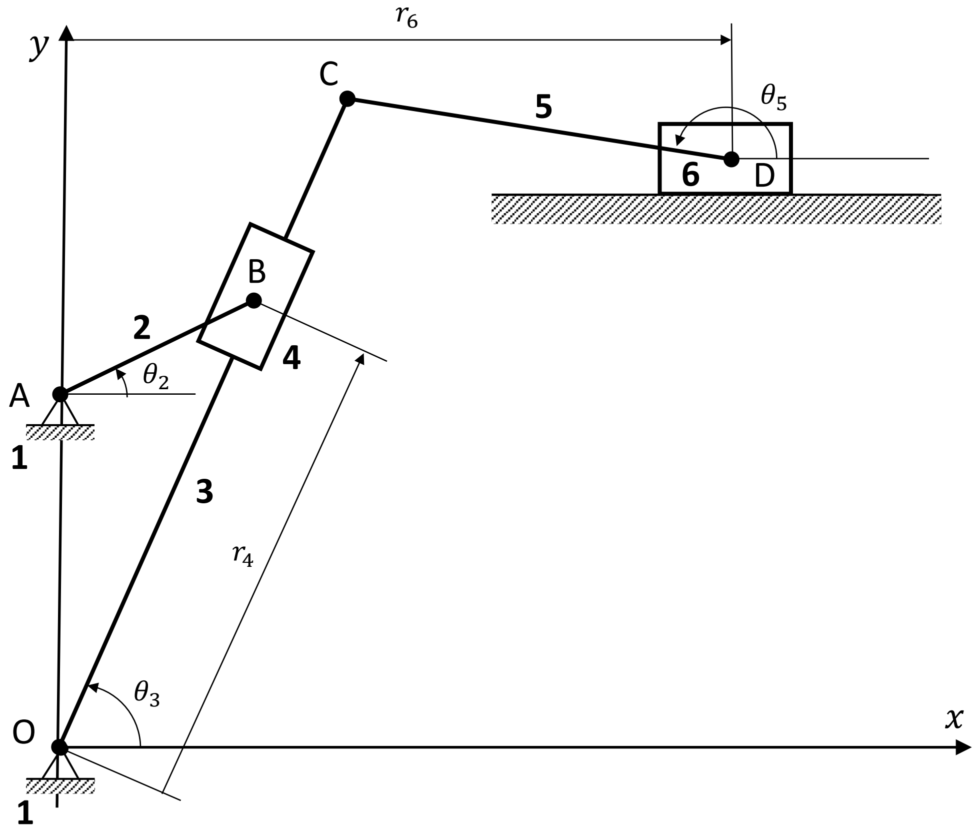
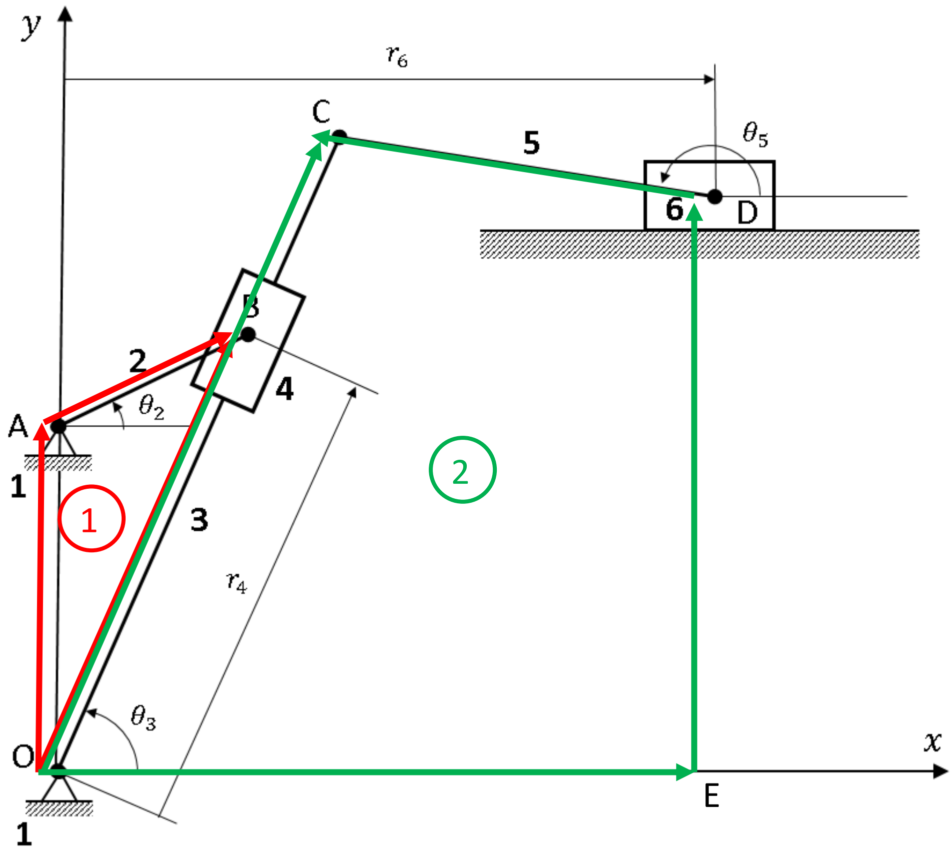
Vargel Mekanizmasının Kinematik Analizi

