0423522 - REINFORCED CONCRETE 1

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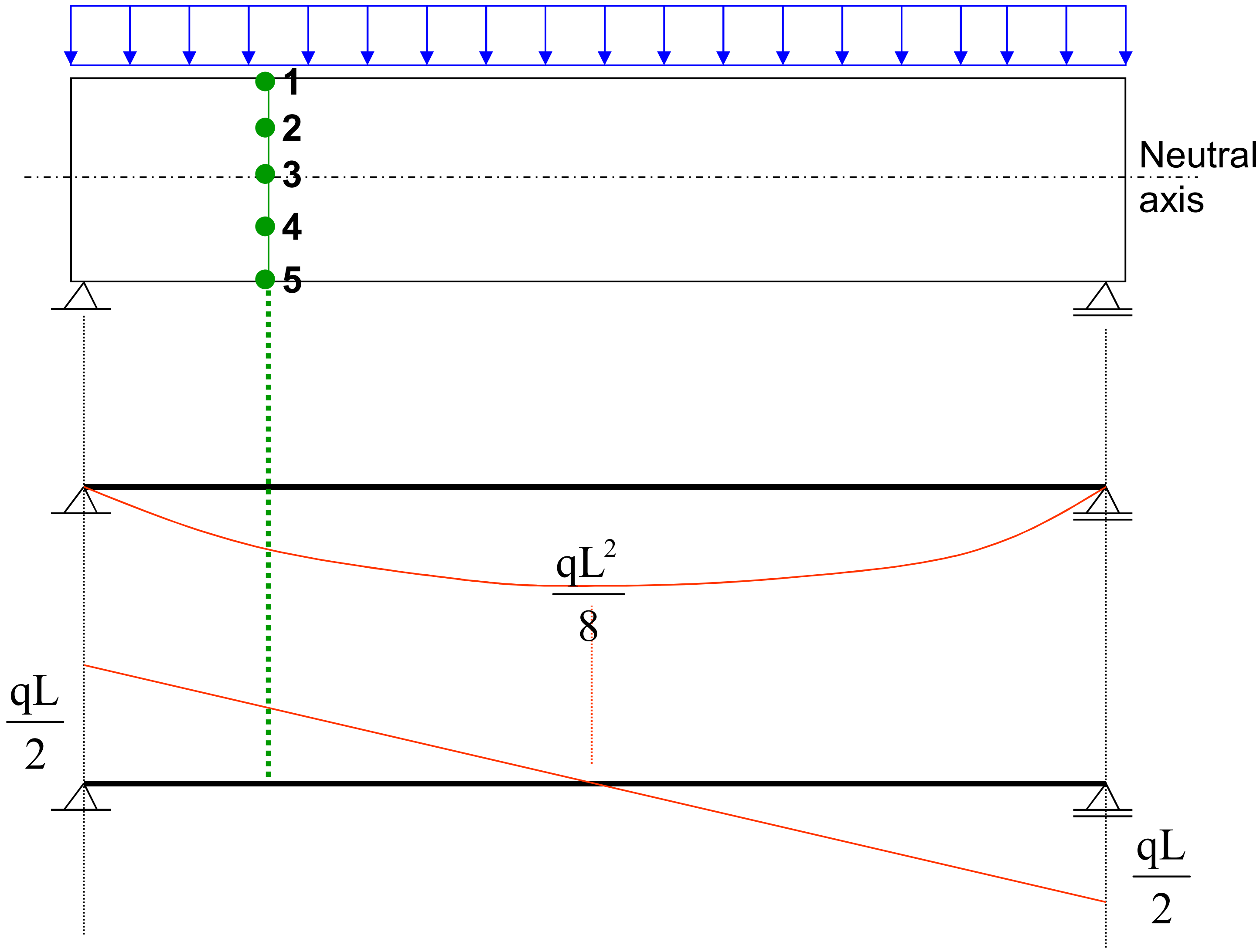
RC members subjected to shear

Members of RC structures are usually subjected to shear in addition to flexure and axial loads. Therefore, design for only flexure is not enough for structural safety. A RC member must also have adequate shear capacity (section dimensions and shear reinforcement) so that shear failure can be prevented. Otherwise premature shear failure can be observed before flexural capacity is reached. The typical shear cracks in RC elements are diagonal cracks with the angle of 45^{o}. They are called shear crack (kesme çatlağı).

RC members subjected to shear

The main reason of shear failure is not shear force itself (concrete has high shear strength)**, but the reason is inclined tensile stresses** (principle tensile stresses) **stemming from shear force** (concrete tensile strength is low)**. Thus, shear cracks are also called as 'inclined tension cracks' in literature.**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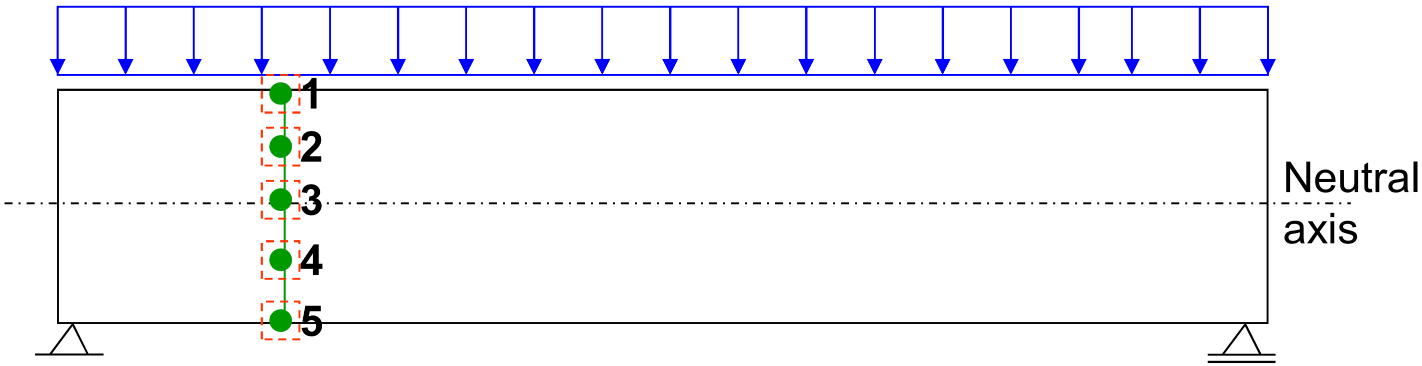
Neutral axis
qL²/8
qL/2
qL/2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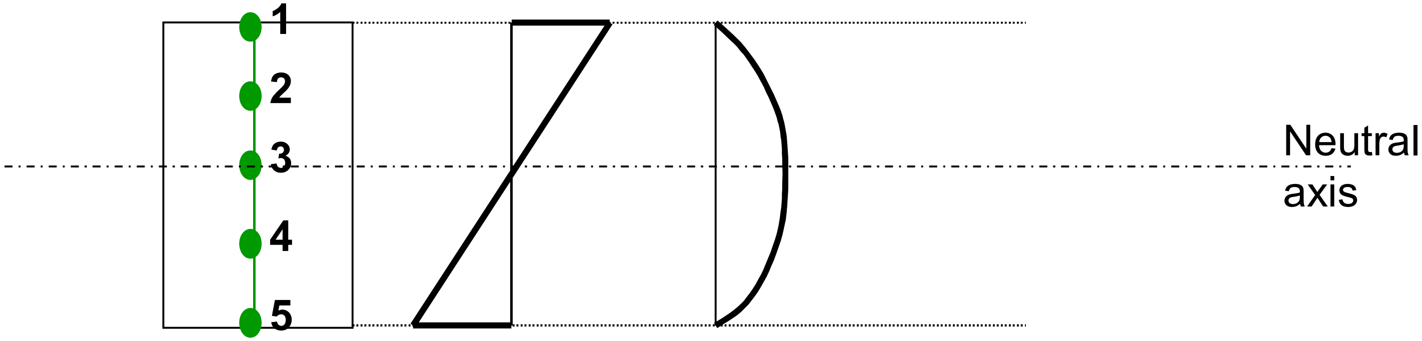
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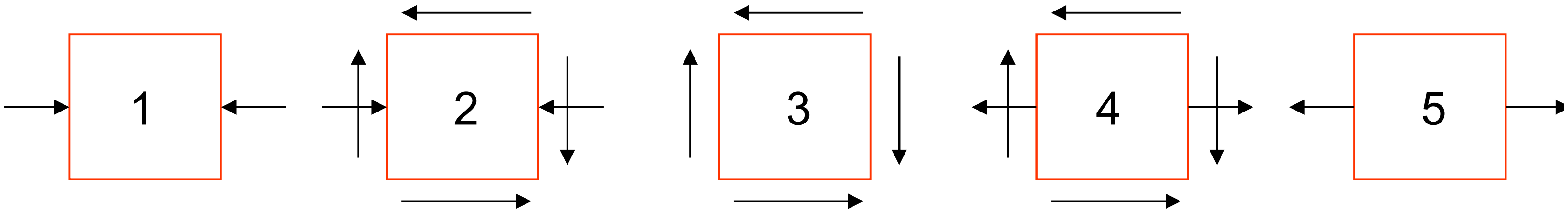
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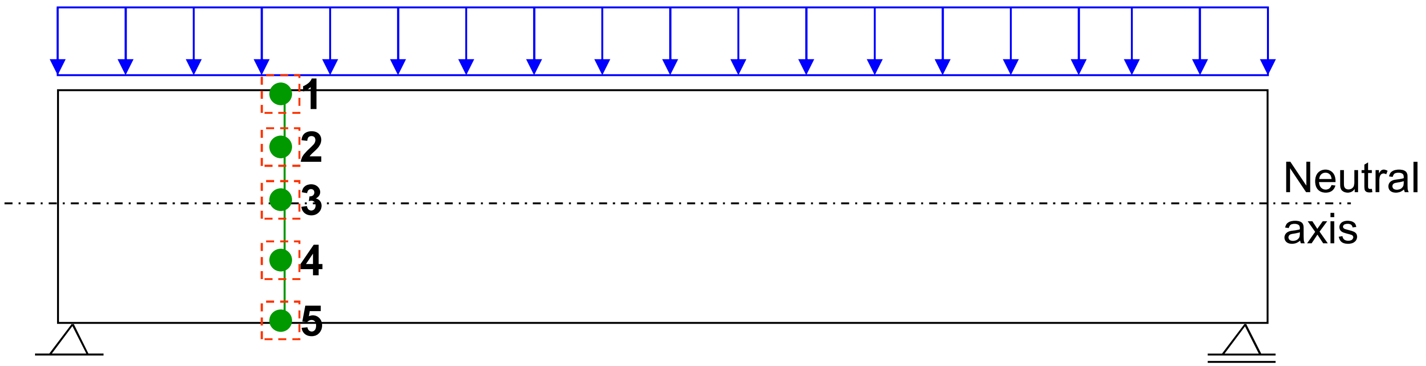
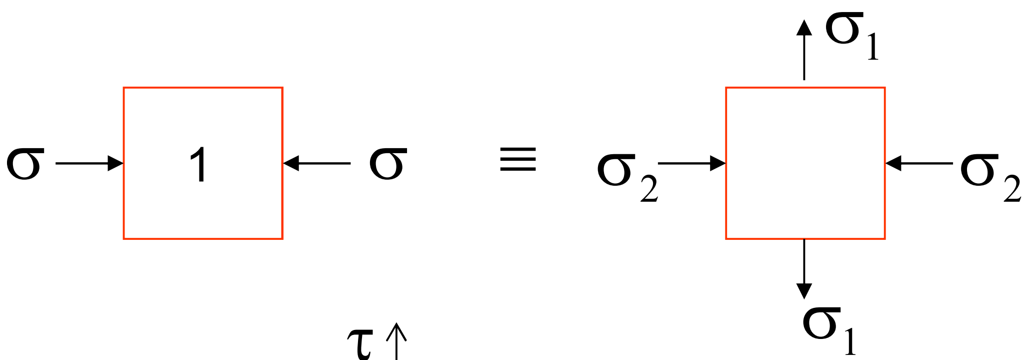
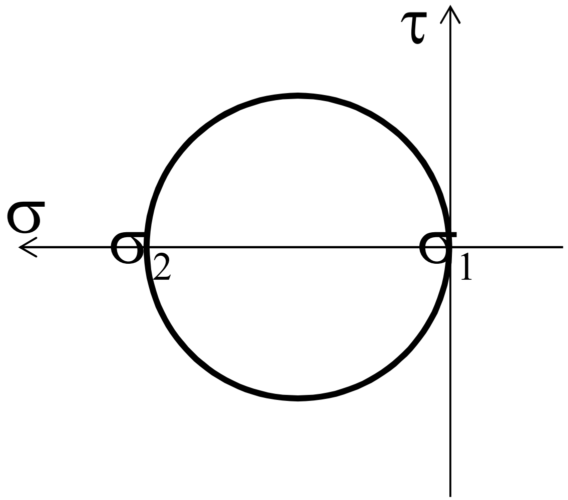
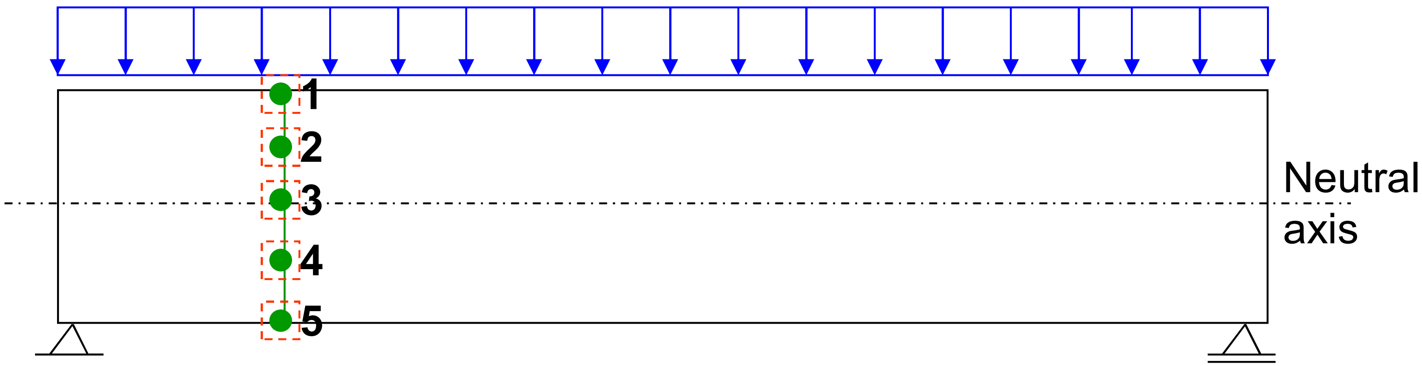


