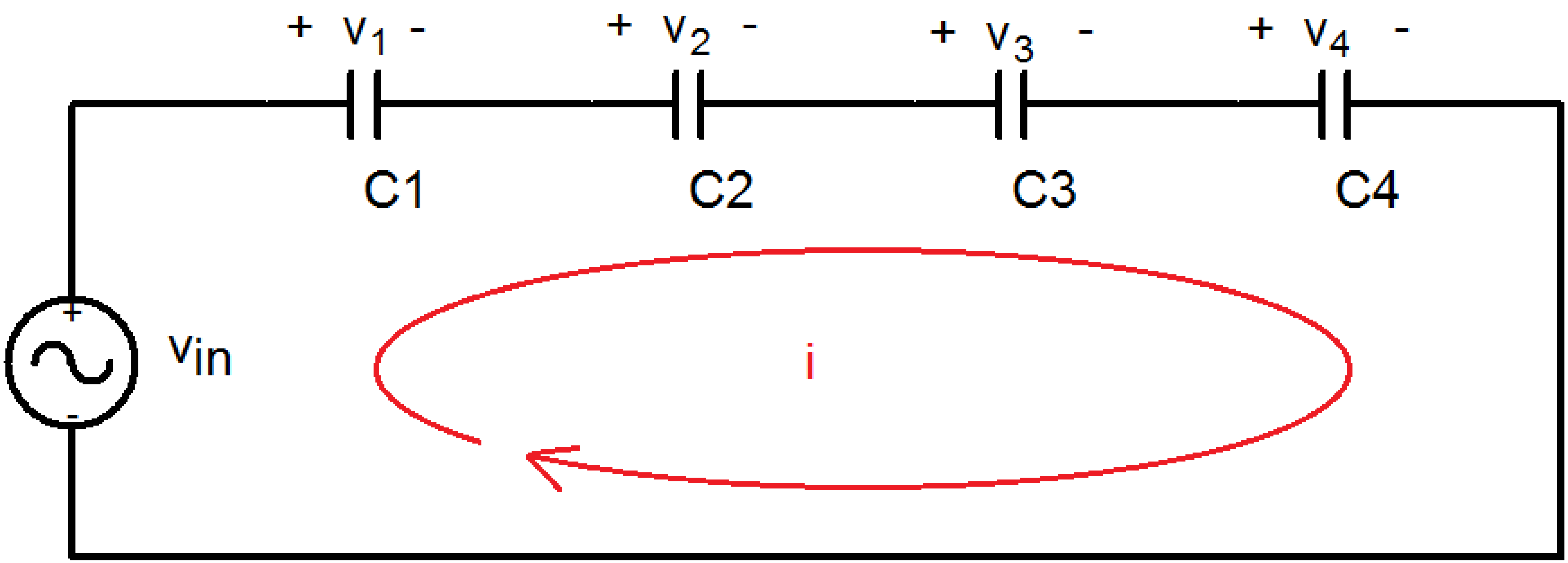
$$i_{in} = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt} + C_4 \frac{dv}{dt}$$

$$i_{in} = C_{eq} \frac{dv}{dt}$$

$$\mathrm{C}_{\mathrm{eq}} = C_1 + C_2 + C_3 + C_4$$

Equivalent Capacitance

- Capacitors in series



C_{eq} for Capacitors in Series

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

$$v_1 = \frac{1}{C_1}\int_{t_o}^{t_1} \mathrm{idt} \qquad v_2 = \frac{1}{C_2}\int_{t_o}^{t_1} \mathrm{idt}$$

$$v_3 = \frac{1}{C_3}\int_{t_o}^{t_1} \mathrm{idt} \qquad i \qquad v_4 = \frac{1}{C_4}\int_{t_o}^{t_1} \mathrm{idt}$$

$$v_{in} = \frac{1}{C_1}\int_{t_o}^{t_1} \mathrm{idt} + \frac{1}{C_2}\int_{t_o}^{t_1} \mathrm{idt} + \frac{1}{C_3}\int_{t_o}^{t_1} \mathrm{idt} + \frac{1}{C_4}\int_{t_o}^{t_1} \mathrm{idt}$$

$$v_{in} = \frac{1}{C_{eq}}\int_{t_o}^{t_1} \mathrm{idt}$$

$$\mathrm{C_{eq}} = \left[\left(1/C_1\right) + \left(1/C_2\right) + \left(1/C_3\right) + \left(1/C_4\right)\right]^{-1}$$

General Equations for C_{eq}

Parallel Combination

- If P capacitors are in parallel, then

$$C_{eq} = \sum_{p=1}^{P} C_P$$

Series Combination

- If S capacitors are in series, then:

$$C_{eq} = \left[\sum_{s=1}^{S} \frac{1}{C_s} \right]^{-1}$$

Summary

- Capacitors are energy storage devices.
- An ideal capacitor act like an open circuits when a DC voltage or current has been applied for at least 5 τ.
- The voltage across a capacitor must be a continuous function; the current flowing across a capacitor can be discontinuous.
- The equation for equivalent capacitance for

capacitors in parallel

$$C_{eq} = \sum_{p=1}^{P} C_P$$

capacitors in series

$$C_{eq} = \left[\sum_{s=1}^{S} \frac{1}{C_s} \right]^{-1}$$

Inductors

Energy Storage Devices

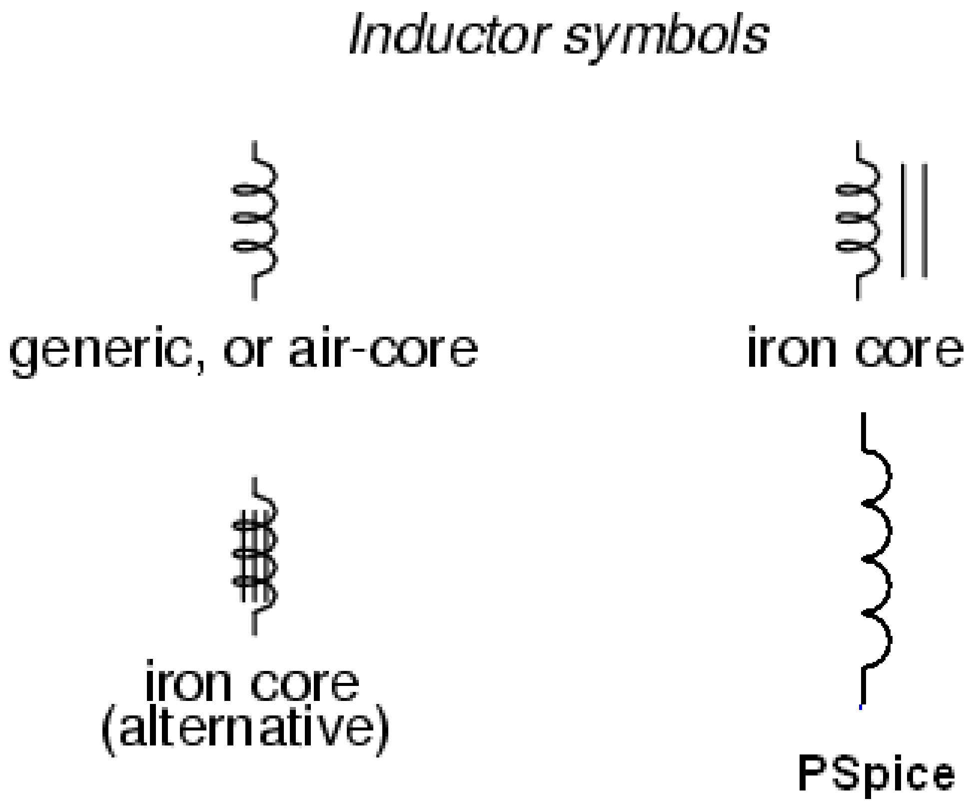
Inductors

- Generally - coil of conducting wire

  - Usually wrapped around a solid core.
  - If no core is used, then the inductor is said to have an 'air core'.

