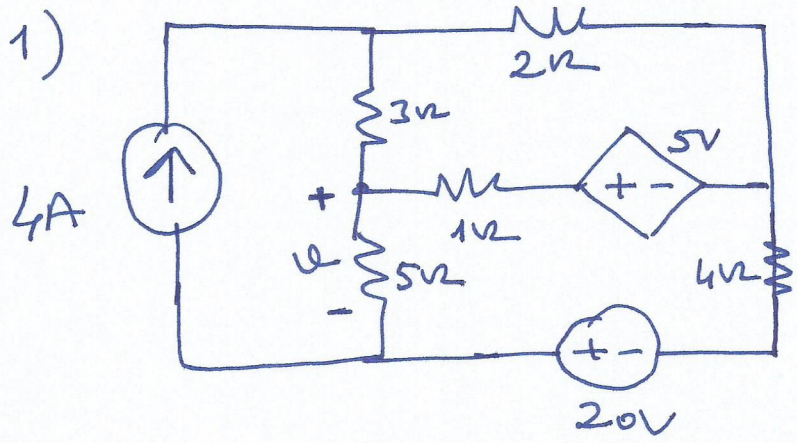
DEVRE TEORİSİ (ÖRNEKLER)

(Ş.i ①)

1)



Verilen devrede v gerilimini

a) Çevre Akımları Yöntemi (ÇSA7 – KVL) ile

b) Süperpozisyon (toplamsallık) teoremi ile

bulunuz.

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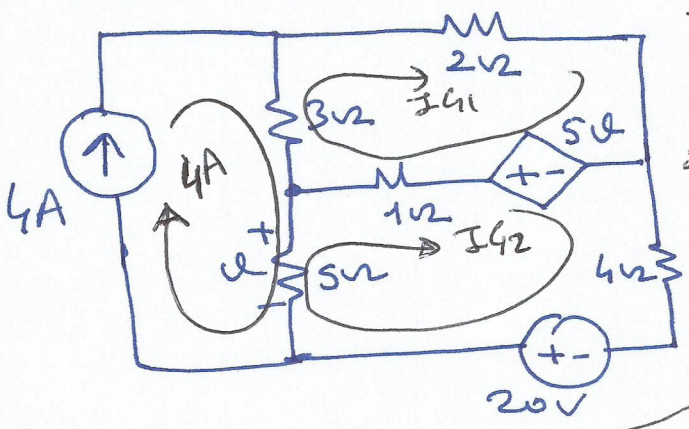
a) KVL (Gerilim Yasası) ile



$v = 5*(4 - I_{G2}) = 20 - 5I_{G2}$

$I_{G1} \rightarrow 2I_{G1} - 5v + 1(I_{G1} - I_{G2}) + 3(I_{G1} - 4$

KVL ↓

$2I_{G1} - 5(20 - 5I_{G2}) + 1(I_{G1} - I_{G2}) + 3(I_{G1} - 4)$

$\boxed{I_{G1}\cdot(6) + I_{G2}(24) = 112}$ 1. denklem

$I_{G2} \rightarrow 4I_{G2} - 20 + 5(I_{G2} - 4) + 1(I_{G2} - I_{G1}) +$

KVL ↓

$4I_{G2} - 20 + 5(I_{G2} - 4) + 1(I_{G2} - I_{G1}) + 5\cdot(20 - 5I_{G2}) = 0$

$\boxed{I_{G1}\cdot(-1) + I_{G2}\cdot(-15) = -60}$ 2. denklem.

Denklemler çözülürse;

1.D → $6I_{G1} + 24I_{G2} = 112$

$6 / \; -I_{G1} - 15I_{G2} = -60$

$-66I_{G2} = -248$

$I_{G2} = \frac{248}{66} \Rightarrow$

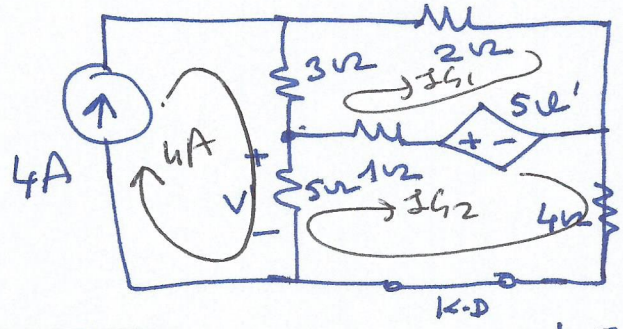
$v = 5*(4 - I_{G2}) = 5*\left(4 - \frac{248}{66}\right) = 5*\frac{16}{66} = 1{,}212$ V

bulunur.

Ş.i

(5) Süperpozisyon (Toplamsallık) Teoremi ile
(Bağımlı kaynaklar daima devrede bırakılacak)

4A lik A.K. devrede (20V → Kısa devre, 5V bağımlı k. devrede)



I_{G1} → KVL

$2I_{G1} - 5V' + 1.(I_{G1} - I_{G2}) + 3(I_{G1} - 4) = 0$

$V' = 5.(4 - I_{G2})$

$20 - 5I_{G2}$

$2I_{G1} - 5(20 - 5I_{G2}) + 1.(I_{G1} - I_{G2}) + 3(I_{G1} - 4) = 0$

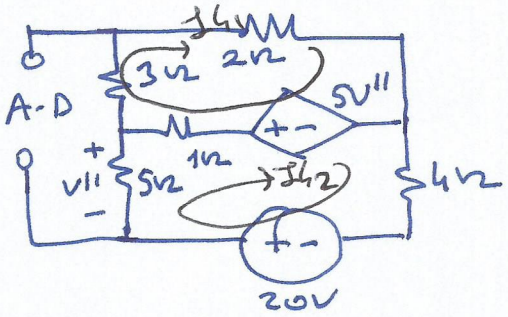
$I_{G1}.(6) + I_{G2}.(24) = 112$ 1. denklem