Analitik Çözüm:

Mekanizmaya ait genelleştirilmiş koordinatlar

$$\underbrace{\theta_2}_{esas\ genelleştirilmiş\ koordinat} \qquad \underbrace{\theta_3,\ r_4,\ \theta_5,\ r_6}_{tali\ genelleştirilmiş\ koordinatlar}$$





1 nolu devre kapalılık denklemini yazalım,

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$

Devre kapalılık denklemini döner vektör formunda yazalım,

$$\overline{OA}e^{i90^\circ} + \overline{AB}e^{i\theta_2} = r_4 e^{i\theta_3}$$

Döner vektörlerin Euler açılımlarını yapalım,

$$\overline{OA}\left(cos90^\circ + isin90^\circ\right) + \overline{AB}(cos\theta_2 + isin\theta_2) = r_4(cos\theta_3 + isin\theta_3)$$

$$\overline{OA}\left(\underbrace{cos90^\circ}_{0} + i\,\underbrace{sin90^\circ}_{1}\right) + \overline{AB}(cos\theta_2 + isin\theta_2) = r_4(cos\theta_3 + isin\theta_3)$$

$$\overline{OA}i + \overline{AB}(cos\theta_2 + isin\theta_2) = r_4(cos\theta_3 + isin\theta_3)$$

Bu kompleks devre kapalılık denklemini reel ve imajiner kısımlarını birbirine eşitleyerek iki adet kısıt denklemini elde edelim.

$$\overline{AB}cos\theta_2 = r_4 cos\theta_3 \qquad 1$$

$$\overline{OA} + \overline{AB}sin\theta_2 = r_4 sin\theta_3 \qquad 2$$

2 nolu devre kapalılık denklemini yazalım,

$$\overrightarrow{OE} + \overrightarrow{ED} + \overrightarrow{DC} = \overrightarrow{OC}$$

Devre kapalılık denklemini döner vektör formunda yazalım,

$$\overline{OE}e^{i0^{\circ}} + \overline{ED}e^{i90^{\circ}} + \overline{DC}e^{i\theta_5} = \overline{OC}e^{i\theta_3}$$

Döner vektörlerin Euler açılımlarını yapalım, burada $\overline{OE} = r_6$ dır.

$$r_6\left(cos0^{\circ} + isin0^{\circ}\right) + \overline{ED}\left(cos90^{\circ} + isin90^{\circ}\right) + \overline{DC}(cos\theta_5 + isin\theta_5) = \overline{OC}(cos\theta_3 + isin\theta_3)$$

$$r_6\left(\underbrace{cos0^{\circ}}_{1} + i\,\underbrace{sin0^{\circ}}_{0}\right) + \overline{ED}\left(\underbrace{cos90^{\circ}}_{0} + i\,\underbrace{sin90^{\circ}}_{1}\right) + \overline{DC}(cos\theta_5 + isin\theta_5) = \overline{OC}(cos\theta_3 + isin\theta_3)$$

$$r_6 + \overline{ED}i + \overline{DC}(cos\theta_5 + isin\theta_5) = \overline{OC}(cos\theta_3 + isin\theta_3)$$

Bu kompleks devre kapalılık denklemini gerçek ve sanal kısımlarını birbirine eşitleyerek iki adet kısıt denklemi elde edelim.

$$r_6 + \overline{DC}cos\theta_5 = \overline{OC}cos\theta_3 \qquad 3$$

$$\overline{ED} + \overline{DC}sin\theta_5 = \overline{OC}sin\theta_3 \qquad 4$$

Şimdi bu dört kısıt denkleminden tali genelleştirilmiş koordinatları esas genelleştirilmiş koordinatlar cinsinden elde edelim.

