Engine dynamics-Ex2
Forces acting on bearings without balancing

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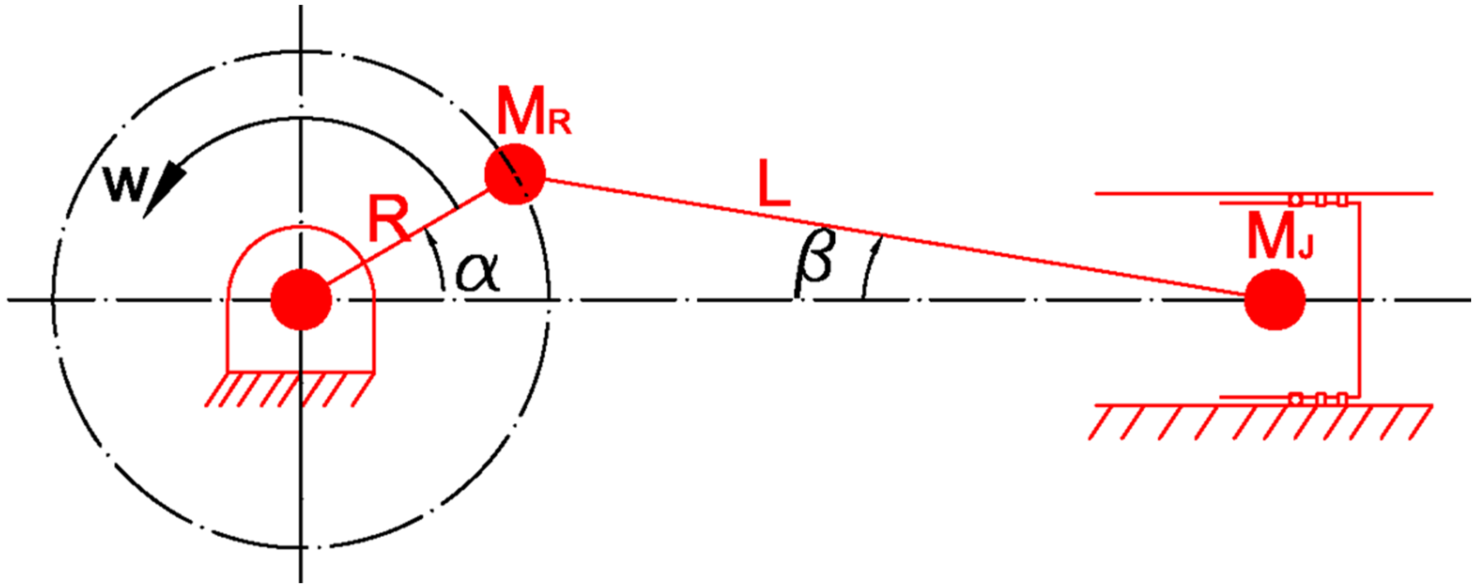


- n=6500 rpm
- R=40.25/1000; %m
- m_{piston}=276.26/1000; %kg
- $m_{ringoil}$=8/1000; %kg
- m_{cp}=225.6/1000; %kg
- ρ=20/1000; %m
- m_b=402.11/1000; %kg
- m_{crp}=0.3*mb %kg

D=79.5/1000 %m

$\lambda=$0.320

$m_{ringcomp}$=4/1000; %kg

m_{wp}=101.68/1000; %kg

m_w=400/1000; %kg

Pg@30°DCA=1 Bar

$\rho_{cw}=R$

$\alpha=$30 DCA

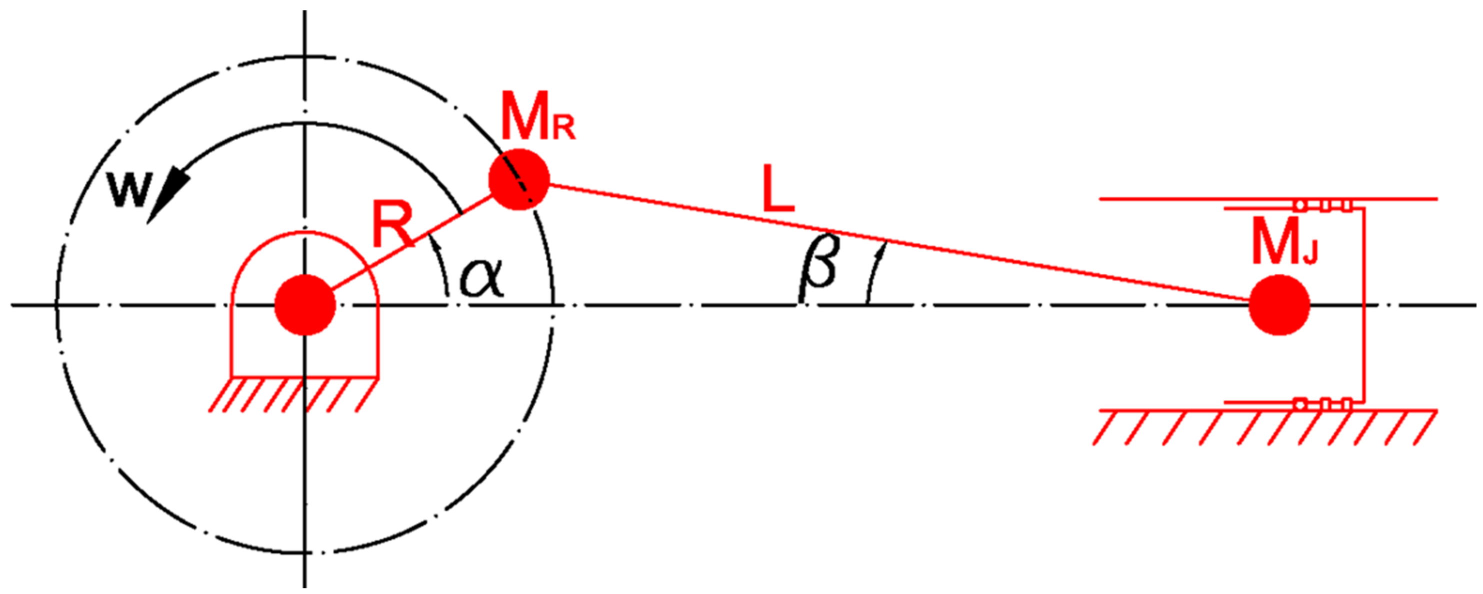
Beta

$Lsin(\beta) = Rsin(\alpha)$
$sin(\beta)$=$\lambda sin(\alpha)$

$\beta = \operatorname{asin}(\lambda \sin\alpha)$

$\beta = \operatorname{asin}(0{,}32 * \sin 30)$

$\beta = 9.20°$



m_j

$$m_p = m_{piston} + m_{wp} + \sum m_{ring}$$

$$m_p = 276.26 + 101.68 + (8 + 2 * 4) / 1000$$

$$m_p = 0.393 kg$$

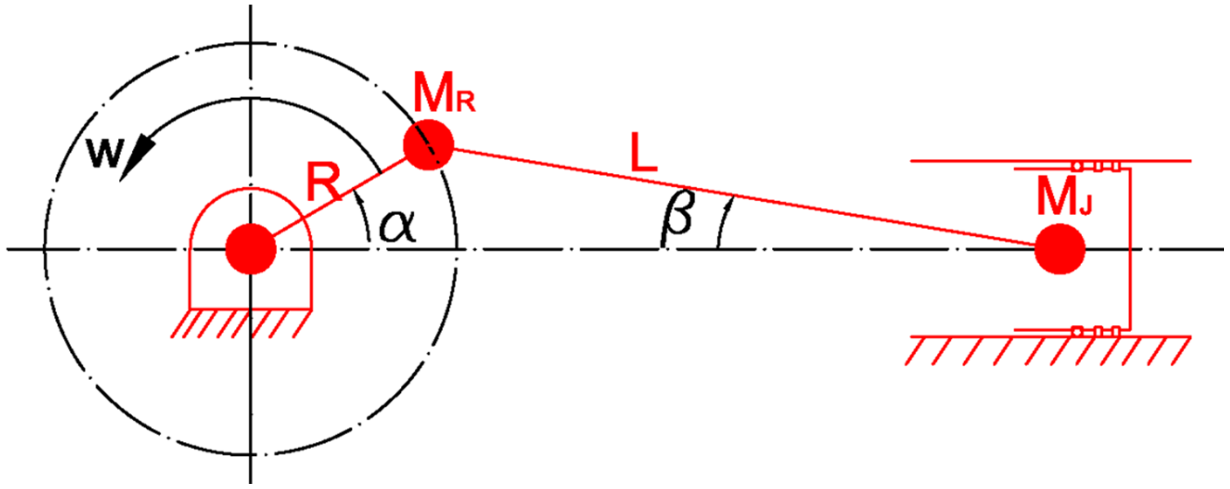
$$m_j = m_{piston} + m_{wp} + \sum m_{ring} + m_{crp}$$