http://bzupages.com/f231/energy-stored-inductor-uzma-noreen-group6-part2-1464/

Symbols



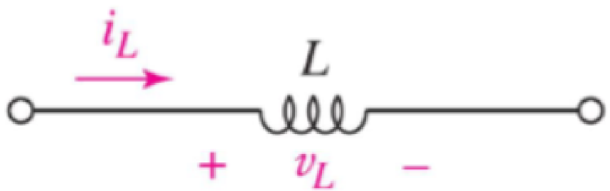
http://www.allaboutcircuits.com/vol_1/chpt_15/1.html

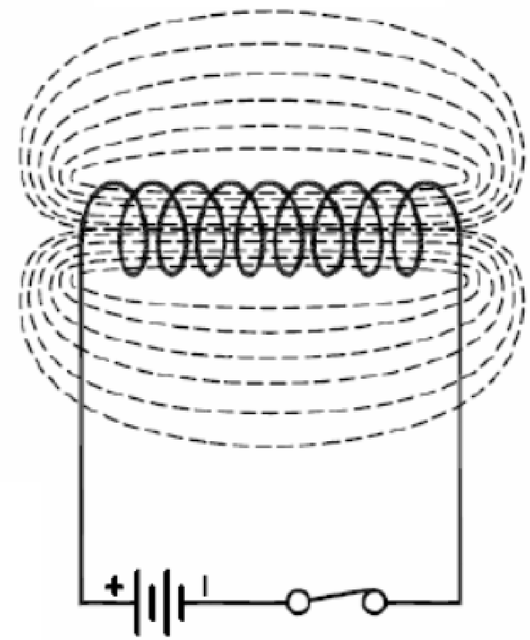
Alternative Names for Inductors

- Reactor
  - inductor in a power grid
- Choke
  - designed to block a particular frequency while allowing currents at lower frequencies or d.c. currents through
    - Commonly used in RF (radio frequency) circuitry
- Coil
  - often coated with varnish and/or wrapped with insulating tape to provide additional insulation and secure them in place
    - A winding is a coil with taps (terminals).
- Solenoid
  - a three dimensional coil.
    - Also used to denote an electromagnet where the magnetic field is generated by current flowing through a toroidal inductor.

Charging - Discharging

An **inductor** is a linear circuit element which stores energy in the **magnetic field** in the space between current-carrying wires occupied by a material with permeability μ .





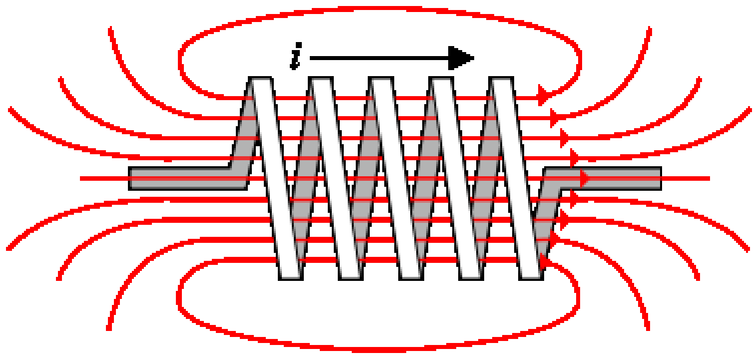
- *charged* by applying current (for a finite amount of time, from another source) to its terminals

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\cdot d\tau + i(t_0)$$

- *discharged* when it provides current (for a finite amount of time, to a circuit) from its terminals

Energy Storage

- The flow of current through an inductor creates a magnetic field (right hand rule).



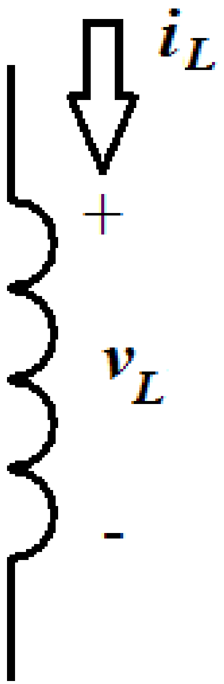
B field

http://en.wikibooks.org/wiki/Circuit_Theory/Mutual_Inductance

- If the current flowing through the inductor drops, the magnetic field will also decrease and energy is released through the generation of a current.

Sign Convention

- The sign convention used with an inductor is the same as for a power dissipating device.



  - When current flows into the positive side of the voltage across the inductor, it is positive and the inductor is dissipating power.
  - When the inductor releases energy back into the circuit, the sign of the current will be negative.

Current and Voltage Relationships

- L , inductance, has the units of Henries (H)

  1 H = 1 V-s/A

$$v_L = L\frac{di}{dt}$$

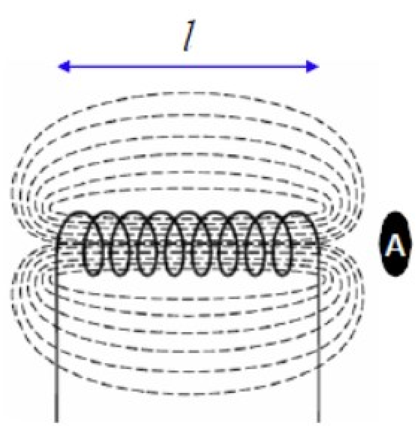
$$i_L = \frac{1}{L}\int_{t_o}^{t_1} v_L dt$$

Current and Voltage Relationships

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(\tau)\cdot d\tau + i(t_0)$$

same equation

$$v(t) = L\frac{di(t)}{dt}$$



$$L = \mu\frac{N^2 A}{l}$$

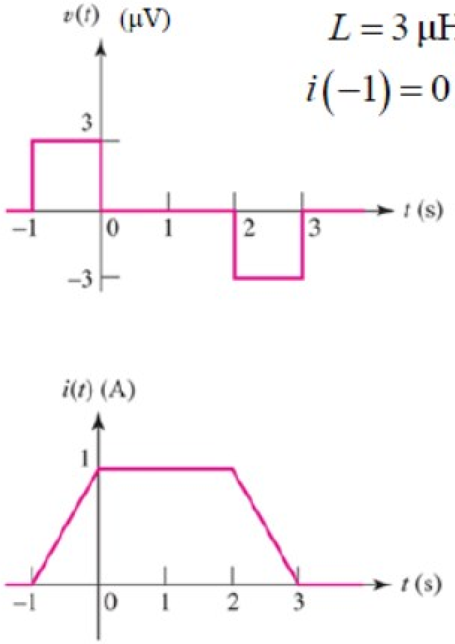
N = # of turns

μ = permeability

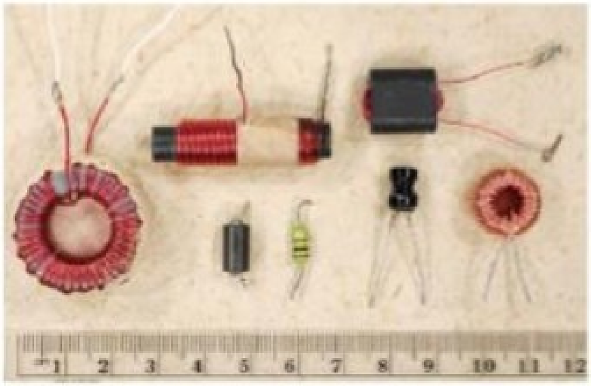
unit of inductance = henry
1 H = 1 V-s / A