I_{G2} → KVL

$4I_{G2} + 5(I_{G2} - 4) + 1.(I_{G2} - I_{G1}) + 5\underbrace{(20 - 5I_{G2})}_{V'} = 0$

$I_{G1}.(-1) + I_{G2}.(-15) = -80$ 2. denklem

20 voltluk K.K devrede
(4A → A-D → 5V devrede)



Denklemler çözülürse;

$6I_{G1} + 24I_{G2} = 112$

$6/ -I_{G1} - 15I_{G2} = -80$

$-66I_{G2} = -368$

$I_{G2} = \frac{368}{66} \Rightarrow V_0' = 5 * (4 - I_{G2}) = 5(4 - \frac{368}{66})$

$V' = -7,878$ volt

$V'' = -5I_{G2}$

I_{G1} → KVL → $5I_{G1} - 5.(\underbrace{-5I_{G2}}_{V''}) + 1.(I_{G1} - I_{G2}) = 0$ → 1. denklem

$I_{G1}.(6) + I_{G2}.(24) = 0 \Rightarrow I_{G1} = -4I_{G2}$ elde edilir

I_{G2} → KVL → $4I_{G2} - 20 + 5I_{G2} + 1.(I_{G2} - I_{G1}) + 5.(-5I_{G2}) = 0$

$I_{G1}.(-1) + I_{G2}.(-15) = 20$ 2. denklem

1. denklemden

$(-4I_{G2})(-1) - 15I_{G2} = 20$

$-11I_{G2} = 20$

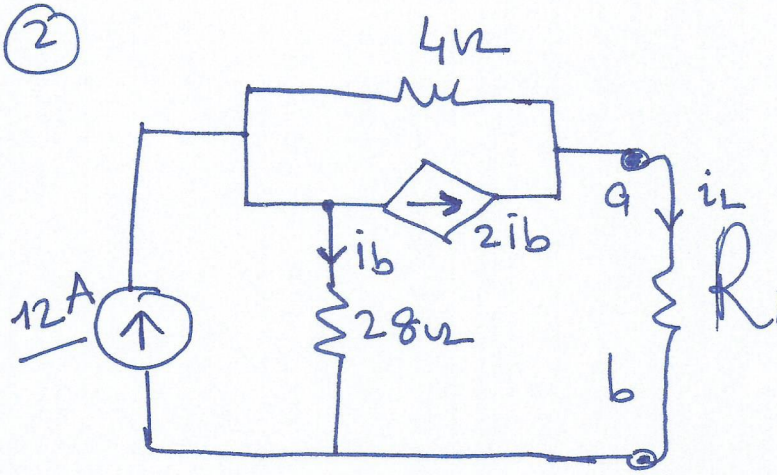
$I_{G2} = -\frac{20}{11}$ A

$V_0'' = -5 * (I_{G2}) = -5 * (-\frac{20}{11}) = \frac{100}{11} = 9,09$ V

Sonuç olarak;

$V = V' + V'' = -7,878 + 9,09 = 1,212$ volt elde edilir.

(2)

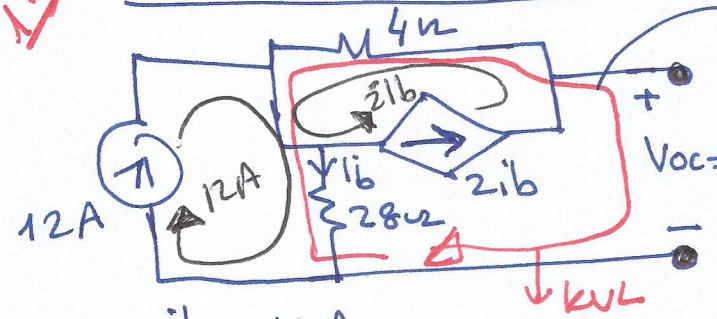


Verilen devrede;
RL yüküne maksimum güç iletebilmek için
RL ne olmalıdır?
$P_{max,RL} = ?$

Çözüm: öncelikle yük (RL) dışındaki devre için Thevenin Eşd. modelini bulalım:.

1-Yol

V+h'nin bulunması:



$V_{th} + 28*(-12) + 4\cdot(-24) = 0$

$V_{th} = 432$ volt

$i_b = 12A$

$2i_b = 24A$

$28*(I_{sc}-12) + 4*(I_{sc}-2i_b)$

$i_b = 12 - I_{sc}$

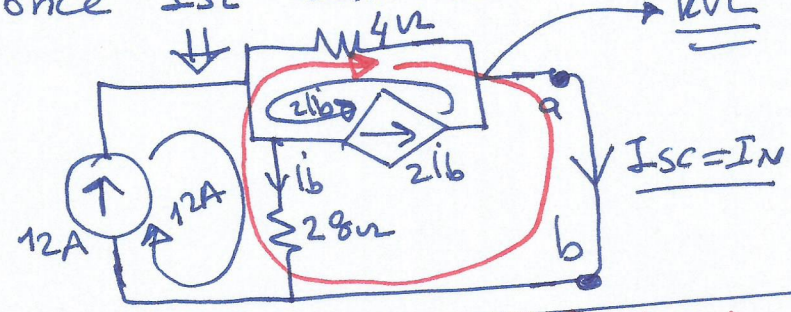
$28*(I_{sc}-12) + 4I_{sc} - 8(12-I_{sc})$

$I_{sc}\cdot(28+4+8) = 432$

$I_{sc} = 432/40$ A

R_{th}'nin bulunması → $\frac{V_{oc}}{I_{sc}} = R_{th}$

önce Isc bulunur.