$$m_{crp} = 0.3 * mb$$

$$m_{crp} = 0.3 * 0.402 = 0.120 kg$$

$$m_j = m_{piston} + m_{wp} + \sum m_{ring} + m_{crp}$$

$$m_j = 0.393 + 0.120 = 0.514 kg$$



m_R

$$m_{crc} = 0.7 * mb$$

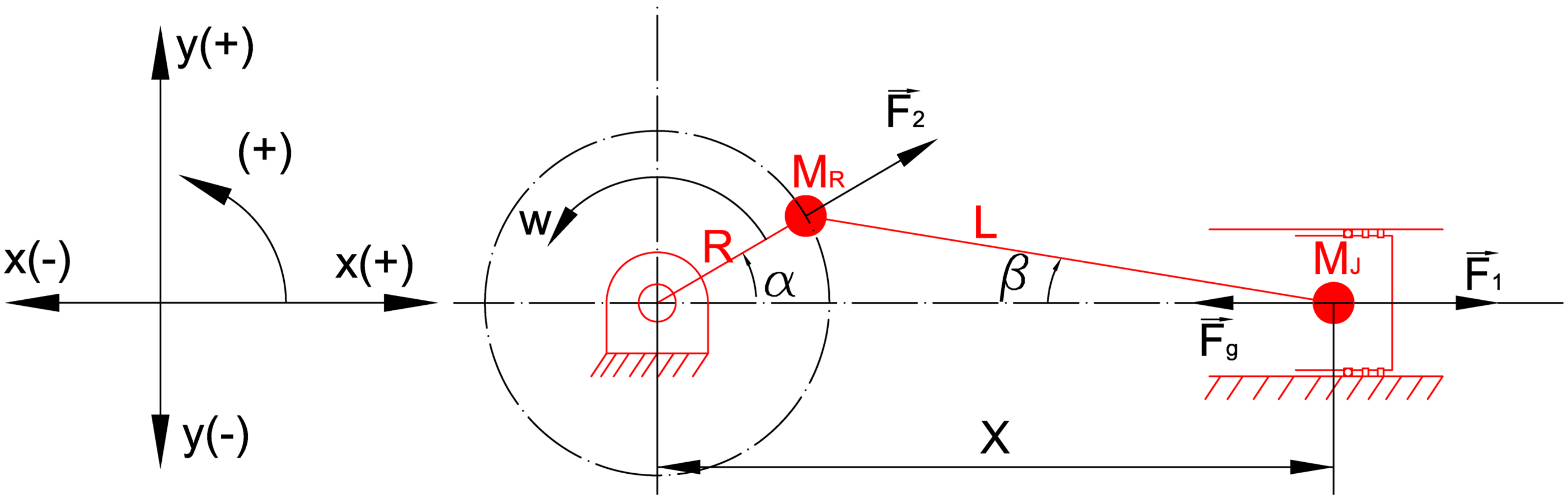
$$m_{crc} = 0.7 * 0.402 = 0.281 kg$$

$$m_R = m_{cp} + 2 m_w \frac{\rho}{R} + m_{crc}$$

$$m_R = 0.2256 + 2 * 400 * \frac{20/1000}{40.25/1000} + 0.281$$

$$m_R = 0.90458 kg$$

Coordinate system and forces



F_g

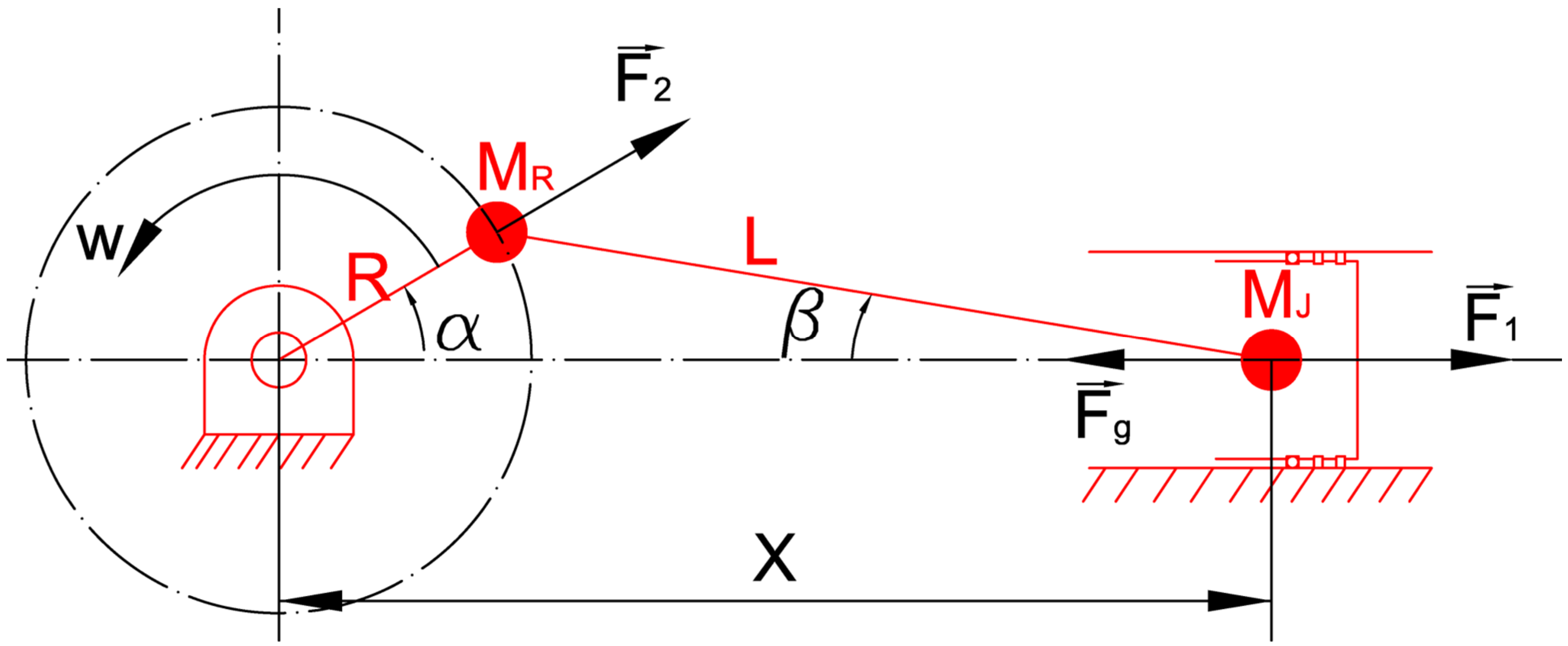
$$P = \frac{F}{A}$$

$$F_g = P(\alpha).\mathrm{A}$$

$$F_g = P(\alpha).\left(\frac{\pi D^2}{4}\right)$$

