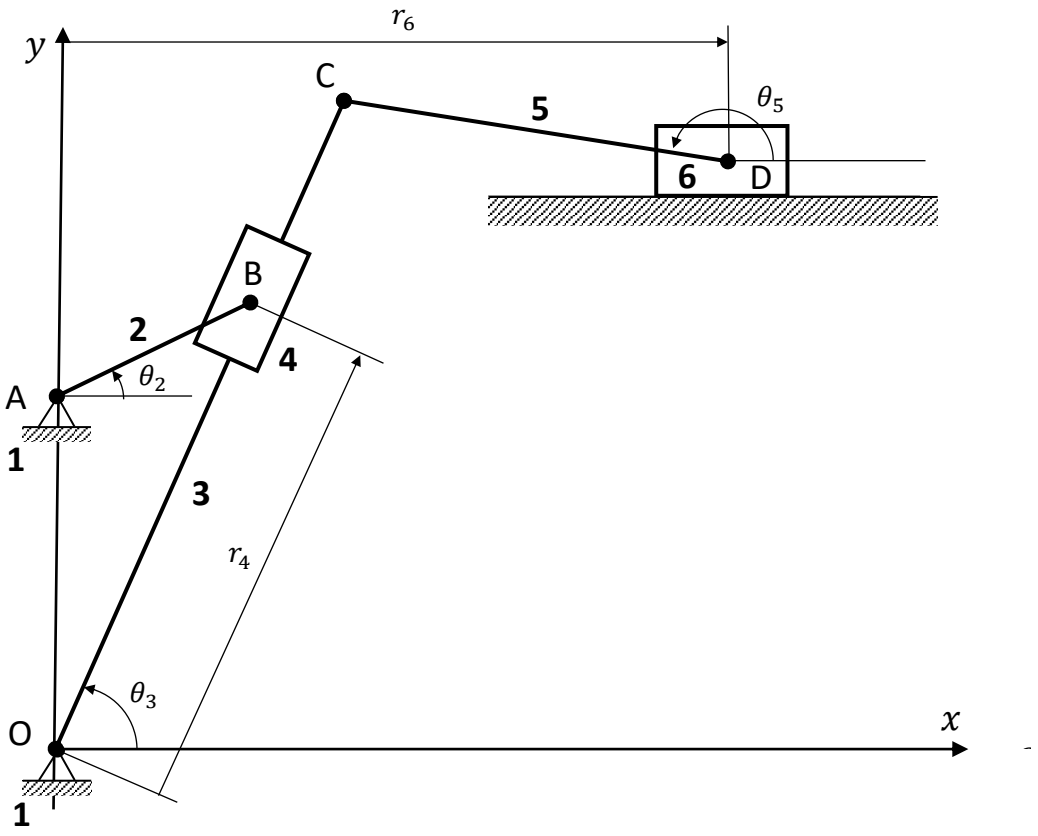
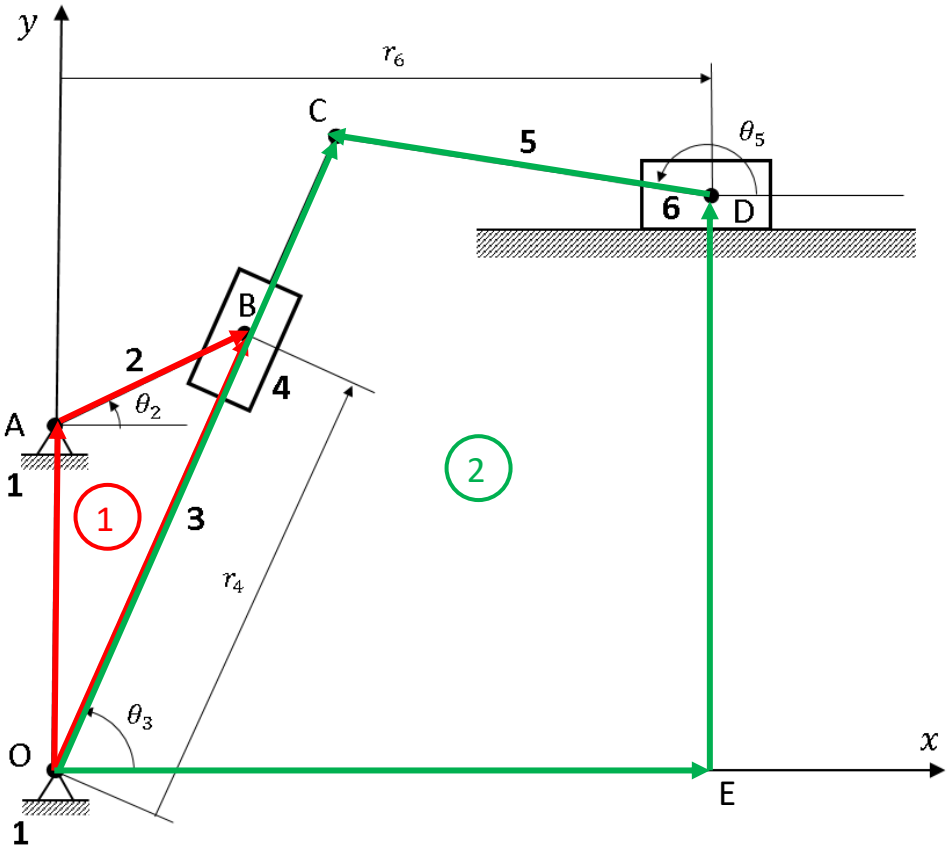
Vargel Mekanizmasının Kinematik Analizi

