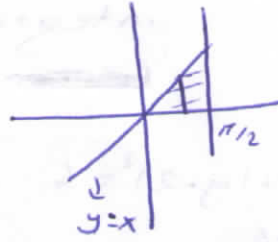


⑧ $\int_0^{\pi/2} \int_0^{\pi/2} y \cos x^3 dx dy = ? \rightarrow$ integrasyon sırası değiştirilmeli

0: $y = \frac{\pi}{2}$ $y = 0$ $x = y$ $x = \frac{\pi}{2}$

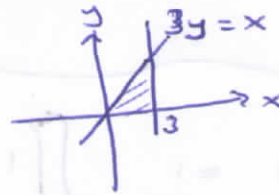


$$I = \int_0^{\pi/2} \int_0^x y \cos x^3 dy dx = \int_0^{\pi/2} \left. \frac{y^2}{2} \cos x^3 \right|_0^x dx = \int_0^{\pi/2} \frac{x^2}{2} \cos x^3 dx$$

$$= \frac{1}{6} \sin x^3 \Big|_0^{\pi/2} = \frac{1}{6} \sin \frac{\pi^3}{8}$$

⑨ $\int_0^1 \int_{3y}^3 e^{x^2} dx dy$ integralini hesaplayın.

0: $y = 0$ $y = 1$ $x = 3$ $x = 3y$



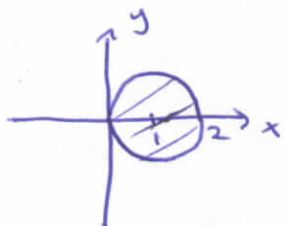
$$I = \int_0^3 \int_0^{x/3} e^{x^2} dy dx = \int_0^3 \left. y e^{x^2} \right|_0^{x/3} dx$$

$$= \int_0^3 \left(\frac{x}{3} e^{x^2} \right) dx = \frac{1}{6} e^{x^2} \Big|_0^3 = \frac{1}{6} (e^9 - 1)$$

⑩ $\iint_D (x^2 + y^2) dx dy$ integralini 0: $x^2 + y^2 = 2x$ bölgesinde kutupsal

Koordinatlara dönüştürerek yazınız.

$$x^2 + y^2 - 2x = 0 \Rightarrow (x-1)^2 + y^2 = 1$$



$$\left. \begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ dx dy &= r dr d\theta \\ x^2 + y^2 &= r^2 \end{aligned} \right\}$$

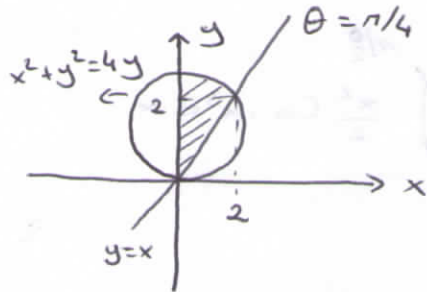
$$I = \int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r^2 \cdot r dr d\theta$$

$$x^2 + y^2 - 2x = 0$$

$$r^2 - 2r \cos \theta = 0 \quad r = 0 \quad r = 2 \cos \theta$$

* D: $\begin{cases} x^2+y^2-4y=0 \\ y=x \\ x=0 \end{cases}$ olmak üzere $I = \iint_D (3y-2) dx dy$
 integralinin ^{sınırlarını} kutupsal dönüşümle yazınız.

$x^2+y^2-4y=0 \rightarrow x^2+(y-2)^2=4 \rightarrow$ Merkezi $(0,2)$ de
 yarıçapı 2 olan çember

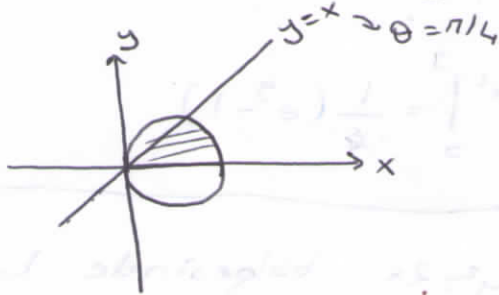


$$\begin{cases} x^2+y^2-4y=0 \\ y=x \end{cases} \Rightarrow \begin{cases} 2x^2-4x=0 \\ x=0 \quad x=2 \end{cases}$$

$$\begin{aligned} &\begin{cases} x=r \cos \theta \\ y=r \sin \theta \\ dx dy = r dr d\theta \end{cases} \rightarrow I = \int_{\pi/4}^{\pi/2} \int_0^{4 \sin \theta} (3r \sin \theta - 2) r dr d\theta \\ &\begin{cases} x^2+y^2-4y=0 \\ r^2-4r \sin \theta=0 \\ r=0 \quad r=4 \sin \theta \end{cases} \end{aligned}$$

* D: $\begin{cases} x^2+y^2-4x=0 \\ y=x \\ y=0 \end{cases} \Rightarrow \iint_D (2x-3) dx dy \rightarrow$ kutupsal dönüşümle
 sınırları yazın.

$x^2+y^2-4x=0 \rightarrow (x-2)^2+y^2=4 \rightarrow$ Merkezi $(2,0)$
 yarıçapı 2 olan çember



$$\begin{cases} x=r \cos \theta \\ y=r \sin \theta \\ dx dy = r dr d\theta \end{cases} \Rightarrow \begin{cases} x^2+y^2-4x=0 \\ r^2-4r \cos \theta=0 \\ r=0 \quad r=4 \cos \theta \end{cases}$$

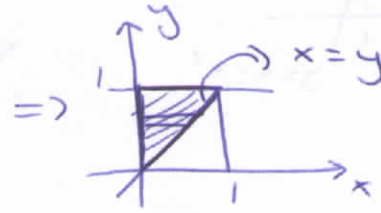
$$I = \int_0^{\pi/4} \int_0^{4 \cos \theta} (2r \cos \theta - 3) r dr d\theta$$

$$*) \int_0^1 \int_x^1 e^{x/y} dy dx = ?$$

integral bu hali ile çözülemez. integrasyon sırasını değiştirmeliyiz.

$$\int_0^1 \int_x^1 e^{x/y} dy dx$$

$$\Rightarrow \begin{matrix} x=1 & x=0 \\ y=1 & y=x \end{matrix}$$



Sınırları bölgeyi
y'ye göre düzgün olarak yazmalıyız.