$$F_g = 1.10^5.\left(\frac{\pi\left(79.5/1000\right)^2}{4}\right)$$

$$\overrightarrow{F_g} = 496\measuredangle 180°\,N$$



w, λ

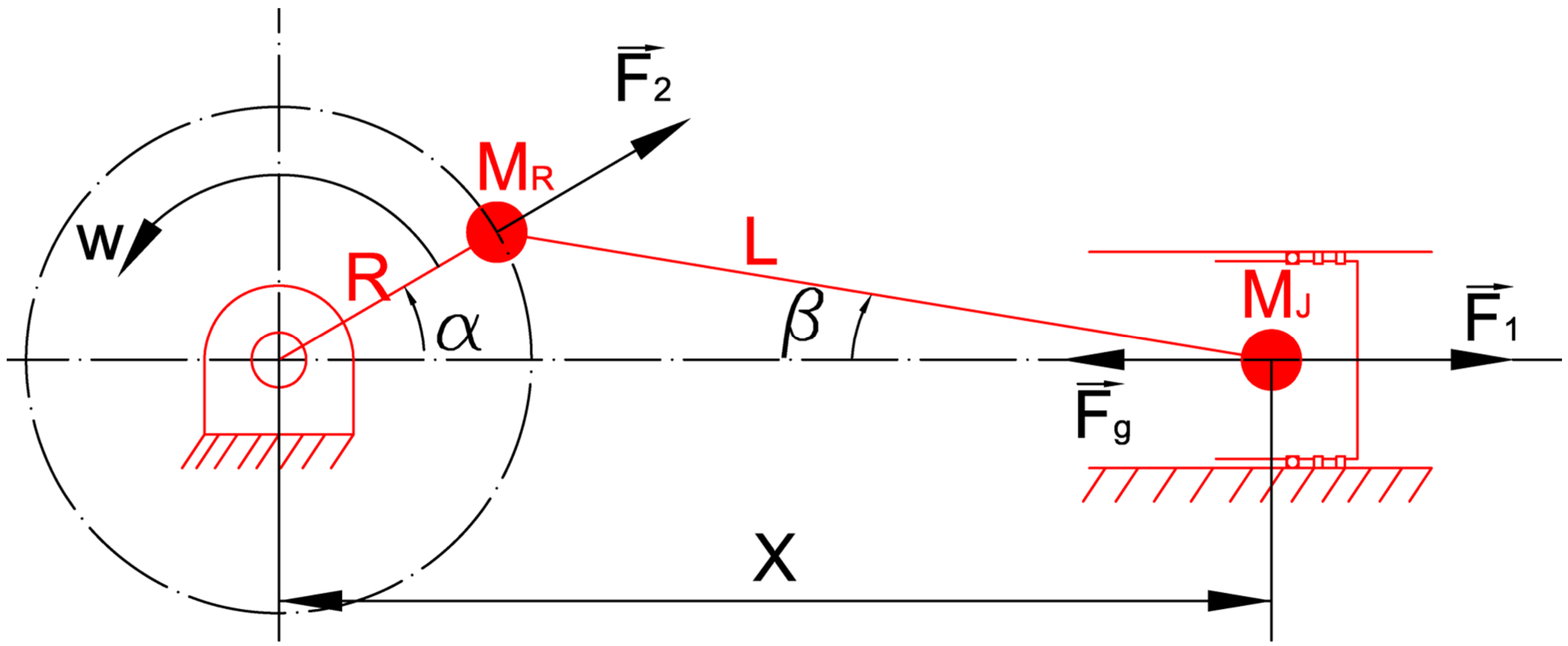
$$w = 6500 * 2\pi / 60$$

$$w = 680.33\, rad / s$$

$$\lambda = \frac{R}{L}$$

$$0.32 = \frac{40.25 / 1000}{L}$$

$$L = 125.78 / 1000\, m$$

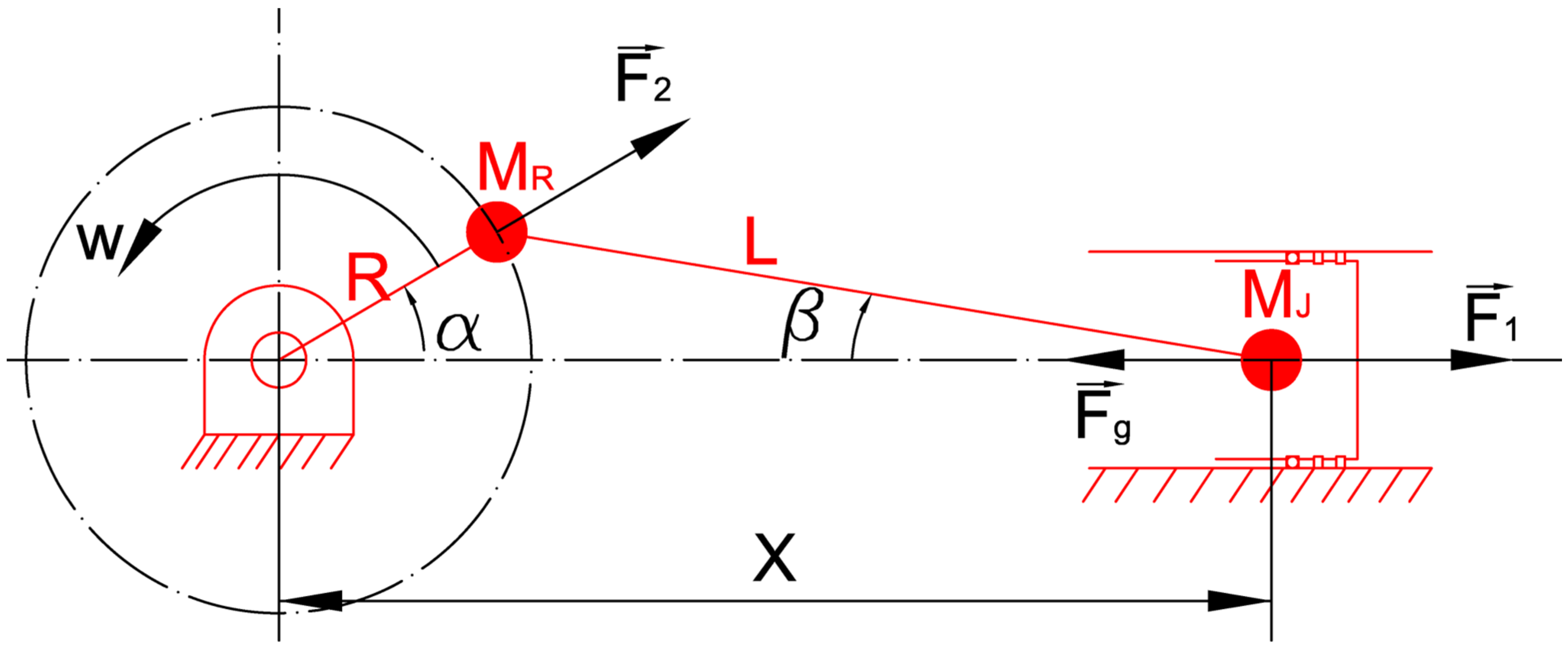


F_1

$$\vec{F_1} = -m_j - w^2 R\left[\cos\alpha + \lambda\cos 2\alpha\right]\measuredangle 0^\circ$$

$$\vec{F_1} = 0.514.(680.33)^2.\frac{40.25}{1000}.\left[\cos 30 + \lambda\cos 60\right]\measuredangle 0^\circ$$

