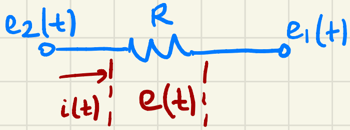
# GMI 3181
# OTOMATİK KONTROL

## 7. HAFTA

ELEKTRİKSEL ELEMANLARIN

BLOK DİYAGRAMLARI İLE GÖST.

## DİRENÇ




$$e(t) = e_2(t) - e_1(t) = R \cdot i(t)$$

$$\mathcal{L}$$

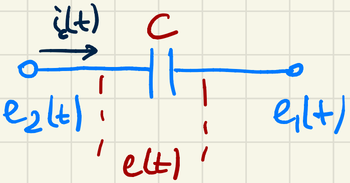
$$E(s) = R \cdot I(s)$$

$$\left.\frac{I(s)}{E(s)} = \frac{1}{R}\right\} \quad E(s) \longrightarrow \boxed{\frac{1}{R}} \longrightarrow I(s)$$

$$E(s) = R \cdot I(s)$$

$$\left.\frac{E(s)}{I(s)} = R\right\} \quad I(s) \longrightarrow \boxed{R} \longrightarrow E(s)$$

## KONDANSATÖR




$$e(0) = 0$$

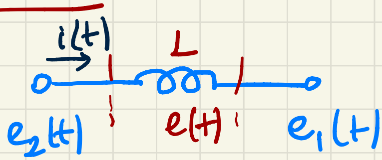
$$i_c(t) = C \cdot \frac{de(t)}{dt}$$

$$\mathcal{L}$$

$$I_c(s) = Cs\,E(s)$$

$$\left.\frac{E(s)}{I_c(s)} = \frac{1}{Cs}\right\} \quad I_c(s) \longrightarrow \boxed{\frac{1}{Cs}} \longrightarrow E(s)$$

## BOBİN



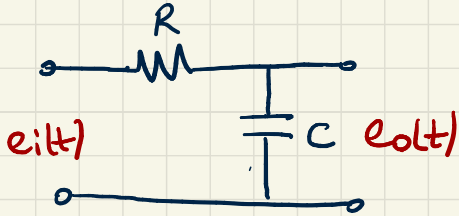

$$e(t) = L \cdot \frac{di(t)}{dt} \qquad i(0) = 0$$

$$\mathcal{L}$$

$$E(s) = Ls\,I(s)$$

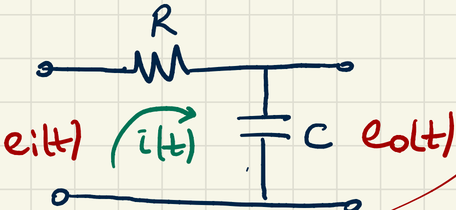
$$\left.\frac{E(s)}{I(s)} = Ls\right\} \quad I(s) \longrightarrow \boxed{Ls} \longrightarrow E(s)$$

Ör:




$\frac{E_o(s)}{E_i(s)}$ transfer fonk. bulunuz.

(Blok diyagramı ile)