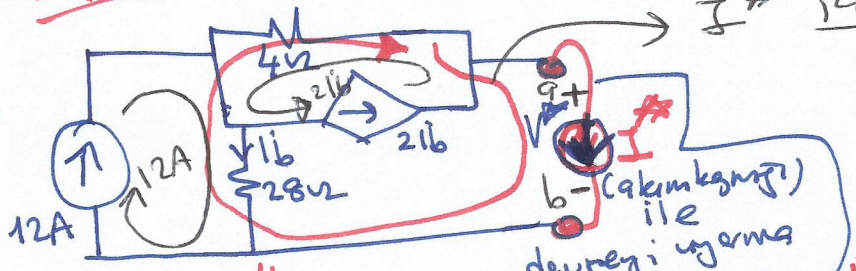


$R_L = R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{432}{432/40} = 40\Omega$

$P_{max,RL} = \frac{V_{th}^2}{4R_{th}} = \frac{(432)^2}{160}$

=

2-Yol → a-b arasına I^* akım kaynağı ile devreyi uyarma:



I^* için KVL

$i_b = 12 - I^*$

$V^* + 28(I^*-12) + 4\cdot(I^*-2i_b) = 0$

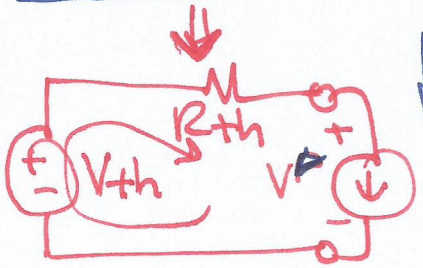
$V^* + 28(I^*-12) + 4I^* - 8(12-I^*) =$

$V^* + I^*\cdot(28+4+8) - 432 = 0$

$V^* = 40\cdot(-I^*) + 432$

$V^* = R_{th}(-I^*) + V_{th}$

$V^* - V_{th} + R_{th}\cdot I^* = 0$



Rth

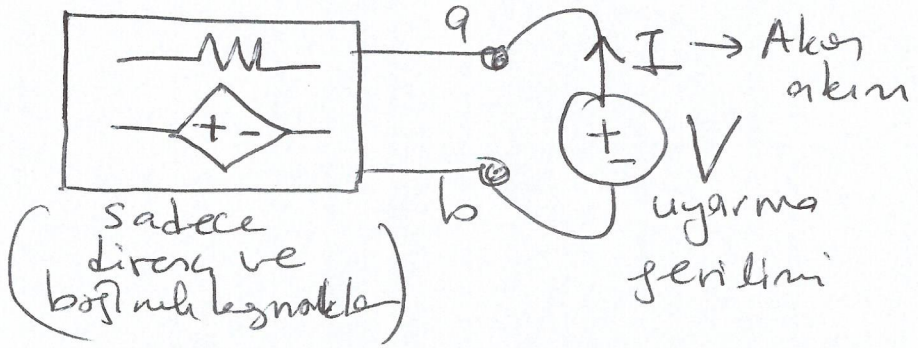
Vth

Thevenin model bileşenleri

Rth için 3. yol :

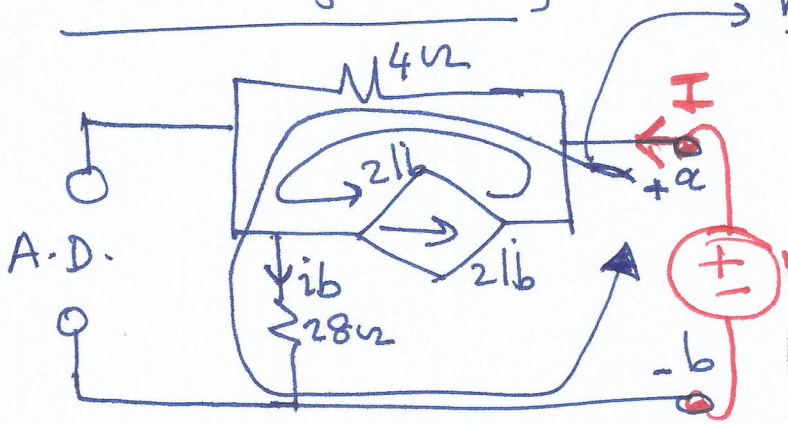
Devrede sadece bağımlı kaynaklar kalacak şekilde diğer kaynaklar (bağımsız) sıfırlanır → Gerilim k → k-d / Akım k → A-d.

a-b arasına V birimli bir gerilim kaynağı bağlanır. Kaynağın çıkışındaki I akımına oranlandığında (V/I) elde edilen değer Rth'dır.



$$\frac{V}{I} = R_{th}$$

Çözüme geçersek;



KVL

$(i_b = I)$'dir.

$$-V + 4\cdot(I + 2i_b) + 28I =$$

$$-V + 4I + 8I + 28I =$$

$$V = 40I$$

$$R_{th} = \frac{V}{I} = 40\ \Omega$$

elde edili

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4. yol (1A)'lik akım kaynağı ile uygarma

(Sadece direnç ve bağımlı kaynaklar devrede)