Analitik Çözüm:

Mekanizmaya ait genelleştirilmiş koordinatlar

$$\underbrace{\theta_2}_{esas\ genelleştirilmiş\ koordinat} \qquad \underbrace{\theta_3,\ r_4,\ \theta_5,\ r_6}_{tali\ genelleştirilmiş\ koordinatlar}$$





1 nolu devre kapalılık denklemini yazalım,

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

Devre kapalılık denklemini döner vektör formunda yazalım,

$$\overline{OA}e^{i90^\circ} + \overline{AB}e^{i\theta_2} = r_4 e^{i\theta_3}$$

Döner vektörlerin Euler açılımlarını yapalım,

$$\overline{OA}(cos90^\circ + isin90^\circ) + \overline{AB}(cos\theta_2 + isin\theta_2) = r_4(cos\theta_3 + isin\theta_3)$$

$$\overline{OA}\left(\underbrace{cos90^\circ}_{0} + i\,\underbrace{sin90^\circ}_{1}\right) + \overline{AB}(cos\theta_2 + isin\theta_2) = r_4(cos\theta_3 + isin\theta_3)$$

$$\overline{OA}i + \overline{AB}(cos\theta_2 + isin\theta_2) = r_4(cos\theta_3 + isin\theta_3)$$

Bu kompleks devre kapalılık denklemini reel ve imajiner kısımlarını birbirine eşitleyerek iki adet kısıt denklemini elde edelim.

$$\overline{AB}cos\theta_2 = r_4 cos\theta_3 \qquad 1$$

$$\overline{OA} + \overline{AB}sin\theta_2 = r_4 sin\theta_3 \qquad 2$$

2 nolu devre kapalılık denklemini yazalım,

$$\overrightarrow{OE} + \overrightarrow{ED} + \overrightarrow{DC} = \overrightarrow{OC}$$

Devre kapalılık denklemini döner vektör formunda yazalım,

$$\overline{OE}e^{i0^\circ} + \overline{ED}e^{i90^\circ} + \overline{DC}e^{i\theta_5} = \overline{OC}e^{i\theta_3}$$

Döner vektörlerin Euler açılımlarını yapalım, burada $\overline{OE} = r_6$ dır.

$$r_6(cos0^\circ + isin0^\circ) + \overline{ED}(cos90^\circ + isin90^\circ) + \overline{DC}(cos\theta_5 + isin\theta_5) = \overline{OC}(cos\theta_3 + isin\theta_3)$$

$$r_6\left(\underbrace{cos0^\circ}_{1} + i\underbrace{sin0^\circ}_{0}\right) + \overline{ED}\left(\underbrace{cos90^\circ}_{0} + i\underbrace{sin90^\circ}_{1}\right) + \overline{DC}(cos\theta_5 + isin\theta_5) = \overline{OC}(cos\theta_3 + isin\theta_3)$$

$$r_6 + \overline{ED}i + \overline{DC}(cos\theta_5 + isin\theta_5) = \overline{OC}(cos\theta_3 + isin\theta_3)$$

Bu kompleks devre kapalılık denklemini gerçek ve sanal kısımlarını birbirine eşitleyerek iki adet kısıt denklemi elde edelim.