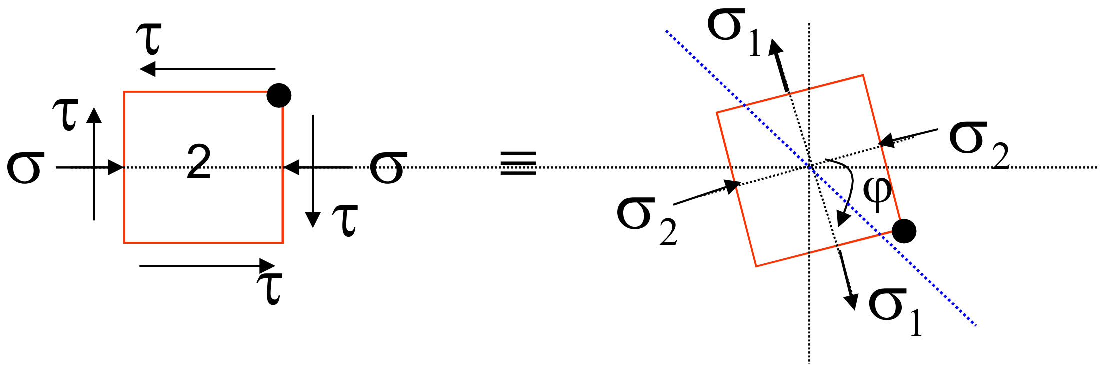


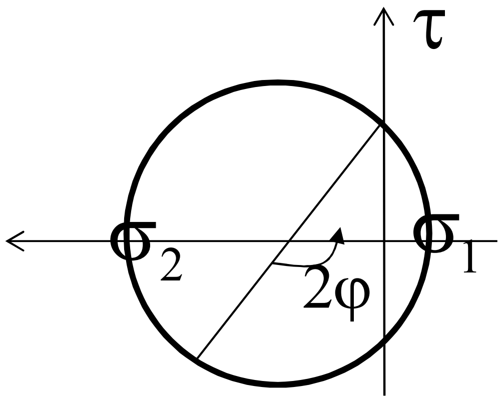


$$\sigma_1 = 0$$

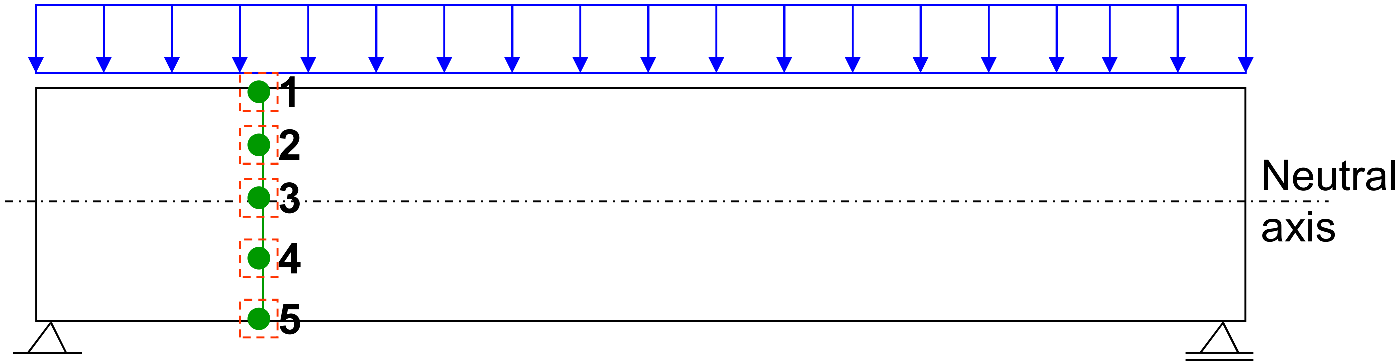
$$\sigma_2 = \sigma$$

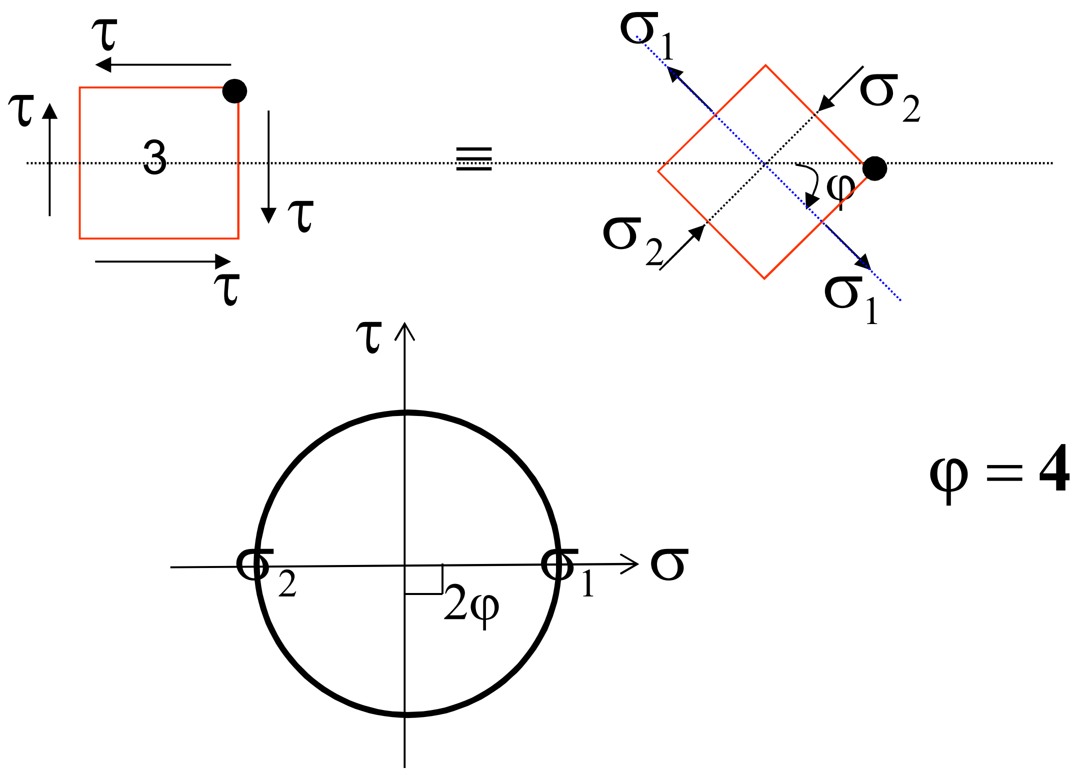




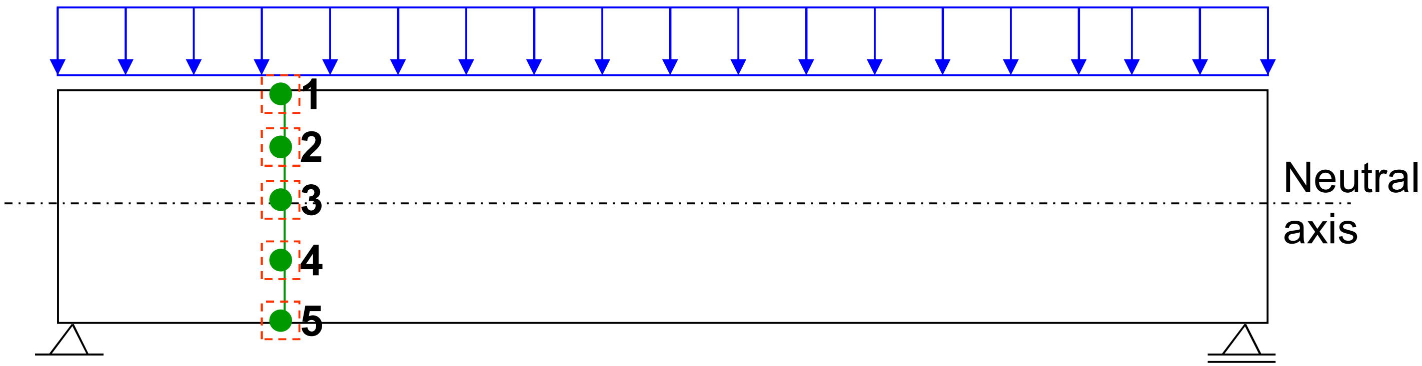


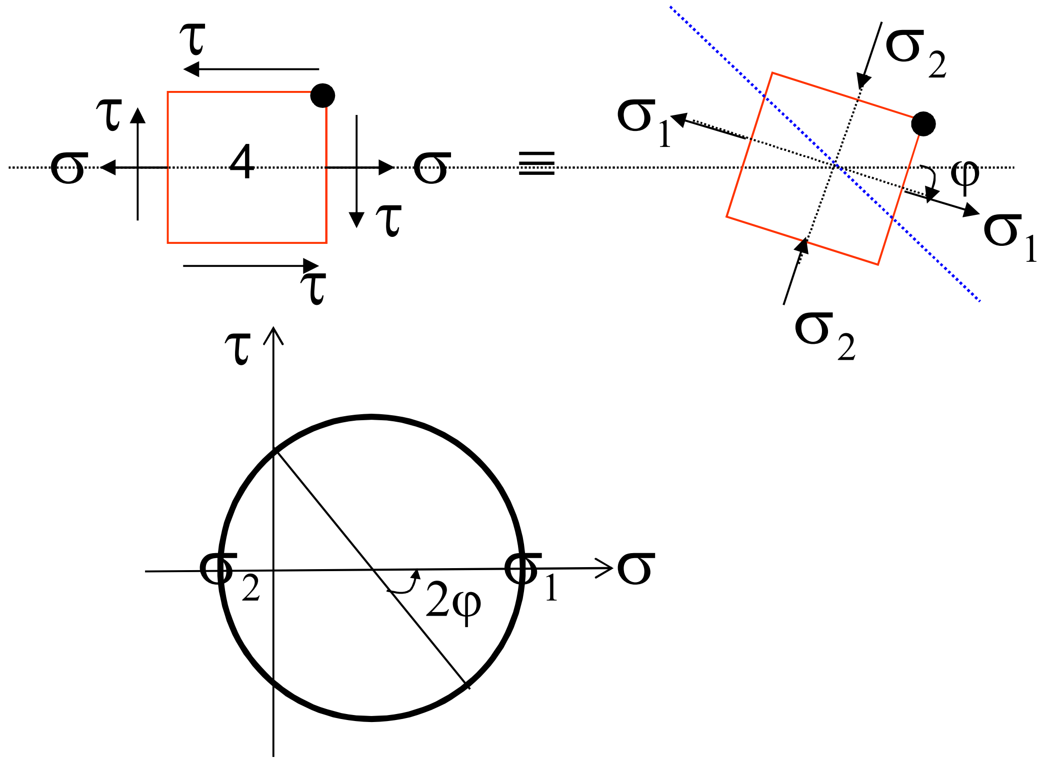
45 < φ < 90



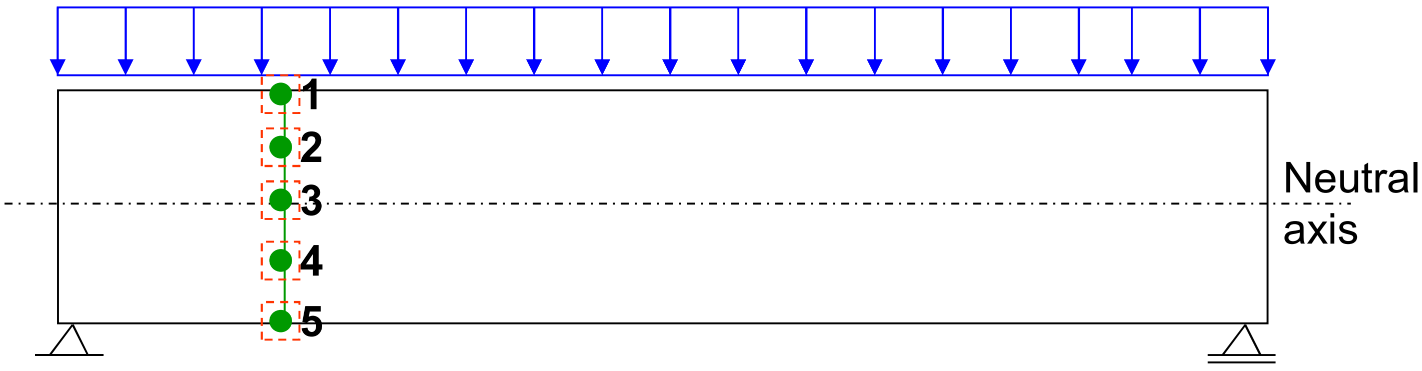


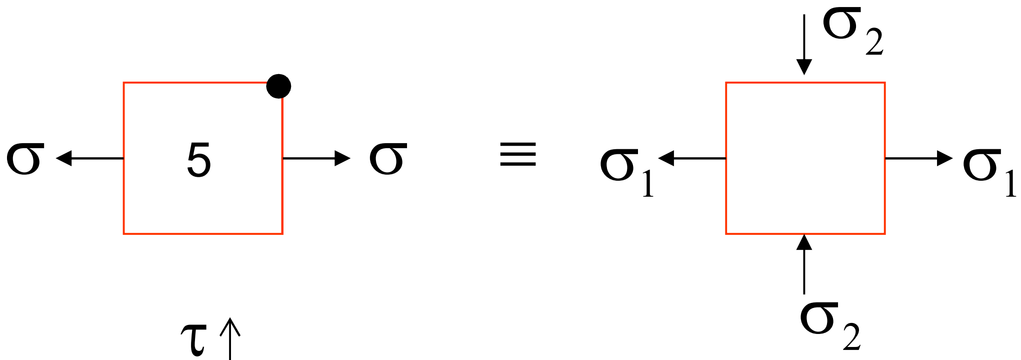
$\varphi = \mathbf{45}$

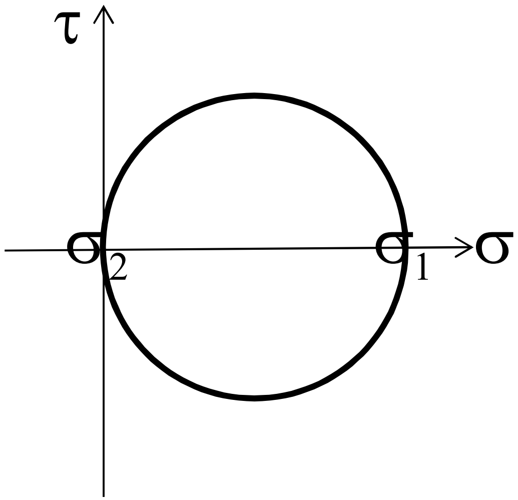




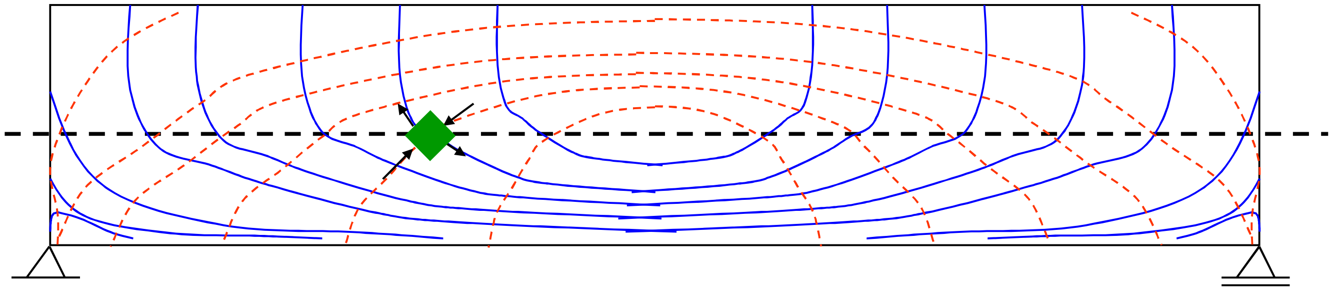
$$0 < \varphi < 45^{\circ}$$

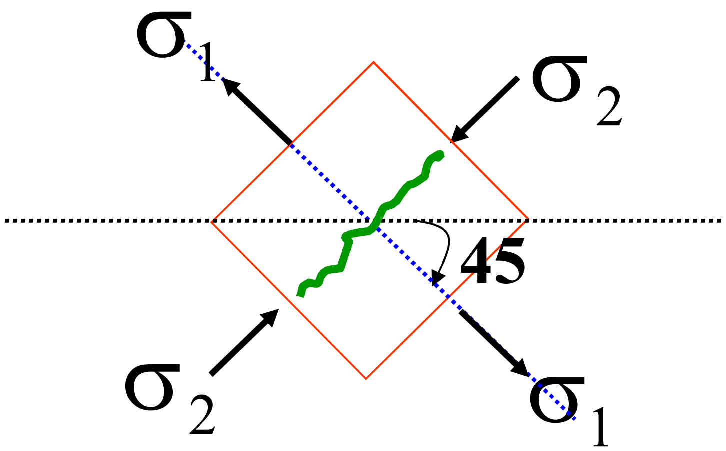




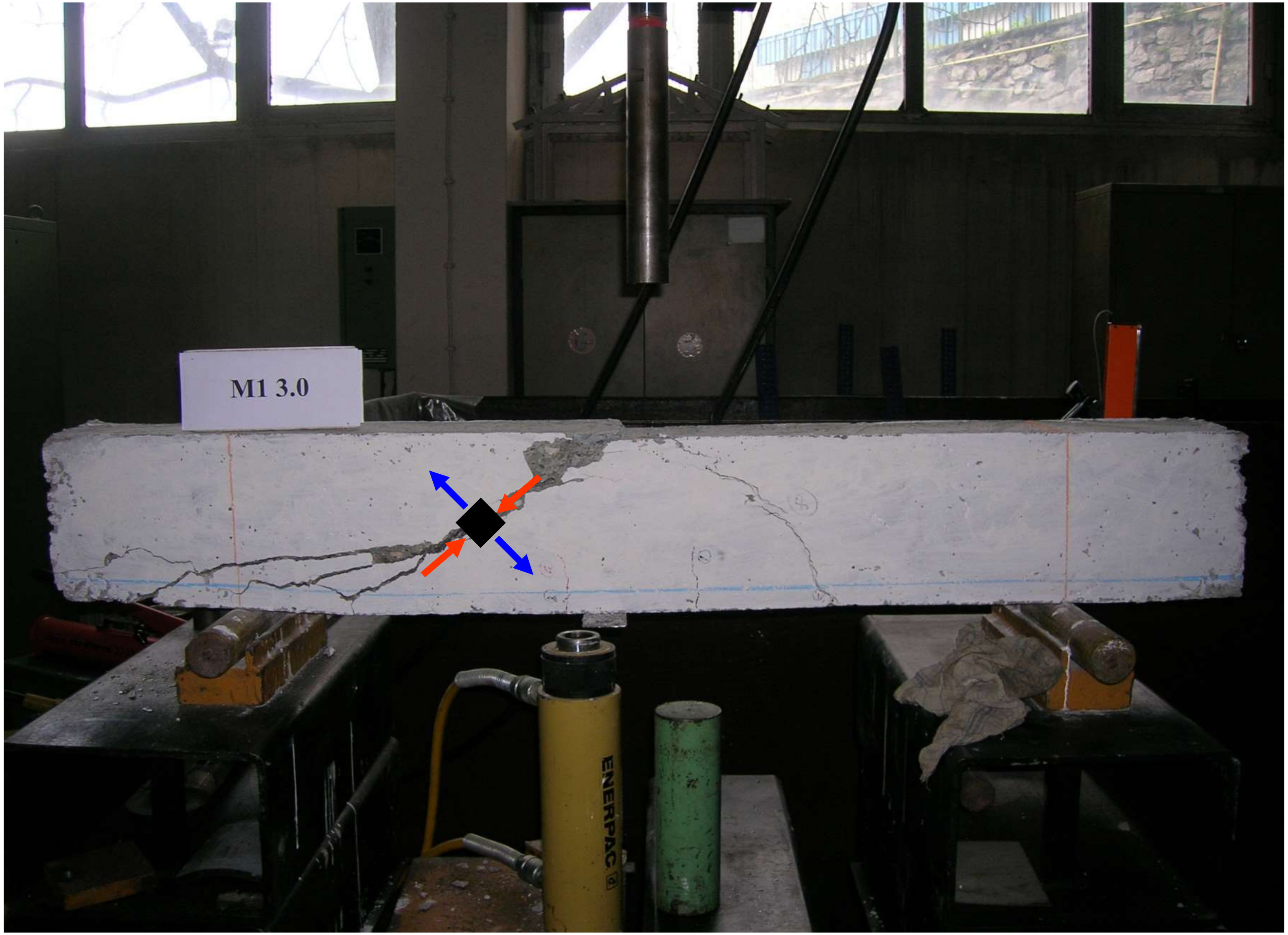


$$\sigma_1 = \sigma$$
$$\sigma_2 = 0$$




σ 1
σ 2
45
σ 2
σ 1

RC members subjected to shear



RC members subjected to shear

