Circuit Theory

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AC Sample Questions

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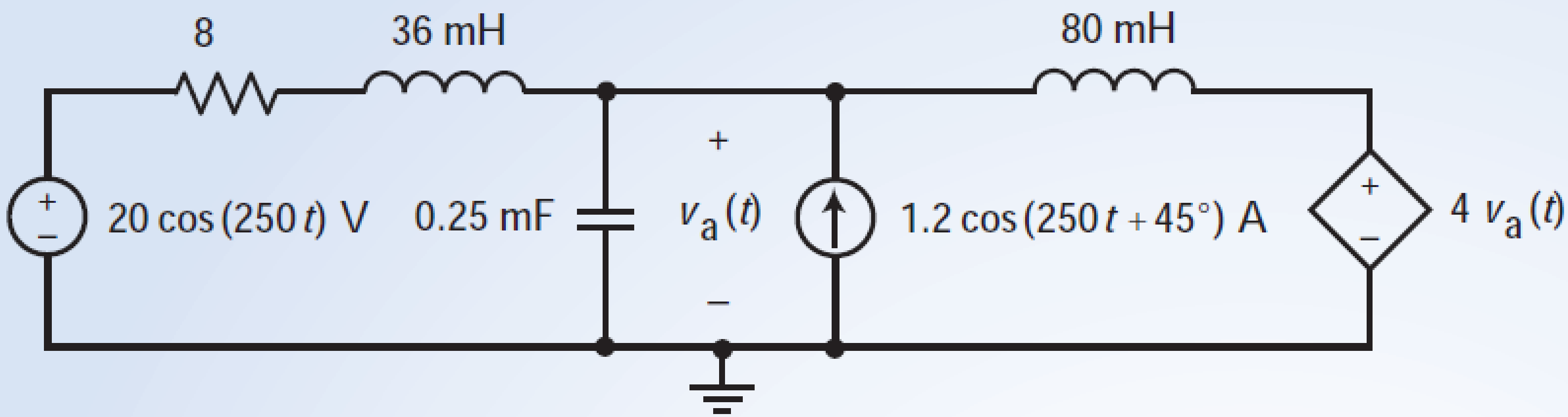
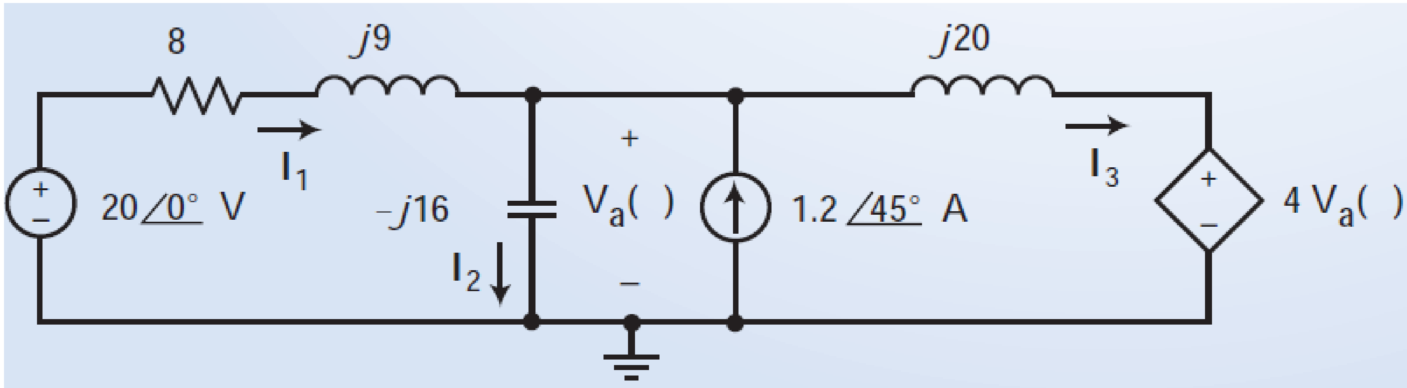
Sample 1

Determine the voltage $v_a(t)$ for the circuit shown in Figure 10.6-2.