$$\theta_3(\theta_2) =? \quad r_4(\theta_2) =? \quad \theta_5(\theta_2) =? \quad r_6(\theta_2) =?$$

1 ve 2 nolu denklemleri taraf tarafa bölelim.

$$\frac{\overline{AB}cos\theta_2}{\overline{OA} + \overline{AB}sin\theta_2} = \frac{r_4cos\theta_3}{r_4sin\theta_3}$$

$$\frac{\overline{AB}cos\theta_2}{\overline{OA} + \overline{AB}sin\theta_2} = \frac{cos\theta_3}{sin\theta_3} \quad \Rightarrow \quad tan\theta_3 = \frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2} \quad \Rightarrow \quad \theta_3 = tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right) \qquad 5$$

5 nolu eşitliği 1 nolu denklemde yerine koyalım.

$$\overline{AB}cos\theta_2 = r_4cos\theta_3 \quad \Rightarrow \quad r_4 = \frac{\overline{AB}cos\theta_2}{cos\theta_3} \quad \Rightarrow \quad r_4 = \frac{\overline{AB}cos\theta_2}{cos\left[tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right]} \qquad 6$$

4 nolu kısıt denklemine 5 nolu ifadeyi koyalım.

$$\overline{ED} + \overline{DC}sin\theta_5 = \overline{OC}sin\theta_3 \quad \Rightarrow \quad sin\theta_5 = \frac{\overline{OC}sin\theta_3 - \overline{ED}}{\overline{DC}} \quad \Rightarrow$$

$$\theta_5 = sin^{-1}\left[\frac{\overline{OC}sin\left[tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right] - \overline{ED}}{\overline{DC}}\right] \qquad 7$$

3 nolu kısıt denklemine 5 ve 7 nolu ifadeyi koyalım.

$$r_6 + \overline{DC}cos\theta_5 = \overline{OC}cos\theta_3 \quad \Rightarrow \quad r_6 = \overline{OC}cos\theta_3 - \overline{DC}cos\theta_5 \quad \Rightarrow$$

$$r_6 = \overline{OC}cos\left[tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right] - \overline{DC}cos\left[\frac{\overline{OC}}{\overline{DC}}\sin\left(tan^{-1}\left(\frac{\overline{OA} + \overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right) - \frac{\overline{ED}}{\overline{DC}}\right] \qquad 8$$

5, 6, 7 ve 8 nolu ifadeler ile mekanizmaya ait konum analizi yapılmış olur.

Nümerik Çözüm:

Devre kapalılık denklemlerinden elde ettiğimiz kısıt denklemlerini alalım,

$$\overline{AB}cos\theta_2 = r_4 cos\theta_3 \qquad 1$$

$$\overline{OA} + \overline{AB}sin\theta_2 = r_4 sin\theta_3 \qquad 2$$

$$r_6 + \overline{DC}cos\theta_5 = \overline{OC}cos\theta_3 \qquad 3$$

$$\overline{ED} + \overline{DC}sin\theta_5 = \overline{OC}sin\theta_3 \qquad 4$$

Kısıt denklemlerinde aşağıdaki yaklaşık ifadeleri kullanalım.

$$cos\theta_3 \cong cos(\bar{\theta}_3 + \Delta\theta_3)$$

$$cos\theta_5 \cong cos(\bar{\theta}_5 + \Delta\theta_5)$$

$$sin\theta_3 \cong sin(\bar{\theta}_3 + \Delta\theta_3)$$

$$sin\theta_5 \cong sin(\bar{\theta}_5 + \Delta\theta_5)$$

$$r_4 \cong \bar{r}_4 + \Delta r_4$$

$$r_6 \cong \bar{r}_6 + \Delta r_6$$

Buradan, kısıt denklemleri aşağıdaki hali alacaktır.

$$\overline{AB}cos\theta_2 = (\bar{r}_4 + \Delta r_4)cos(\bar{\theta}_3 + \Delta\theta_3) \qquad 5$$

$$\overline{OA} + \overline{AB}sin\theta_2 = (\bar{r}_4 + \Delta r_4)sin(\bar{\theta}_3 + \Delta\theta_3) \qquad 6$$

$$(\bar{r}_6 + \Delta r_6) + \overline{DC}cos(\bar{\theta}_5 + \Delta\theta_5) = \overline{OC}cos(\bar{\theta}_3 + \Delta\theta_3) \qquad 7$$

$$\overline{ED} + \overline{DC}sin(\bar{\theta}_5 + \Delta\theta_5) = \overline{OC}sin(\bar{\theta}_3 + \Delta\theta_3) \qquad 8$$