$$r_6 + \overline{DC}cos\theta_5 = \overline{OC}cos\theta_3 \qquad 3$$

$$\overline{ED} + \overline{DC}sin\theta_5 = \overline{OC}sin\theta_3 \qquad 4$$

Şimdi bu dört kısıt denkleminden tali genelleştirilmiş koordinatları esas genelleştirilmiş koordinatlar cinsinden elde edelim.

$$\theta_3(\theta_2) =? \quad r_4(\theta_2) =? \quad \theta_5(\theta_2) =? \quad r_6(\theta_2) =?$$

1 ve 2 nolu denklemleri taraf tarafa bölelim.

$$\frac{\overline{AB}cos\theta_2}{\overline{OA} + \overline{AB}sin\theta_2} = \frac{r_4cos\theta_3}{r_4sin\theta_3}$$

$$\frac{\overline{AB}cos\theta_2}{\overline{OA} + \overline{AB}sin\theta_2} = \frac{cos\theta_3}{sin\theta_3} \quad \Rightarrow \quad tan\theta_3 = \frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2} \quad \Rightarrow \quad \theta_3 = tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right) \quad 5$$

5 nolu eşitliği 1 nolu denklemde yerine koyalım.

$$\overline{AB}cos\theta_2 = r_4cos\theta_3 \quad \Rightarrow \quad r_4 = \frac{\overline{AB}cos\theta_2}{cos\theta_3} \quad \Rightarrow \quad r_4 = \frac{\overline{AB}cos\theta_2}{cos\left[tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right]} \qquad 6$$

4 nolu kısıt denklemine 5 nolu ifadeyi koyalım.

$$\overline{ED} + \overline{DC}sin\theta_5 = \overline{OC}sin\theta_3 \quad \Rightarrow \quad sin\theta_5 = \frac{\overline{OC}sin\theta_3 - \overline{ED}}{\overline{DC}} \quad \Rightarrow$$

$$\theta_5 = sin^{-1}\left[\frac{\overline{OC}sin\left[tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right] - \overline{ED}}{\overline{DC}}\right] \qquad 7$$

3 nolu kısıt denklemine 5 ve 7 nolu ifadeyi koyalım.

$$r_6 + \overline{DC}cos\theta_5 = \overline{OC}cos\theta_3 \quad \Rightarrow \quad r_6 = \overline{OC}cos\theta_3 - \overline{DC}cos\theta_5 \quad \Rightarrow$$

$$r_6 = \overline{OC}cos\left[tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right] - \overline{DC}cos\left[\frac{\overline{OC}\sin\left(tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right) - \overline{ED}}{\overline{DC}}\right] \qquad 8$$

5, 6, 7 ve 8 nolu ifadeler ile mekanizmaya ait konum analizi yapılmış olur.

Nümerik Çözüm:

Devre kapalılık denklemlerinden elde ettiğimiz kısıt denklemlerini alalım,

$$\overline{AB}cos\theta_2 = r_4 cos\theta_3 \qquad 1$$

$$\overline{OA} + \overline{AB}sin\theta_2 = r_4 sin\theta_3 \qquad 2$$

$$r_6 + \overline{DC}cos\theta_5 = \overline{OC}cos\theta_3 \qquad 3$$

$$\overline{ED} + \overline{DC}sin\theta_5 = \overline{OC}sin\theta_3 \qquad 4$$

Kısıt denklemlerinde aşağıdaki yaklaşık ifadeleri kullanalım.

$$cos\theta_3 \cong cos(\bar{\theta}_3 + \Delta\theta_3)$$

$$cos\theta_5 \cong cos(\bar{\theta}_5 + \Delta\theta_5)$$

$$sin\theta_3 \cong sin(\bar{\theta}_3 + \Delta\theta_3)$$

$$sin\theta_5 \cong sin(\bar{\theta}_5 + \Delta\theta_5)$$

$$r_4 \cong \bar{r}_4 + \Delta r_4$$

$$r_6 \cong \bar{r}_6 + \Delta r_6$$

Buradan, kısıt denklemleri aşağıdaki hali alacaktır.

$$\overline{AB}cos\theta_2 = (\bar{r}_4 + \Delta r_4)cos(\bar{\theta}_3 + \Delta\theta_3) \qquad 5$$

$$\overline{OA} + \overline{AB}sin\theta_2 = (\bar{r}_4 + \Delta r_4)sin(\bar{\theta}_3 + \Delta\theta_3) \qquad 6$$

$$(\bar{r}_6 + \Delta r_6) + \overline{DC}cos(\bar{\theta}_5 + \Delta\theta_5) = \overline{OC}cos(\bar{\theta}_3 + \Delta\theta_3) \qquad 7$$

$$\overline{ED} + \overline{DC}sin(\bar{\theta}_5 + \Delta\theta_5) = \overline{OC}sin(\bar{\theta}_3 + \Delta\theta_3) \qquad 8$$