The RC elements should have **enough shear reinforcement to carry the principle tensile stesses; otherwise** shear cracks get wider and **sudden and brittle failure is observed.** If an RC element has enough amount of shear reinforcement but **with smaller section dimensions than required**, then **the principle compression stresses, which are higher than the compression strength of concrete, cause failure. Failure is sudden and brittle in both case.**

The shear failure is always sudden and brittle. This failure is called as shear failure although the main reason is inclined tension stresses. Because those stresses are caused by shear force.

RC members subjected to shear

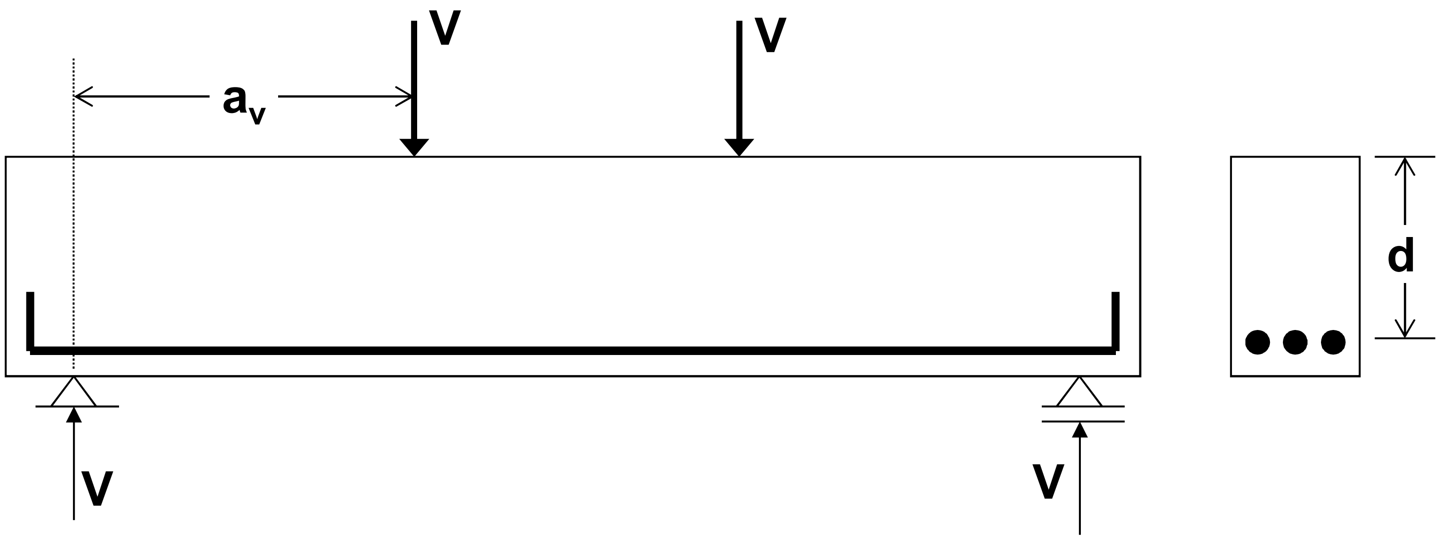