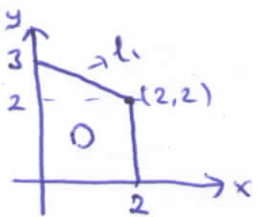
$$\begin{aligned} I &= \int_0^1 \int_0^y e^{x/y} dx dy = \int_0^1 \frac{e^{x/y}}{\frac{1}{y}} \bigg|_0^y dy = \int_0^1 y e^{x/y} \bigg|_0^y dy \\ &= \int_0^1 (y e - y) dy = \frac{y^2}{2} e - \frac{y^2}{2} \bigg|_0^1 \\ &= \frac{e}{2} - \frac{1}{2} \end{aligned}$$

*) $z = x + y$ yüzeyinin altında $R = [0, 1] \times [0, 2]$ dikdörtgeni üstünde bulunan cismin hacmi?

$$\begin{aligned} V &= \iint_R (x+y) dx dy = \int_0^1 \int_0^2 (x+y) dy dx = \int_0^1 \left(xy + \frac{y^2}{2} \right) \bigg|_0^2 dx = \int_0^1 (2x+2) dx \\ &= x^2 + 2x \bigg|_0^1 = 3 \end{aligned}$$

*) $f(x,y) = 2x - y$ fonksiyonunu köşeleri $(0,0), (2,0), (2,2), (0,3)$ koordinatları üzerinde olan yamuk üzerinde integre ediniz.

l_1 doğrusu: $(0,3), (2,2)$

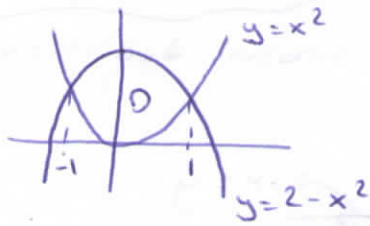


$$\frac{x-x_1}{y-y_1} = \frac{x-x_2}{y-y_2} \Rightarrow \frac{x-0}{y-3} = \frac{0-2}{3-2} \Rightarrow x+2y=6$$

$$\boxed{y = 3 - \frac{x}{2}}$$

$$I = \iint_0^2 (2x-y) dx dy = \int_0^2 \int_0^{3-x/2} (2x-y) dy dx = 3$$

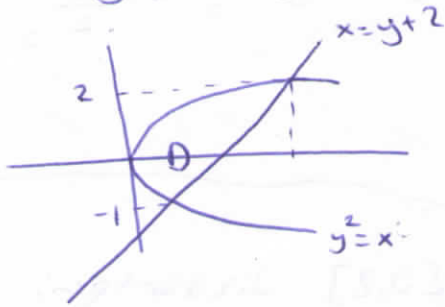
* $z = 1 + 2xy$ nin altında ve $y = x^2$, $y = 2 - x^2$ parabolleri ile sınırlı bölgenin üstünde bulunan T cisminin hacmi?



$$y = x^2, y = 2 - x^2 \Rightarrow x^2 = 2 - x^2 \Rightarrow x = \pm 1$$

$$\begin{aligned} V &= \iiint_D (1 + 2xy) dx dy = \int_{-1}^1 \int_{x^2}^{2-x^2} (1 + 2xy) dy dx \\ &= \int_{-1}^1 y + xy^2 \Big|_{x^2}^{2-x^2} dx \\ &= \int_{-1}^1 (2 - x^2 + x(2 - x^2)^2 - x^2 - x \cdot x^4) dx = \frac{8}{3} \end{aligned}$$

* $x = y^2$, $x = y + 2$ ile sınırlı bölgenin alanı?

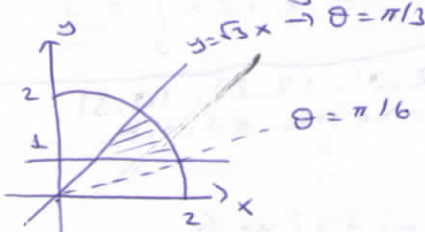


$$\begin{aligned} A &= \iint_D dx dy = \int_{-1}^2 \int_{y^2}^{y+2} dx dy = \int_{-1}^2 (y + 2 - y^2) dy \\ &= \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_{-1}^2 \\ &= \frac{9}{2} \end{aligned}$$

$$x = y + 2, y^2 = x \Rightarrow y^2 = y + 2 \Rightarrow y = -1, y = 2$$

* 2 katlı kutupsal integrasyon ile $y = 1$, $y = \sqrt{3}x$ in altında

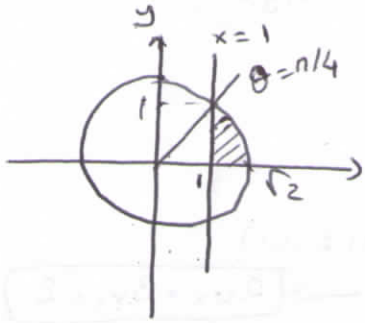
üzerinde kalan ve $y^2 + x^2 = 4$ çemberi ile sınırlı alanı bulun.



$$\begin{aligned} y = 1 &\Rightarrow r \sin \theta = 1 \\ r &= \frac{1}{\sin \theta} \end{aligned}$$

$$\begin{aligned} A &= \int_{\pi/6}^{\pi/3} \int_{1/\sin \theta}^2 r dr d\theta = \int_{\pi/6}^{\pi/3} \frac{r^2}{2} \Big|_{1/\sin \theta}^2 d\theta \\ &= \int_{\pi/6}^{\pi/3} \frac{1}{2} [4 - \text{Cosec}^2 \theta] d\theta \\ &= \frac{1}{2} (4\theta + \cot \theta) \Big|_{\pi/6}^{\pi/3} = \frac{\pi - \sqrt{3}}{6} \end{aligned}$$

* $D: \begin{cases} x^2 + y^2 \leq 2 \\ x > 1 \\ y > 0 \end{cases} \Rightarrow I = \iint_D x \cdot dA$ integralinin sınırlarını kutupsal dönüşüm ile yazınız.

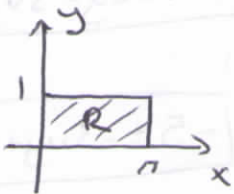


$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ dx dy = r dr d\theta \end{cases} \begin{cases} x=1 \Rightarrow r \cos \theta = 1 \rightarrow \boxed{r = \frac{1}{\cos \theta}} \\ x^2 + y^2 = 2 \Rightarrow r^2 = 2 \rightarrow \boxed{r = \sqrt{2}} \end{cases}$$

$$I = \int_0^{\pi/4} \int_{1/\cos \theta}^{\sqrt{2}} r \cos \theta \cdot r \cdot dr d\theta$$

* $f(x,y) = x \cos xy$ fonksiyonunun $R: \begin{cases} 0 \leq x \leq \pi \\ 0 \leq y \leq 1 \end{cases}$ bölgesindeki ortalama değerini bulunuz.

$$\bar{f} = \frac{1}{R \text{ nin Alanı}} \cdot \iint_R f(x,y) dA$$



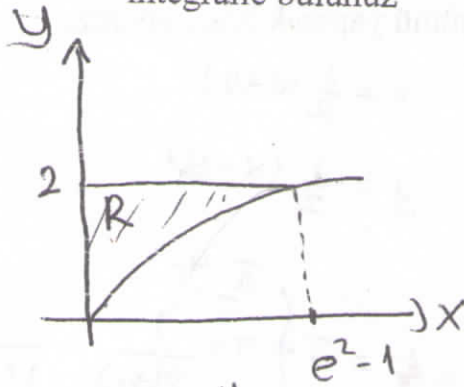
$$R \text{ 'nin Alanı} = \pi$$

$$\iint_R f(x,y) dA = \int_0^{\pi} \int_0^1 x \cos xy \cdot dy dx$$

$$= \int_0^{\pi} \sin xy \Big|_0^1 dx = \int_0^{\pi} \sin x dx = (-\cos x) \Big|_0^{\pi} = -(-1-1) = 2$$

$$\bar{f} = \frac{1}{\pi} \cdot 2 = \frac{2}{\pi}$$

S.3-a) $y = \ln(1+x)$ eğrisi ve $x=0$, $y=2$ doğruları ile sınırlanmış bölgenin alanını iki katlı integrale bulunuz