$L = 3\ \mu\text{H}$

$i(-1) = 0\ \text{A}$

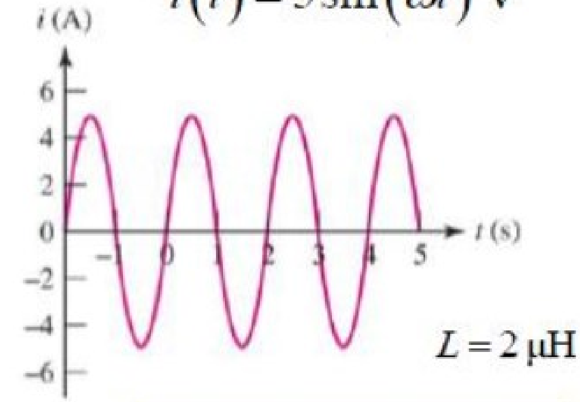


Inductor Characteristics

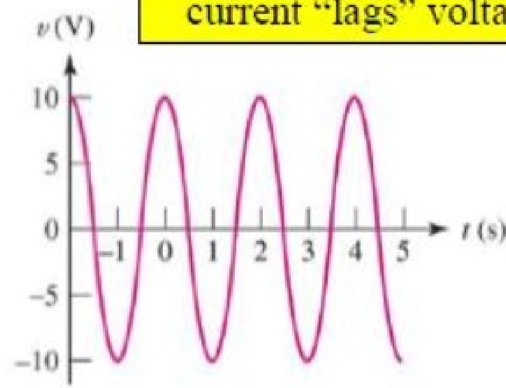


$$v = L\frac{di}{dt}$$

$$i(t) = 5\sin(\omega t)\ \text{V}$$



voltage "leads" current,
current "lags" voltage



$$v(t) = 10\cos(\omega t)\ \mu\text{A}$$

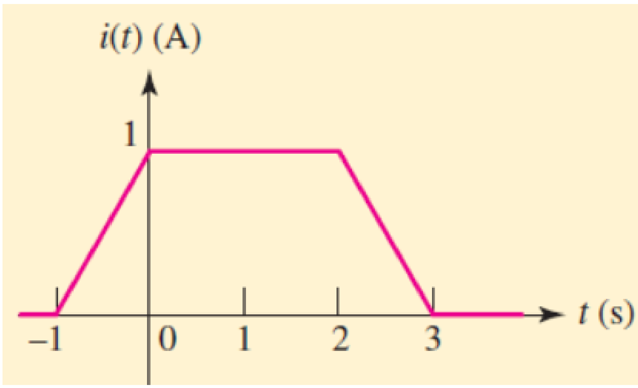
similar to the capacitor, but the relationships between voltage & current are reversed

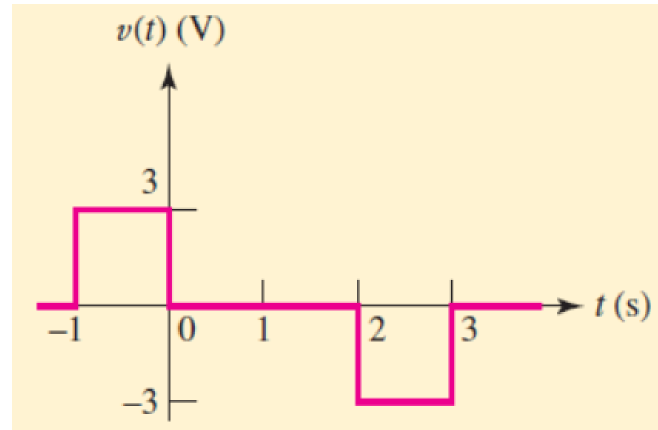
$$v_L = L\frac{di_L}{dt}$$

$$i_C = C\frac{dv_C}{dt}$$

Example 3

- Given the waveform of the current in a 3 H inductor as shown in below Figure. Determine the inductor voltage and sketch it.





$$v = 3\frac{di}{dt}$$

t < -1 sec i = 0 ampere

0 sec < t < 2 sec i = 1 ampere

t > 3 sec i = 0 ampere

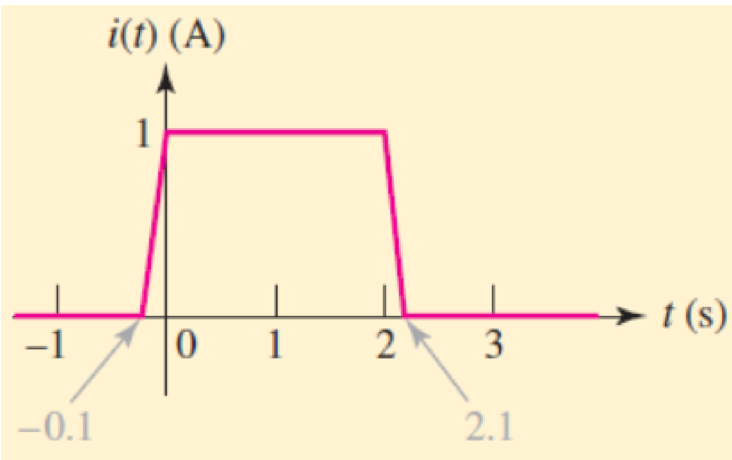
the current is constant, v = 0

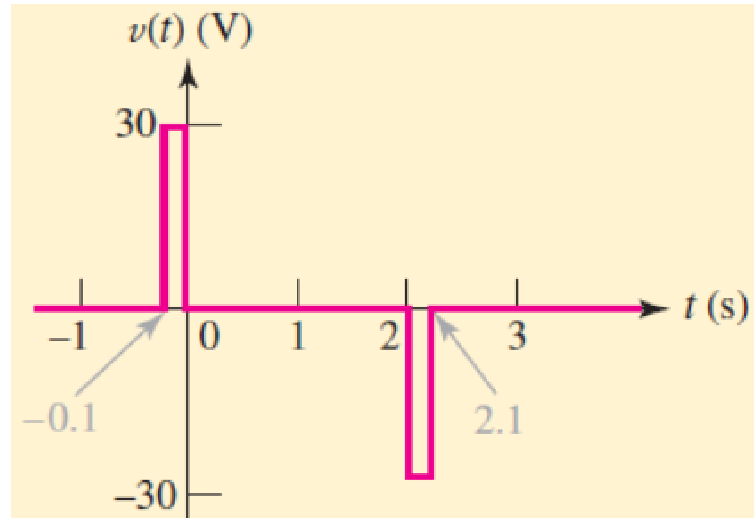
-1 sec <= t <= 0 sec di/dt = 1 A/s
v = 3 volt

2 sec <= t <= 3 sec di/dt = -1 A/s
v = -3 volt

…Example 3…

• Given the waveform of the current in a 3 H inductor as shown in below Figure. Determine the inductor voltage and sketch it.