1A'lik akım kaynağının gerilimi V olsun, $R_{th} = \frac{V}{1}$

$i_b = 1A$

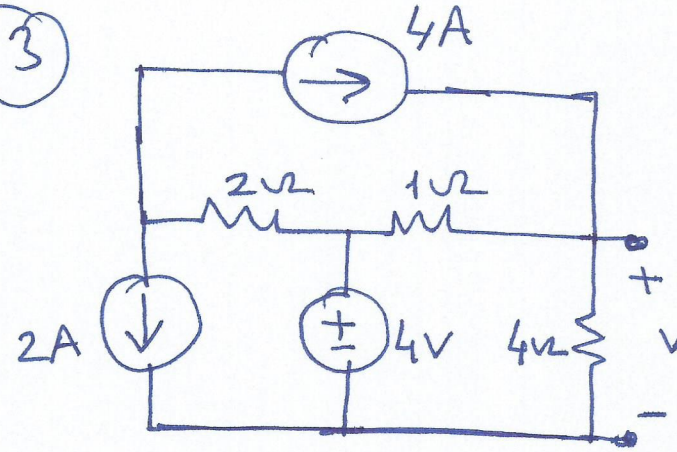
$$-V + 3.4 + 28.1 = 0$$

$$V = 40 \text{ volt}$$

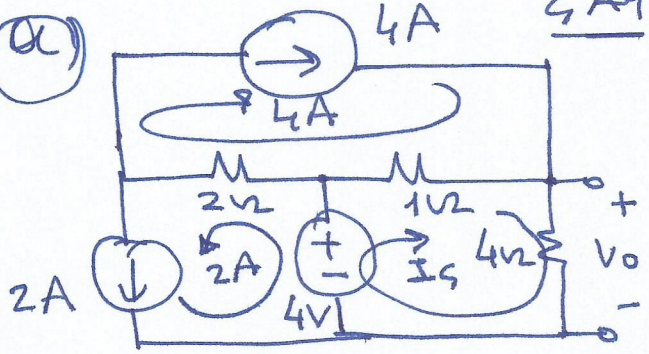
$$R_{th} = \frac{V_{volt}}{1 amper} = \frac{40}{1} = 40\ \Omega$$

elde edildi

(3) Verilen devrede 4Ω'luk direncin gerilimini a) ÇAM (KVL) ile b) DÜĞÜM (KCL) ile c) Thevenin ile d) Norton ile e) Süperpozisyon ile f) 4Ω yerine ne olmalıki max güç iletilsin bulunuz



(a) ÇAM



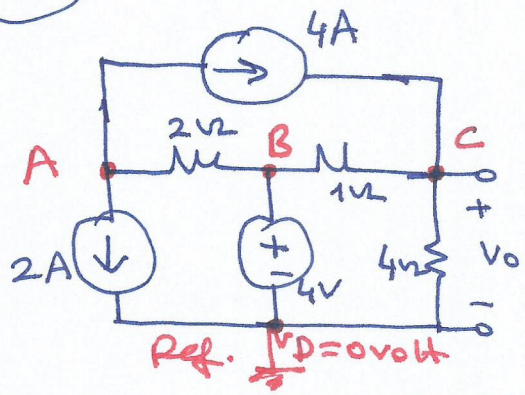
Ig → KVL →

$4I_g - 4 + 1.(I_g - 4) = 0$

$5I_g = 8 \Rightarrow I_g = \frac{8}{5}$ A

$V_o = 4 * I_g = \frac{32}{5}$ Volt

(b) DÜĞÜM



V_A, V_C → bilinmiyor (2 adet KCL denklemi)

$V_B = 4V$ → bilinmiyor.

A → KCL → $2 + 4 + \frac{V_A - 4}{2} = 0$

$\frac{V_A}{2} = -4 \Rightarrow V_A = -8$ volt

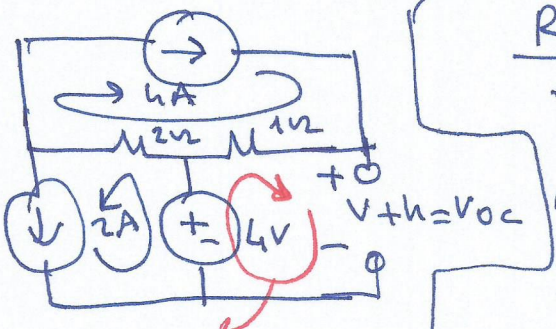
C → KCL → $\frac{V_C}{4} + \frac{V_C - 4}{1} - 4 = 0$

$V_C(\frac{1}{4} + 1) = 4 + 4 = 8$

$V_o = V_C = \frac{32}{5}$ volt

(c) Thevenin ile

Vth 'yi bulalım.

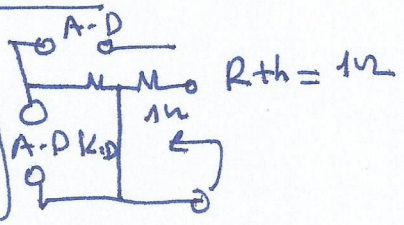


KVL

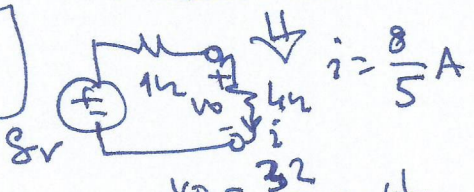
$V_{th} - 4 + 1.(-4) = 0$

$V_{th} = 8$ volt

Rth



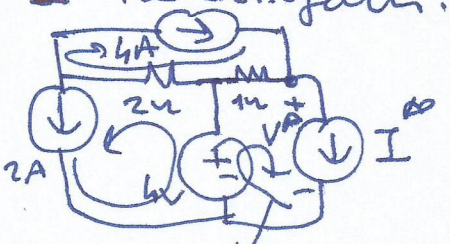
Thevenin Eşd. Devresi



$i = \frac{8}{5}$ A

$V_o = \frac{32}{5}$ volt

I* ile deneyelim.