How can an engineer prevent shear failure during design?

The easiest way to prevent the shear failure is providing shear strength to a beam which is higher than its flexural strength. Therefore, the failure will be flexural and ductile if the beam is designed to exhibit ductile flexural failure.

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

Experiments show that the most effective parameter on the behavior of beams without web reinforcement is **shear span (a_v/d)**.

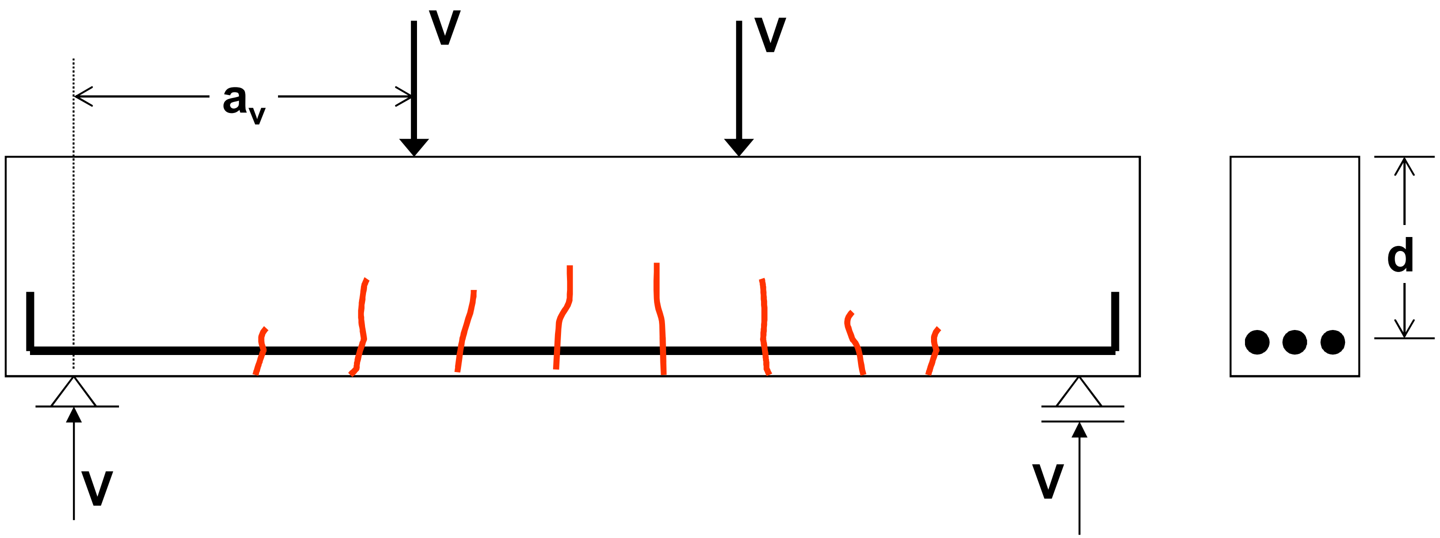


RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

$$\frac{a_v}{d} > 6$$

Tension failure / flexural failure

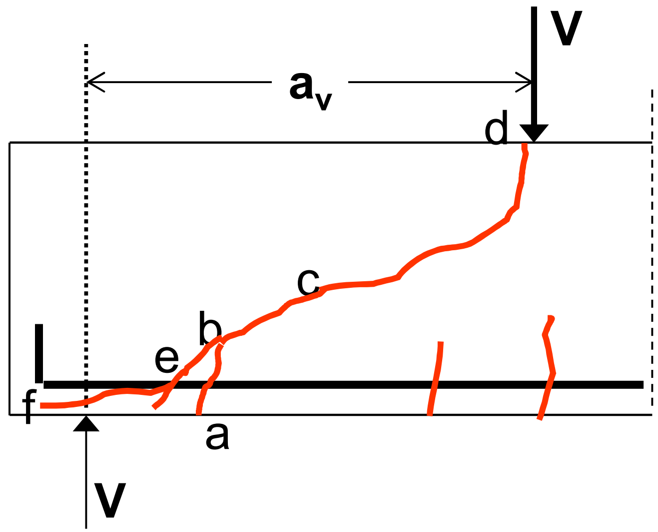


The behavior is dominated by flexure. In such a beam no diagonal cracks form.