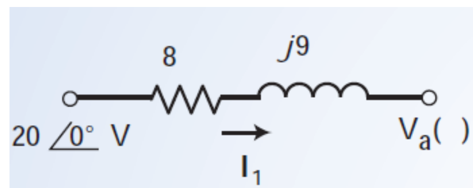


Solution 1

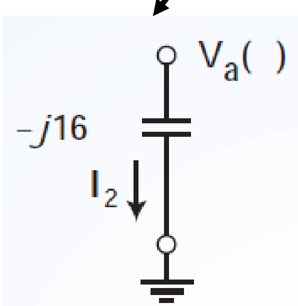
Represent the circuit in Frequency Domain



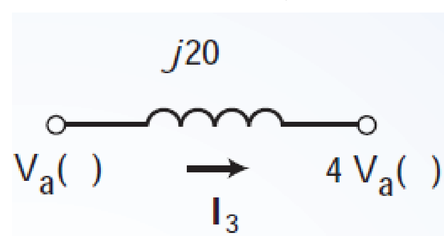
$$\mathbf{I}_1 = \frac{20\angle 0^\circ - \mathbf{V}_a(\omega)}{8 + j9}$$



$$\mathbf{I}_2 = \frac{\mathbf{V}_a(\omega) - 0}{-j16} = \frac{\mathbf{V}_a(\omega)}{-j16}$$



$$\mathbf{I}_3 = \frac{\mathbf{V}_a(\omega) - 4\,\mathbf{V}_a(\omega)}{j20} = -\frac{3\,\mathbf{V}_a(\omega)}{j20}$$



Solution 1

Applying KCL at the top node of the capacitor gives

$$\mathbf{I}_1 + 1.2\angle 45^\circ = \mathbf{I}_2 + \mathbf{I}_3$$

Substituting for $\mathbf{I}_1$, $\mathbf{I}_2$, and $\mathbf{I}_3$ gives

$$\frac{20\angle 0^\circ - \mathbf{V}_a(\omega)}{8 + j9} + 1.2\angle 45^\circ = \frac{\mathbf{V}_a(\omega)}{-j16} + \left(-\frac{3\mathbf{V}_a(\omega)}{j20}\right)$$

Collecting the terms involving $\mathbf{V}_a(\omega)$ gives

$$\frac{20\angle 0^\circ}{8 + j9} + 1.2\angle 45^\circ = \left(\frac{1}{8 + j9} + \frac{1}{-j16} - \frac{3}{j20}\right)\mathbf{V}_a(\omega)$$

Solving for $\mathbf{V}_a(\omega)$, perhaps using MATLAB (see Figure 10.6-5), gives