Bu denklemleri aşağıdaki eşitlikleri kullanarak tekrar düzenleyelim.

$$cos(\bar{\theta}_3 + \Delta\theta_3) = cos\bar{\theta}_3 cos\Delta\theta_3 - sin\bar{\theta}_3 sin\Delta\theta_3$$

$$cos(\bar{\theta}_5 + \Delta\theta_5) = cos\bar{\theta}_5 cos\Delta\theta_5 - sin\bar{\theta}_5 sin\Delta\theta_5$$

$$sin(\bar{\theta}_3 + \Delta\theta_3) = sin\bar{\theta}_3 cos\Delta\theta_3 + cos\bar{\theta}_3 sin\Delta\theta_3$$

$$sin(\bar{\theta}_5 + \Delta\theta_5) = sin\bar{\theta}_5 cos\Delta\theta_5 + cos\bar{\theta}_5 sin\Delta\theta_5$$

5, 6, 7 ve 8 nolu denkler aşağıdaki hali alır.

$$\overline{AB}cos\theta_2 = (\bar{r}_4 + \Delta r_4)(cos\bar{\theta}_3 cos\Delta\theta_3 - sin\bar{\theta}_3 sin\Delta\theta_3) \qquad 9$$

$$\overline{OA} + \overline{AB}sin\theta_2 = (\bar{r}_4 + \Delta r_4)(sin\bar{\theta}_3 cos\Delta\theta_3 + cos\bar{\theta}_3 sin\Delta\theta_3) \qquad 10$$

$$(\bar{r}_6 + \Delta r_6) + \overline{DC}(cos\bar{\theta}_5 cos\Delta\theta_5 - sin\bar{\theta}_5 sin\Delta\theta_5) = \overline{OC}(cos\bar{\theta}_3 cos\Delta\theta_3 - sin\bar{\theta}_3 sin\Delta\theta_3) \qquad 11$$

$$\overline{ED} + \overline{DC}(sin\bar{\theta}_5 cos\Delta\theta_5 + cos\bar{\theta}_5 sin\Delta\theta_5) = \overline{OC}(sin\bar{\theta}_3 cos\Delta\theta_3 + cos\bar{\theta}_3 sin\Delta\theta_3) \qquad 12$$

Aşağıdaki kabulleri 9, 10, 11 ve 12 nolu denklemlerde yerine koyup tekrar düzenleyelim.

$$cos\Delta\theta_3 \cong 1, \quad sin\Delta\theta_3 \cong \Delta\theta_3, \quad cos\Delta\theta_5 \cong 1, \quad sin\Delta\theta_5 \cong \Delta\theta_5$$

$$\overline{AB}cos\theta_2 = (\bar{r}_4 + \Delta r_4)\left(cos\bar{\theta}_3\underbrace{cos\Delta\theta_3}_{1} - sin\bar{\theta}_3\underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

$$\overline{OA} + \overline{AB}sin\theta_2 = (\bar{r}_4 + \Delta r_4)\left(sin\bar{\theta}_3\underbrace{cos\Delta\theta_3}_{1} + cos\bar{\theta}_3\underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

$$(\bar{r}_6 + \Delta r_6) + \overline{DC}\left(cos\bar{\theta}_5\underbrace{cos\Delta\theta_5}_{1} - sin\bar{\theta}_5\underbrace{sin\Delta\theta_5}_{\Delta\theta_5}\right) = \overline{OC}\left(cos\bar{\theta}_3\underbrace{cos\Delta\theta_3}_{1} - sin\bar{\theta}_3\underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

$$\overline{ED} + \overline{DC}\left(sin\bar{\theta}_5\underbrace{cos\Delta\theta_5}_{1} + cos\bar{\theta}_5\underbrace{sin\Delta\theta_5}_{\Delta\theta_5}\right) = \overline{OC}\left(sin\bar{\theta}_3\underbrace{cos\Delta\theta_3}_{1} + cos\bar{\theta}_3\underbrace{sin\Delta\theta_3}_{\Delta\theta_3}\right)$$

Yukarıdaki denklemleri aşağıdaki gibi açalım.