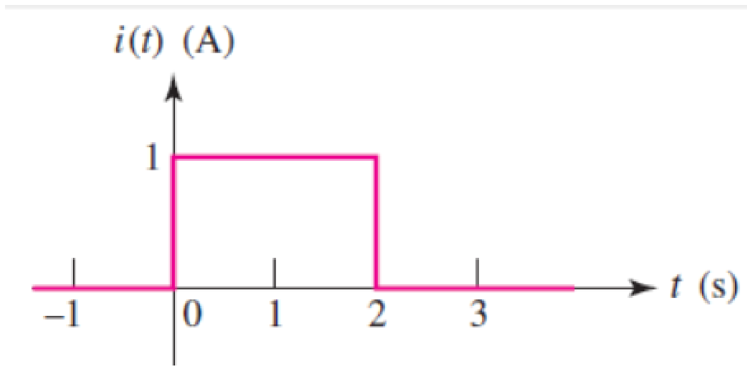


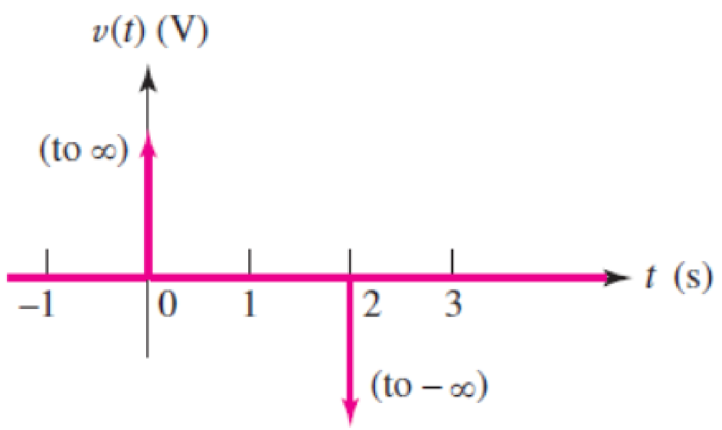
• The intervals for the rise and fall have decreased to 0.1 s. Thus, the magnitude of each derivative will be 10 times larger.

• It is interesting to note that the area under each voltage pulse is 3 V× s.

…Example 3

- A further decrease in the rise and fall times of the current waveform will produce a proportionally larger voltage magnitude, but only within the interval in which the current is increasing or decreasing.
- An abrupt change in the current will cause the infinite voltage "spikes" (each having an area of 3 V×s)





- This is useful in the **ignition system of some automobiles**, where the current through the spark coil is interrupted by the distributor and the arc appears across the spark plug.

Power and Energy

$$p = i \cdot v$$

$$v = L\frac{di}{dt}$$

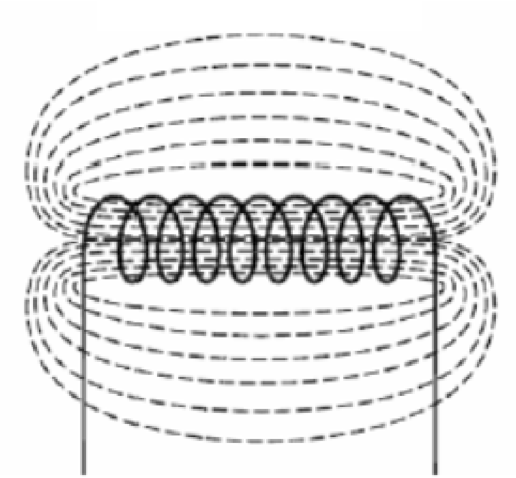
$$w = \int_0^t p(\tau)\cdot d\tau$$

$$= \int_{v(0)}^{v(t)} i \cdot L\frac{di}{d\tau}\cdot d\tau$$

$$= L \cdot \int_0^t i \cdot di$$

$$= L \cdot \frac{1}{2} i^2 \Big|_0^{\mathrm{t}}$$

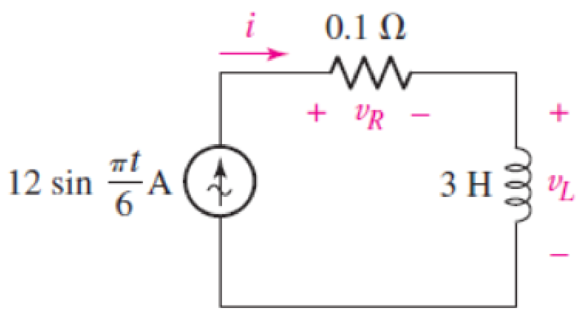
$$= L \cdot \frac{1}{2}\left[i(t)^2 - i(0)^2 \right] \rightarrow \boxed{w = \frac{1}{2} L \cdot i^2}$$



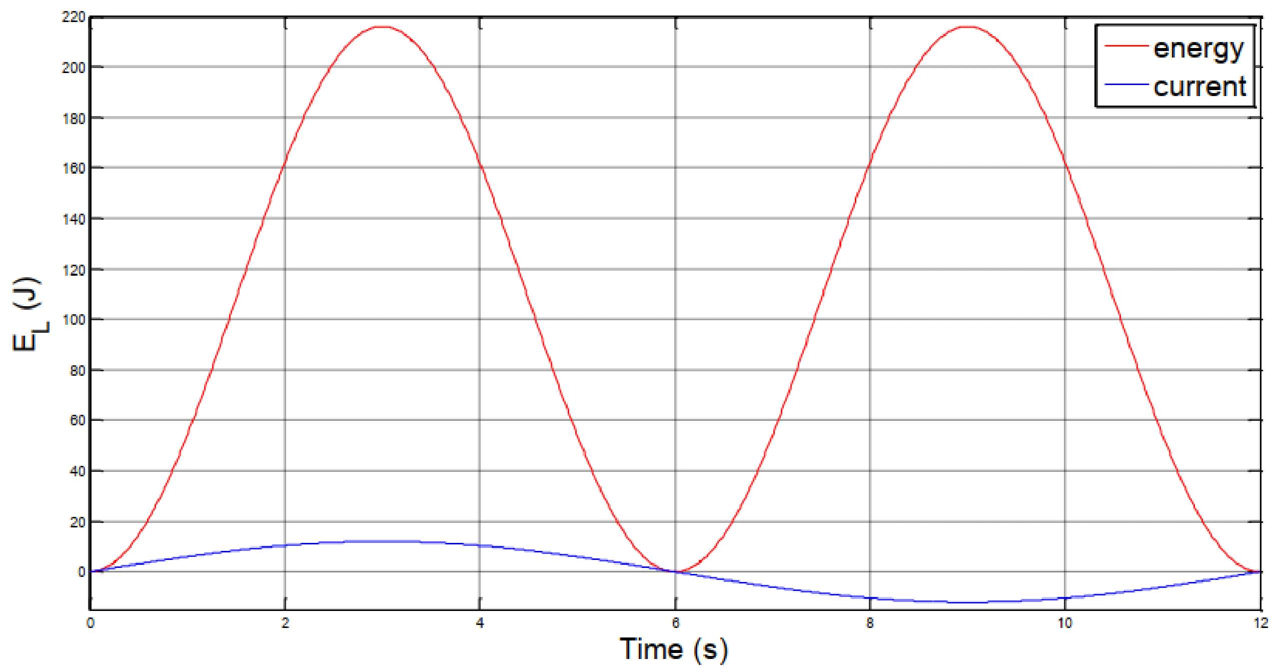
magnetic energy stored in an inductor with current *i* through its coils:

Example 4

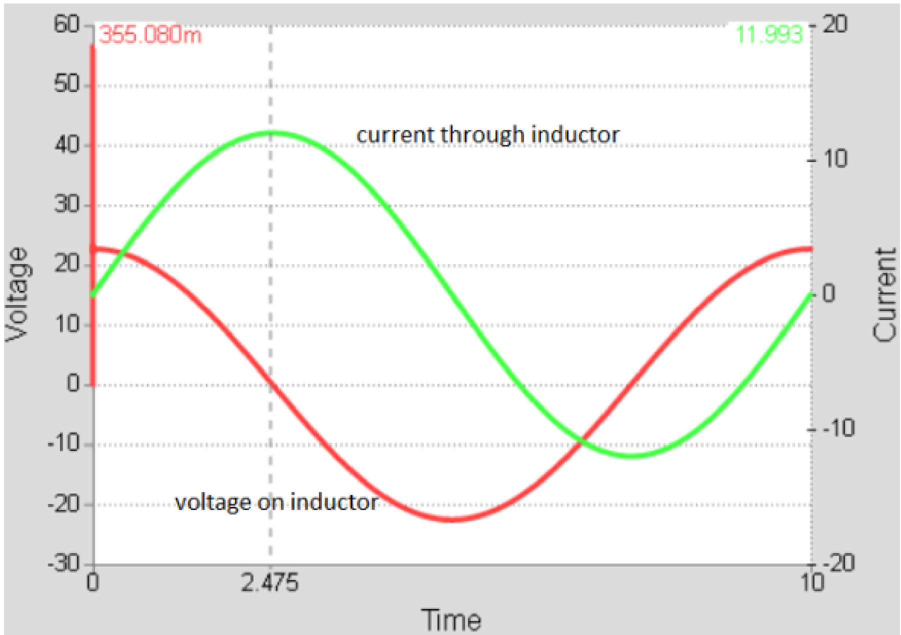
- Find the maximum energy stored in the and calculate how much energy is dissipated in the resistor in the time during which the energy is being stored in, and then recovered from, the inductor.

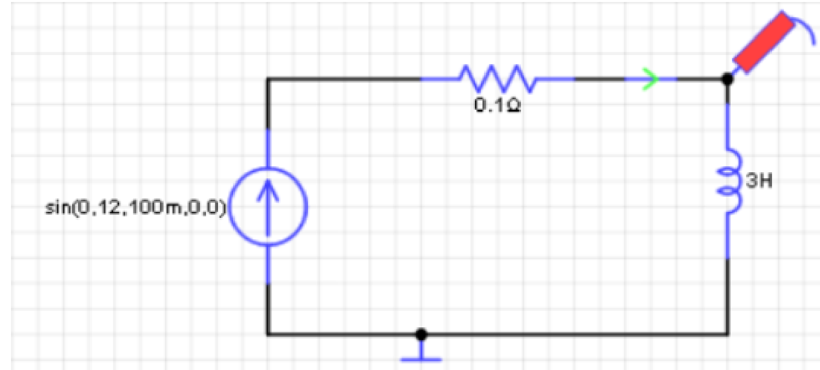


$$w_L = \frac{1}{2}Li^2 = 216 \sin^2 \frac{\pi t}{6} \quad \text{J}$$



…Example 4…





Current and voltage has 90 degrees phase shift. Voltage **is leading** the current.

…Example 4

- The power dissipated in the resistor is easily found as

$$p_R = i^2 R = 14.4 \sin^2 \frac{\pi t}{6} \qquad \text{W}$$

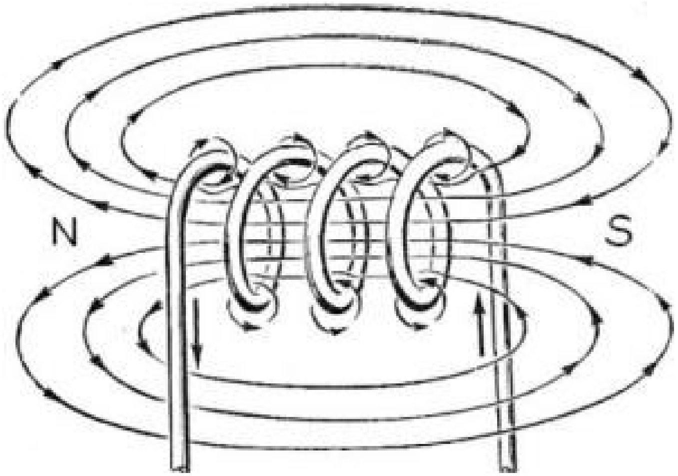
- the energy converted into heat in the resistor within this 6 s interval is

$$w_R = \int_0^6 p_R \, dt = \int_0^6 14.4 \sin^2 \frac{\pi}{6} t \, dt$$

$$w_R = \int_0^6 14.4 \left(\frac{1}{2}\right)\left(1 - \cos \frac{\pi}{3} t\right) dt = 43.2 \text{ J}$$

Inductors

- Stores energy in an magnetic field created by the electric current flowing through it.
  - Inductor opposes change in current flowing through it.
    - Current through an inductor is continuous; voltage can be discontinuous.



http://www.rfcafe.com/references/electrical/Electricity%20-%20Basic%20Navy%20Training%20Courses/electricity%20-%20basic%20navy%20training%20courses%20-%20chapter%2012.htm

Calculations of L

- For a solenoid (toroidal inductor)

$$L = \frac{N^2 \mu A}{\ell} = \frac{N^2 \mu_r \mu_o A}{\ell}$$

N is the number of turns of wire

A is the cross-sectional area of the toroid in m^2.

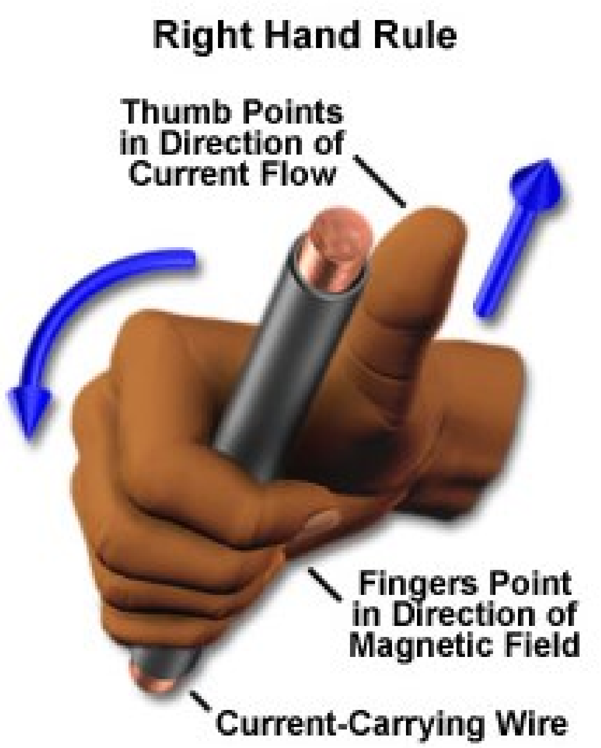
μ_r is the relative permeability of the core material

μ_o is the vacuum permeability ($4\pi \times 10^{-7}$ H/m)

ℓ is the length of the wire used to wrap the toroid in meters

Wire

- Unfortunately, even bare wire has inductance.



$$L = \ell\left[\ln\left(4\frac{\ell}{d}\right)-1\right]\left(2x10^{-7}\right)H$$

  - d is the diameter of the wire in meters.

Properties of an Inductor

1. There is no voltage across an inductor if the current through it is not changing with time. An inductor is therefore a ***short circuit*** to dc.

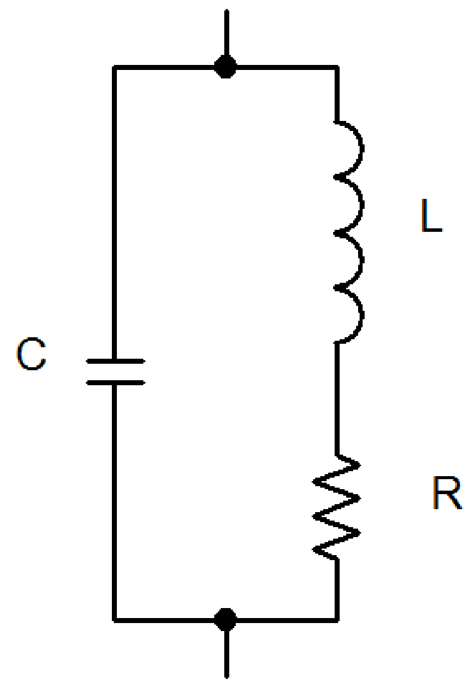
2. A finite amount of energy can be stored in an inductor even if the voltage across the inductor is zero, such as when the current through it is constant.