$$A = \int_0^2 \int_0^{e^y-1} dx dy$$

$$= \int_0^2 (e^y - 1) dy$$

$$= (e^y - y) \Big|_0^2$$

$$= e^2 - 3 \text{ br}^2$$

$$\ln(1+x) = 2 \Rightarrow x = e^2 - 1$$

$$R = \{ (x,y) : 0 \leq x \leq e^2 - 1, \ln(1+x) \leq y \leq 2 \}$$

$$A = \int_0^{e^2-1} \int_{\ln(1+x)}^2 dy dx$$

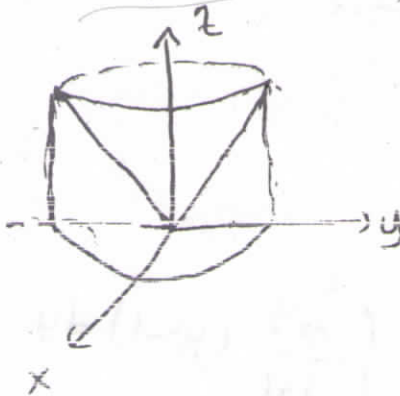
$$= \int_0^{e^2-1} [2 - \ln(1+x)] dx$$

$$= [2x - ((1+x)\ln(1+x) - x)] \Big|_0^{e^2-1}$$

$$= e^2 - 3 \text{ br}^2$$

$$\begin{cases} u = \ln(1+x) \\ du = dx \end{cases}$$

-b) Üstten $z = \sqrt{x^2 + y^2}$ konisi, alttan $z=0$ düzlemi ve yandan $x^2 + y^2 = 1$ silindiri ile sınırlanan cismin hacmini bulunuz. (Yol Gösterme: R bölgesi, xy -düzleminde $x^2 + y^2 = 1$ dairesidir.)



$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \Rightarrow$$

$$x^2 + y^2 = 1$$

$$0 \leq r \leq 1$$

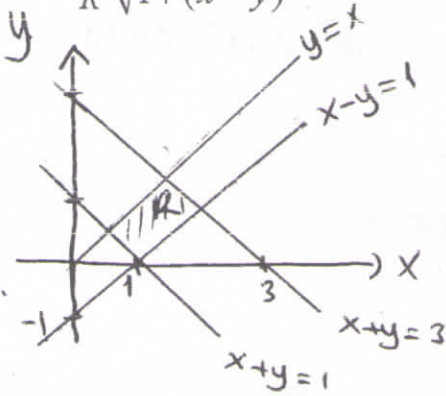
$$0 \leq \theta \leq 2\pi$$

$$V = \iint_R f(x,y) dA = \int_0^{2\pi} \int_0^1 \sqrt{r^2} r dr d\theta$$

$$= \int_0^{2\pi} \left[\frac{1}{3} r^3 \right]_0^1 d\theta = \frac{2\pi}{3} \text{ br}^3$$

S.4-a) Eğer R , $x+y=1$, $x+y=3$, $x-y=0$ ve $x-y=1$ doğruları ile sınırlanmış bölge ise,

$\iint_R \frac{x^2 - y^2}{\sqrt{1+(x-y)^2}} dA$ integralini $u=x-y$, $v=x+y$ dönüşümü yaparak hesaplayınız.



$$x-y=0 \rightarrow u=0$$

$$x = \frac{1}{2}(u+v)$$

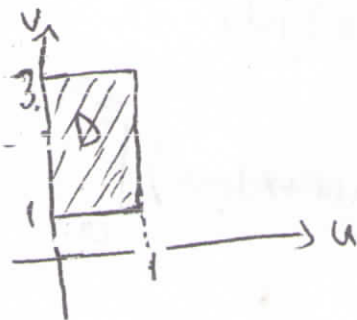
$$x-y=1 \rightarrow u=1$$

$$y = \frac{1}{2}(v-u)$$

$$x+y=1 \rightarrow v=1$$

$$x+y=3 \rightarrow v=3$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2} = J$$

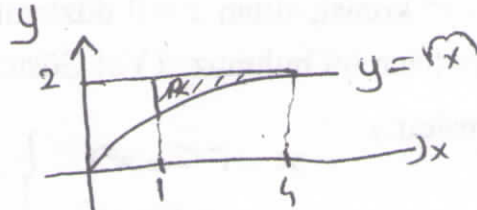


$$I = \int_0^1 \int_1^3 \frac{1}{2} \frac{uv}{\sqrt{1+u^2}} dv du = \frac{1}{2} \int_0^1 \frac{1}{\sqrt{1+u^2}} \left[\frac{v^2}{2} \right]_1^3 du$$

$$= 2 \sqrt{1+u^2} \Big|_0^1 = 2 //$$

-b) $\int_1^4 \int_{\sqrt{x}}^2 \frac{e^y}{y+1} dy dx$ integralini hesaplayınız

$$R: \begin{cases} y=\sqrt{x}, y=2 \\ x=1, x=4 \end{cases}$$



$$R = \{ (x,y) : 1 \leq x \leq 4, \sqrt{x} \leq y \leq 2 \}$$

$$R' = \{ (x,y) : 1 \leq y \leq 2, 1 \leq x \leq y^2 \}$$

$$I = \int_1^4 \int_{\sqrt{x}}^2 \frac{e^y}{y+1} dy dx = \int_1^2 \int_1^{y^2} \frac{e^y}{y+1} dx dy = \int_1^2 \frac{e^y}{y+1} (y^2-1) dy$$

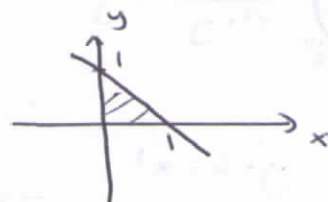
$$= \int_1^2 (y-1) e^y dy = (y-2) e^y \Big|_1^2 = e$$

$$\left[\begin{array}{l} u=y-1 \\ dv=e^y dy \end{array} \right] \rightarrow du=dy \Rightarrow uv - \int v du = (y-1) e^y - \int e^y dy = (y-2) e^y$$

* $I = \int_0^1 \int_0^{1-y} e^{\frac{x-y}{x+y}} dx dy = ?$

$u = x - y$
 $v = x + y$

dönüşümü yapılmalı



0: $y=1$ $y=0$ $x=0$ $x=1-y \rightarrow$

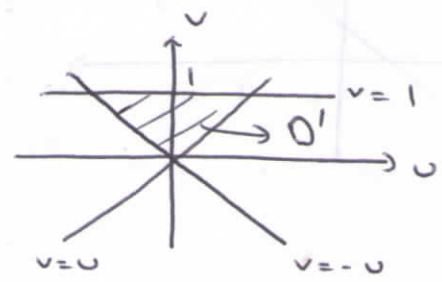
Bölgeyi sınırlayan: $x=1-y$, $x=0$, $y=0$

$\frac{0}{x+y=1} \rightarrow \frac{0'}{v=1}$

$\begin{cases} u = x - y \\ v = x + y \end{cases} \rightarrow \begin{cases} u + v = 2x \\ u - v = -2y \end{cases}$

$x=0 \rightarrow u+v=0$

$y=0 \rightarrow u-v=0$



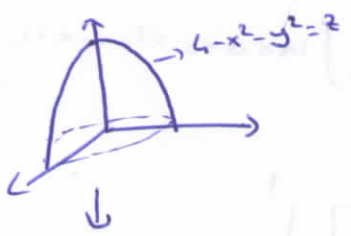
$J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$

$$I = \iint_{0'} e^{u/v} \cdot \frac{1}{2} du dv = \int_0^1 \int_{-v}^v e^{u/v} \frac{1}{2} du dv = \frac{1}{2} \int_0^1 \left(\frac{e^{u/v}}{\frac{1}{v}} \Big|_{-v}^v \right) dv$$

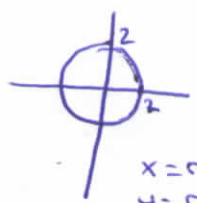
$$= \frac{1}{2} \int_0^1 v \cdot (e - e^{-1}) dv = \frac{1}{2} (e - \frac{1}{e}) \cdot \frac{v^2}{2} \Big|_0^1$$

$$= \frac{1}{4} (e - \frac{1}{e})$$

* $z = 4 - x^2 - y^2$ paraboloidi ve $z \geq 0$ in sınırladığı bölgenin hacmini 2 katlı integral ile hesaplayın.