$$\overline{AB}cos\theta_2 = \bar{r}_4cos\bar{\theta}_3 - \bar{r}_4sin\bar{\theta}_3\Delta\theta_3 + \Delta r_4cos\bar{\theta}_3 - \underbrace{\Delta r_4sin\bar{\theta}_3\Delta\theta_3}_{0}$$

$$\overline{OA} + \overline{AB}sin\theta_2 = \bar{r}_4sin\bar{\theta}_3 + \bar{r}_4cos\bar{\theta}_3\Delta\theta_3 + \Delta r_4sin\bar{\theta}_3 + \underbrace{\Delta r_4cos\bar{\theta}_3\Delta\theta_3}_{0}$$

$$\bar{r}_6 + \Delta r_6 + \overline{DC}cos\bar{\theta}_5 - \overline{DC}sin\bar{\theta}_5\Delta\theta_5 = \overline{OC}cos\bar{\theta}_3 - \overline{OC}sin\bar{\theta}_3\Delta\theta_3$$

$$\overline{ED} + \overline{DC}sin\bar{\theta}_5 + \overline{DC}cos\bar{\theta}_5\Delta\theta_5 = \overline{OC}sin\bar{\theta}_3 + \overline{OC}cos\bar{\theta}_3\Delta\theta_3$$

Elde ettiğimiz denklemleri son bir düzenleyelim.

$$-\bar{r}_4sin\bar{\theta}_3\Delta\theta_3 + cos\bar{\theta}_3\Delta r_4 = \overline{AB}cos\theta_2 - \bar{r}_4cos\bar{\theta}_3 \qquad 13$$

$$\bar{r}_4cos\bar{\theta}_3\Delta\theta_3 + sin\bar{\theta}_3\Delta r_4 = \overline{OA} + \overline{AB}sin\theta_2 - \bar{r}_4sin\bar{\theta}_3 \qquad 14$$

$$\overline{OC}sin\bar{\theta}_3\Delta\theta_3 - \overline{DC}sin\bar{\theta}_5\Delta\theta_5 + \Delta r_6 = \overline{OC}cos\bar{\theta}_3 - \bar{r}_6 - \overline{DC}cos\bar{\theta}_5 \qquad 15$$

$$-\overline{OC}cos\bar{\theta}_3\Delta\theta_3 + \overline{DC}cos\bar{\theta}_5\Delta\theta_5 = \overline{OC}sin\bar{\theta}_3 - \overline{ED} - \overline{DC}sin\bar{\theta}_5 \qquad 16$$

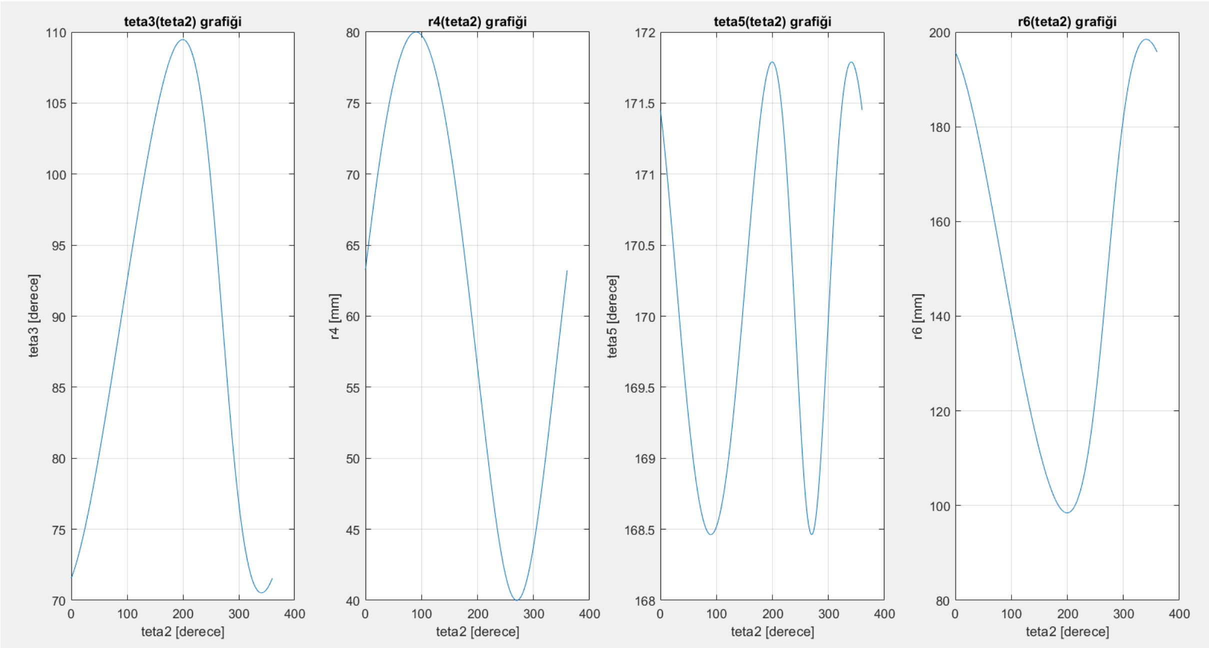
13, 14, 15 ve 16 nolu denklemler aşağıdaki gibi matris formunda yazabiliriz.