3. It is impossible to change the current through an inductor by a finite amount in zero time, for this requires an infinite voltage across the inductor.

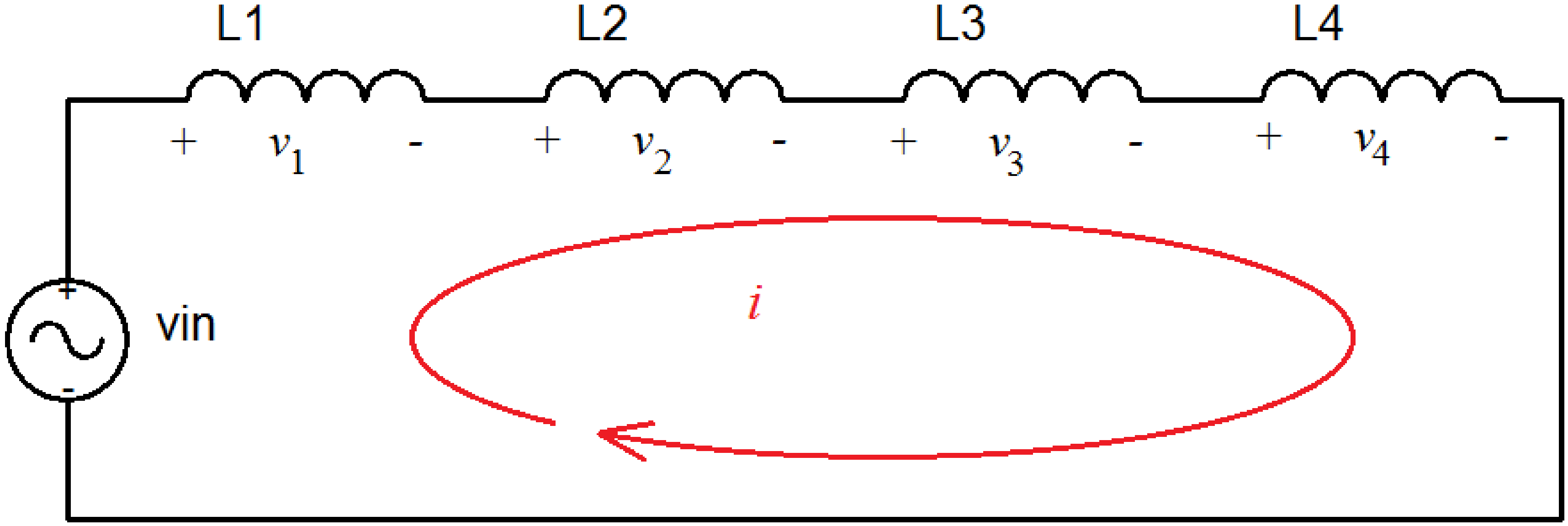
4. The inductor never dissipates energy, but only stores it. Although this is true for the *mathematical model, it is not true for a physical* inductor due to series resistances.

Properties of a Real Inductor

- Real inductors do dissipate energy due to resistive losses in the length of wire and capacitive coupling between turns of the wire.



Inductors in Series



L_{eq} for Inductors in Series

$$v_{in} = v_1 + v_2 + v_3 + v_4$$

$$v_1 = L_1 \frac{\mathrm{di}}{\mathrm{dt}} \qquad v_2 = L_2 \frac{\mathrm{di}}{\mathrm{dt}}$$

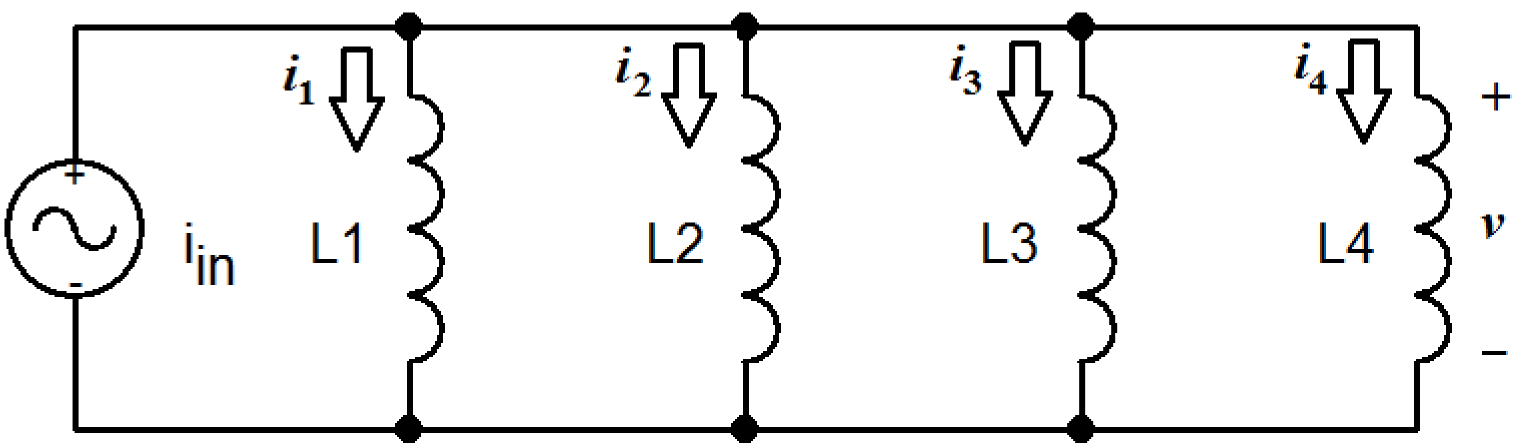
$$v_3 = L_3 \frac{\mathrm{di}}{\mathrm{dt}} \qquad i \qquad v_4 = L_4 \frac{\mathrm{di}}{\mathrm{dt}}$$

$$v_{in} = L_1 \frac{\mathrm{di}}{\mathrm{dt}} + L_2 \frac{\mathrm{di}}{\mathrm{dt}} + L_3 \frac{\mathrm{di}}{\mathrm{dt}} + L_4 \frac{\mathrm{di}}{\mathrm{dt}}$$

$$v_{in} = L_{eq} \frac{\mathrm{di}}{\mathrm{dt}}$$

$$\mathrm{L_{eq}} = L_1 + L_2 + L_3 + L_4$$

Inductors in Parallel



L_{eq} for Inductors in Parallel

$$i_{in} = i_1 + i_2 + i_3 + i_4$$

$$i_1 = \frac{1}{L_1}\int_{t_o}^{t_1} vdt \qquad i_2 = \frac{1}{L_2}\int_{t_o}^{t_1} vdt$$

$$i_3 = \frac{1}{L_3}\int_{t_o}^{t_1} vdt \qquad {}_{i}\, i_4 = \frac{1}{L_4}\int_{t_o}^{t_1} vdt$$

$$i_{in} = \frac{1}{L_1}\int_{t_o}^{t_1} vdt + \frac{1}{L_2}\int_{t_o}^{t_1} vdt + \frac{1}{L_3}\int_{t_o}^{t_1} vdt + \frac{1}{L_4}\int_{t_o}^{t_1} vdt$$

$$i_{in} = \frac{1}{L_{eq}}\int_{t_o}^{t_1} vdt$$

$$L_{eq} = \left[\left(1/L_1\right) + \left(1/L_2\right) + \left(1/L_3\right) + \left(1/L_4\right)\right]^{-1}$$