0: $x^2 + y^2 = 4$



$x = r \cos \theta$
 $y = r \sin \theta$
 $|z| = r$
 $dx dy = r dr d\theta$

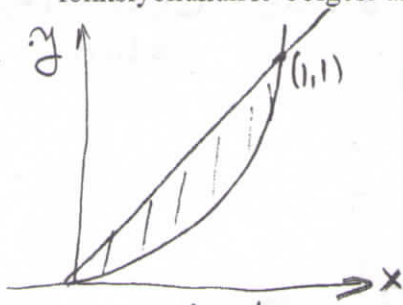
$$V = \iint_0 (4 - x^2 - y^2) dx dy = \iint_0 (4 - r^2) r dr d\theta$$

$$= \int_0^{2\pi} \int_0^2 (4r - r^3) dr d\theta$$

$$= \int_0^{2\pi} \left(2r^2 - \frac{r^4}{4} \Big|_0^2 \right) d\theta$$

$$= \frac{8\pi}{1}$$

4-a) R bölgesi, xy -düzleminde, $y=x$, $y=x^3$, $x \geq 0, y \geq 0$ ile sınırlı bir bölge olsun. $f(x,y) = e^{2x^2-x^4}$ fonksiyonunun R bölgesi üzerinde ortalama değerini hesaplayınız. (R bölgesini çiziniz) (12 Puan)



$$\text{Ort. Değer} = \frac{1}{R \text{nin Alanı}} \iint_R f \, dA$$

$$\iint_R dA = \int_0^1 \int_{x^3}^x dy \, dx = \int_0^1 \int_y^{\sqrt[3]{y}} dx \, dy = \frac{1}{4} \quad (5)$$

$$\iint_R e^{2x^2-x^4} dA = \int_0^1 \int_{x^3}^x e^{2x^2-x^4} dy \, dx = \int_0^1 e^{2x^2-x^4} (x-x^3) dx = \int_0^1 \frac{1}{4} e^u du = \frac{1}{4}(e-1) \quad (5)$$

$$\text{Ortalama Değer} = \frac{1}{\frac{1}{4}} \cdot \frac{1}{4}(e-1) = e-1 \quad (2)$$

4-b) R bölgesi, $2x^2+6xy+5y^2=1$ ile sınırlı bir bölge ise, $x=2u-v$ ve $y=-u+v$ dönüşümünü yaparak,

$\iint_R \sqrt{2x^2+6xy+5y^2} \, dx \, dy$ integralini hesaplayınız. (13 Puan)

$$2x^2+6xy+5y^2=1$$

$$2(2u-v)^2+6(2u-v)(-u+v)+5(-u+v)^2=1$$

$$2(4u^2-4uv+v^2)+6(-2u^2+2uv+uv-v^2)+5(u^2-2uv+v^2)=1$$

$$8u^2-8uv+2v^2-12u^2+18uv+6v^2+5u^2-10uv+5v^2=1$$

R' , $u^2+v^2=1$, R eliptik bölge, yarıçapı 1 olan daireye dönüştü. (2)

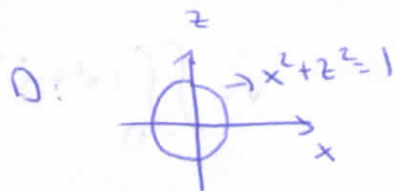
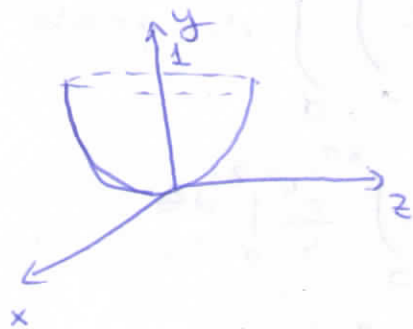
$$J = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix} = 1 \quad dx \, dy = |J| \, du \, dv \Rightarrow dx \, dy = du \, dv \quad (2)$$

$$\iint_R \sqrt{2x^2+6xy+5y^2} \, dx \, dy = \iint_{R'} \sqrt{u^2+v^2} \, du \, dv = \int_{\theta=0}^{2\pi} \int_{r=0}^1 r \cdot r \, dr \, d\theta$$

$$u = r \cos \theta \\ v = r \sin \theta$$

$$= \frac{2\pi}{3} \quad (1)$$

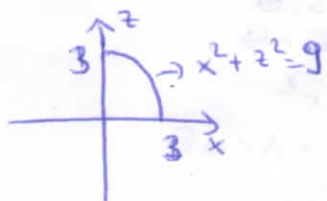
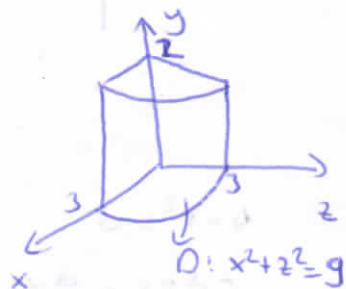
* $y = x^2 + z^2$ paraboloidinin $y=1$ ile kesilmesi ile oluşan cismin hacmi?



$$\begin{aligned} x &= r \cos \theta \\ z &= r \sin \theta \\ dx dz &= r dr d\theta \\ x^2 + z^2 &= r^2 \end{aligned}$$

$$\begin{aligned} V &= \int_0^1 \int_0^{2\pi} (1 - (x^2 + z^2)) dx dz \\ &= \int_0^{2\pi} \int_0^1 (1 - r^2) r dr d\theta = \int_0^{2\pi} \left(\frac{r^2}{2} - \frac{r^4}{4} \right) \Big|_0^1 d\theta \\ &= \int_0^{2\pi} \frac{1}{4} d\theta = \frac{\pi}{2} \end{aligned}$$

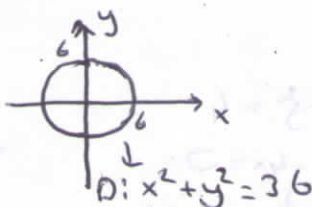
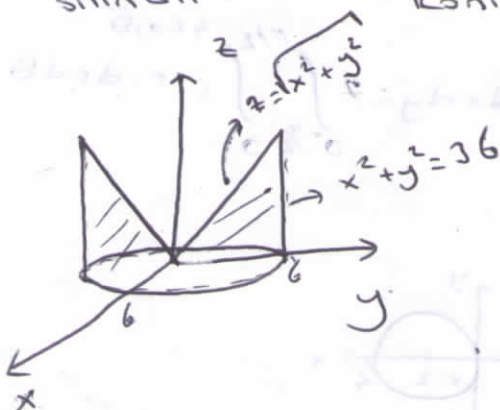
* $x^2 + z^2 = 9$, $y=2$, $y=0$, $z=0$, $x=0$ arasındaki cismin hacmi?