$$\mathbf{V}_a(\omega) = 12.43\angle -81.2^\circ \text{ V}$$

The corresponding sinusoid is

$$v_a(t) = 12.43\cos(250t - 81.2^\circ) \text{ V}$$

```
>> Vs = 20;
>> Is = 1.2*exp(j*45*pi/180);
>> Va = (Vs/(8+9j) + Is) / (1/(8+9j) + 1/-16j + -3/20j)

Va =

   1.8929 -12.2816i
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>> abs(Va)

ans =

   12.4267

>> angle(Va)*180/pi

ans =

  -81.2381
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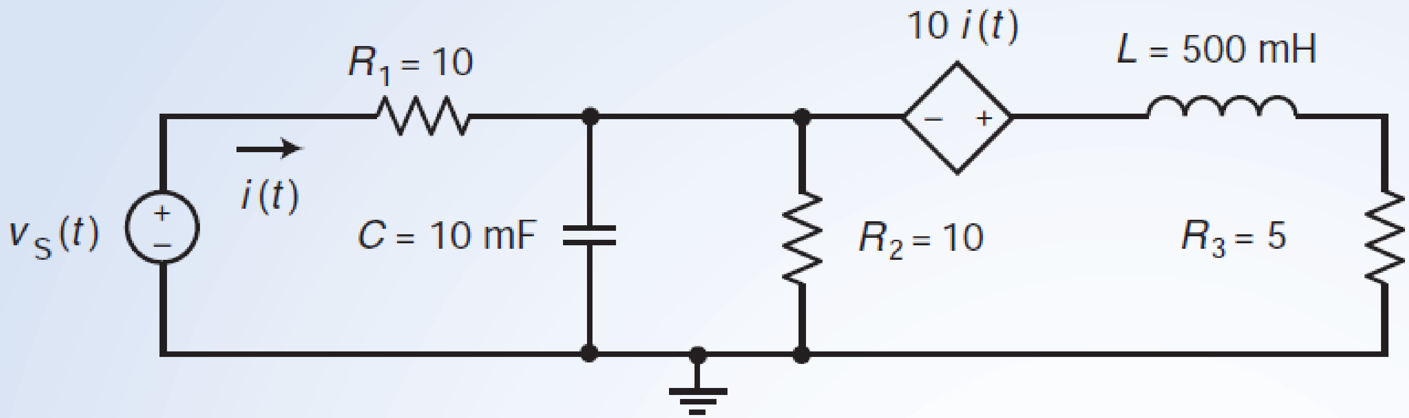
Sample 2

The input to the circuit shown in Figure 10.6-10 is the voltage source voltage

$$v_s(t) = 10\cos(10t)\ \text{V}$$

The output is the current $i(t)$ in resistor R_1. Determine $i(t)$.



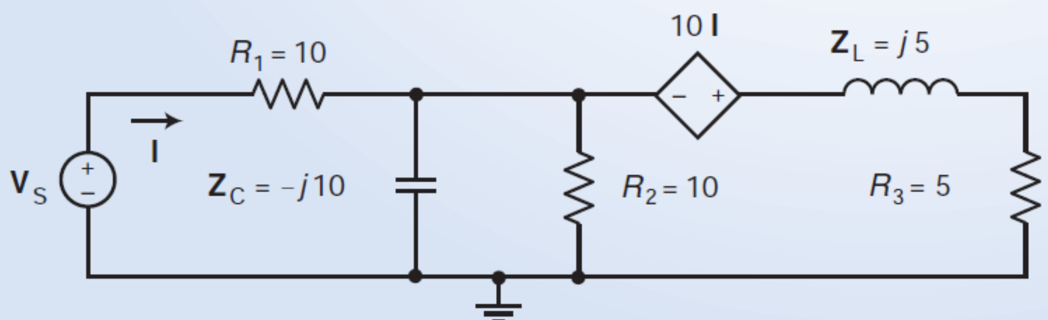
Solution 2

First, we will represent the circuit in the frequency domain using phasors and impedances. The impedances of the capacitor and inductor are

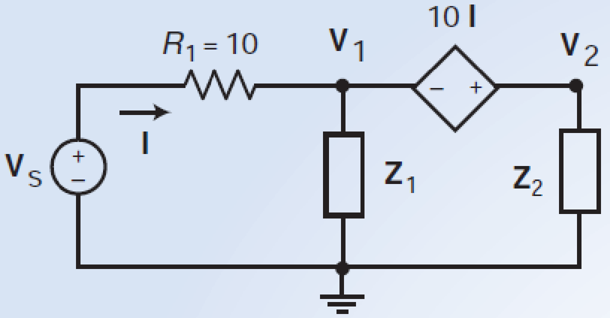
$$\mathbf{Z}_c = -j\frac{1}{10(0.010)} = -j10\ \Omega \text{ and } \mathbf{Z}_L = j10(0.5) = j5\ \Omega$$

The frequency domain representation of the circuit is shown in Figure 10.6-11. We can analyze this circuit by writing and solving node equations. To simply this process, we can first replace series and parallel impedances by equivalent impedances as shown in Figure 10.6-12. Impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in Figure 10.6-12 are given by

$$\mathbf{Z}_1 = 10\|(-j10) = \frac{10(-j10)}{10-j10} = 5 - j5\ \Omega \text{ and } \mathbf{Z}_2 = 5 + j5\ \Omega$$



Solution 2



Next, consider the dependent source in Figure 10.6-12. We can use Ohm's law to express the controlling current **I** as

$$\mathbf{I} = \frac{\mathbf{V}_s - \mathbf{V}_1}{R_1} \tag{10.6-7}$$

Using KVL, we can express the dependent source voltage as

$$10\mathbf{I} = \mathbf{V}_2 - \mathbf{V}_1$$

Apply KCL to the supernode identified in Figure 10.6-12 to get