General Equations for L_{eq}

Series Combination

- If S inductors are in series, then

$$L_{eq} = \sum_{s=1}^{S} L_s$$

Parallel Combination

- If P inductors are in parallel, then:

$$L_{eq} = \left[\sum_{p=1}^{P} \frac{1}{L_p} \right]^{-1}$$

Summary

- Inductors are energy storage devices.
- An ideal inductor act like a short circuit at steady state when a DC voltage or current has been applied.
- The current through an inductor must be a continuous function; the voltage across an inductor can be discontinuous.
- The equation for equivalent inductance for

inductors in series

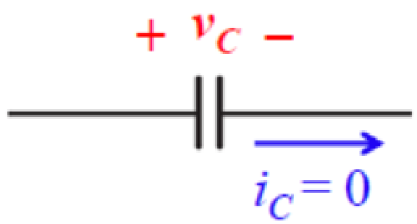
$$L_{eq} = \sum_{s=1}^{S} L_s$$

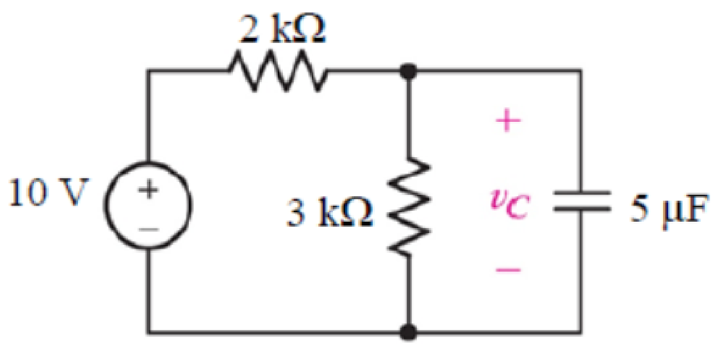
inductors in parallel

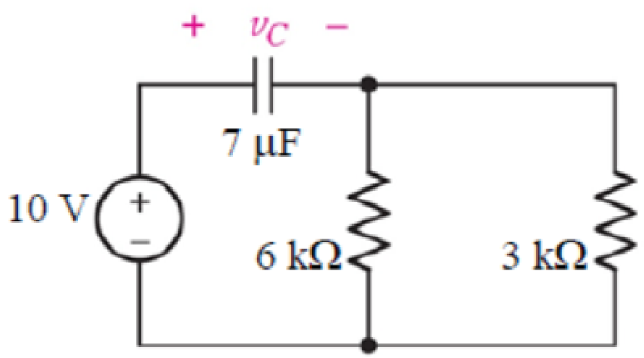
$$L_{eq} = \left[\sum_{p=1}^{P} \frac{1}{L_p} \right]^{-1}$$

Example 5

A **capacitor** that has been sitting in a DC circuit for a long time acts as an **open circuit**.