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

$$6 > \frac{a_v}{d} > 2.5$$ Diagonal tension failure



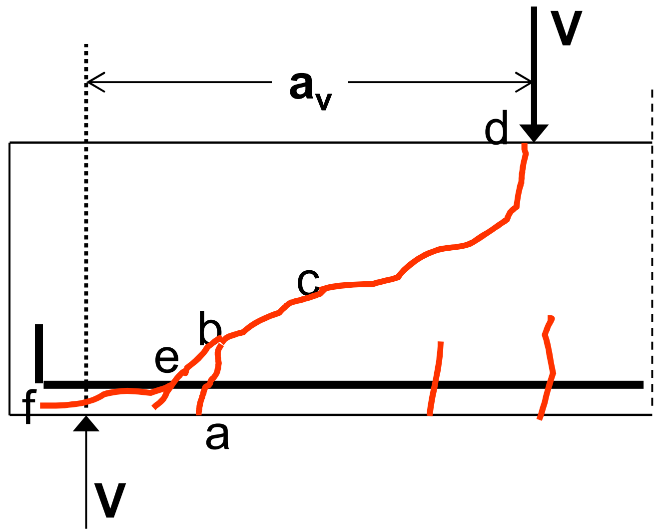
At early stages of loading, flexural cracks appear at zones of maximum moment **(a-b)**. As the load is increased, some of these flexural cracks are inclined and reach **point c (b-c).**

İf shear span (a_v/d) is close to 2.5 then upward progress of crack is prevented by compression stresses stemming from the concentrated load. The main reason of failure is loss of adherence near the support of the beam **(e-f)**.

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

$$6 > \frac{a_v}{d} > 2.5$$ Diagonal tension failure

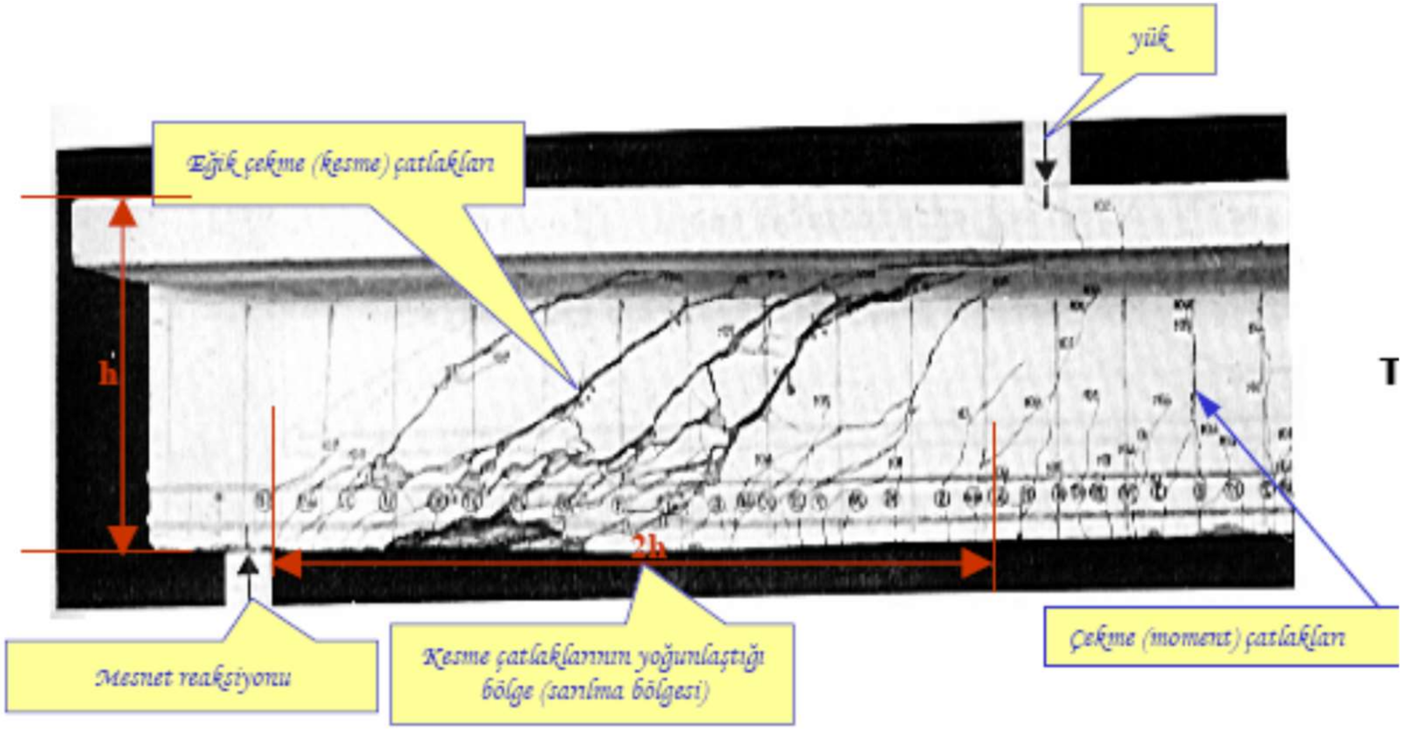


If shear span (a_v/d) is close to 6 then crack extends to **point d**

The difference between cracking load and failure load is little.

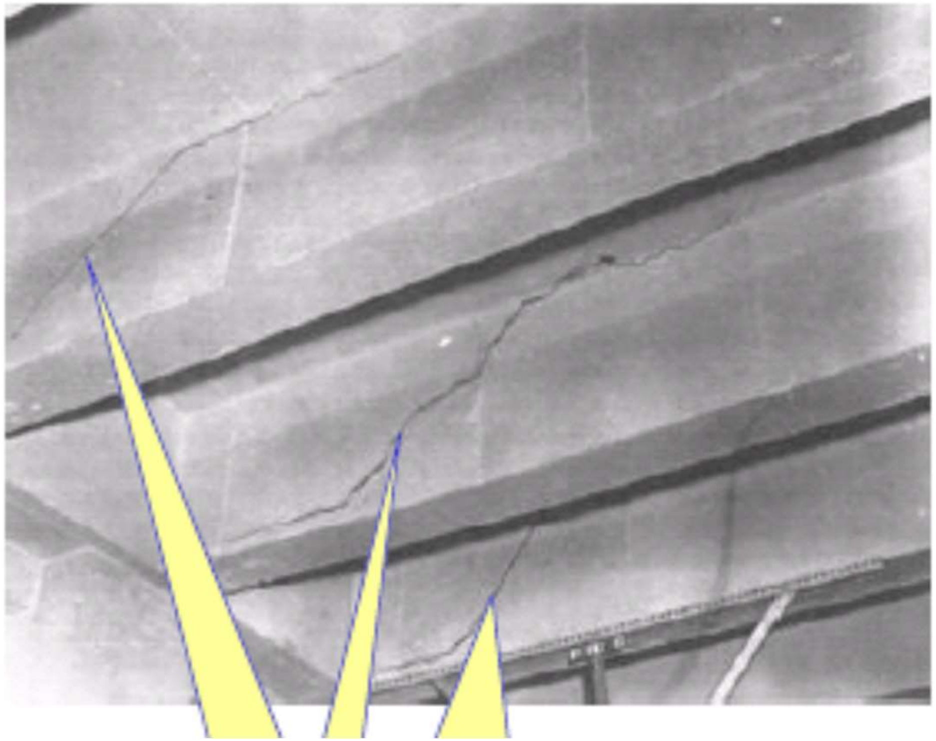
RC members subjected to shear

RC members subjected to shear



Fotoğraf Prof.Dr.Ahmet Topçu'nun ders notlarından alınmıştır.

RC members subjected to shear



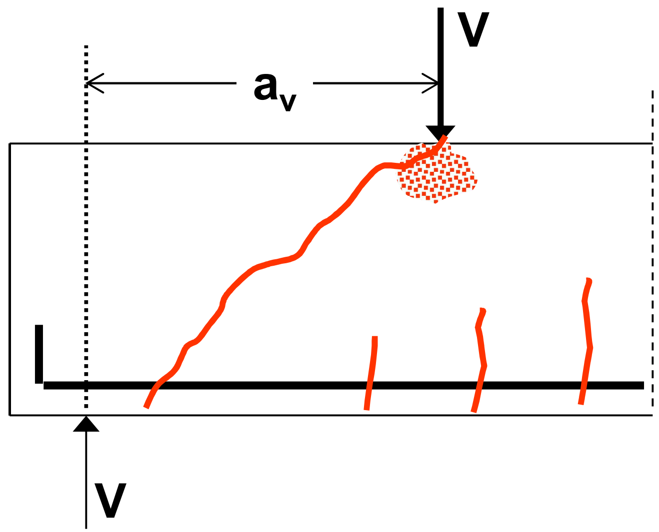
Fotoğraf Prof.Dr.Ahmet Topçu'nun ders notlarından alınmıştır.

RC members subjected to shear

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

$$2.5 > \frac{a_v}{d} > 1.0$$ Shear-compression failure

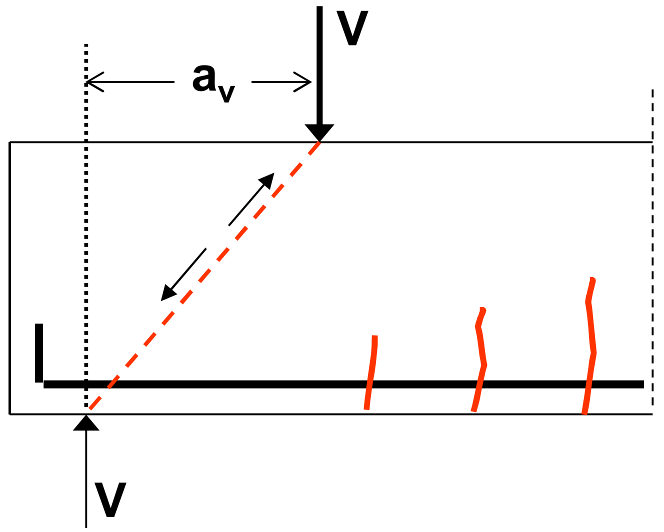


Failure takes place by crushing of concrete in the compression zone. That's why it is called 'Shear-compression failure' and failure load is higher than the cracking load.