$$\mathbf{I} = \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_2}{\mathbf{Z}_3} = \frac{\mathbf{V}_1}{\mathbf{Z}_2} + \frac{\mathbf{V}_1 + 10\mathbf{I}}{\mathbf{Z}_3} \quad \Rightarrow \quad (\mathbf{Z}_2 + \mathbf{Z}_3)\mathbf{V}_1 + \mathbf{Z}_2(10 - \mathbf{Z}_3)\mathbf{I} = 0 \tag{10.6-8}$$

Organizing Eqs. 10.6-7 and 10.6-8 into matrix form, we get

$$\begin{bmatrix} 1 & R_1 \\ \mathbf{Z}_2 + \mathbf{Z}_3 & \mathbf{Z}_2(10 - \mathbf{Z}_3) \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_s \\ 0 \end{bmatrix}$$

Solving these equations, perhaps using MATLAB, gives

$$\mathbf{V}_1 = 4.4721\,\angle 63.4^\circ\text{ V and }\mathbf{I} = 0.89443\,\angle -26.6^\circ\text{ A}$$

Back in the time domain, the output current is

$$i(t) = 0.89443\ \cos(10t - 26.6^\circ)\text{ A}$$

Sample 3/Solution 3

The input to the ac circuit shown in Figure 10.6-13 is the voltage source voltage

$$v_s(t) = 125\cos(500t + 15°)\ \text{mV}$$

Determine the output voltage $v_o(t)$.

The impedances of the capacitor and inductor are

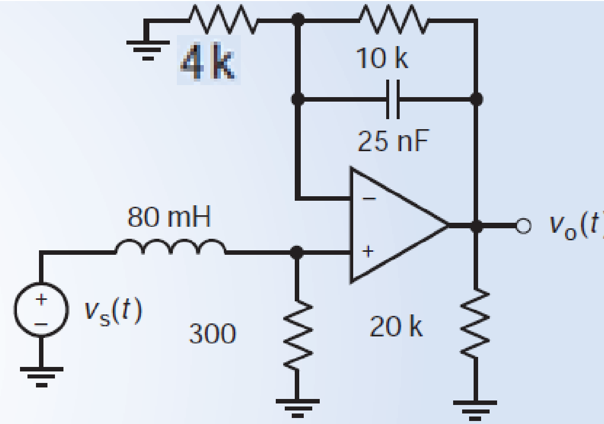


FIGURE 10.6-13 The circuit considered in Example 10.6-4.

<u>Solution</u>

The impedances of the capacitor and inductor are

$$\mathbf{Z}_C = -j\frac{1}{5000(25\times10^{-9})} = -j8000\ \Omega \text{ and } \mathbf{Z}_L = j5000(80\times10^{-3}) = j400\ \Omega$$

Figure 10.6-14 show the circuit represented in the frequency domain using phasors and impedances.

Solution 3

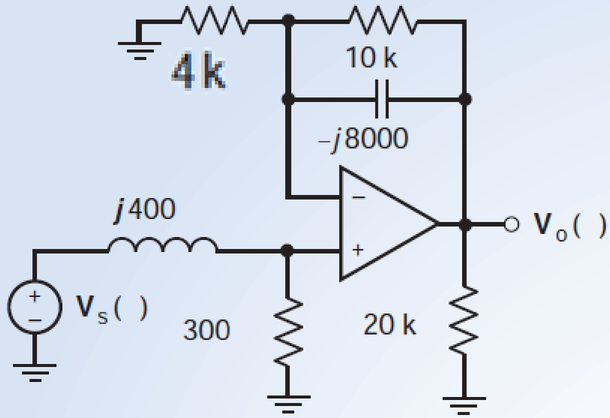


FIGURE 10.6-14 The frequency domain representation of the circuit from Figure 10.6-13.

Applying KCL at the noninverting node of the op amp, we get

$$\frac{\mathbf{V}_s - \mathbf{V}_a}{j400} = \frac{\mathbf{V}_a}{300} + 0 \quad \Rightarrow \quad \mathbf{V}_s = \mathbf{V}_a\left(1 + \frac{j400}{300}\right)$$

Solving for $\mathbf{V}_a$ gives