$$\begin{bmatrix} -\bar{r}_4sin\bar{\theta}_3 & cos\bar{\theta}_3 & 0 & 0 \\ \bar{r}_4cos\bar{\theta}_3 & sin\bar{\theta}_3 & 0 & 0 \\ \overline{OC}sin\bar{\theta}_3 & 0 & -\overline{DC}sin\bar{\theta}_5 & 1 \\ -\overline{OC}cos\bar{\theta}_3 & 0 & \overline{DC}cos\bar{\theta}_5 & 0 \end{bmatrix} . \begin{bmatrix} \Delta\theta_3 \\ \Delta r_4 \\ \Delta\theta_5 \\ \Delta r_6 \end{bmatrix} = \begin{bmatrix} \overline{AB}cos\theta_2 - \bar{r}_4cos\bar{\theta}_3 \\ \overline{OA} + \overline{AB}sin\theta_2 - \bar{r}_4sin\bar{\theta}_3 \\ \overline{OC}cos\bar{\theta}_3 - \bar{r}_6 - \overline{DC}cos\bar{\theta}_5 \\ \overline{OC}sin\bar{\theta}_3 - \overline{ED} - \overline{DC}sin\bar{\theta}_5 \end{bmatrix} \qquad 17$$

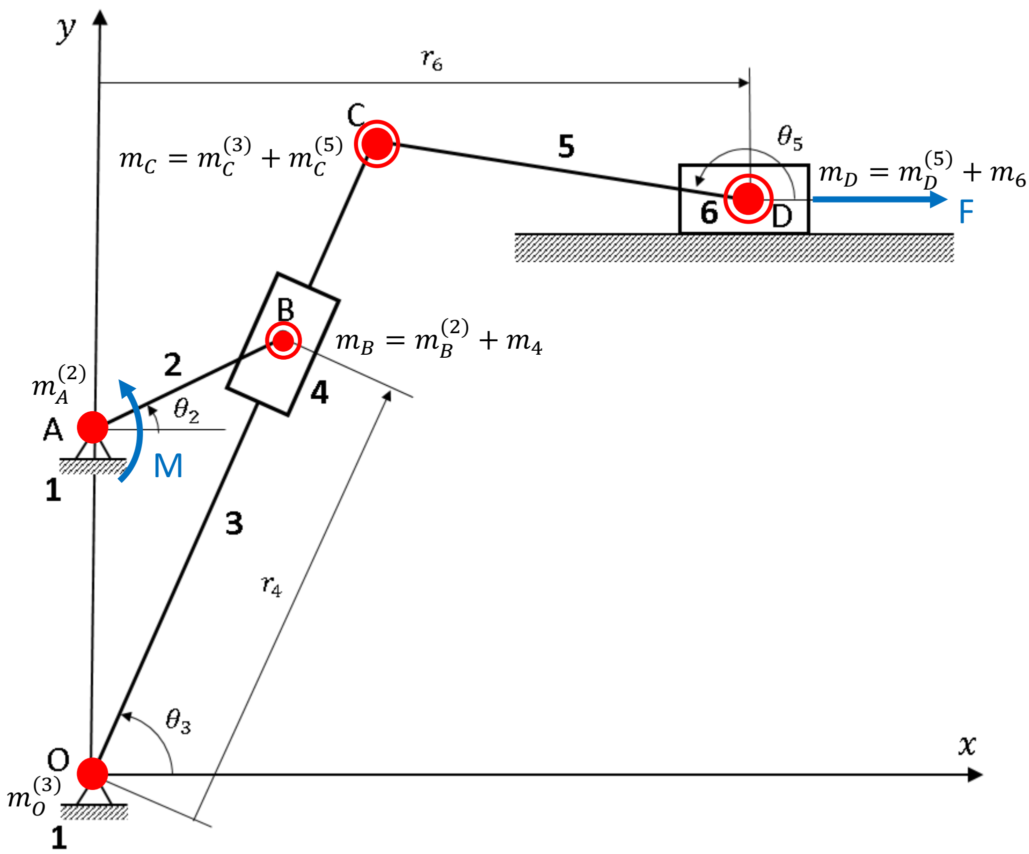
17 nolu denklem takımı aşağıda MATLAB kodları verilen program ile çözüldü.

```
clc
clear
% Uzuvların Başlangıç Konumları
teta2=0:0.001:2*pi;
teta3c(1)=71.57*pi/180;
r4c(1)=63.25;
teta5c(1)=171.45*pi/180;
r6c(1)=195.77;
% Uzuv Boyutları
AB=20;
OA=60;
OC=150;
```

```
DC=150;
ED=120;
for i=1:length(teta2)-1
    K=[-r4c(i)*sin(teta3c(i))    cos(teta3c(i))              0                 0;
          r4c(i)*cos(teta3c(i))    sin(teta3c(i))              0                 0;
          OC*sin(teta3c(i))              0              -DC*sin(teta5c(i))      1;
         -OC*cos(teta3c(i))              0               DC*cos(teta5c(i))      0];
    F=[AB*cos(teta2(i))-r4c(i)*cos(teta3c(i));
        OA+AB*sin(teta2(i))-r4c(i)*sin(teta3c(i));
