PROBLEM SET FOR CHAPTER 7

Chemical Engineering Fluid Mechanics

7–3 Write the primary dimensions of the *universal ideal gas constant* R_u. (*Hint*: Use the *ideal gas law*, $PV = nR_uT$ where P is pressure, V is volume, T is absolute temperature, and n is the number of moles of the gas.) *Answer:* $\{m^1L^2t^{-2}T^{-1}N^{-1}\}$

7–4 Write the primary dimensions of each of the following variables from the field of thermodynamics, showing all your work: (*a*) energy E; (*b*) specific energy $e = E/m$; (*c*) power $\dot{W}$.
Answers: (*a*) $\{m^1L^2t^{-2}\}$, (*b*) $\{L^2t^{-2}\}$, (*c*) $\{m^1L^2t^{-3}\}$

7–6 Consider the table of Prob. 7–5 where the primary dimensions of several variables are listed in the mass–length–time system. Some engineers prefer the force–length–time system (force replaces mass as one of the primary dimensions). Write the primary dimensions of three of these (density, surface tension, and viscosity) in the force–length–time system.

7–9 We define the *specific ideal gas constant* R_{gas} for a particular gas as the ratio of the universal gas constant and the molar mass (also called *molecular weight*) of the gas, $R_{gas} = R_u/M$. For a particular gas, then, the ideal gas law is written as follows:

$$PV = mR_{gas}T \quad \text{or} \quad P = \rho R_{gas}T$$

where P is pressure, V is volume, m is mass, T is absolute temperature, and ρ is the density of the particular gas. What are the primary dimensions of R_{gas}? For air, $R_{air} = 287.0$ J/kg·K in standard SI units. Verify that these units agree with your result.

7–17 Write the primary dimensions of each of the following variables from the study of convection heat transfer (Fig. P7–17), showing all your work: (*a*) heat generation rate $\dot{g}$ (*Hint*: rate of conversion of thermal energy per unit volume); (*b*) heat flux $\dot{q}$ (*Hint*: rate of heat transfer per unit area); (*c*) heat transfer coefficient h (*Hint*: heat flux per unit temperature difference).

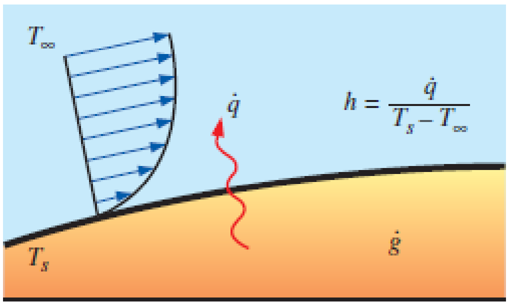

FIGURE P7–17

7–21 In Chap. 4, we defined the *material acceleration*, which is the acceleration following a fluid particle,

$$\vec{a}(x, y, z, t) = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla})\vec{V}$$

(*a*) What are the primary dimensions of the gradient operator $\vec{\nabla}$? (*b*) Verify that each additive term in the equation has the same dimensions. *Answers:* (*a*) $\{L^{-1}\}$; (*b*) $\{L^1 t^{-2}\}$

7–27 Cold water enters a pipe, where it is heated by an external heat source (Fig. P7–27). The inlet and outlet water temperatures are T_{in} and T_{out}, respectively. The total rate of heat transfer $\dot{Q}$ from the surroundings into the water in the pipe is

$$\dot{Q} = \dot{m}c_p(T_{out} - T_{in})$$

where $\dot{m}$ is the mass flow rate of water through the pipe, and c_p is the specific heat of the water. Write the primary dimensions of each additive term in the equation, and verify that the equation is dimensionally homogeneous. Show all your work.

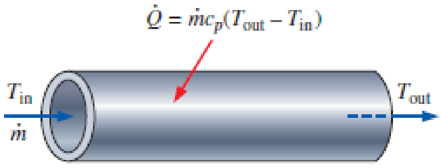

FIGURE P7–27

7–34 Consider ventilation of a well-mixed room as in Fig. P7–25. The differential equation for mass concentration in the room as a function of time is given in Prob. 7–25 and is repeated here for convenience,

$$V\frac{dc}{dt} = S - \dot{V}c - cA_s k_w$$

There are three characteristic parameters in such a situation: L, a characteristic length scale of the room (assume $L = V^{1/3}$); $\dot{V}$, the volume flow rate of fresh air into the room, and c_{limit}, the maximum mass concentration that is not harmful. (*a*) Using these three characteristic parameters, define dimensionless forms of all the variables in the equation. (*Hint*: For example, define $c^* = c/c_{limit}$.) (*b*) Rewrite the equation in dimensionless form, and identify any established dimensionless groups that may appear.