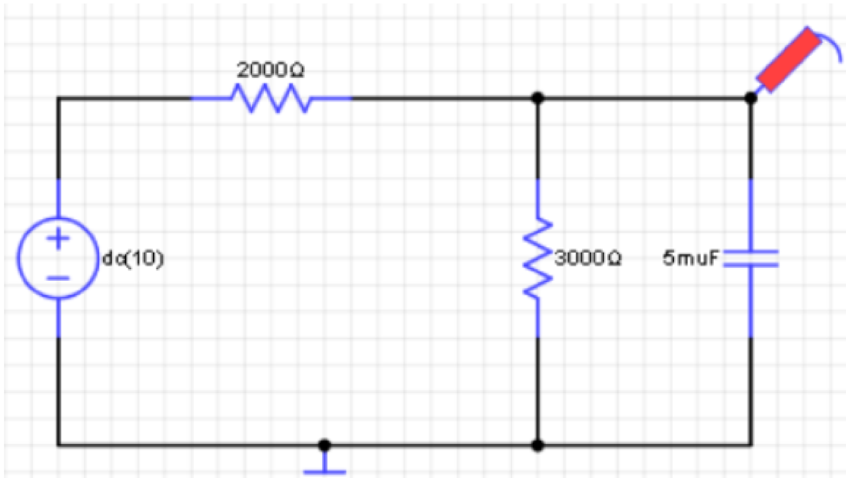


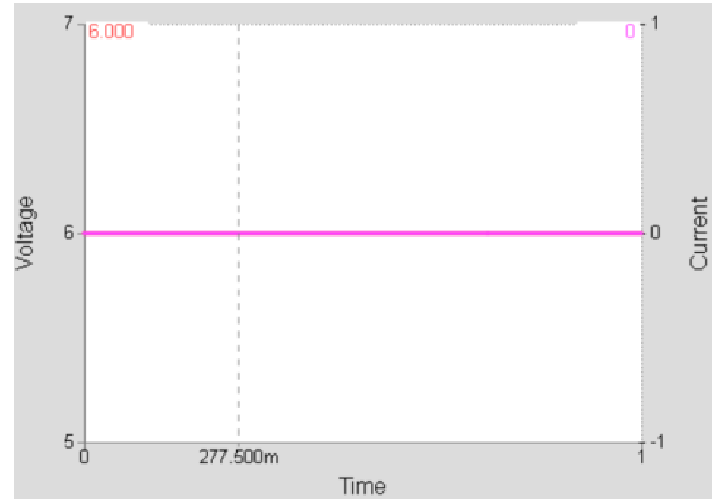


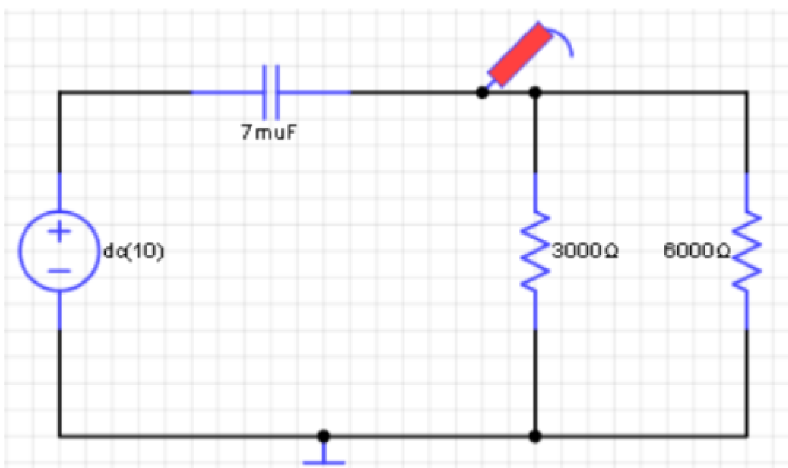


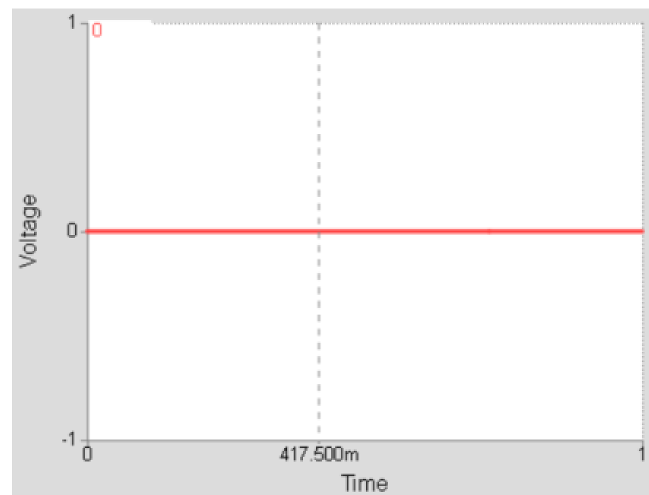
…Example 5

-


2000Ω
dc(10)
3000Ω
5muF

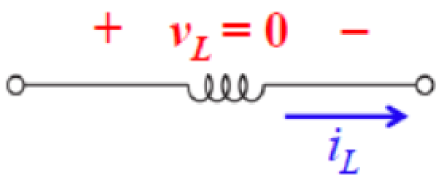

7
6.000
0
1
Voltage
Current
6
0
5
-1
0
277.500m
1
Time

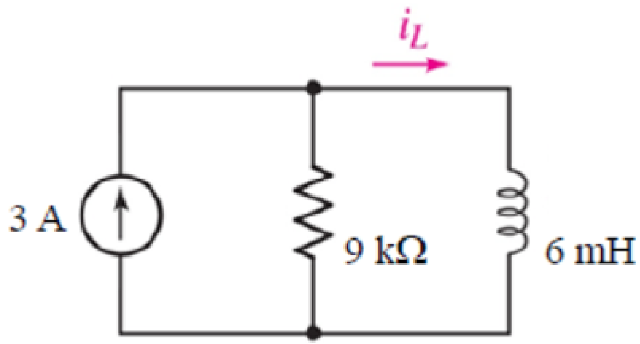

7muF
dc(10)
3000Ω
6000Ω

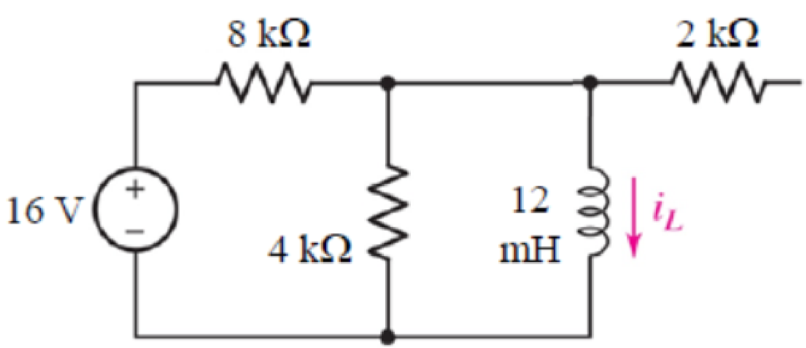

1
0
Voltage
0
-1
0
417.500m
1
Time

Example 6

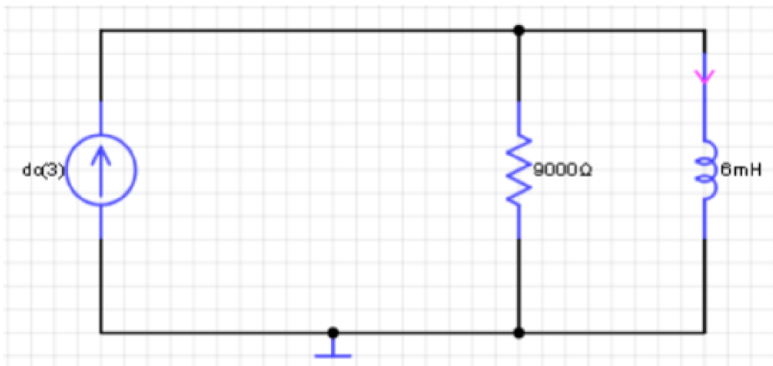
An **inductor** that has been sitting in a DC circuit for a long time acts as a **short circuit**.



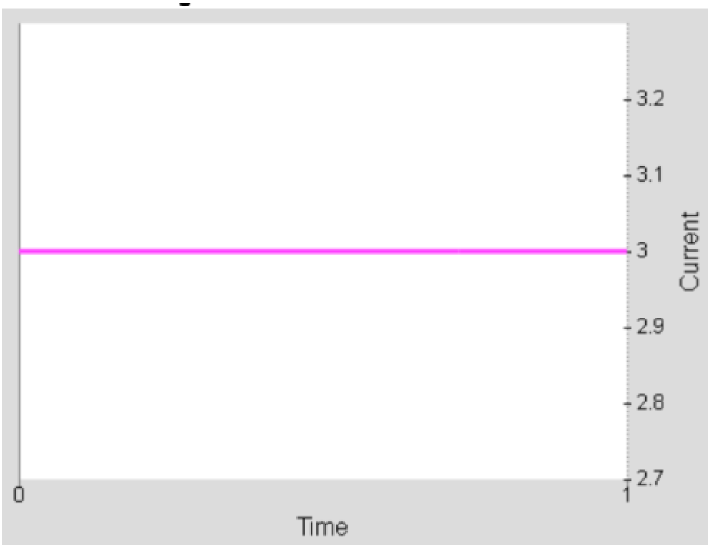




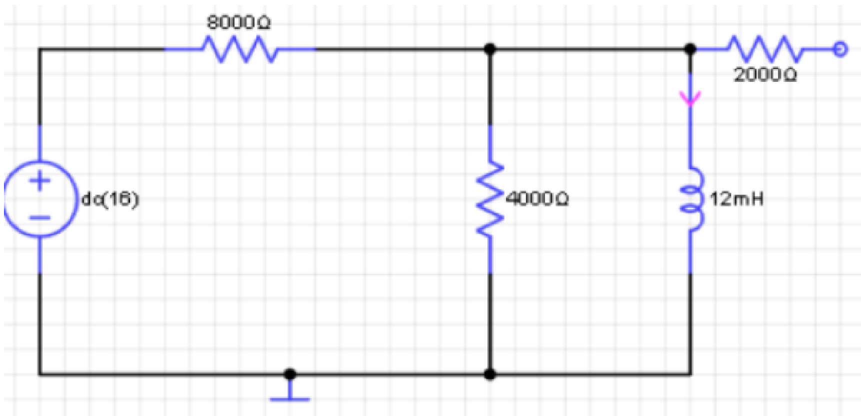
…Example 6



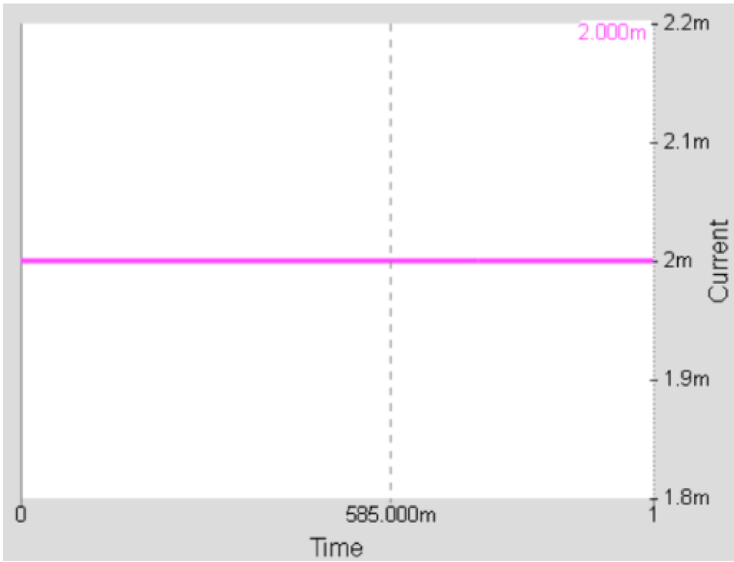
dc(3)
9000Ω
6mH



3.2
3.1
3
2.9
2.8
2.7
Current
0
1
Time



8000Ω
2000Ω
dc(16)
4000Ω
12mH



2.000m
2.2m
2.1m
2m
1.9m
1.8m
Current
0
585.000m
1
Time