KVL

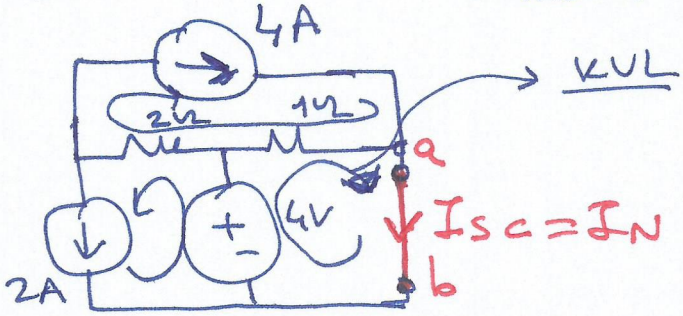
$V^* - 4 + 1.(I^* - 4)$

$V^* = 1.(-I^*) + 8$

↓Rth ↓Vth

bulunur

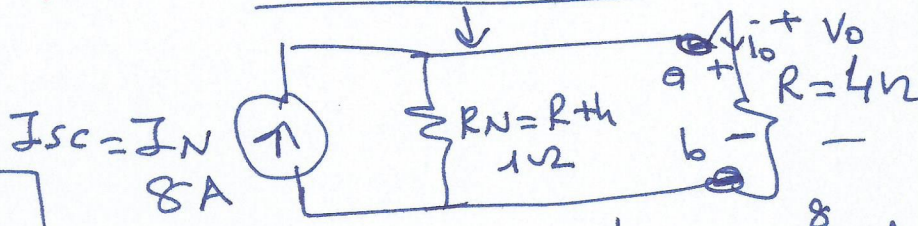
(d) I_N (Norton akımını) bulalım. → "I_{sc}" → Kısa devre akımı



KVL → $-4+1.(I_{sc}-4)=0$

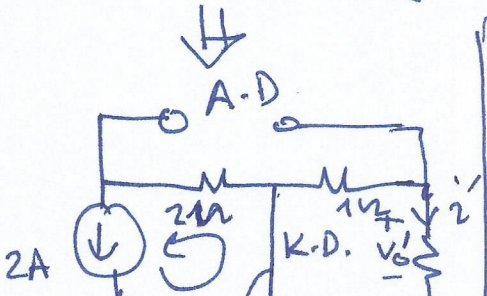
$I_{sc} = 8\ A$

Norton Eşd. Devresi



(e) Süperpozisyon T. ile bulalım. (Kaynaklar tek-tek devrede olacak)

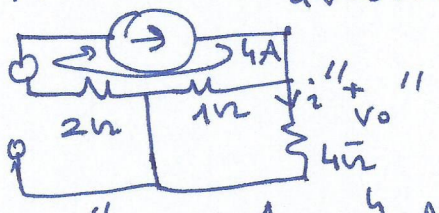
2A varken (4A → A.D, 4V → K.D.)



i' doğru
$i' = 0\ A$

$V_o' = 0$ volt.

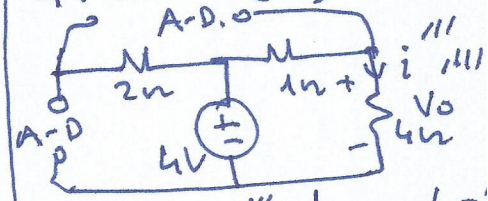
4A varken (2A → A.D., 4V → K.D)



$i'' = 4 * \frac{1}{5} = \frac{4}{5}\ A$

$V_o'' = 4 * \frac{4}{5} = \frac{16}{5}$ volt

4V varken (2A, 4A → A.D.)



$V_o''' = \frac{4}{5} * 4 =$

$i_o = 8 * \frac{1}{1+4} = \frac{8}{5}\ A$

$V_o = \frac{8}{5} * 4 = \frac{32}{5}$ volt bulunur

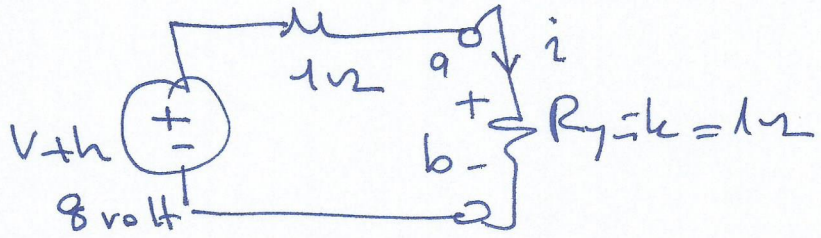
Sonuç olarak,

$V_o = V_o' + V_o'' + V_o'''$

$= 0 + \frac{16}{5} + \frac{16}{5} = \frac{32}{5} =$

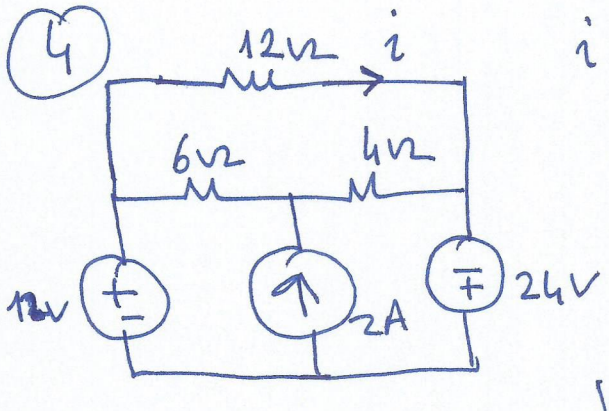
(f) max. güç için $R_{yük} = R_{th}$ olmalı;

$R_{th} = 1\Omega$, $R_{yük} = 1\Omega$ olur.