RC members subjected to shear

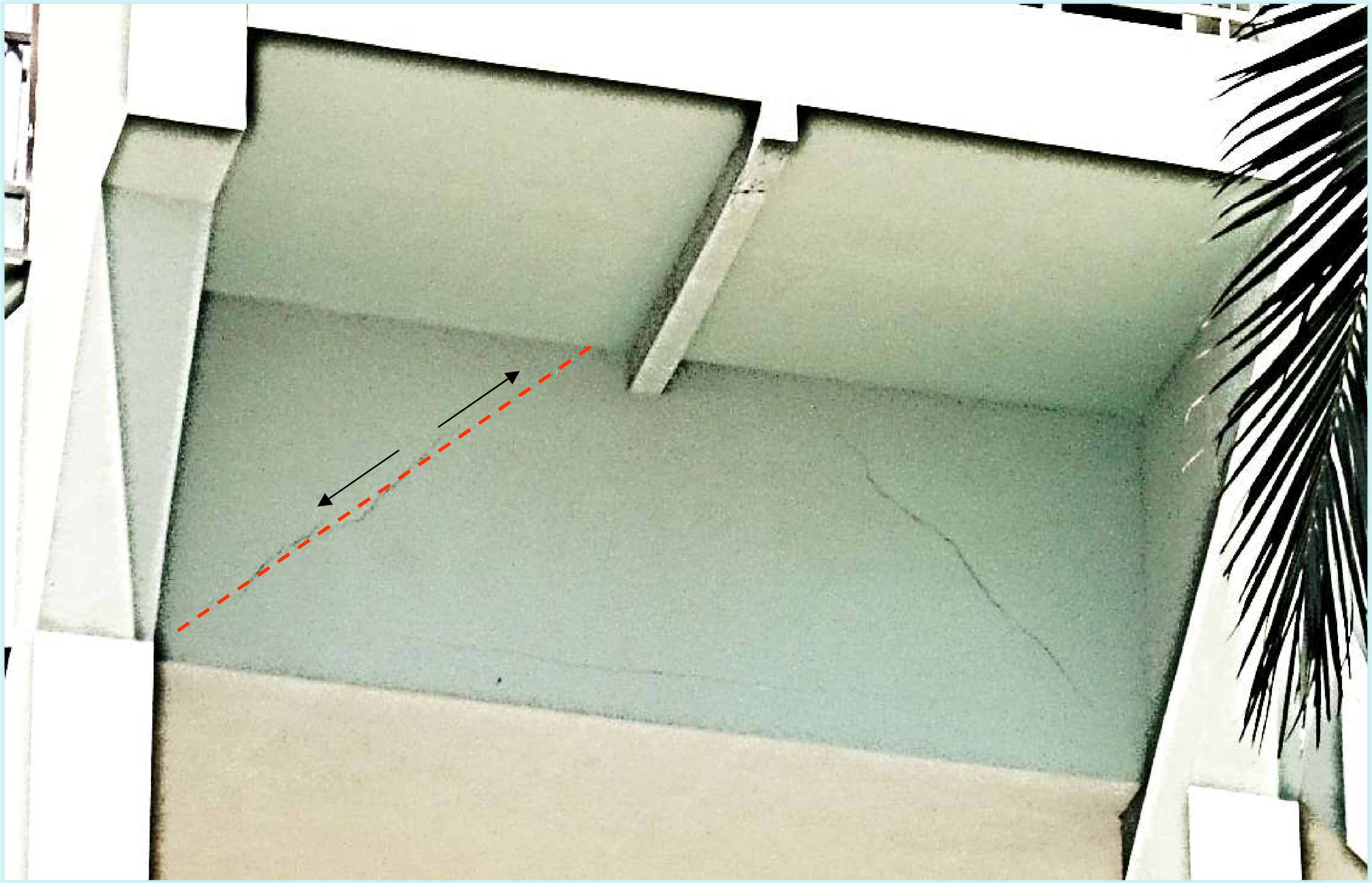
Mechanism of shear failure for beams without web reinforcement

$$1.0 > \frac{a_v}{d}$$ Deep beam failure



In beams with very short shear spans failure due to principle tensile stresses is not possible. Load is directly transferred to the support with a compression strut forming in the web. Beams with such a shorter span behave like a tied arch rather than a beam. Therefore longitudinal steel acts as a tie rod. The failure is caused by either crushing of concrete or loss of adherence of reinforcement. **The failure load may be few times larger than cracking load.**

RC members subjected to shear



RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

The experimental studies show that the most effective parameter on the behavior of an RC beam without web reinforcement is **shear span ratio.**

The shear span ratio can be generalized as (M/V_d) so that it can be applied to beams subjected to distributed loads.

$$\frac{a_v}{d} = \frac{Va_v}{Vd} = \frac{M}{Vd}$$

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

Experimental studies have shown that after the formation of diagonal crack, the shear force is mainly resisted by

V_{cz} = Shear transferred through the uncracked portion of the concrete,

$$V_{cz} = (\%20\sim40)\ V_c$$

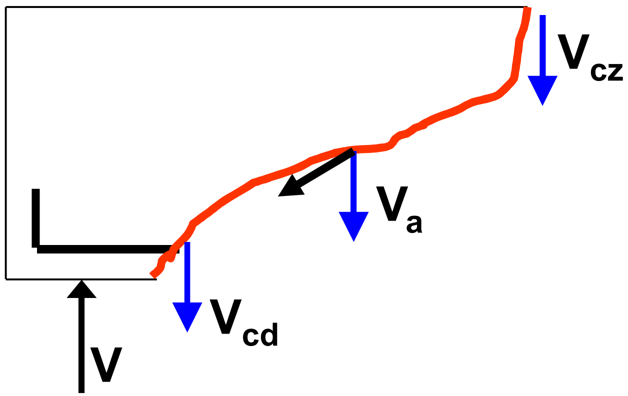
V_a = Vertical component of the aggregate interlocking force in the cracked portion of the concrete

$$V_a = (\%35\sim50)\ V_c$$

V_{cd} = Shear force carried through the dowel action of the longitudinal steel

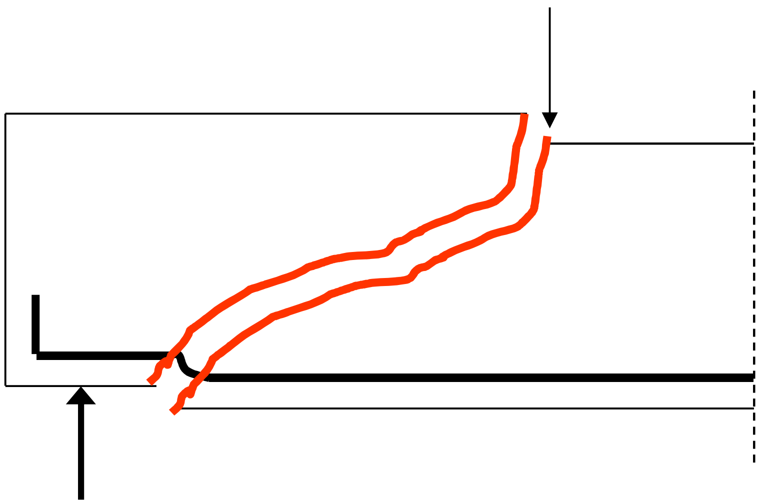
$$V_{cd} = (\%15\sim25)\ V_c$$

$$V_c = V_{cd} + V_a + V_{cz}$$



RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement



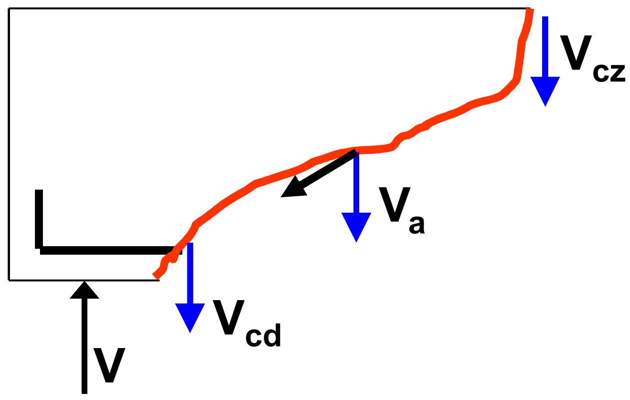
Dowel action (Kaldıraç etkisi): Reinforcement bars provide shear strength by resisting the relative vertical displacement of the beam parts which are taken place at the both side of the crack.

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

Experimental studies have shown that after the formation of diagonal crack, the shear force is mainly resisted by

$$V_c = V_{cd} + V_a + V_{cz}$$



As the shear force increases dowel action loose its effectiveness and then strength provided by inclined shear stresses along the face of the crack is lost.

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

The diagonal crack strength given by ACI 318

$$V_c = \left(\frac{\sqrt{f_{ck}}}{6}\right) b_w d$$

TS500 uses %85 of the strength given by ACI 318

$$V_c = 0.85 \left(\frac{\sqrt{f_{ck}}}{6}\right) b_w d$$

$$f_{ctk} = 0.35 \sqrt{f_{ck}} \qquad f_{ctd} = \frac{0.35}{1.5} \sqrt{f_{ck}}$$

$$V_{cr} = 0.61 f_{ctd} b_w d \longrightarrow$$ TS500 $$V_{cr} = 0.65 f_{ctd} b_w d$$

Shear force which causes diagonal crack

$$V_c = 0.80 V_{cr}$$

The shear strength of the concrete section

RC members subjected to shear

Mechanism of shear failure for beams without web reinforcement

$$V_c = 0.80\, V_{cr}$$

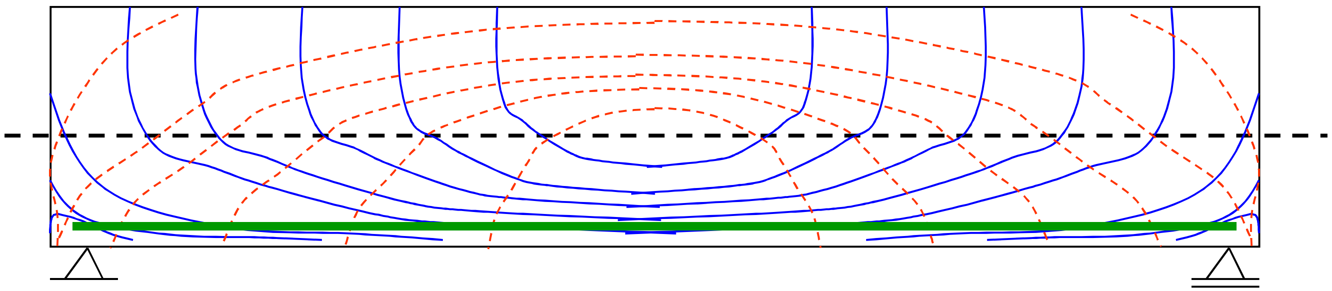
The shear strength of the concrete section

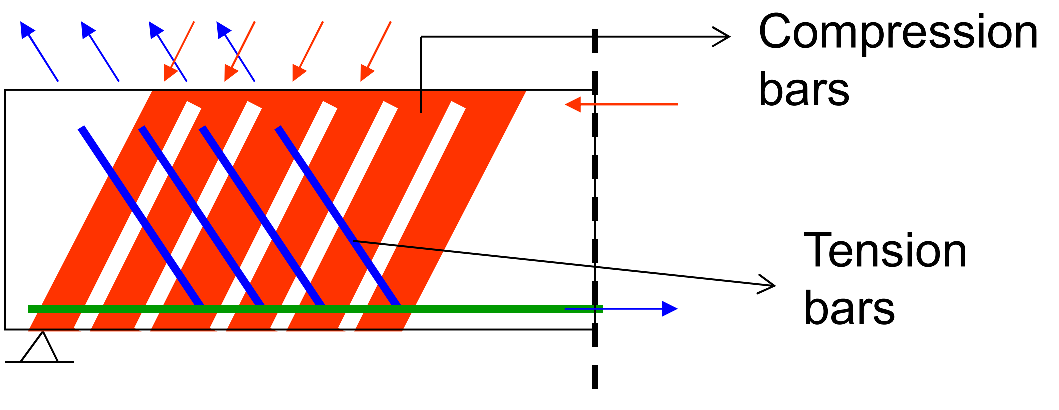
If the quality control is not provided for concrete then coefficient can be assumed less than 0.80.