7–37 A student team is to design a human-powered submarine for a design competition. The overall length of the prototype submarine is 4.85 m, and its student designers hope that it can travel fully submerged through water at 0.440 m/s. The water is freshwater (a lake) at T = 15°C. The design team builds a one-fifth scale model to test in their university's wind tunnel (Fig. P7–37). A shield surrounds the drag balance strut so that the aerodynamic drag of the strut itself does not influence the measured drag. The air in the wind tunnel is at

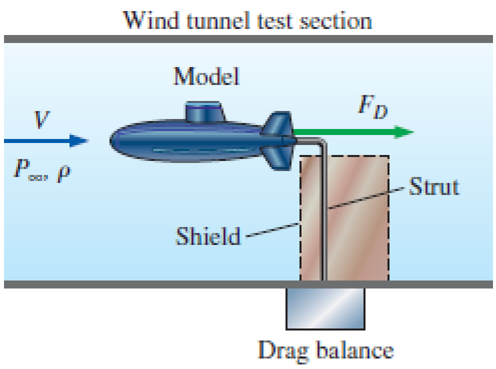

FIGURE P7–37

7–38 Repeat Prob. 7–37 with all the same conditions except that the only facility available to the students is a much smaller wind tunnel. Their model submarine is a one-twenty-fourth scale model instead of a one-fifth scale model. At what air speed do they need to run the wind tunnel in order to achieve similarity? Do you notice anything disturbing or suspicious about your result? Discuss your results.

7–39 This is a follow-up to Prob. 7–37. The students measure the aerodynamic drag on their model submarine in the wind tunnel (Fig. P7–37). They are careful to run the wind tunnel at conditions that ensure similarity with the prototype submarine. Their measured drag force is 5.70 N. Estimate the drag force on the prototype submarine at the conditions given in Prob. 7–37. *Answer:* 25.5 N

7–46 Using primary dimensions, verify that the Archimedes number (Table 7–5) is indeed dimensionless.

7–47 Using primary dimensions, verify that the Grashof number (Table 7–5) is indeed dimensionless.

7–48 Using primary dimensions, verify that the Rayleigh number (Table 7–5) is indeed dimensionless. What other established nondimensional parameter is formed by the ratio of Ra and Gr? *Answer:* the Prandtl number

7–49 A periodic *Kármán vortex street* is formed when a uniform stream flows over a circular cylinder (Fig. P7–49). Use the method of repeating variables to generate a dimensionless relationship for Kármán vortex shedding frequency f_k as a function of free-stream speed V, fluid density ρ, fluid viscosity μ, and cylinder diameter D. Show all your work. *Answer:* St $= f$ (Re)

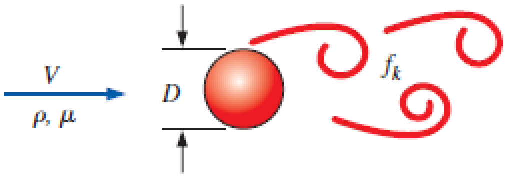

FIGURE P7–49

7–50 Repeat Prob. 7–49, but with an additional independent parameter included, namely, the speed of sound c in the fluid. Use the method of repeating variables to generate a dimensionless relationship for Kármán vortex shedding frequency f_k as a function of free-stream speed V, fluid density ρ, fluid viscosity μ, cylinder diameter D, and speed of sound c. Show all your work.

7–55 Consider fully developed **Couette flow**—flow between two infinite parallel plates separated by distance h, with the top plate moving and the bottom plate stationary as illustrated in Fig. P7–55. The flow is steady, incompressible, and two-dimensional in the xy-plane. Use the method of repeating variables to generate a dimensionless relationship for the x-component of fluid velocity u as a function of fluid viscosity μ, top plate speed V, distance h, fluid density ρ, and distance y. Show all your work. *Answer:* $u/V = f(\text{Re}, y/h)$

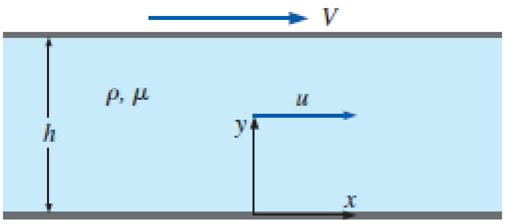

FIGURE P7–55

7–56 Consider *developing* Couette flow—the same flow as Prob. 7–55 except that the flow is not yet steady-state, but is developing with time. In other words, time t is an additional parameter in the problem. Generate a dimensionless relationship between all the variables.