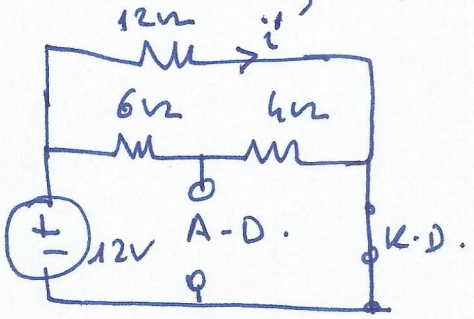
$i = \frac{8}{2} = 4\ A$

$P_{max, R_{yük}} = R_y i^2 = 1 * 4^2 = 16\ W$ elde edilir.

4) 

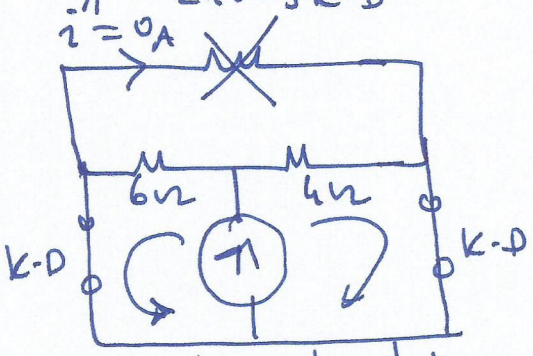
i akımını;
a) süperpozisyon t. ile
b) Thevenin t. ile
bulunuz.

a) 12 V varken
(2A → A.D, 24V → K.D.)



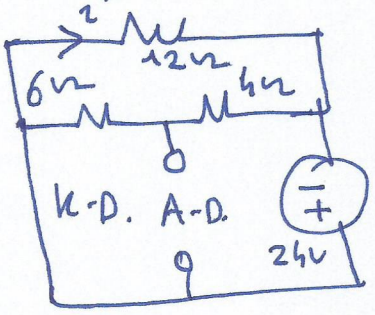
$i' = \frac{12}{12} = 1A$

2A varken
(12V → K.D.
24V → K.D)



Kısa devreden dolayı
$i'' = 0A$

24 V varken
(2A → A-D
12V → K.D.)



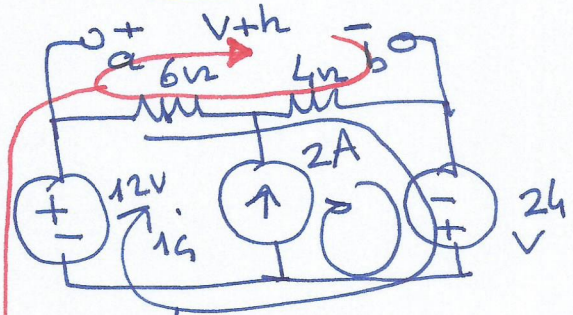
$i''' = \frac{24}{12} = 2A$

$i = i' + i'' + i'''$

$i = 1 + 0 + 2 = 3A$ elde edilir

b) Thevenin Teoremi ile

V+h



$i_G \rightarrow$ KVL

$-24 - 12 + 6i_G + 4\cdot(i_G + 2) = 0$

$10 i_G = 28 \Rightarrow$

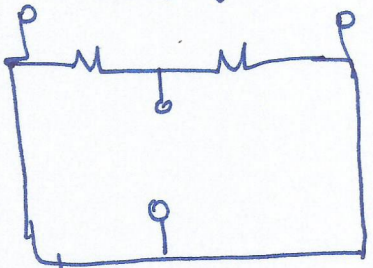
$i_G = \frac{14}{5} A.$

KVL

$V_{+h} + 4\cdot(-i_G - 2) + 6\cdot(-i_G) = 0$

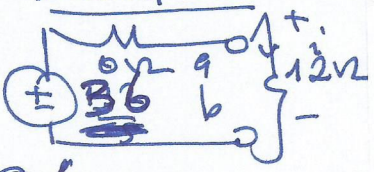
$V_{+h} = 10 i_G + 8 = 36$ volt

R+h



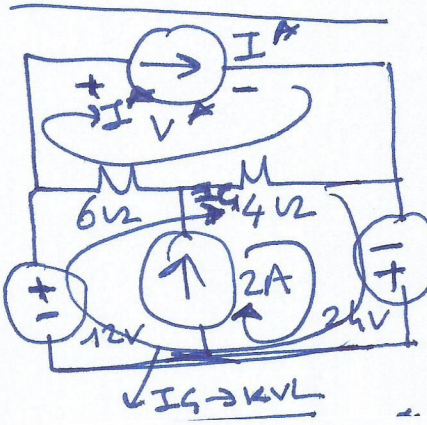
$R_{+h} = 0\,\Omega$
(Kısa Devre)

Sonuç olarak



$i = \frac{36 \text{ volt}}{12\,\Omega} = 3A$ elde edilir

I^* ile bulalım:



$I_G \rightarrow$ KVL

$-24 - 12 + 6(I_G - I^*) + 4(2 + I_G - I^*) = 0$

$10 I_G = 28 + 10 I^*$

$I_G = 2{,}8 + 1\cdot I^*$

$V^* \rightarrow$ KVL

$V^* + 4\cdot(-2 - I_G + I^*) + 6(-I_G + I^*) = 0$

$V^* + 4(-2 - 2{,}8 - I^* + I^*) + 6(-2{,}8 - I^* + I^*) = 0$

$V^* + (0)\cdot(I^*) - 36 = 0$

$V^* = (0)\cdot(-I^*) + 36$

V_{+h}

(5) ETKİN DEĞER (EFEKTİF DEĞER) - ORTALAMA DEĞER

Bilgi Notu:

(1) Periyodik bir fonksiyona ilişkin efektif değer

$$x_{ef} = \sqrt{\frac{1}{T}\int_0^{+} [x(t)]^2 dt}$$

(2) Ortalama değer

$$x_{ort} = \frac{1}{T}\int_0^{t} x(t)\,dt$$

parçalı fonksiyon durumunda;

$$x_{ort} = \frac{1}{T}\left[\int_0^{t_1} x_1(t)\,dt + \int_{t_1}^{t_2} x_2(t)\,dt + \dots\right]$$ şeklinde olacaktır.

Not: parçalı fonksiyonlarda her kısım için ayrı ayrı işlem yapılır.

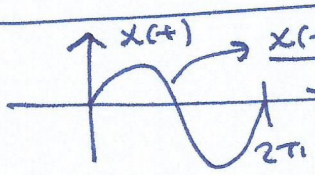
$$x(t) = \begin{cases} x_1(t) & 0 < t < t_1 \\ x_2(t) & t_1 < t < t_2 \end{cases}$$

$$x_{ef} = \sqrt{\frac{1}{T}\left[\int_0^{t_1} x_1^2(t)\,dt + \int_{t_1}^{t_2} x_2^2(t)\,dt \dots\right.}$$

(3) $x(t) = x_1(t) + x_2(t) + \dots x_n(t) = x_{m1}\cdot\sin\omega_1 t + x_{m2}\cdot\sin\omega_2 t + x_{mn}$ şeklindeki sinüsoidal fonksiyonların efektif değeri:

$$x_{ef} = \sqrt{\left(\frac{x_{m1}}{\sqrt{2}}\right)^2 + \left(\frac{x_{m2}}{\sqrt{2}}\right)^2 + \dots \left(\frac{x_{mn}}{\sqrt{2}}\right)^2}$$ şeklinde hesaplanır.

Örnek (5) $x(t) = x_m\cdot\sin\omega t$ fonksiyonun efektif değeri bulunuz.



$$\sin^2\omega t = \frac{1-2\cos 2\omega t}{2}$$

$$x_{ef} = \sqrt{\frac{1}{2\pi}\cdot\int_0^{2\pi}(x_m\cdot\sin\omega t)^2\,dt}$$

$$x_{ef}^2 = \frac{x_m^2}{2\pi}\cdot\int_0^{2\pi}\left(\frac{1-2\cos 2\omega t}{2}\right)dt = \frac{x_m^2}{2\pi}\cdot\int_0^{2\pi}\left(\frac{1}{2}-\cos 2\omega t\right)dt$$

$$= \frac{x_m^2}{2\pi}\cdot\int_0^{2\pi}\left[\frac{1}{2}-\cos 2\omega t\right]dt = \frac{x_m^2}{2\pi}\cdot\left[\frac{\omega t}{2}+\frac{1}{2}\sin 2\omega t\right]\Bigg|_0^{2\pi}$$

$$x_{ef}^2 = \frac{x_m^2}{2\pi}\cdot\left[\frac{2\pi}{2}-0\right] = \frac{x_m^2}{2}$$

$$x_{ef} = \frac{x_m}{\sqrt{2}} \qquad x_m = \sqrt{2}\cdot x_{ef}$$

(Sinüsoidal işaretler için;)

($\sqrt{2}$) kavramının önemi

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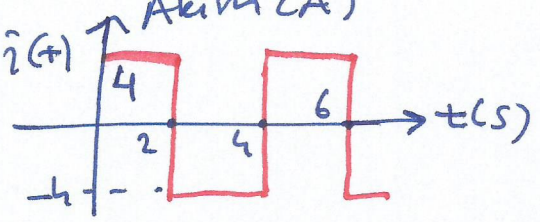

a) verilen dalga şekli için
$i_{ef} = ?$ $i_{ort} = ?$

b) i_{ef} → R = 10Ω $P_R = ?$
$E_{R(24 saat)} = ?$

a) $i(t) = \begin{cases} 4 & 0 < t \leq 2 \\ -4 & 2 < t \leq 4 \end{cases}$

T (periyot) = 4 s.

$$x_{ef} = i_{ef} = \sqrt{\frac{1}{4}\left[\int_0^2 4^2\,dt + \int_2^4 (-4)^2\,dt\right]}$$

$$x_{ef}^2 = i_{ef}^2 = \frac{1}{4}\cdot\left[16t\Big|_0^2 + 16t\Big|_2^4\right] \Rightarrow$$

$$i_{ef}^2 = \frac{1}{4}\cdot[32+32] = 16$$

$i_{ef} = 4$ A ede edilir.