Bu denklemleri aşağıdaki eşitlikleri kullanarak tekrar düzenleyelim.

$$cos(\bar{\theta}_3 + \Delta\theta_3) = cos\bar{\theta}_3 cos\Delta\theta_3 - sin\bar{\theta}_3 sin\Delta\theta_3$$

$$cos(\bar{\theta}_5 + \Delta\theta_5) = cos\bar{\theta}_5 cos\Delta\theta_5 - sin\bar{\theta}_5 sin\Delta\theta_5$$

$$sin(\bar{\theta}_3 + \Delta\theta_3) = sin\bar{\theta}_3 cos\Delta\theta_3 + cos\bar{\theta}_3 sin\Delta\theta_3$$

$$sin(\bar{\theta}_5 + \Delta\theta_5) = sin\bar{\theta}_5 cos\Delta\theta_5 + cos\bar{\theta}_5 sin\Delta\theta_5$$

5, 6, 7 ve 8 nolu denkler aşağıdaki hali alır.

$$\overline{AB}cos\theta_2 = (\bar{r}_4 + \Delta r_4)(cos\bar{\theta}_3 cos\Delta\theta_3 - sin\bar{\theta}_3 sin\Delta\theta_3) \qquad 9$$

$$\overline{OA} + \overline{AB}sin\theta_2 = (\bar{r}_4 + \Delta r_4)(sin\bar{\theta}_3 cos\Delta\theta_3 + cos\bar{\theta}_3 sin\Delta\theta_3) \qquad 10$$

$$(\bar{r}_6 + \Delta r_6) + \overline{DC}(cos\bar{\theta}_5 cos\Delta\theta_5 - sin\bar{\theta}_5 sin\Delta\theta_5) = \overline{OC}(cos\bar{\theta}_3 cos\Delta\theta_3 - sin\bar{\theta}_3 sin\Delta\theta_3) \qquad 11$$

$$\overline{ED} + \overline{DC}(sin\bar{\theta}_5 cos\Delta\theta_5 + cos\bar{\theta}_5 sin\Delta\theta_5) = \overline{OC}(sin\bar{\theta}_3 cos\Delta\theta_3 + cos\bar{\theta}_3 sin\Delta\theta_3) \qquad 12$$

Aşağıdaki kabulleri 9, 10, 11 ve 12 nolu denklemlerde yerine koyup tekrar düzenleyelim.

$$cos\Delta\theta_3 \cong 1, \quad sin\Delta\theta_3 \cong \Delta\theta_3, \quad cos\Delta\theta_5 \cong 1, \quad sin\Delta\theta_5 \cong \Delta\theta_5$$

$$\overline{AB}cos\theta_2 = (\bar{r}_4 + \Delta r_4)\left(cos\bar{\theta}_3 \underbrace{cos\Delta\theta_3}_{1} - sin\bar{\theta}_3 \underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

$$\overline{OA} + \overline{AB}sin\theta_2 = (\bar{r}_4 + \Delta r_4)\left(sin\bar{\theta}_3 \underbrace{cos\Delta\theta_3}_{1} + cos\bar{\theta}_3 \underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

$$(\bar{r}_6 + \Delta r_6) + \overline{DC}\left(cos\bar{\theta}_5 \underbrace{cos\Delta\theta_5}_{1} - sin\bar{\theta}_5 \underbrace{sin\Delta\theta_5}_{\Delta\theta_5}\right) = \overline{OC}\left(cos\bar{\theta}_3 \underbrace{cos\Delta\theta_3}_{1} - sin\bar{\theta}_3 \underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

$$\overline{ED} + \overline{DC}\left(sin\bar{\theta}_5 \underbrace{cos\Delta\theta_5}_{1} + cos\bar{\theta}_5 \underbrace{sin\Delta\theta_5}_{\Delta\theta_5}\right) = \overline{OC}\left(sin\bar{\theta}_3 \underbrace{cos\Delta\theta_3}_{1} + cos\bar{\theta}_3 \underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

Yukarıdaki denklemleri aşağıdaki gibi açalım.