        OC*cos(teta3c(i))-r6c(i)-DC*cos(teta5c(i));
        OC*sin(teta3c(i))-ED-DC*sin(teta5c(i))];
    B=inv(K)*F;
    teta3c(i+1)=teta3c(i)+B(1);
    r4c(i+1)=r4c(i)+B(2);
    teta5c(i+1)=teta5c(i)+B(3);
    r6c(i+1)=r6c(i)+B(4);
end
subplot(1,4,1)
plot(teta2*180/pi,teta3c*180/pi)
grid
title('teta3(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('teta3 [derece]')
subplot(1,4,2)
plot(teta2*180/pi,r4c)
grid
title('r4(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('r4 [mm]')
subplot(1,4,3)
plot(teta2*180/pi,teta5c*180/pi)
grid
title('teta5(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('teta5 [derece]')
subplot(1,4,4)
plot(teta2*180/pi,r6c)
grid
title('r6(teta2) grafiği')
xlabel('teta2 [derece]')
ylabel('r6 [mm]')
```



Vargel Mekanizmasının Kütle İndirgemesi



m_2 kütlesinin indirgenmesi : $\overline{AB} = 20\,mm,\;\; m_2 = 0{,}5\,kg,\;\; L_A = 8\,m,\;\; i_S^2 = 96\,mm^2$

$$y_B = 0$$

$$x_B = L_B = \frac{i_S^2}{L_A} = \frac{96}{8} = 12\,mm$$

$$m_A^{(2)} = \frac{L_B}{L_A + L_B} m = \frac{L_B}{L} m = \frac{12}{20} 0{,}5 = 0{,}3\,kg$$

$$m_B^{(2)} = \frac{L_A}{L_A + L_B} m = \frac{L_A}{L} m = \frac{8}{20} 0{,}5 = 0{,}2\,kg$$

m_3 kütlesinin indirgenmesi : $\overline{OC} = 150\,mm,\;\; m_2 = 2\,kg,\;\; L_O = 75\,mm,\;\; i_S^2 = 5.625\,mm^2$

$$y_C = 0$$

$$x_C = L_C = \frac{i_S^2}{L_O} = \frac{5.625}{75} = 75\,mm$$

$$m_O^{(3)} = \frac{L_C}{L_O + L_C} m = \frac{L_C}{L} m = \frac{75}{150} 2 = 1\,kg$$

$$m_C^{(3)} = \frac{L_O}{L_O + L_C} m = \frac{L_O}{L} m = \frac{75}{150} 2 = 1\,kg$$