$i_{ort} = 0$ A [Alan eşitliği]

b) $P_R = R \cdot i_{ef}^2$
$= 10 * (4)^2 = 160$ W

$E_{R(24 saat)} = P \cdot t$
$= 160 * 24$
$= 3840$ W·saat
$= 3,84$ kWh

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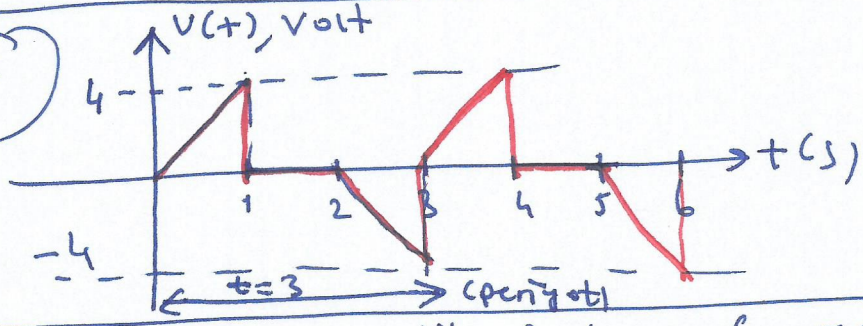


a) $V_{ef} = ?$
b) i_{ef} → V_{ef} } R = 15Ω
$P_R = ?$
$E_R = ?$ (1 gün)

Dikkat edilirse periyodik fonksiyon (parçalı)

a) $V(t) = \begin{cases} 4t & 0 < t \leq 1\,s \\ 0 & 1 < t \leq 2\,s \\ -4t+8 & 2 < t \leq 3\,s \end{cases}$

$$V_{ef}^2 = \frac{1}{3}\cdot\left[\int_0^1 (4t)^2\,dt + \int_2^3 (-4t+8)^2\,dt\right]$$

$$V_{ef}^2 = \frac{1}{3}\cdot\left[16\frac{t^3}{3}\Big|_0^1 + \left(16\frac{t^3}{3} - 32t^2 + 64t\right)\Big|_2^3\right]$$

$$V_{ef}^2 = \frac{1}{3}\left[\frac{16}{3} + (144 - 288 + 192) - \left(\frac{128}{3} - 128 + 128\right)\right]$$

$V_{ef}^2 = 3,55 \Rightarrow V_{ef} = 1,89$ volt

b) $P_R = R \cdot i^2 = V^2/R = \frac{(1,89)^2}{15} = 0,236$ W

$E_{R(1 gün)} = 0,236 * 24 = 5,68$ W.h ede edilir.

$y = mx + n$ ile denklem bulunması:

0-1 için — (1,4) noktası için x, y — m = (eğim) : 4

$4 = 4 \cdot (1) + n$
$n = 0 \Rightarrow y = 4x$ | $V(t) = 4t$ bulunur.

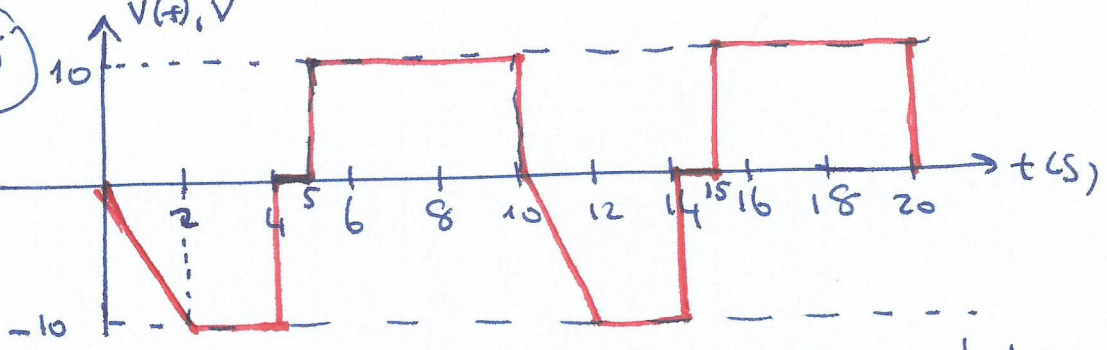
2-3 için — (3,-4) noktası

$y = mx + n$ — $m = -4$

$-4 = -4 \cdot (3) + n$
$n = 8$ $y = -4x + 8$
$V(t) = -4t + 8$ bulunur.

(5-i)

(8)



2Ω'luk bir direnç üzerindeki gerilim dalga edönüşümünü kullanarak bulunuz.

Direnç tarafından çekilen ortalama güçü bulunuz.

$$V(t)=\begin{cases} -5t & 0<t\le 2 \\ -10 & 2<t\le 4 \\ 0 & 4<t\le 5 \\ 10 & 5<t\le 10 \end{cases}$$

periyot(T): 10

$$V_{ort}=\frac{1}{T}\int_0 V(t)\,dt$$

$$V_{ort}=\frac{1}{10}\cdot\left[\int_0^2(-5t)\,dt+\int_2^4(-10)\,dt+\int_5^{10}10\,dt\right]$$

$$V_{ort}=\frac{1}{10}\cdot\left[-5\frac{t^2}{2}\Big|_0^2-10t\Big|_2^4+10t\Big|_5^{10}\right]$$

$$V_{ort}=\frac{1}{10}\cdot\left[-10-40+20+100-50\right]=2\ \text{volt}$$

$$P_{ort}=\frac{V_{ort}^2}{R}=\frac{2^2}{2}=2\ W$$

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(9) $i(t)=10\sin t+5\sin 2t+2\sin 3t$ A ise

$i_{ef}=?$

$$i_{ef}=\sqrt{\left(\frac{10}{\sqrt2}\right)^2+\left(\frac{5}{\sqrt2}\right)^2+\left(\frac{2}{\sqrt2}\right)^2}=8{,}03\ \text{A}$$ elde edilir.

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(10) $V(t)=4+3\cos 3t+2\cos 5t$ V $\Rightarrow$

$V_{ef}=?$

$$V_{ef}=\sqrt{4^2+\left(\frac{3}{\sqrt2}\right)^2+\left(\frac{2}{\sqrt2}\right)^2}=4{,}74\ \text{V}$$ elde edilir.

Σ.1