$$\begin{aligned} x &= r \cos \theta \\ z &= r \sin \theta \\ dx dz &= r dr d\theta \end{aligned}$$

$$\begin{aligned} V &= \int_0^2 \int_0^{\pi/2} 2 dx dz = \int_0^2 \int_0^{\pi/2} 2r dr d\theta \\ &= \int_0^2 \left(r^2 \right) \Big|_0^{\pi/2} d\theta = \int_0^2 9 d\theta = \frac{9\pi}{2} \end{aligned}$$

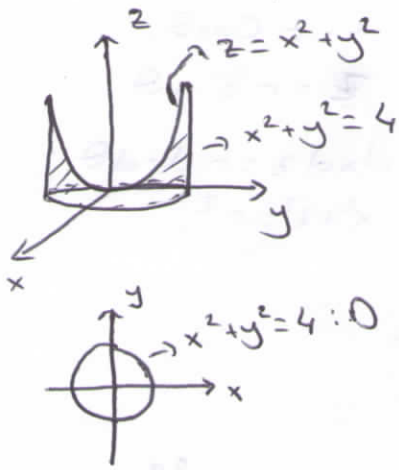
* $x^2 + y^2 = 36$ silindir, $z^2 = x^2 + y^2$ koni, $z \geq 0$ arasında kalan hacim?



$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ x^2 + y^2 &= r^2 \\ dx dy &= r dr d\theta \end{aligned}$$

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^6 \sqrt{x^2 + y^2} dx dy = \int_0^{2\pi} \int_0^6 r^2 \cdot r dr d\theta \\ &= \int_0^{2\pi} \left(\frac{r^4}{4} \right) \Big|_0^6 d\theta \\ &= \frac{6^4}{4} \cdot 2\pi \\ &= 144\pi \end{aligned}$$

* xy düzleminin üstünde, $z = x^2 + y^2$ paraboloidi ve $x^2 + y^2 = 4$ silindiri arasındaki cismin hacmi?



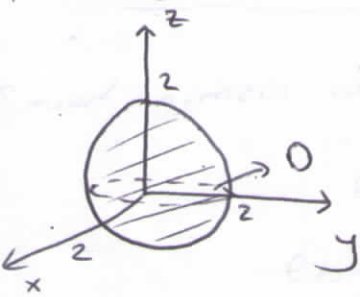
$$V = \iint_D (x^2 + y^2) dA = \int_0^{2\pi} \int_0^2 r^2 \cdot r dr d\theta$$

$$= \int_0^{2\pi} \left[\frac{r^4}{4} \right]_0^2 d\theta$$

$$= \underline{\underline{8\pi}}$$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ x^2 + y^2 = r^2 \\ dx dy = r dr d\theta \end{cases}$$

* $x^2 + y^2 + z^2 = 4$ küresinin hacmini bulunuz.

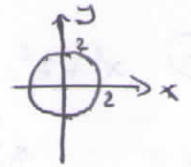


izdüşüm bölgesi: $D: x^2 + y^2 = 4$

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases}$$

$$x^2 + y^2 = r^2$$

$$dx dy = r dr d\theta$$



$$\frac{V}{2} = \iint_D \sqrt{4 - x^2 - y^2} dx dy = \int_0^{2\pi} \int_0^2 \sqrt{4 - r^2} \cdot r dr d\theta$$

$$4 - r^2 = u$$

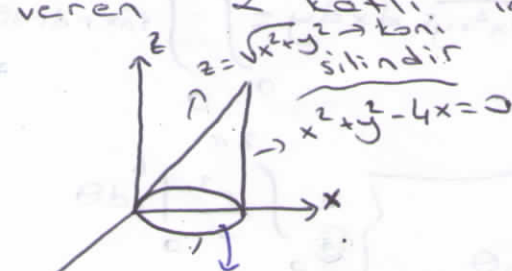
$$r dr = -\frac{du}{2}$$

$$r = 2 \rightarrow u = 0$$

$$r = 0 \rightarrow u = 4$$

$$= \int_0^{2\pi} \int_0^4 -\frac{r}{2} du d\theta = \int_0^{2\pi} \frac{8}{3} d\theta = \frac{16\pi}{3} \Rightarrow V = \frac{32}{3}\pi$$

* $x^2 + y^2 - 4x = 0$, $z \geq 0$, $x^2 + y^2 = z^2$ arasında kalan hacmi veren 2 katlı integrali kutupsal dönüşüm ile yazınız.



$$V = \iint_D \sqrt{x^2 + y^2} dx dy = \int_{-\pi/2}^{\pi/2} \int_0^{4 \cos \theta} r \cdot r dr d\theta$$

$$D: x^2 + y^2 - 4x = 0 \rightarrow (x-2)^2 + y^2 = 4$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

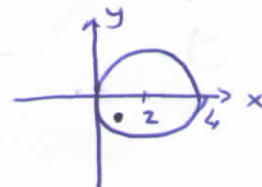
$$dx dy = r dr d\theta$$

$$x^2 + y^2 - 4x = 0$$

$$\downarrow$$

$$r^2 = 4r \cos \theta$$

$$r = 0 \quad r = 4 \cos \theta$$

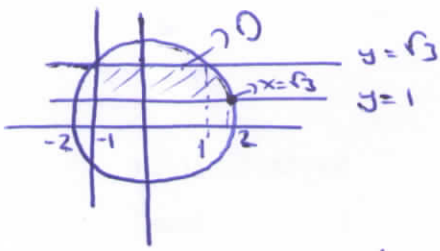


$$*) \int_{-1}^{\sqrt{3}} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \frac{x}{y} dx dy$$

a) integrasyon sırasını değiştir

b) $x^2 + y^2 = u$
 $y^2 = v$ dönüşümü ile çözün.

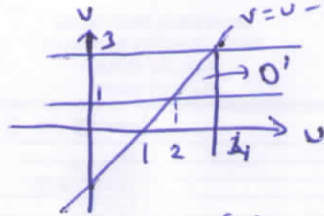
$$\left. \begin{array}{l} y = \sqrt{3} \\ y = 1 \\ x = \sqrt{4-y^2} \\ x = -1 \end{array} \right\} \text{O bölgesi}$$



$$\Rightarrow \int_{-1}^{\sqrt{3}} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \frac{x}{y} dy dx + \int_{-1}^{\sqrt{3}} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{x}{y} dy dx$$

$$b) \begin{array}{l} \text{O} \\ y = \sqrt{3} \rightarrow v = 3 \\ y = 1 \rightarrow v = 1 \\ x^2 + y^2 = 4 \rightarrow u = 4 \\ x = -1 \rightarrow u = v + 1 \end{array}$$

$$|J| = \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \frac{1}{\left| \frac{\partial(u,v)}{\partial(x,y)} \right|} = \frac{1}{\begin{vmatrix} 2x & 2y \\ 0 & 2y \end{vmatrix}} = \frac{1}{4xy}$$



$$\iint_{O'} \frac{x}{y} \cdot \frac{1}{4xy} du dv = \iint_{O'} \frac{1}{4y^2} du dv = \iint_{O'} \frac{1}{4v} du dv = \frac{1}{4} \int_2^4 \int_1^{v-1} \frac{1}{v} dv du = -\frac{1}{2} (1 - 3 \ln \sqrt{3})$$