$$\vec{F_1} = 9824.85\measuredangle 0^\circ\ N$$

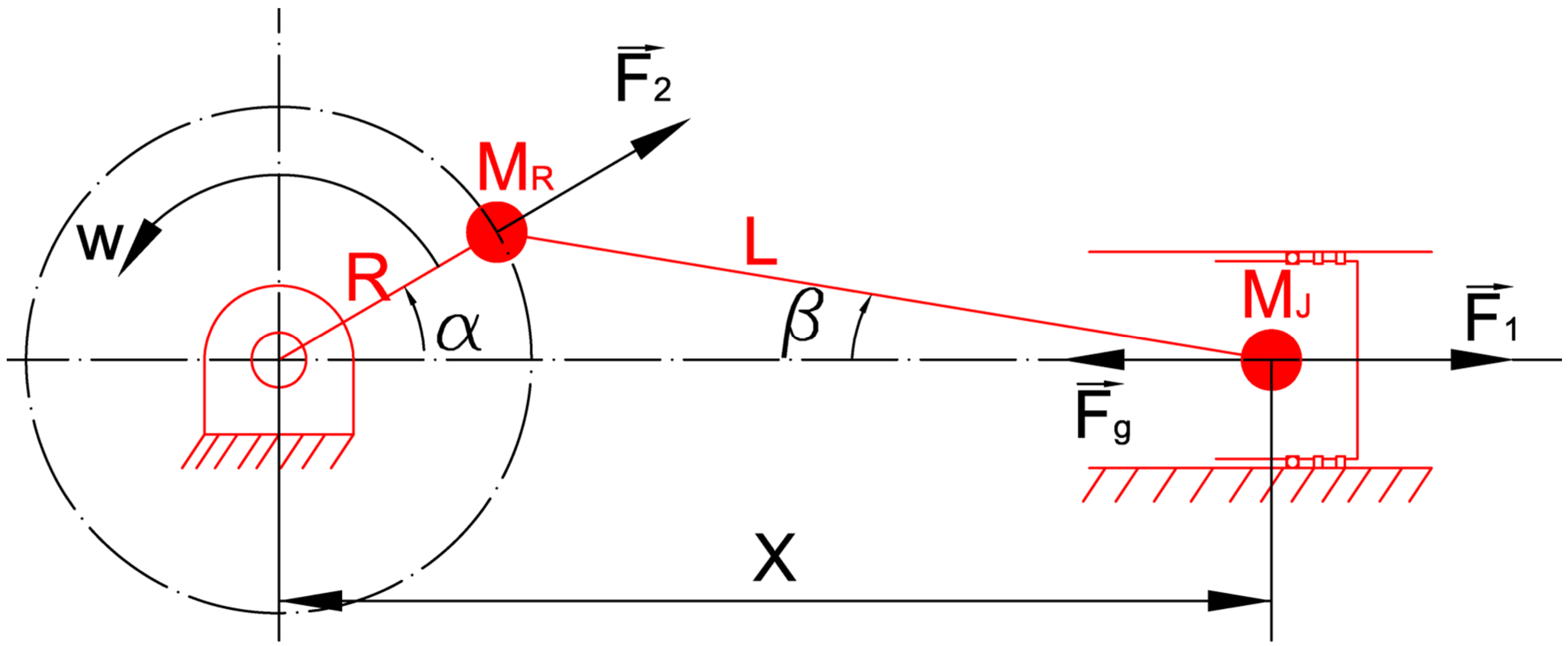


F_2

$$\overrightarrow{F_2} = m_R.R.w^2 \measuredangle \alpha$$

$$\overrightarrow{F_2} = (0.904).\frac{40.25}{1000}.(680.33)^2 \measuredangle \alpha$$

$$\overrightarrow{F_2} = 16852.02 \measuredangle \alpha \; N$$



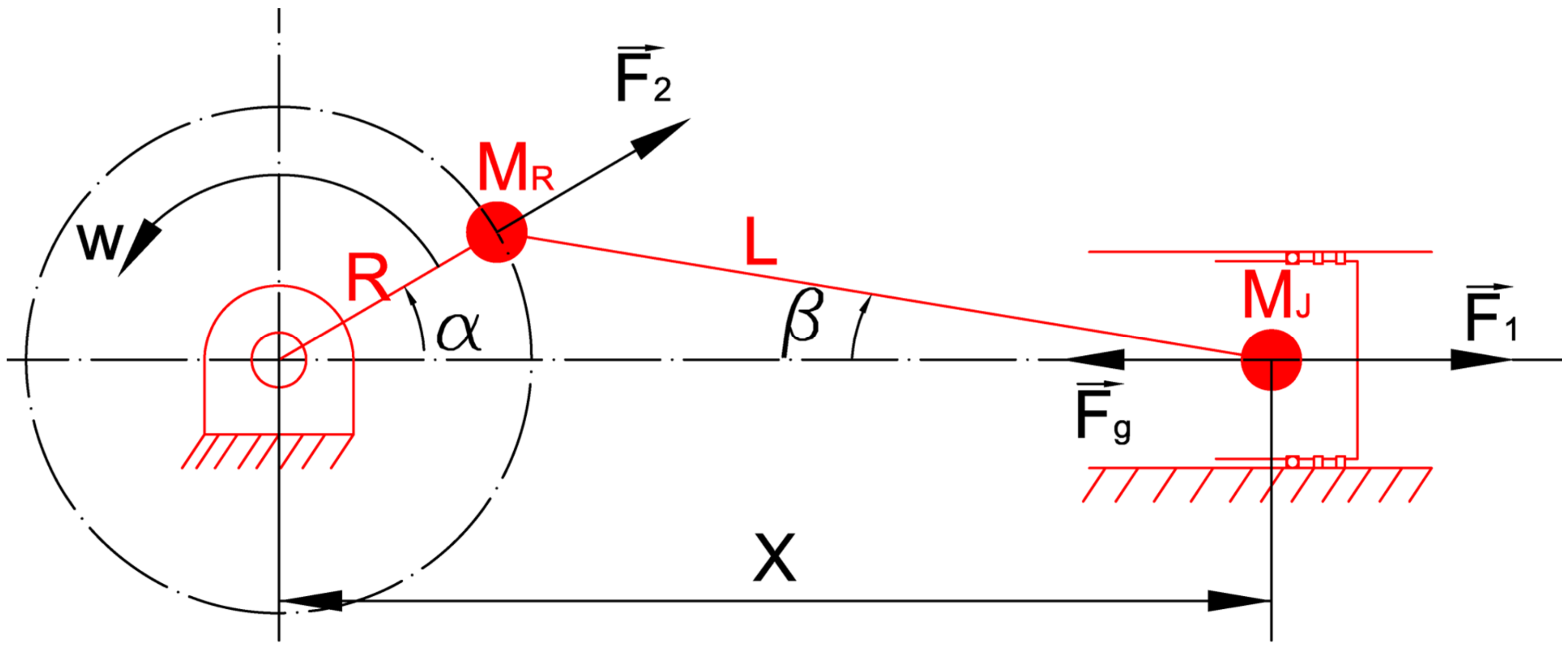
Total force acting on main bearings w/o balance

