SORU ÖNERİ (B. Ali İBRAHİMOĞLU) MTM1501 ANALİZ 1 Dersi Final (14/01/2021)	NOT TABLOSU			

[35p] S1) $f^{-1}(x)=\dfrac{e^{x}-e^{-x}}{2}$ ise, $f\left(\dfrac{1}{3}\right)$ **'ün yaklaşık değerini lineer yaklaşım** veya **diferansiyel hesap** kullanarak bulunuz. (Çözümünüzün aşamalarını açıklayınız.) (35p)

ÇÖZÜM

$$f^{-1}(x)=\frac{e^{x}-e^{-x}}{2}=\sinh(x)\Rightarrow f(x)=\sinh^{-1}(x)$$

$f(x)=\sinh^{-1}(x)=y$ olsun.

$$y=\sinh^{-1}(x)\Rightarrow \sinh(y)=x$$

$$\cosh(y)\,y'=1$$

$$f'(x)=\frac{1}{\cosh(y)}=\frac{1}{\sqrt{\sinh^{2}(y)+1}}=\frac{1}{\sqrt{x^{2}+1}}\quad (\text{Açıklama:}\cosh^{2}(y)-\sinh^{2}(y)=1)$$

$$\left(\text{veya }\left(f^{-1}\right)'(x)=\frac{1}{f'\left(f^{-1}(x)\right)}\Rightarrow \cosh(x)=\frac{1}{f'\left(\sinh(x)\right)}\right)$$

$$f(x)\approx L(x)=f(a)+f'(a)(x-a),$$

a=0 olsun. Bu durumda, $\underbrace{f(0)=0}_{\text{açıklanacak}}$, $\underbrace{f'(0)=1}_{\text{açıklanacak}}$ olur.

$$f(x)\approx L(x)=0+1(x-0)=x$$

$$\boxed{f\left(\frac{1}{3}\right)\approx\frac{1}{3}}$$

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Let $f(x) = x.e^{-(1/x)}$

a) Determine whether f has any asymptotes and give their equations

to find the horizontol asymptotes, compute

$\lim_{x\to\infty} x.e^{-1/x}$ and $\lim_{x\to-\infty} x.e^{-1/x}$

$\lim_{x\to\infty} x = \infty$ and $\lim_{x\to\infty}(-\frac{1}{x}) = 0$, Hence $\lim_{x\to\infty} e^{-1/x} = e^0 = 1$

$\lim_{x\to\infty} x.e^{-1/x} = \infty$

Similorly, $\lim_{x\to-\infty} x.e^{-1/x} = -\infty$

f has <u>no</u> horizontal asymtotes.

f is continuous at every point of its domoin.

The domoin of f is $\{x \mid x \neq 0\}$

Thus, the only point we need to check for a <u>vertical</u> asymptote of f at $x=0$

$\lim_{x\to0^+} x e^{-1/x} = 0$, since $\lim_{x\to0^+}\frac{1}{x} = \infty$

$\lim_{x\to0^+} e^{1/x} = \infty$

$\lim_{x\to0^+}\frac{1}{e^{1/x}} = 0$

ond $\lim_{x\to0^+} x = 0$

to find

$\lim_{x\to0^-} x e^{-1/x}$; $\lim_{x\to0^-}\frac{1}{x} = -\infty$, $\lim_{x\to0^-}\left(-\frac{1}{x}\right) = \infty$ ond

$\lim_{x\to0^-} e^{-1/x} = \infty$. (indeterminote type $0.\infty$ for $\lim_{x\to0^-} x.e^{-1/x}$)

let $t = -1/x$, so as $x\to0^-$, we have $t\to\infty$

$$\lim_{x\to 0^-} x e^{-1/x} = \lim_{t\to\infty} \frac{e^t}{t} = -\lim_{t\to\infty} \frac{e^t}{t} \quad \left(\frac{\infty}{\infty}\right)$$

$$-\lim_{t\to\infty} \frac{e^t}{1} = -\infty$$

Since $\lim_{x\to 0^-} x e^{-1/x} = -\infty$, $\boxed{x=0}$ is a vertical asymptote of f.

To find the oblique asymptote,

$$m = \lim_{x\to\infty} \frac{f(x)}{x} = 1$$

$$n = \lim_{x\to\infty} (-x + x e^{-1/x}) = \lim_{x\to\infty} x(e^{-1/x} - 1)$$

$$= \lim_{x\to\infty} \frac{e^{-1/x} - 1}{\frac{1}{x}} \quad \left(\frac{0}{0}\right) \stackrel{L}{=}$$

$$\stackrel{L}{=} \lim_{x\to\infty} \frac{\left(\frac{1}{x^2}\right) \cdot e^{-1/x}}{-\frac{1}{x^2}} = -1$$

So, $y = x - 1 \Rightarrow$ oblique asymptote

Note that it can be found by using series

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b) On what Interval(s) is f increasing? decreasing?