$$\mathbf{V}_a = \left(\frac{300}{300 + j\,400}\right)\mathbf{V}_s = \left(0.6\,\angle{-53.1^\circ}\right)\left(0.125\,\angle{15^\circ}\right) = 0.075\,\angle{-38.1^\circ}\ \mathbf{V}$$

Next, apply KCL at the inverting node of the op amp to get

$$\frac{\mathbf{V}_a}{4000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{10,000} + \frac{\mathbf{V}_a - \mathbf{V}_o}{-j8000} = 0$$

Multiplying by 80,000 gives

$$0 = 20\,\mathbf{V}_a + 8(\mathbf{V}_a - \mathbf{V}_o) + j10(\mathbf{V}_a - \mathbf{V}_o)$$

Solving for $\mathbf{V}_o$ gives

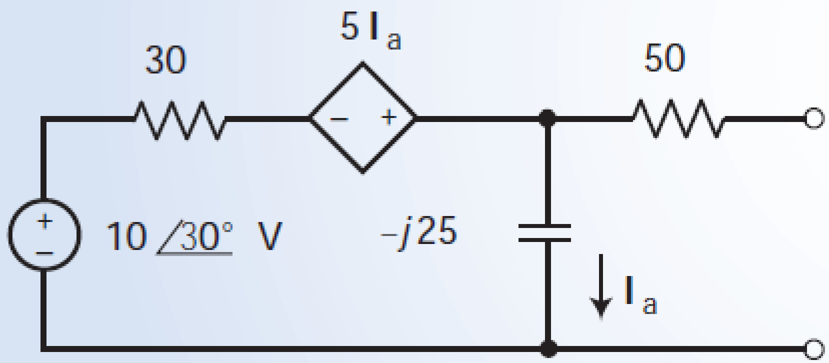
$$\mathbf{V}_o = \frac{28 + j\,10}{8 + j\,10}\,\mathbf{V}_a = \frac{29.73\,\angle{19.65^\circ}}{12.81\,\angle{51.34^\circ}}\left(0.075\,\angle{-38.1^\circ}\right) = 0.174\,\angle{-69.79^\circ}$$

In the time domain, the output voltage is

$$v_o(t) = 174\cos(500\,t - 69.79^\circ)\ \text{mV}$$

Sample 4

Find the Norton equivalent circuit of the ac circuit in Figure 10.7-7.



Solution 4

In Figure 10.7-8, an open circuit is connected across the terminals of circuit. The voltage across that open circuit is the open-circuit voltage $\mathbf{V}_{oc}$. (Notice that there is no current in the 50-Ω impedance due to the open circuit.) Apply KVL to the left mesh to get

$$30\,\mathbf{I}_a - 5\,\mathbf{I}_a + (-j25)\,\mathbf{I}_a = 10\,\underline{/30^\circ}$$

Solving for $\mathbf{I}_a$, we get

$$\mathbf{I}_a = \frac{10\,\underline{/30^\circ}}{25 - j25} = 0.2828\,\underline{/75^\circ}\text{ A}$$

Apply KVL to the right mesh to get

$$\mathbf{V}_{oc} = -j25\,\mathbf{I}_a = \left(25\,\underline{/-90^\circ}\right)0.2828\,\underline{/75^\circ} = 7.071\,\underline{/-15^\circ}\text{ V}$$

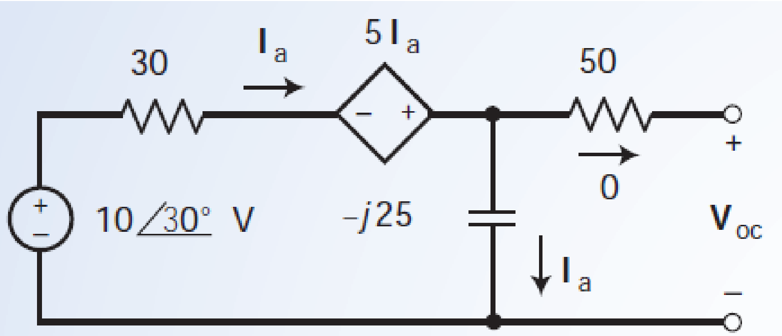


FIGURE 10.7-8 The circuit used to determine the open circuit voltage of the circuit in Figure 10.7-7.

Solution 4