$$\overrightarrow{F_g} = 496\measuredangle 180°\,N$$

$$\overrightarrow{F_1} = 9824.85\measuredangle 0°\,N$$

$$\overrightarrow{F_2} = 16852.02\measuredangle \alpha\,N$$

$$\sum \vec{F} = \vec{F}_g + \vec{F}_1 + \vec{F}_2$$



Total force acting on main bearings w/o balance

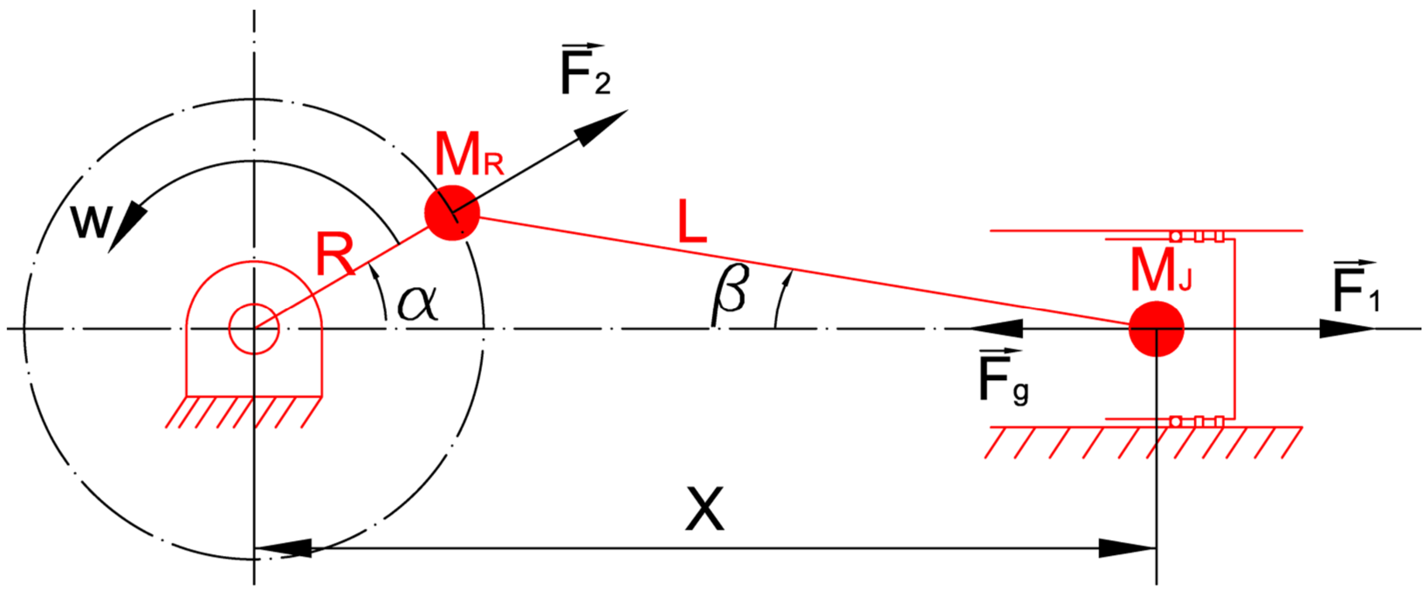
$$\sum \vec{F} = 496\measuredangle 180° + 9824.85\measuredangle 0° + 16852.02\measuredangle \alpha$$

$$\sum \vec{F} = 25363\measuredangle 19.4°\ N$$

Total resultant force is calculated, but each bearing carries the load equally hence the force acting on each bearing is obtained then,

$$\vec{F} = 25363\measuredangle 19.4° / 2$$

$$\vec{F} = 12681\measuredangle 19.4°\ N$$



$$X = R\cos\alpha + L\cos\beta$$

$$\ddot{X} = -w^2 R[cos\alpha + \lambda cos2\alpha]$$

$$\ddot{X} = -\left(w^2\right) R\left[\cos\alpha + \lambda\cos 2\alpha\right]$$

$$\ddot{X} = -\left(680.33^2\right)\left(\frac{40.25}{1000}\right)\left[\cos 30 + 0.32\cos 60\right]$$

$$\ddot{X} = -19114,03\, m/s^2$$

F3

$$\overrightarrow{F_3} = \left|F_1 / \cos\beta\right| \measuredangle (360-\beta)^\circ \left\{\ddot{X} < 0 \;\&\; \beta > 0\right\}$$

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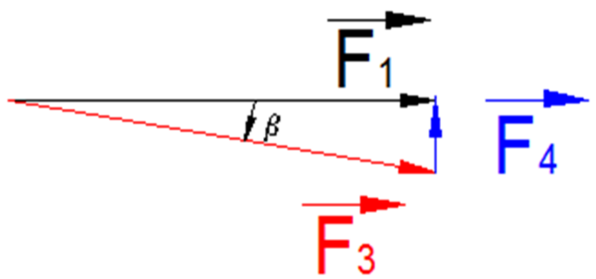
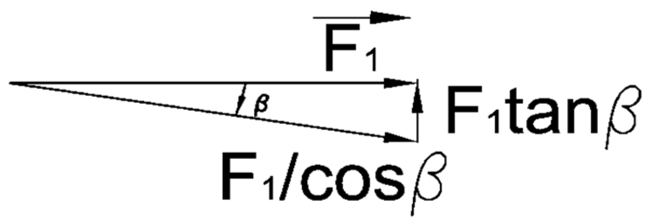
$$\overrightarrow{F_3} = \left|F_1 / \cos\beta\right| \measuredangle (180-\beta)^\circ \left\{\ddot{X} > 0 \;\&\; \beta > 0\right\}$$

$$\overrightarrow{F_3} = \left|F_1 / \cos\beta\right| \measuredangle (180-\beta)^\circ \left\{\ddot{X} > 0 \;\&\; \beta < 0\right\}$$



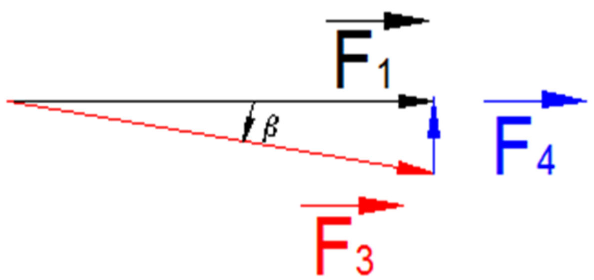


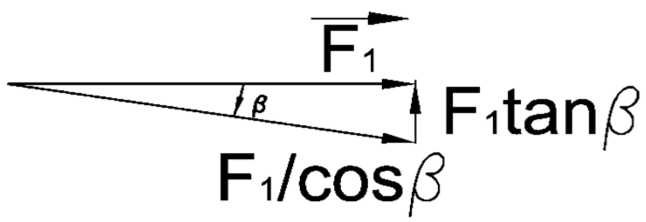
F3

$$\overrightarrow{F_3} = \left|F_1 / \cos\beta\right| \measuredangle (360-\beta)^\circ \left\{\ddot{X} < 0 \;\&\; \beta > 0\right\}$$

$$\overrightarrow{F_3} = \left|9824.85 / \cos 9.2\right| \measuredangle (360-\beta)^\circ$$

$$\overrightarrow{F_3} = 9953.24 \measuredangle (350.8)^\circ \text{ N}$$



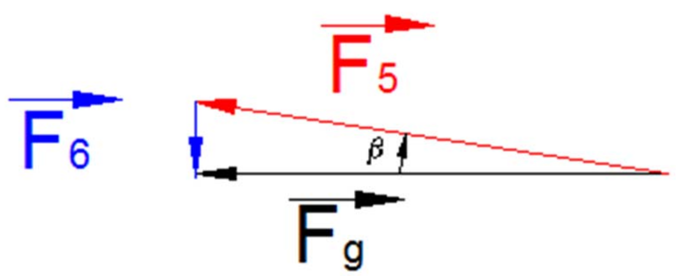


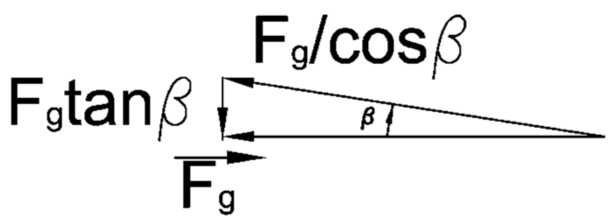
F5

$$\overrightarrow{F_5} = \left| F_g / \cos\beta \right| \measuredangle (180 - \beta)^\circ$$

$$\overrightarrow{F_5} = \left| 496 / \cos 9.2 \right| \measuredangle (180 - 9.2)^\circ$$