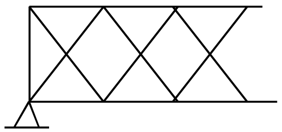
RC members subjected to shear

Shear strength of an RC beam with web reinforcement

The truss analogy is useful for calculation of shear strength provided by web reinforcement (shear reinforcement). It is assumed that the beam with shear reinforcement behaves like a truss while the transmission of the load to the support of beam.



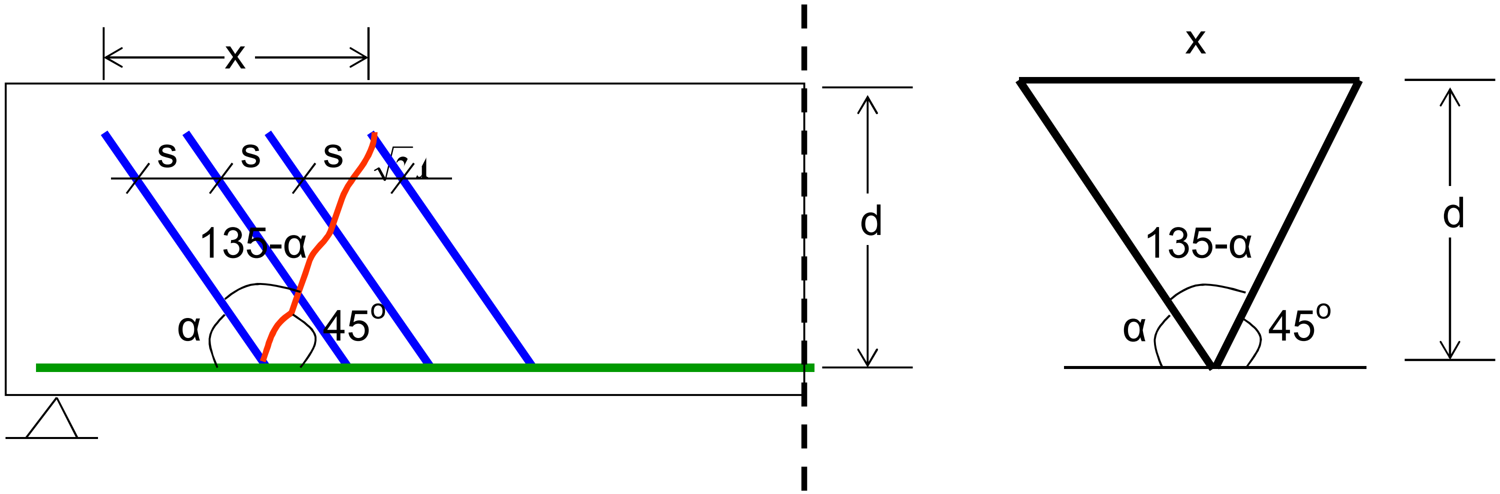




RC members subjected to shear

Shear strength of an RC beam with web reinforcement

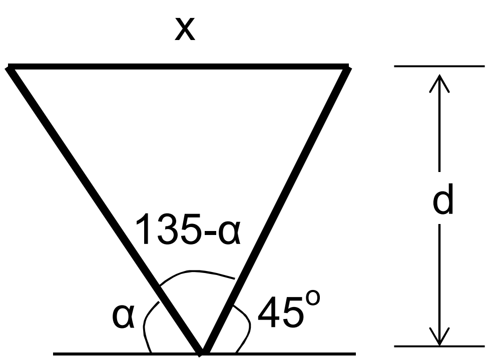
The truss analogy is useful for calculation of shear strength provided by web reinforcement (shear reinforcement). It is assumed that the beam with shear reinforcement behaves like a truss while the transmission of the load to the support of beam.



$$V_w = A_{sw} f_{ywd} \frac{x}{s} \sin \alpha$$

RC members subjected to shear

Shear strength of an RC beam with web reinforcement



$$\frac{x}{\sin(135-\alpha)} = \frac{\sqrt{2}d}{\sin\alpha}$$

$$\sin(135-\alpha) = \sin 135\cos\alpha - \cos 135\sin\alpha$$

$$V_w = A_{sw} f_{ywd} \frac{x}{s} \sin\alpha$$

$$x = \frac{\sqrt{2}d}{\sin\alpha}\sin(135-\alpha)$$

$$x = \sqrt{2}d\,\frac{\sqrt{2}}{2}\,\frac{\sin\alpha + \cos\alpha}{\sin\alpha}$$

$$x = \frac{\sin\alpha + \cos\alpha}{\sin\alpha}d$$

$$V_w = A_{sw} f_{ywd} \frac{d}{s}(\sin\alpha + \cos\alpha)$$

RC members subjected to shear

Shear strength of an RC beam with web reinforcement

$$V_w = A_{sw} f_{ywd} \frac{d}{s} (\sin\alpha + \cos\alpha)$$

V_w= The shear strength provided by shear reinfocement

f_{ywd}= Design yield strength of shear reinforcement

α= The angle of the shear reinforcement

s= The space interval of shear reinforcement

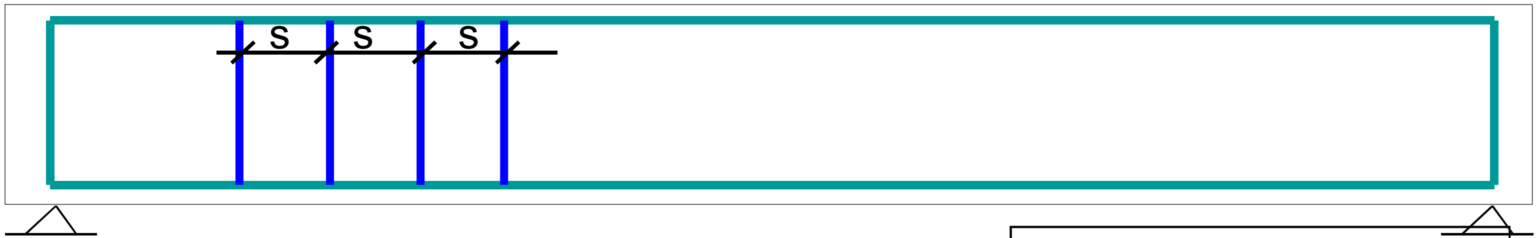
d= Effective depth of beam

RC members subjected to shear

Shear strength of an RC beam with web reinforcement

$$V_w = A_{sw} f_{ywd} \frac{d}{s} (\sin \alpha + \cos \alpha)$$

Shear reinforcement is vertical stirrup α= 90º

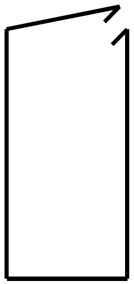


$$V_w = A_{sw} f_{ywd} \frac{d}{s_{etr}}$$

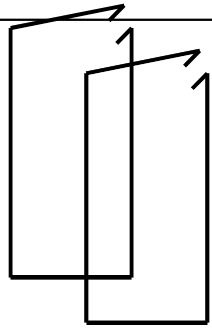
$$A_{sw} = nA_0$$

n = Number of arms of stirrup

A_o= The area of one arm of stirrup



$$n = 2$$



$$n = 4$$

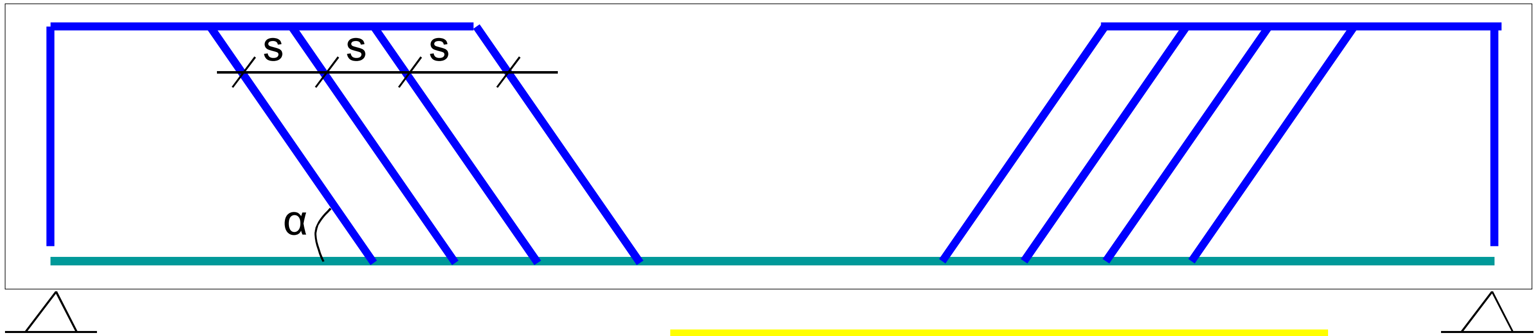
RC members subjected to shear

Shear strength of an RC beam with web reinforcement

$$V_w = A_{sw} f_{ywd} \frac{d}{s} (\sin \alpha + \cos \alpha)$$

Shear reinforcement is only bent up bars

(TS500 doesn't allow to use bent up bars as shear reinforcement!!!)



$$\alpha = 45°$$

$$V_w = 1.414 A_{sw} f_{ywd} \frac{d}{s_{pl}}$$

$$\alpha = 60°$$

$$V_w = 1.366 A_{sw} f_{ywd} \frac{d}{s_{pl}}$$

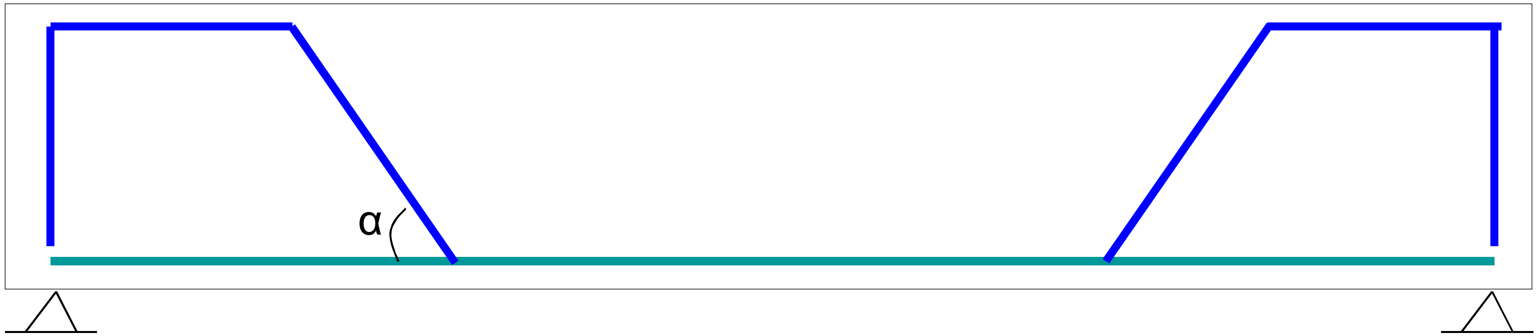
RC members subjected to shear

Shear strength of an RC beam with web reinforcement

$$V_w = A_{sw} f_{ywd} \sin \alpha$$

If Shear reinforcement is only ONE bent up bar

(TS500 doesn't allow to use bent up bars as shear reinforcement!!!)