Next, we determine the short-circuit current using the circuit shown in Figure 10.7-9. In Figure 10.7-9, a short circuit is connected across the terminals of circuit. The current in that open circuit is the short-circuit current $\mathbf{I}_{sc}$. In Figure 10.7-9, the controlling current of the dependent source is related to the mesh currents by

$$\mathbf{I}_a = \mathbf{I}_1 - \mathbf{I}_{sc}$$

Apply KVL to the left mesh to get

$$30\,\mathbf{I}_1 - 5(\mathbf{I}_1 - \mathbf{I}_{sc}) - j25(\mathbf{I}_1 - \mathbf{I}_{sc}) = 10\,\underline{/30^\circ}$$

Apply KVL to the left mesh to get

$$50\,\mathbf{I}_{sc} - (-j25)(\mathbf{I}_1 - \mathbf{I}_{sc}) = 0$$

Organize these equations in matrix form to get

$$\begin{bmatrix} 25 - j25 & 5 + j25 \\ j25 & 50 - j25 \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_{sc} \end{bmatrix} = \begin{bmatrix} 10\,\underline{/30^\circ} \\ 0 \end{bmatrix}$$

Solving using MATLAB gives
$$\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_{sc} \end{bmatrix} = \begin{bmatrix} 0.2370\,\underline{/61.4^\circ} \\ 0.1060\,\underline{/-2^\circ} \end{bmatrix}$$

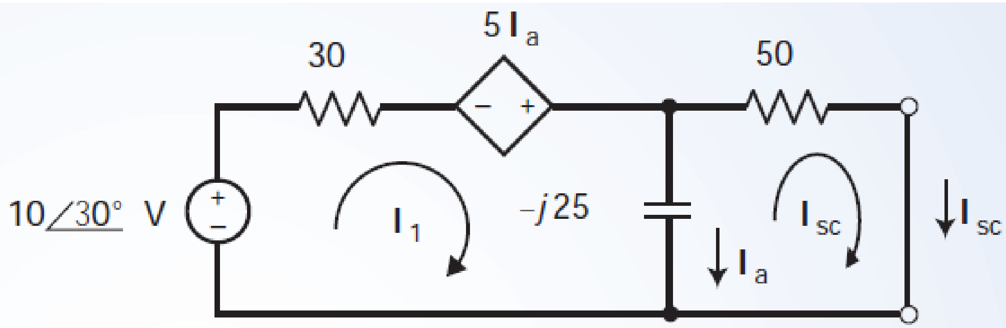


FIGURE 10.7-9 The circuit used to determine the short circuit current of the circuit in Figure 10.7-7.

Solution 4

The Thévenin impedance is

$$\mathbf{Z}_{\mathrm{t}} = \frac{\mathbf{V}_{\mathrm{oc}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{7.071\angle{-15^\circ}}{0.1060\angle{-2^\circ}} = 66.71\angle{-13^\circ}\ \Omega$$

Finally, Figure 10.7.10 shows the Norton equivalent circuit, which consists of a current source in parallel with an impedance. The current source current is the short-circuit voltage $\mathbf{I}_{\mathrm{sc}}$. The impedance is the Thévenin impedance $\mathbf{Z}_{\mathrm{t}}$.

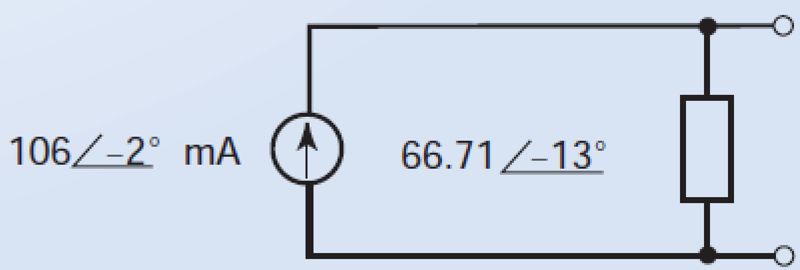


FIGURE 10.7-10 The Norton equivalent circuit of the circuit in Figure 10.7-7.