$$\overline{AB}cos\theta_2 = \bar{r}_4 cos\bar{\theta}_3 - \bar{r}_4 sin\bar{\theta}_3\Delta\theta_3 + \Delta r_4 cos\bar{\theta}_3 - \underbrace{\Delta r_4 sin\bar{\theta}_3\Delta\theta_3}_{0}$$

$$\overline{OA} + \overline{AB}sin\theta_2 = \bar{r}_4 sin\bar{\theta}_3 + \bar{r}_4 cos\bar{\theta}_3\Delta\theta_3 + \Delta r_4 sin\bar{\theta}_3 + \underbrace{\Delta r_4 cos\bar{\theta}_3\Delta\theta_3}_{0}$$

$$\bar{r}_6 + \Delta r_6 + \overline{DC}cos\bar{\theta}_5 - \overline{DC}sin\bar{\theta}_5\Delta\theta_5 = \overline{OC}cos\bar{\theta}_3 - \overline{OC}sin\bar{\theta}_3\Delta\theta_3$$

$$\overline{ED} + \overline{DC}sin\bar{\theta}_5 + \overline{DC}cos\bar{\theta}_5\Delta\theta_5 = \overline{OC}sin\bar{\theta}_3 + \overline{OC}cos\bar{\theta}_3\Delta\theta_3$$

Elde ettiğimiz denklemleri son bir düzenleyelim.

$$-\bar{r}_4 sin\bar{\theta}_3\Delta\theta_3 + cos\bar{\theta}_3\Delta r_4 = \overline{AB}cos\theta_2 - \bar{r}_4 cos\bar{\theta}_3 \qquad 13$$

$$\bar{r}_4 cos\bar{\theta}_3\Delta\theta_3 + sin\bar{\theta}_3\Delta r_4 = \overline{OA} + \overline{AB}sin\theta_2 - \bar{r}_4 sin\bar{\theta}_3 \qquad 14$$

$$\overline{OC}sin\bar{\theta}_3\Delta\theta_3 - \overline{DC}sin\bar{\theta}_5\Delta\theta_5 + \Delta r_6 = \overline{OC}cos\bar{\theta}_3 - \bar{r}_6 - \overline{DC}cos\bar{\theta}_5 \qquad 15$$

$$-\overline{OC}cos\bar{\theta}_3\Delta\theta_3 + \overline{DC}cos\bar{\theta}_5\Delta\theta_5 = \overline{OC}sin\bar{\theta}_3 - \overline{ED} - \overline{DC}sin\bar{\theta}_5 \qquad 16$$

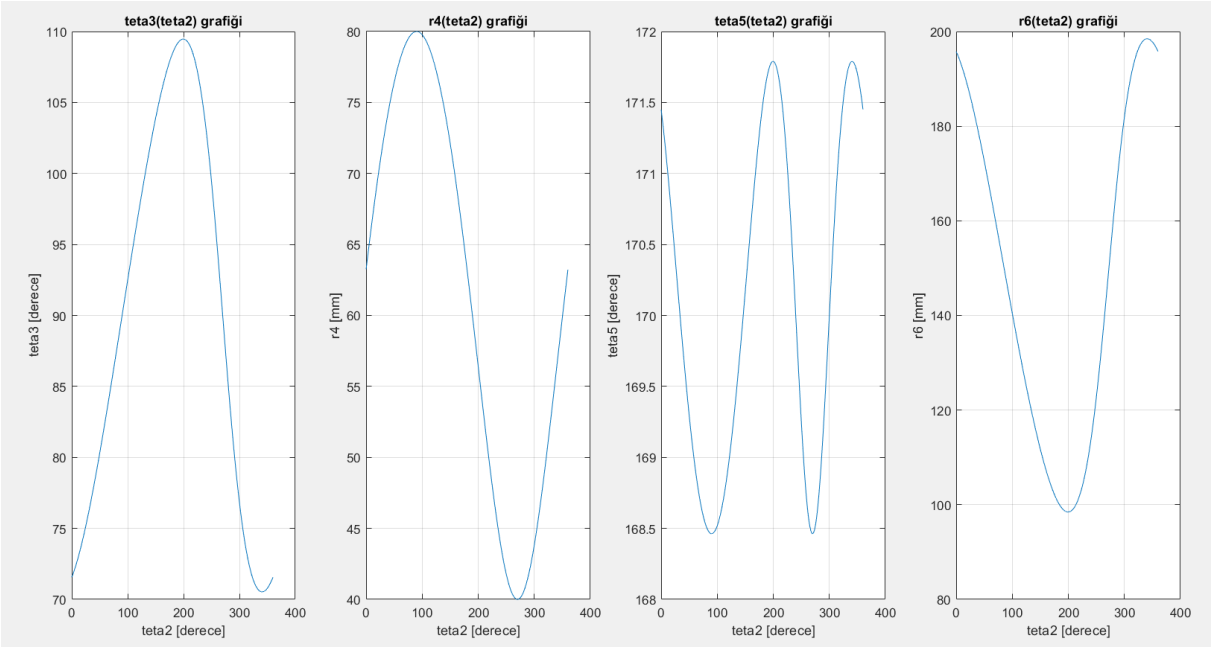
13, 14, 15 ve 16 nolu denklemler aşağıdaki gibi matris formunda yazabiliriz.