*) $\sqrt{(2,01)^2 + (1,95)^2}$ sayısını 2. mertebe Taylor açılımı ile yaklaşık olarak hesaplayın.

$$f(x,y) = \sqrt{x^2 + y^2} \quad (a,b) = (2,1) \text{ olsun.}$$

$$f_x = \frac{3x^2}{2\sqrt{x^2 + y^2}} \quad f_y = \frac{2y}{2\sqrt{x^2 + y^2}} \quad f_{xx} = \frac{6x \cdot 2\sqrt{x^2 + y^2} - 3x^2 \cdot 2}{2\sqrt{x^2 + y^2}^3} = \frac{6x \cdot 2\sqrt{x^2 + y^2} - 3x^2 \cdot 2}{4(x^2 + y^2)^{3/2}}$$

$$f_{yy} = \frac{\sqrt{x^2 + y^2} - y \cdot \frac{2y}{2\sqrt{x^2 + y^2}}}{(x^2 + y^2)^{3/2}} \quad f_{yx} = -\frac{y \cdot \frac{3x^2}{2\sqrt{x^2 + y^2}}}{x^2 + y^2}$$

$$f_x(2,1) = 2 \quad f_y(2,1) = \frac{1}{3} \quad f_{xx}(2,1) = \frac{2}{3} \quad f_{yx}(2,1) = -\frac{2}{9} \quad f_{yy}(2,1) = \frac{8}{27}$$

$$f(x,y) \approx 3 + 2(x-2) + \frac{1}{3}(y-1) + \frac{1}{2} \left(\frac{2}{3} \cdot (x-2)^2 + 2 \cdot \left(-\frac{2}{9}\right) \cdot (y-1)(x-2) + \frac{8}{27} (y-1)^2 \right)$$

$$f(2,01,1,95) \approx 3 + 2 \cdot (0,01) + \frac{1}{3} \cdot (0,95) + \frac{1}{2} \left(\frac{2}{3} \cdot (0,01)^2 - \frac{4}{9} (0,01) \cdot (0,95) + \frac{8}{27} (0,95)^2 \right) = 3,45$$

3-a) $\sin[\pi(0.01)(1.05) + \ln(1.05)]$ nin yaklaşık değerini toplam diferansiyel veya lineer yaklaşım kullanarak hesaplayınız. (12 Puan)

$$f(x,y) = \sin(\pi xy + \ln y) \quad f(0,1) = 0 \quad h = \Delta x = 0.01 \\ k = \Delta y = 0.05$$

$$\frac{\partial f}{\partial x} = \pi y \cos(\pi xy + \ln y), \quad \frac{\partial f(0,1)}{\partial x} = \pi$$

$$\frac{\partial f}{\partial y} = \left(\pi x + \frac{1}{y}\right) \cos(\pi xy + \ln y) \quad \frac{\partial f(0,1)}{\partial y} = 1$$

$$f(0.01; 1.05) \approx f(0,1) + 0.01 \cdot \pi + 0.05 \cdot 1 \approx 0.081416$$

3-b) $f(x,y) = xy + \frac{1}{x} + \frac{8}{y}$ fonksiyonunun kritik noktalarını bulunuz ve sınıflandırınız. (13 Puan)

bulunuz. (13 Puan)

$$\begin{aligned} f_x = y - \frac{1}{x^2} = 0 &\Rightarrow y = \frac{1}{x^2} \\ f_y = x - \frac{8}{y^2} = 0 &\Rightarrow x = \frac{8}{y^2} \end{aligned} \Rightarrow x = \frac{8}{(\frac{1}{x^2})^2} \Rightarrow x - 8x^4 = 0 \\ x=0 \quad x = \frac{1}{2} \text{ Kritik N.}$$

$$x=0 \notin D_f$$

$$P(\frac{1}{2}, 4) \text{ için}$$

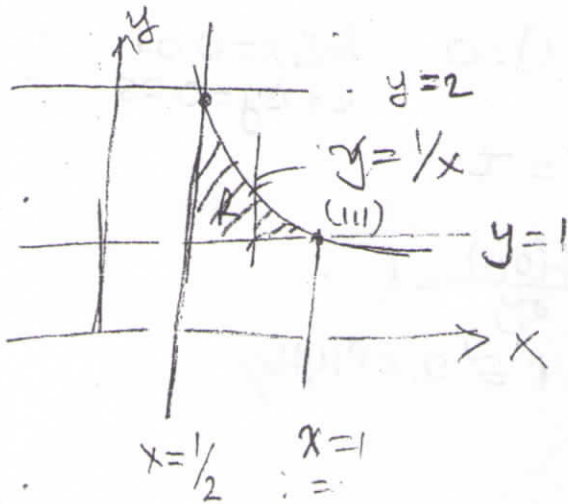
$$f_{xx} = \frac{2}{x^3} \quad f_{yy} = \frac{16}{y^3} \quad f_{xy} = 1$$

$$f_{xx} \cdot f_{yy} - f_{xy}^2 = \frac{1}{(\frac{1}{2})^3} \cdot \frac{16}{4^3} - 1 = 3 > 0$$

$$f_{xx} = 16 > 0$$

$$\Rightarrow P(\frac{1}{2}, 4) \text{ lokal min noktadır.}$$

4-a) $\int_1^2 \int_{1/2}^{1/y} e^{\ln x - x} dx dy$ integralini, integral sırasını değiştirerek hesaplayınız. (12 Puan)



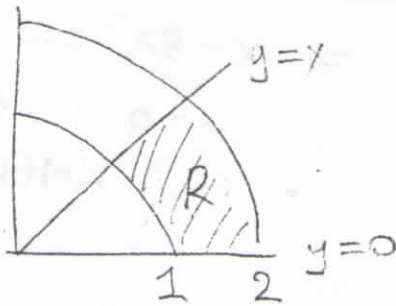
$$I = \int_{x=1/2}^1 \left(\int_{y=1}^{1/x} e^{\ln x - x} dy \right) dx$$

$$= \int_{1/2}^1 e^{\ln x - x} \left. y \right|_1^{1/x} dx$$

$$= \int_{1/2}^1 \left(\frac{1}{x} - 1 \right) e^{\ln x - x} dx = e^{\ln x - x} \Big|_{1/2}^1 = 1 \cdot e^{-1} - \frac{1}{2} e^{-1/2} = \frac{1}{e} - \frac{1}{2\sqrt{e}}$$

4-b) R bölgesi $\{(x, y) \in \mathbb{R}^2 \mid 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$ olmak üzere $\iint_R \arctan\left(\frac{y}{x}\right) dA$ integralini hesaplayınız.