$$\overrightarrow{F_5} = 502.48 \measuredangle (170.8)^\circ \text{ N}$$



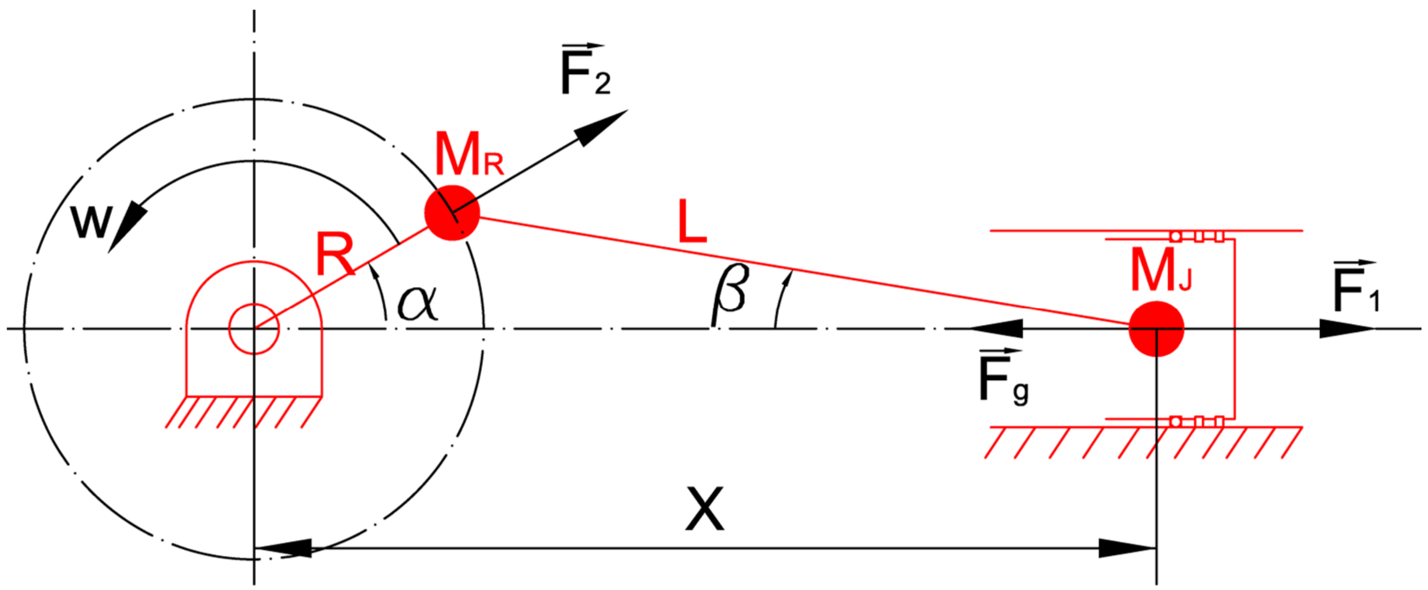


Total force acting on conrod bearing w/o balance

$$\sum \overrightarrow{F} = \overrightarrow{F_R} = \overrightarrow{F_2} \measuredangle \alpha° + \overrightarrow{F_5} \measuredangle (180 - \beta) + \overrightarrow{F_3} \measuredangle (360 - \beta)$$

$$\sum \overrightarrow{F} = \overrightarrow{F_R} = 16852.02 \measuredangle 30° + 502.48 \measuredangle (170.8) + 9953.24 \measuredangle (350.8)$$

$$\sum \overrightarrow{F} = \overrightarrow{F_R} = 24902.73 \measuredangle 16.12°\ N$$



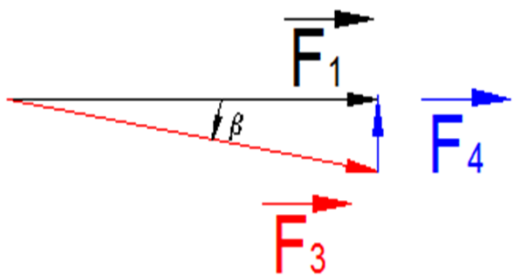
F4

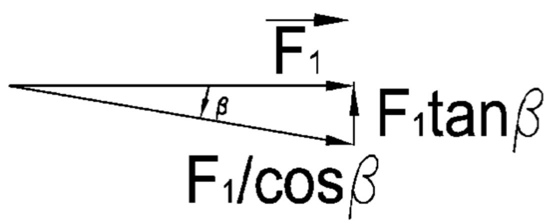
$$\boxed{\overrightarrow{F_4} = \left|F_1 \tan\beta\right| \measuredangle 90^\circ \left\{\ddot{X} < 0 \;\&\; \beta > 0\right\}}$$

$$\overrightarrow{F_4} = \left|F_1 \tan\beta\right| \measuredangle 270^\circ \left\{\ddot{X} < 0 \;\&\; \beta < 0\right\}$$

$$\overrightarrow{F_4} = \left|F_1 \tan\beta\right| \measuredangle 270^\circ \left\{\ddot{X} > 0 \;\&\; \beta > 0\right\}$$

$$\overrightarrow{F_4} = \left|F_1 \tan\beta\right| \measuredangle 90^\circ \left\{\ddot{X} > 0 \;\&\; \beta < 0\right\}$$

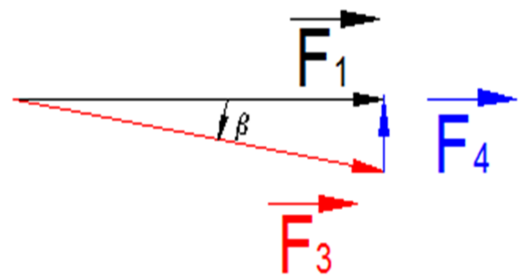


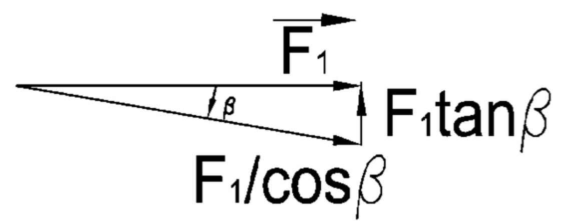


F4

$$\overrightarrow{F_4} = \left|9824{,}85 \tan 9.2\right| \measuredangle 90°$$

$$\overrightarrow{F_4} = 1591.27 \measuredangle 90°\ \text{N}$$

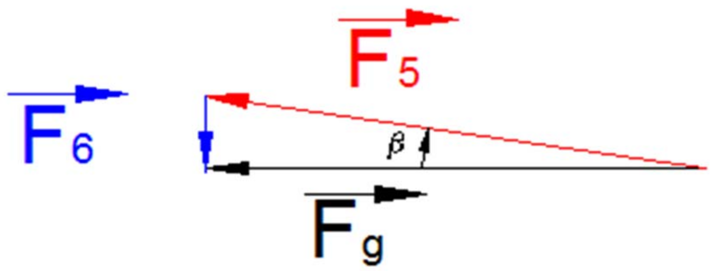


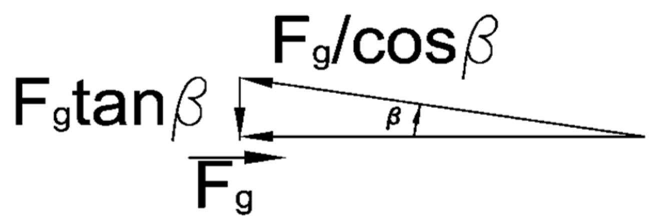


F6

$$\overrightarrow{F_6} = \left| F_g \tan\beta \right| \measuredangle 270^\circ \{\beta > 0\}$$

$$\overrightarrow{F_6} = \left| F_g \tan\beta \right| \measuredangle 90^\circ \{\beta < 0\}$$