$f'(x) = \left(1+\frac{1}{x}\right) \cdot e^{-1/x}$

since $e^{-1/x} > 0$ for all $x \neq 0$, the sign of $f'(x)$ is the same as the sign of $1+\frac{1}{x}$.

$1+\frac{1}{x} = 0$ for $x = -1$

when $x < -1$, $\frac{1}{x} > -1$ so $\frac{1}{x}+1 > 0$ and $f'(x) > 0$

when $-1 < x < 0$, $\frac{1}{x} < -1$, so $\frac{1}{x}+1 < 0$ and $f'(x) < 0$

when $x > 0$, $\frac{1}{x} > 0$, so $\frac{1}{x}+1 > 1 > 0$ and $f'(x) > 0$

Thus, f is increasing on $(-\infty,-1) \cup (0,\infty)$ and f is decreasing on the Interval $(-1,0)$

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c) On what interval(s) is f concave upward? downward?

$$f''(x) = \left(-\frac{1}{x^2}\right)e^{-1/x} + \left(1+\frac{1}{x}\right)e^{-1/x}\cdot\frac{1}{x^2}$$

$$f''(x) = e^{-1/x}\left(-\frac{1}{x^2} + \left(1+\frac{1}{x}\right)\frac{1}{x^2}\right) = e^{-1/x}\cdot\frac{1}{x^3} = \frac{1}{x^3 \cdot e^{1/x}}$$

since $e^{1/x} > 0$ for all $x \neq 0$, the sign of $f''(x)$ is the same as the sign of x. Thus,

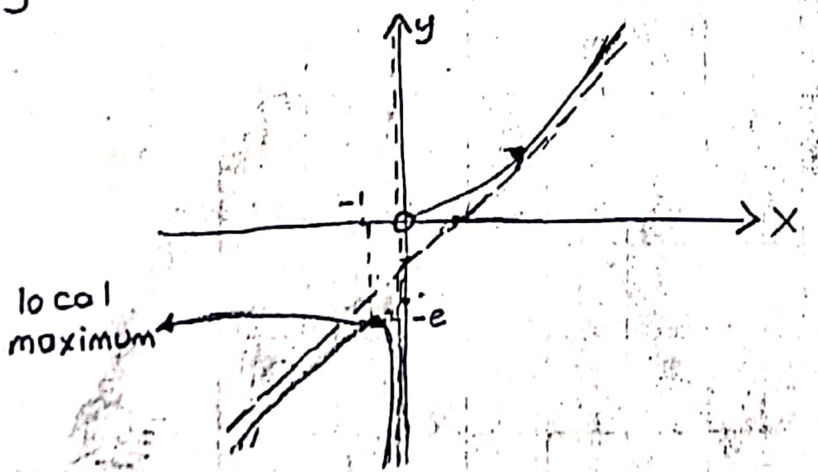
for $x>0$, $f''>0$, for $x<0$, $f''<0$. That is,

f is concave up on $(0,\infty)$

f is concave down on $(-\infty,0)$

d) Using the information that you have found sketch the graph of $y=f(x)$.

Label and provide (x,y) coordinates of any local extrema and inflection points.



$(0,0)$ is not an inflection point, because 0 is not in the domain of f.

$$\lim_{x\to 0^+} f(x) = 0$$

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Teklif edilen sorular.

Final.

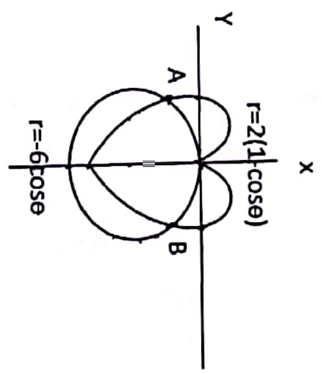
S3

a. $r = 2(1 - \cos \propto)$ ve $r = -6\cos \propto$ denklemleri ile verilen ifadelerin grafiklerini çiziniz. Bu iki eğrinin kesim noktalarını bulunuz.

Denklemlerde $\propto$ açısına bilinen değerler verilerek r bulunur.

2(1-cos$\propto$) =-6cos$\propto$ eşitliğinden cos$\propto$=-1/2 veya $\propto=\frac{2\pi}{3}$ $ve\ \alpha=\frac{4\pi}{3}$ bulunur. Buradan her iki denklemden de r=3 bulunur.

A=$(3,\frac{2\pi}{3})$ ve B=$(3,\frac{4\pi}{3})$ şeklindedir.



b. Aşağıda kutupsal koordinat sisteminde verilen eğrinin dik koordinat sistemindeki denklemini bulunuz.

$$r^2 cos2\theta = a^2$$

Çözüm:

$$r^2 cos2\theta = a^2$$

$$r^2(\quad \cos\theta^2 - \sin\theta^2) = a^2$$

olur.

$x = rcos\theta\ ve\ y = rsin\theta$ olduğundan;

$$r^2\ \cos\theta^2 - r^2\sin\theta^2 = a^2$$

$x^2 - y^2 = a^2$ bulunur.