(R bölgesinin şeklini çiziniz). (13 Puan)



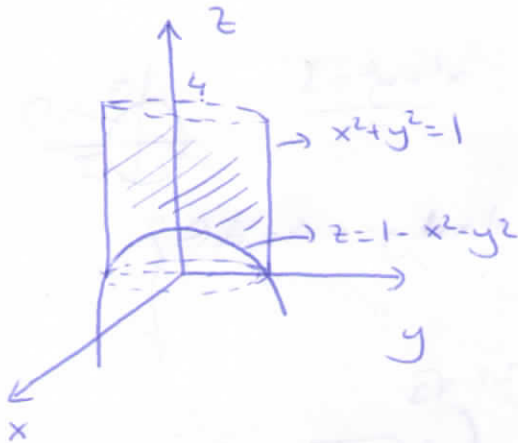
$$\iint_R \arctan\left(\frac{y}{x}\right) dx dy = \int_{\theta=0}^{\pi/4} \theta d\theta \int_{r=1}^2 r dr = \frac{3\pi^2}{64}$$

$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \end{aligned} \Rightarrow r = \sqrt{x^2 + y^2}$$

$$\theta =$$

* E cismi $x^2+y^2=1$ silindirin içinde, $z=4$ düzleminin altında, $z=1-x^2-y^2$ paraboloidinin üstündedir. Cismin hacmini bulunuz.

izdüşüm bölgesi: $x^2+y^2=1$



$$D: x^2+y^2=1$$

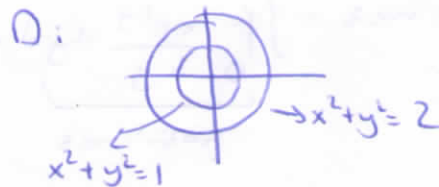
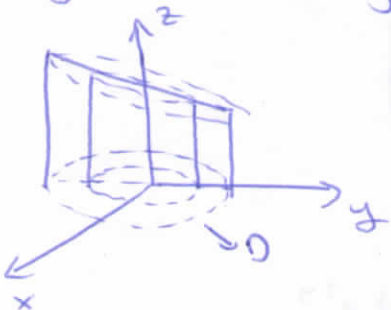
$$\left\{ \begin{array}{l} x=r \cos \theta \\ y=r \sin \theta \\ dx dy = r dr d\theta \\ x^2+y^2=r^2 \end{array} \right.$$

$$V = \iint_D (4 - (1-x^2-y^2)) dx dy = \iint_D (3 + r^2) r dr d\theta$$

$$= \int_0^{2\pi} \int_0^1 (3r + r^3) dr d\theta = \int_0^{2\pi} \left[\frac{3}{2} r^2 + \frac{r^4}{4} \right]_0^1 d\theta$$

$$= \int_0^{2\pi} \frac{7}{4} d\theta = \frac{7}{2} \pi$$

* $5=z+y$ düzleminin altında, xy düzleminin üstünde, $x^2+y^2=2$ ve $x^2+y^2=1$ silindirleri arasında kalan cismin hacmi?

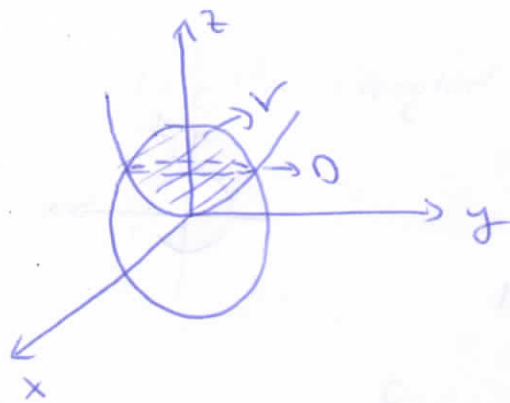


$$\left\{ \begin{array}{l} x=r \cos \theta \\ y=r \sin \theta \\ dx dy = r dr d\theta \end{array} \right.$$

$$V = \iint_D (5-y) dx dy = \int_0^{2\pi} \int_1^{\sqrt{2}} (5-r \sin \theta) r dr d\theta = \int_0^{2\pi} \left(\frac{5}{2} + \frac{1-2\sqrt{2}}{3} \sin \theta \right) d\theta$$

$$= \frac{5}{2} \theta - \frac{1-2\sqrt{2}}{3} \cos \theta \Big|_0^{2\pi} = \frac{5\pi}{2}$$

* $x^2 + y^2 + z^2 \leq 4$, $3z \geq x^2 + y^2$ arasında kalan hacim?



$$z^2 + 3z = 4 \rightarrow \boxed{z=1} \\ z=-4 \times$$

$$D: z=1 \rightarrow \underline{x^2 + y^2 = 3}$$



$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ x^2 + y^2 = r^2 \end{cases} \quad dx dy = r dr d\theta$$

$$V = \iint_D \left(\sqrt{4-x^2-y^2} - \frac{x^2+y^2}{3} \right) dx dy = \int_0^{2\pi} \int_0^{\sqrt{3}} \left(\sqrt{4-r^2} - \frac{r^2}{3} \right) r dr d\theta$$

$$= \int_0^{2\pi} \left[-\frac{(4-r^2)^{3/2}}{3} - \frac{r^4}{12} \right]_0^{\sqrt{3}} d\theta = \int_0^{2\pi} \left(-\frac{1}{3} - \frac{3}{4} + \frac{8}{3} \right) d\theta = \frac{19}{12} \cdot 2\pi = \underline{\underline{\frac{19}{6}\pi}}$$

* D: 1. bölgede $y = 4 - x^2$ parabolü, $x=0$ ve $y=0$ doğruları arasında kalan bölge ise $I = \iint_D \frac{x e^{2y}}{4-y} dA = ?$



$$I = \iint_D \frac{x e^{2y}}{4-y} dx dy \Rightarrow$$

$$\left(\begin{array}{l} (x' \text{ e eğri düğün} \\ \text{bölge alınırsa integral} \\ \text{çözülmez: } \underbrace{\int_0^4 \frac{x e^{2y}}{4-y} dy dx}_{\text{çözülmez}} \end{array} \right)$$

$$I = \int_0^4 \int_0^{\sqrt{4-y}} \frac{x e^{2y}}{4-y} dx dy = \int_0^4 \frac{x^2 e^{2y}}{2(4-y)} \Big|_0^{\sqrt{4-y}} dy = \int_0^4 \frac{(4-y) e^{2y}}{2(4-y)} dy$$

$$= \frac{e^{2y}}{2} \Big|_0^4 = \frac{e^8 - 1}{2}$$

	YTÜ - Final Sınav Soru ve Cevap Kağıdı				NOT TABLOSU				
					1. S	2. S	3. S	4. S	Σ
Adı Soyadı									
Öğrenci Numarası		Grup No							
Bölümü						Sınav Tarihi		05.06.2017	
Dersin Adı		MAT1072 MATEMATİK II			Sınav Süresi		90dk	Sınav Yeri	
Dersi veren Öğretim Üyesinin Adı Soyadı							İmza		
YÖK nun 2547 sayılı Kanunun Öğrenci Disiplin Yönetmeliğinin 9. Maddesi olan “Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek” fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.									