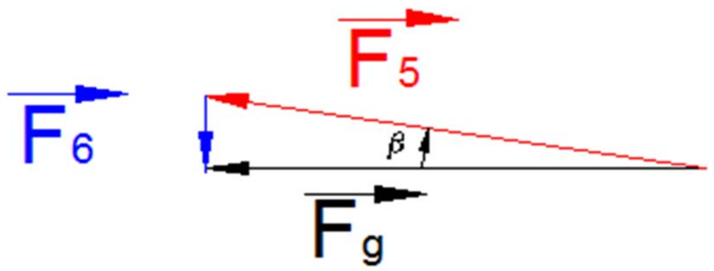


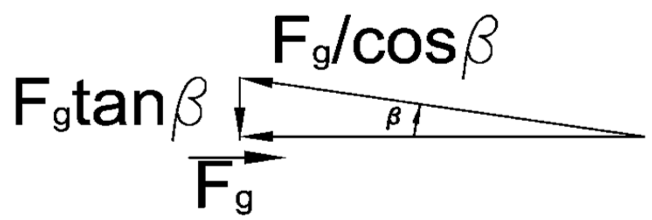


F6

$$\overrightarrow{F_6} = |496 \tan 9.2| \measuredangle 270°$$

$$\overrightarrow{F_6} = 80.33 \measuredangle 270°\ N$$





Moment created by reciprocating movement

$$X = R\cos\alpha + L\cos\beta$$

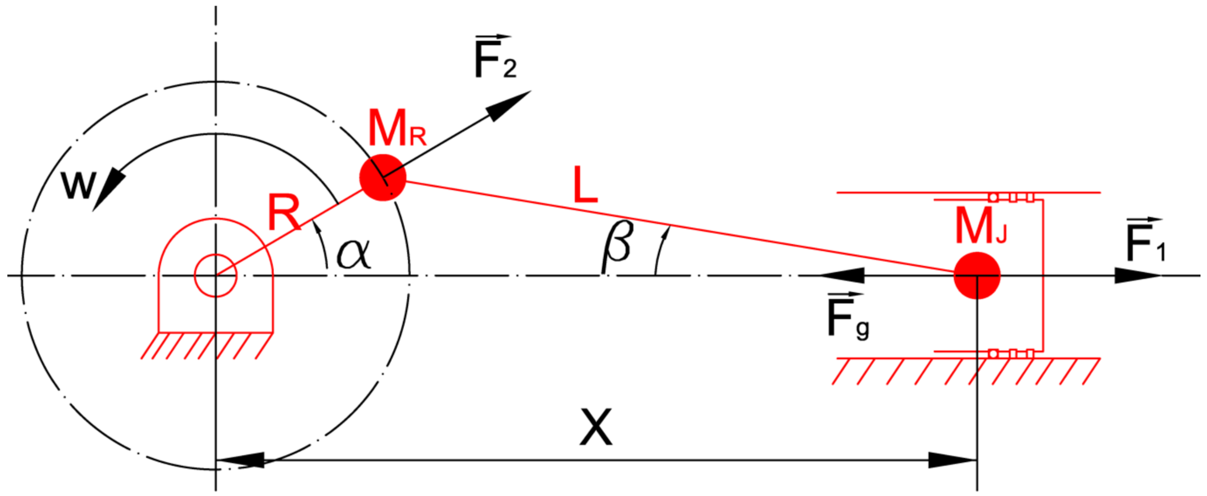
$$X = \frac{40.25}{1000}.\cos(30) + \frac{125.78}{1000}\cos(9.2)$$

$$X = 0.1590\,m$$

$$\sum M = X.\left[\vec{F_4} + \vec{F_6}\right]$$

$$\sum M = 0.1590.\left[1591.27\measuredangle 90 + 80.33\measuredangle 270\right]$$

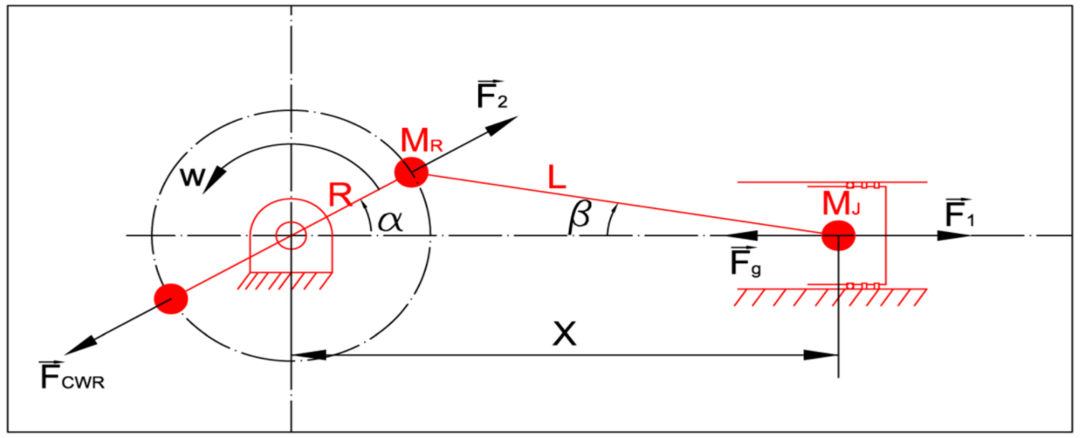
$$\sum M = 240.23\,Nm$$



FcwR

$$F_{cwR} = m_{cwR}.R.\omega^{2}$$

$$\overrightarrow{F_2} = \overrightarrow{F_{CWR}}$$



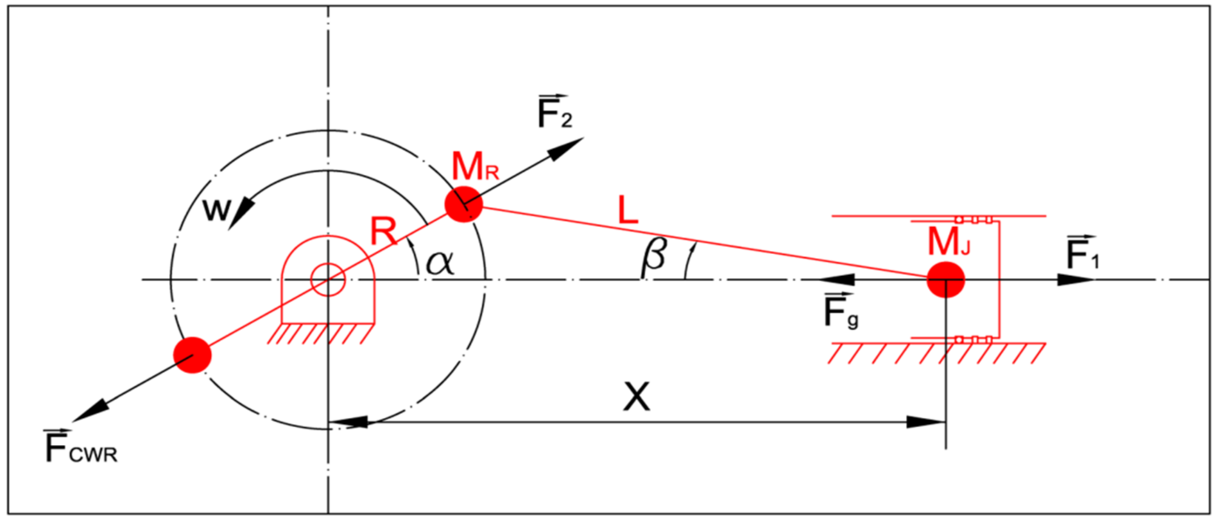
$$\overrightarrow{F_{cwR}} = \left|F_{cwR}\right| \measuredangle \alpha + 180°$$