$m_4 = 3\,kg$

m_5 kütlesinin indirgenmesi : $\overline{CD} = 150\,mm,\;\; m_5 = 2\,kg,\;\; L_C = 75\,mm,\;\; i_S^2 = 5.625\,mm^2$

$$y_D = 0$$

$$x_D = L_D = \frac{i_S^2}{L_C} = \frac{5.625}{75} = 75\,mm$$

$$m_C^{(5)} = \frac{L_D}{L_C + L_D} m = \frac{L_D}{L} m = \frac{75}{150} 2 = 1\,kg$$

$$m_D^{(5)} = \frac{L_C}{L_C + L_D} m = \frac{L_C}{L} m = \frac{75}{150} 2 = 1\,kg$$

$m_6 = 5\,kg$

Vargel Mekanizmasının Hareket Denklemi

Mekanizmanın toplam kinetik enerjisi aşağıdaki gibi yazılabilir.

$$T = \frac{1}{2} m_B V_B^2 + \frac{1}{2} m_C V_C^2 + \frac{1}{2} m_D V_D^2$$

$$V_B = \overline{AB}\dot{\theta}_2 \quad \Rightarrow \quad V_B^2 = \overline{AB}^2 \dot{\theta}_2^2$$

$$\theta_3 = tan^{-1}\left(\frac{\overline{OA} + \overline{AB} sin\theta_2}{\overline{AB} cos\theta_2}\right) \quad \Rightarrow \quad \dot{\theta}_3 = \frac{\overline{AB}^2 \dot{\theta}_2 + \overline{OA}\,\overline{AB}\dot{\theta}_2 sin\theta_2}{\overline{OA}^2 + \overline{AB}^2 + 2\overline{OA}\,\overline{AB}\, sin\theta_2}$$

$$V_C = \overline{OC}\dot{\theta}_3 \quad \Rightarrow \quad V_C^2 = \overline{OC}^2\dot{\theta}_3^2 = \left[\frac{\overline{OC}^2\,\overline{AB}^4 + (\overline{OC}^2\overline{OA}^2\,\overline{AB}^2 + 2\overline{OC}^2\overline{OA}\,\overline{AB}^3)sin^2\theta_2}{(\overline{OA}^2 + \overline{AB}^2 + 2\overline{OA}\,\overline{AB}\,sin\theta_2)^2}\right]\dot{\theta}_2^2$$

$$V_D = \dot{r}_6 \quad \Rightarrow \quad V_D^2 = \left\{-\overline{OC}sin\left[tan^{-1}\left(\frac{\overline{OA}+\overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right]\right.$$

$$\left.+\overline{DC}sin\left[\frac{\overline{OC}}{\overline{DC}}\sin\left(tan^{-1}\left(\frac{\overline{OA}+\overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right)-\frac{\overline{ED}}{\overline{DC}}\right]\frac{\overline{OC}}{\overline{DC}}cos\left(tan^{-1}\left(\frac{\overline{OA}+\overline{AB}sin\theta_2}{\overline{AB}cos\theta_2}\right)\right)\right\}^2$$

$$\left(\frac{\overline{AB}^2 + \overline{OA}\,\overline{AB}sin\theta_2}{\overline{OA}^2 + \overline{AB}^2 + 2\overline{OA}\,\overline{AB}\,sin\theta_2}\right)^2 \dot{\theta}_2^2$$

Yukarıdaki ifadeleri kinetik enerji ifadesinde yerine koyalım. Elde ettiğimiz kinetik enerji ifadesi tamamen θ_2 'nin fonksiyonu olacaktır.

$$T = \frac{1}{2}m_B V(\theta_2)_B^2 + \frac{1}{2}m_C V(\theta_2)_C^2 + \frac{1}{2}m_D V(\theta_2)_D^2$$

$$T = \frac{1}{2}(\mathfrak{I})\dot{\theta}_2^2$$

Burada, $\mathfrak{I}$ hareket miline indirgenmiş kütlesel atalet momenti ifadesidir. Bu ifadenin aşağıdaki makine hareket denkleminde yerine konması ile mekanizmanın hareket veya dinamik denklemi elde edilir.

$$\mathfrak{I}(\theta_2)\,\ddot{\theta}_2 + \frac{1}{2}\frac{d\mathfrak{I}(\theta_2)}{d\theta_2}\dot{\theta}_2^2 = Q(\theta_2, \dot{\theta}_2, t)$$

$$Q(\theta_2, \dot{\theta}_2, t) = M\frac{\dot{\theta}_2}{\dot{\theta}_2} - F\frac{\dot{r}_6}{\dot{\theta}_2} = M - F\frac{\dot{r}_6}{\dot{\theta}_2}$$