S. 1-a) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$ serisinin toplamını bulunuz. (12 P)

$$\begin{aligned}
 S_n &= \sum_{k=1}^n \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}} \right) \\
 &= \left(1 - \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right) + \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} \right) + \dots + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \\
 &= 1 - \frac{1}{\sqrt{n+1}}
 \end{aligned}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) = \lim_{n \rightarrow \infty} \left(1 - \frac{1}{\sqrt{n+1}} \right) = 1 //$$

S. 1-b) Diferanşiyel yaklaşımı kullanarak, $\sqrt{(2.06)^2 + 5(0.97)^4}$ değerini yaklaşık olarak hesaplayınız. (13 P)

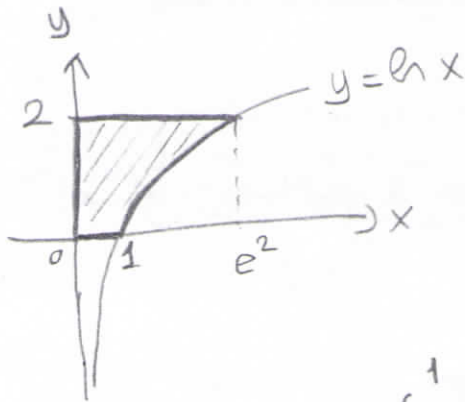
$$\begin{aligned}
 f(x,y) &= \sqrt{x^2 + 5y^4} \\
 a &= 2, \quad \Delta x = 0.06 \\
 b &= 1, \quad \Delta y = -0.03 \\
 f(2,1) &= \sqrt{4+5} = 3
 \end{aligned}
 \quad \left\{ \begin{aligned} f_x &= \frac{x}{\sqrt{x^2 + 5y^4}} \Big|_{(2,1)} = \frac{2}{3} \\ f_y &= \frac{10y^3}{\sqrt{x^2 + 5y^4}} \Big|_{(2,1)} = \frac{10}{3} \end{aligned} \right.$$

$$f(a + \Delta x, b + \Delta y) \approx f(a,b) + f_x(a,b) \Delta x + f_y(a,b) \Delta y$$

$$\sqrt{(2.06)^2 + 5(0.97)^4} \approx 3 + \frac{2}{3}(0.06) + \frac{10}{3}(-0.03) = 2.94 //$$

2017 final

S. 4-a) $y = \ln x$ eğrisi, $y=0$, $y=2$ doğruları ve y -ekseni ile sınırlanmış bölgenin **alanını**, *iki katlı integral* ile hesaplayınız. (13 P)



1.40L : $A = \int_0^2 \int_0^{e^y} dx dy$

$A = \int_0^2 e^y dy = (e^2 - 1) br^2 //$

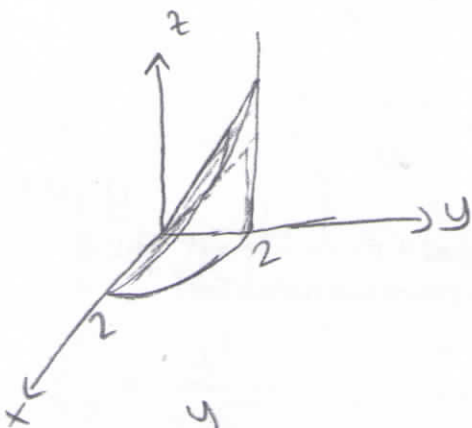
2.40L : $A = \int_0^1 \int_0^2 dy dx + \int_1^{e^2} \int_{\ln x}^2 dy dx = 2 + \int_1^{e^2} (2 - \ln x) dx$

$A = 2 + [x(2 - \ln x) + x] \Big|_1^{e^2}$

$A = 2 + [e^2(2 - 2) + e^2 - 2 - 1] = e^2 - 1 //$

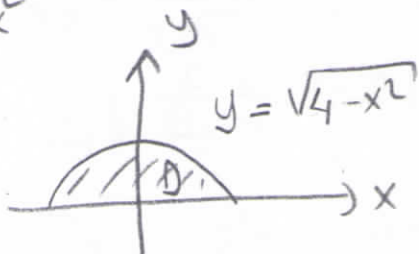
$$\begin{aligned} \int (2 - \ln x) dx &= x(2 - \ln x) + \int dx \\ &= x(2 - \ln x) + x + C \\ 2 - \ln x &= u \\ -\frac{1}{x} dx &= du \end{aligned} \quad ; \quad \begin{aligned} dv &= dx \\ v &= x \end{aligned}$$

S. 4-b) $z=0$ ve $z=y$ düzlemleri ve $x^2 + y^2 = 4$ silindiri ile oluşturulmuş cismin, xy -düzleminin üstünde kalan kısmının **hacmini** *iki katlı integral* ile bulunuz. (12 P)



$V = \iint_D y dA = \int_{-2}^2 \int_0^{\sqrt{4-x^2}} y dy dx = \int_{-2}^2 \left[\frac{y^2}{2} \right]_0^{\sqrt{4-x^2}} dx$

$V = \frac{1}{2} \int_{-2}^2 (4 - x^2) dx = \frac{1}{2} \left(4x - \frac{x^3}{3} \right) \Big|_{-2}^2 = \frac{16}{3} br^3 //$



2.40L $V = \iint_D y dA = \int_0^\pi \int_0^2 r \sin \theta r dr d\theta$

$V = \int_0^\pi \left[\frac{r^3}{3} \right]_0^2 \sin \theta d\theta = \frac{8}{3} (-\cos \theta) \Big|_0^\pi$

$V = \frac{8}{3} (1 + 1) = \frac{16}{3} br^3 //$

S. 3-a) $f(x, y) = e^y \sin(y^2 - x)$ ile tanımlı f fonksiyonunun $(\pi, 0)$ noktasındaki **yönlü** türevi, hangi yönlerde **sıfır** olur? (11 P)

$$\nabla f = \langle f_x, f_y \rangle$$

$$\nabla f = \langle -e^y \cos(y^2 - x), e^y \sin(y^2 - x) + 2ye^y \cos(y^2 - x) \rangle$$

$$\nabla f(\pi, 0) = \langle 1, 0 \rangle$$

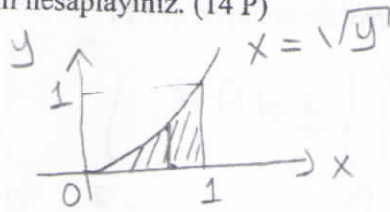
$$\vec{u} = \langle a, b \rangle \text{ ve } a^2 + b^2 = 1 \text{ olsun.}$$

$$D_{\vec{u}} f(\pi, 0) = \nabla f(\pi, 0) \cdot \vec{u} = a = 0 \Rightarrow b = \pm 1$$

$$\vec{u} = \langle 0, \pm 1 \rangle \text{ veya } \vec{u} = \pm \vec{j}$$

S. 3-b) $\int_0^1 \int_{\sqrt{y}}^1 \frac{dx}{\sqrt{1+3x^3}} dy$ integralini hesaplayınız. (14 P)

$$\begin{cases} y=0, y=1 \\ x=\sqrt{y}, x=1 \end{cases}$$



$$\int_0^1 \int_{\sqrt{y}}^1 \frac{dx}{\sqrt{1+3x^3}} dy = \int_0^1 \int_0^{x^2} \frac{1}{\sqrt{1+3x^3}} dy dx = \int_0^1 \frac{1}{\sqrt{1+3x^3}} y \Big|_0^{x^2} dx$$

$$= \int_0^1 \frac{x^2}{\sqrt{1+3x^3}} dx = \frac{2}{9} \sqrt{1+3x^3} \Big|_0^1 = \frac{2}{9} (2-1) = \frac{2}{9} //$$

$$\left[\begin{aligned} 1+3x^3 &= t^2 \\ 9x^2 dx &= 2t dt \end{aligned} ; \int \frac{x^2}{\sqrt{1+3x^3}} dx = \frac{2}{9} \int \frac{t dt}{t} = \frac{2}{9} t + C \right]$$

$$= \frac{2}{9} \sqrt{1+3x^3} + C$$