FcwR

$$F_{cwR} = m_{cwR}.R.\omega^{2}$$

$$\overrightarrow{F_2} = \overrightarrow{F_{CWR}}$$

$$m_{cwR} = m_R$$



$$\overrightarrow{F_{cwR}} = \left|F_{cwR}\right| \measuredangle \alpha + 180°$$

Balancing centrifugal force created by MR

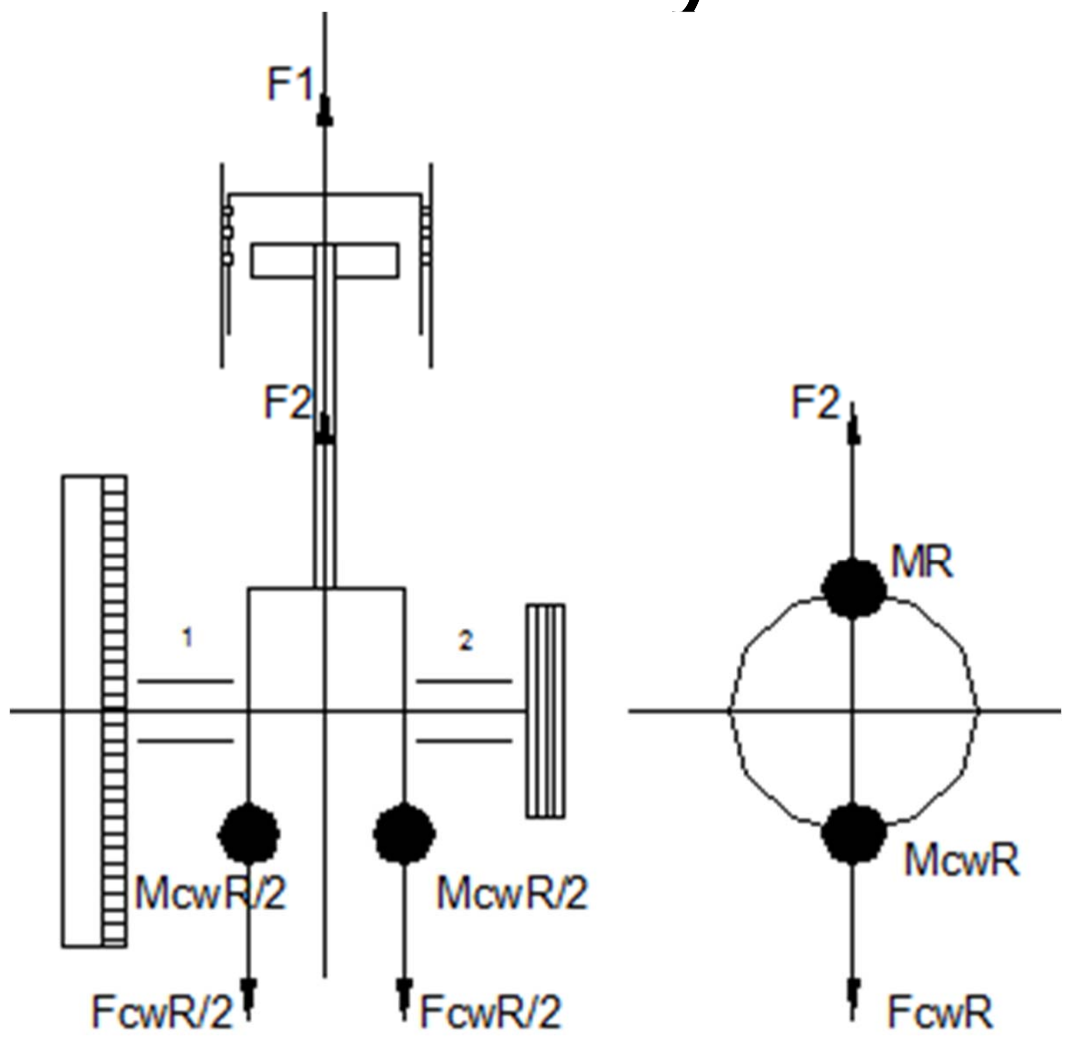
$$F_2 = m_R.R.\omega^2$$

$$F_{cwR} = m_{cwR}.\rho_{cw}.\omega^2$$

$$\boxed{\rho_{cw} = R}$$

May not be equal

$$m_{cwR} = m_R$$



First and second order inertia forces

$$\ddot{X} = -w^2 R[cos\alpha + \lambda cos2\alpha]$$

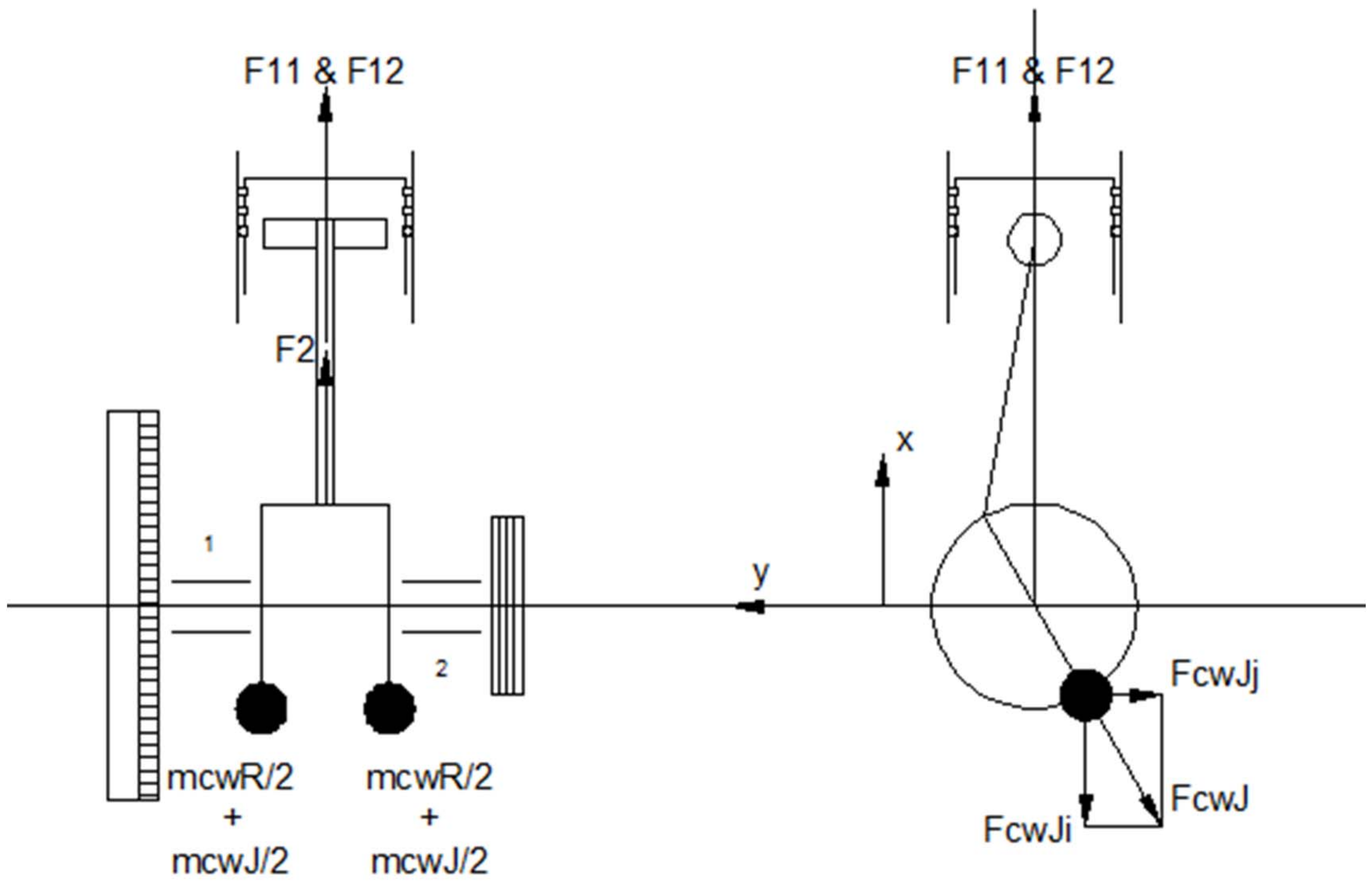
$$\overrightarrow{F_1} = -m_j \dot{X}$$

$$\overrightarrow{F_1} = -m_j(-w^2 R[cos\alpha + \lambda cos2\alpha]\,)$$

$$\overrightarrow{F_{11}} = -m_j(-w^2 Rcos\alpha\,)$$

$$\overrightarrow{F_{12}} = -m_j(-w^2 R\lambda cos2\alpha\,)$$

Balancing first order inertia force



Balancing first order inertia force

$$\overrightarrow{F}_{11} = m_j.R.\omega^2.\cos\alpha$$

$$\overrightarrow{F}_{cwJ} = m_{cwj}\rho\omega^2$$

$$\rho = R$$

$$\overrightarrow{F}_{cwJ} = m_{cwj}R\omega^2$$

$$\overrightarrow{F}_{cwJi} = m_{cwj}R\omega^2\cos\alpha$$

$$\overrightarrow{F_{11}} = \overrightarrow{F}_{cwJi}$$

$$m_jR\omega^2\cos\alpha = m_{cwj}R\omega^2\cos\alpha$$

$$m_j = m_{cwj}$$

Balancing first order inertia force

For a single cylinder engine, mcwj can be chosen as %50 percent of the calculated value which is called as partial balancing. In case of multi-cylinder engines, balancing the first order inertia forces have to be done by considering each individual cylinder hence partial balancing can not be applied.

$$m_{cwj} = 0.5 * m_{cwj}$$