$$\alpha = 45°$$ $$V_w = 0.707 A_{sw} f_{ywd}$$

$$\alpha = 60°$$ $$V_w = 0.866 A_{sw} f_{ywd}$$

RC members subjected to shear

Shear strength of an RC beam with web reinforcement

The total shear strength of an RC beam

$$V_r = V_c + V_w$$

V_w: Shear strength provided by shear reinforcement

V_c: Shear strength provided by concrete

RC members subjected to shear

Shear strength of an RC beam with web reinforcement

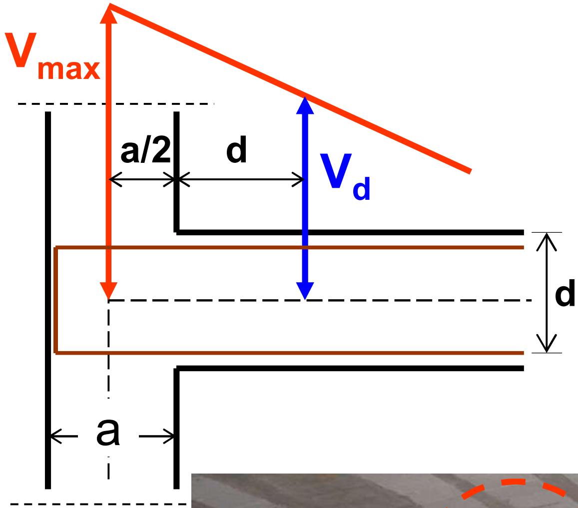
PRINCIPLE COMPRESSION STRESSES

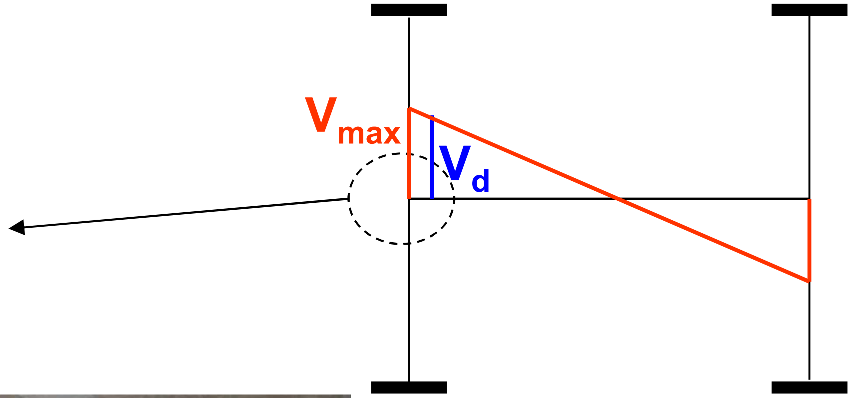
The upper limit for principle compression stresses

$$V_{r,max} = 0.22 f_{cd} b_w d$$

The principle tensile stresses is carried by shear reinforcement while principle compression stresses is resisted by concrete section. If the compression stress is higher than the section capacity then sudden and brittle failure will be observed.

Design shear force for direct supports





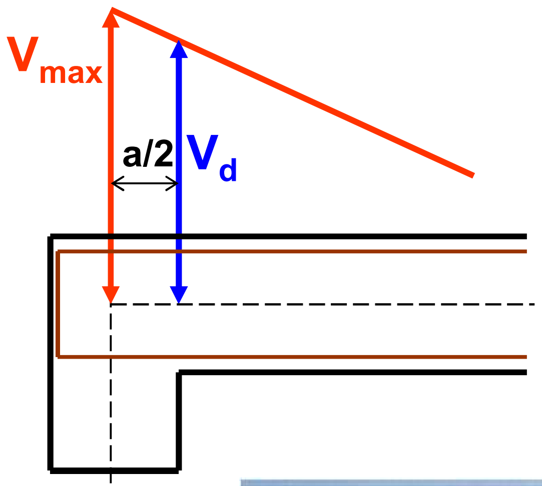
d:Effective beam depth

V_d: Design shear force

RC members subjected to shear

Shear design of RC beams

Design shear force for indirect supports



V_d: Design shear force

RC members subjected to shear

Shear design of RC beams

Known parameters: $\mathbf{b_w}$, $\mathbf{d}$, $\mathbf{f_{cd}}$, $\mathbf{f_{ctd}}$, $\mathbf{f_{ywd}}$, $\mathbf{f_{yd}}$, $\mathbf{V_d}$

Parameters to be calculated: **Ø**, **s**

Diagonal crack strength $V_{cr} = 0.65 f_{ctd} b_w d$ $\quad V_{r,max} = 0.22 f_{cd} b_w d$

$V_d < V_{cr}$: The concrete section can resist the design shear force itself. However min. shear reinforcement must be used to provide ductility.

$V_{cr} < V_d < V_{r,max}$: The strength provided by concrete is only not enough, the required shear reinforcement must be calculated.

$V_{r,max} < V_d$: The concrete section is not sufficient to resist the principle compression stresses. It must be enlarged.

RC members subjected to shear

Shear design of RC beams

Known parameters: $\mathbf{b_w}$, **d**, $\mathbf{f_{cd}}$, $\mathbf{f_{ctd}}$, $\mathbf{f_{ywd}}$, $\mathbf{f_{yd}}$, $\mathbf{V_d}$

Parameters to be calculated: **Ø**, **s**

Diagonal crack strength $$V_{cr} = 0.65 f_{ctd} b_w d$$

$$V_d < V_{cr}$$

The concrete section can resist the design shear force itself. However min. shear reinforcement must be used to provide ductility.

$$\min \rho_w = 0.30 \frac{f_{ctd}}{f_{yd}}$$ TS500

$$\frac{A_{sw}}{b_w s} = 0.30 \frac{f_{ctd}}{f_{yd}} \qquad A_{sw} = nA_0 \qquad \frac{nA_0}{b_w s} = 0.30 \frac{f_{ctd}}{f_{yd}}$$

Select the diameter of reinforcement bar and calculate its area (A_0)

Find the required spacing of stirrups (s)

RC members subjected to shear

Shear design of RC beams

Known parameters: b_w, d, f_{cd}, f_{ctd}, f_{ywd}, f_{yd}, V_d

Parameters to be calculated: **Ø**, **s**

Eğik çatlama dayanımı $V_{cr} = 0.65 f_{ctd} b_w d$ $\quad V_{r,max} = 0.22 f_{cd} b_w d$

$$V_{cr} < V_d < V_{r,max}$$

The strength provided by concrete is only not enough, the required shear reinforcement must be calculated.

$V_c = 0.8 V_{cr}$ The shear strength of the concrete section

$V_w = V_d - V_c$ The shear strength provided by shear reinforcement

$$V_w = A_{sw} f_{ywd} \frac{d}{s} \qquad A_{sw} = nA_0 \qquad V_w = nA_0 f_{ywd} \frac{d}{s}$$

$$\frac{A_0}{s} = \frac{V_w}{n f_{ywd} d}$$

Select the diameter of reinforcement bar and calculate its area (A_0)

Find the required spacing of stirrups (s)

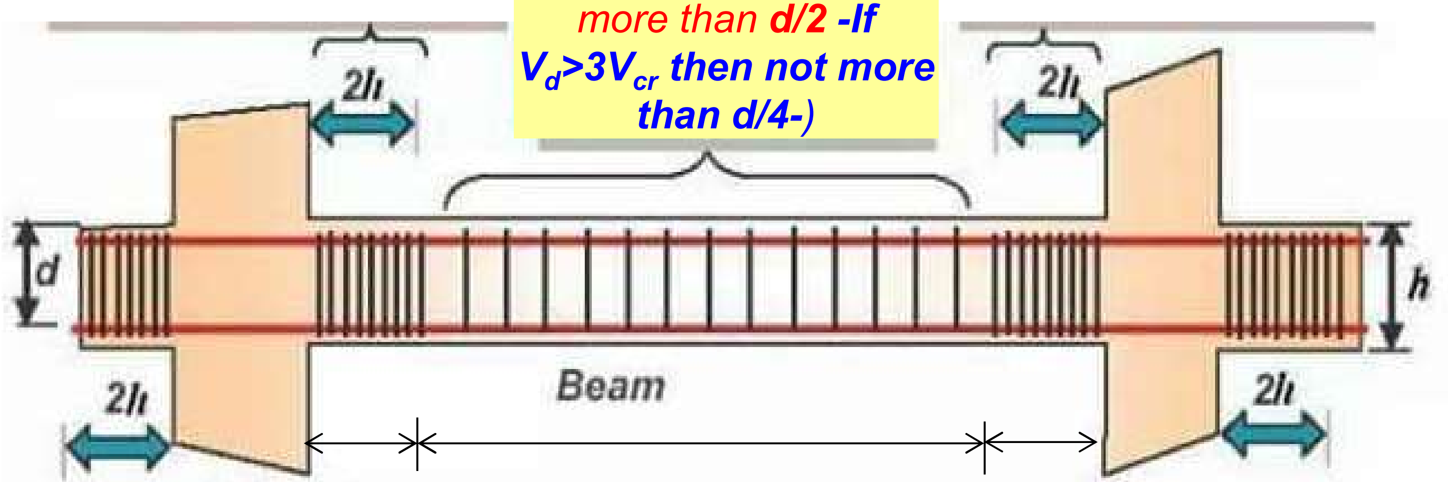
RC members subjected to shear

Requirements for shear reinforcement (TS500, DBYBHY)

*Spacing of stirrups as calculated (but not more than **d/4**, **8 times min. beam bar diameter** and **150mm**)*

*Spacing of stirrups as calculated (but not more than **d/2 -If $V_d > 3V_{cr}$ then not more than d/4-**)*

*Spacing of stirrups as calculated (but not more than **d/4**, **8 times min. beam bar diameter** and **150mm**)*



Confined zone / plastic hinge zone

Confined zone / plastic hinge zone

d = Effective beam depth