$$\begin{bmatrix} -\bar{r}_4 sin\bar{\theta}_3 & cos\bar{\theta}_3 & 0 & 0 \\ \bar{r}_4 cos\bar{\theta}_3 & sin\bar{\theta}_3 & 0 & 0 \\ \overline{OC}sin\bar{\theta}_3 & 0 & -\overline{DC}sin\bar{\theta}_5 & 1 \\ -\overline{OC}cos\bar{\theta}_3 & 0 & \overline{DC}cos\bar{\theta}_5 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta\theta_3 \\ \Delta r_4 \\ \Delta\theta_5 \\ \Delta r_6 \end{bmatrix} = \begin{bmatrix} \overline{AB}cos\theta_2 - \bar{r}_4 cos\bar{\theta}_3 \\ \overline{OA} + \overline{AB}sin\theta_2 - \bar{r}_4 sin\bar{\theta}_3 \\ \overline{OC}cos\bar{\theta}_3 - \bar{r}_6 - \overline{DC}cos\bar{\theta}_5 \\ \overline{OC}sin\bar{\theta}_3 - \overline{ED} - \overline{DC}sin\bar{\theta}_5 \end{bmatrix} \qquad 17$$

17 nolu denklem takımı aşağıda MATLAB kodları verilen program ile çözüldü.

```
clc
clear
% Uzuvların Başlangıç Konumları
teta2=0:0.001:2*pi;
teta3c(1)=71.57*pi/180;
r4c(1)=63.25;
teta5c(1)=171.45*pi/180;
r6c(1)=195.77;
% Uzuv Boyutları
AB=20;
OA=60;
OC=150;
```

```
DC=150;
ED=120;
for i=1:length(teta2)-1
    K=[-r4c(i)*sin(teta3c(i))    cos(teta3c(i))              0               0;
         r4c(i)*cos(teta3c(i))    sin(teta3c(i))              0               0;
         OC*sin(teta3c(i))             0            -DC*sin(teta5c(i))     1;
        -OC*cos(teta3c(i))             0            DC*cos(teta5c(i))      0];
    F=[AB*cos(teta2(i))-r4c(i)*cos(teta3c(i));
       OA+AB*sin(teta2(i))-r4c(i)*sin(teta3c(i));
       OC*cos(teta3c(i))-r6c(i)-DC*cos(teta5c(i));
       OC*sin(teta3c(i))-ED-DC*sin(teta5c(i))];
    B=inv(K)*F;
    teta3c(i+1)=teta3c(i)+B(1);
    r4c(i+1)=r4c(i)+B(2);
    teta5c(i+1)=teta5c(i)+B(3);
    r6c(i+1)=r6c(i)+B(4);
end
subplot(1,4,1)
plot(teta2*180/pi,teta3c*180/pi)
grid
title('teta3(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('teta3 [derece]')
subplot(1,4,2)
plot(teta2*180/pi,r4c)
grid
title('r4(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('r4 [mm]')
subplot(1,4,3)
plot(teta2*180/pi,teta5c*180/pi)
grid
title('teta5(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('teta5 [derece]')
subplot(1,4,4)
plot(teta2*180/pi,r6c)
grid
title('r6(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('r6 [mm]')
```


teta3(teta2) grafiği
teta3 [derece]
teta2 [derece]
r4(teta2) grafiği
r4 [mm]
teta2 [derece]
teta5(teta2) grafiği
teta5 [derece]
teta2 [derece]
r6(teta2) grafiği
r6 [mm]
teta2 [derece]