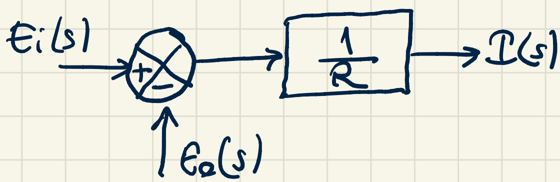



★ $i(t)=\dfrac{e_i(t)-e_o(t)}{R}$

★ $i(t)=C\cdot\dfrac{de_o(t)}{dt}$

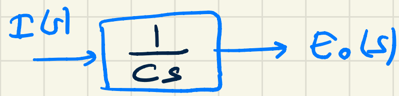
$\mathcal{L}$

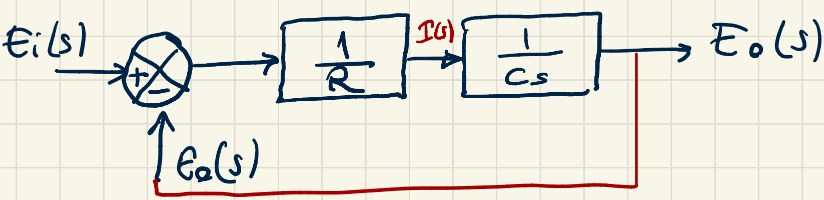
$$I(s)=\frac{E_i(s)-E_o(s)}{R}$$




$$i(t)=C\cdot\frac{de_o(t)}{dt}$$

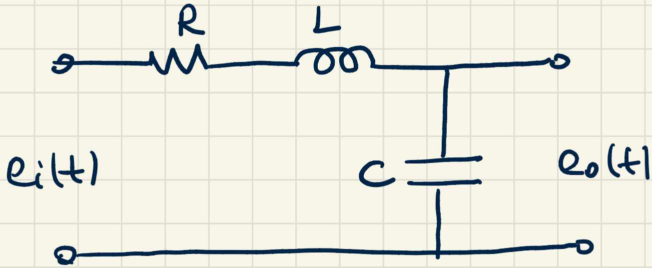
$$I(s)=Cs\,E_o(s)$$



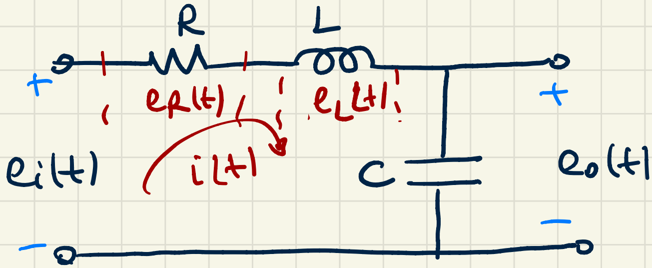




$$\frac{E_o(s)}{E_i(s)}=\frac{\dfrac{1}{RCs}}{1+\dfrac{1}{RCs}}=\frac{1}{1+RCs}$$

**Ör:**



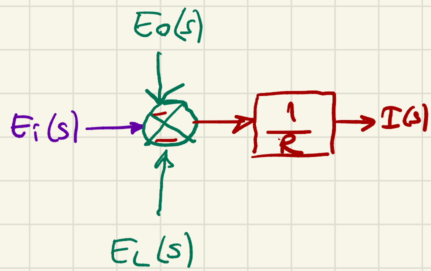

$$\frac{E_o(s)}{E_i(s)} = ?$$




① $-e_i(t) + e_R(t) + e_L(t) + e_o(t) = 0$

$\mathcal{L}$

$E_i(s) = E_R(s) + E_L(s) + E_o(s)$




$e_R(t) = R.i(t)$

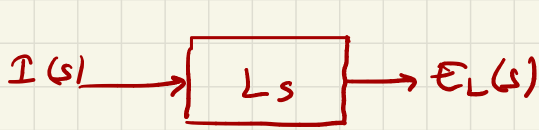
$E_R(s) = R\,I(s)$

$E_i(s) - E_L(s) - E_o(s) = \underbrace{R.I(s)}_{E_R(s)}$

② $e_L(t) = L \cdot \frac{di(t)}{dt}$

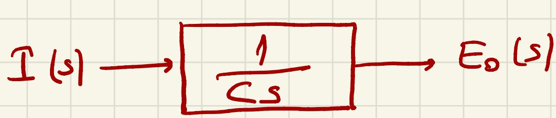
$\mathcal{L}$

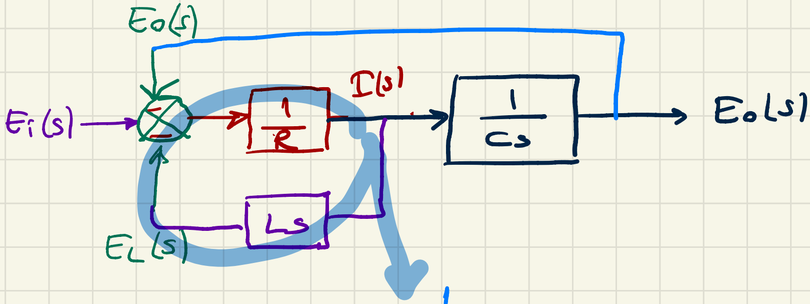
$E_L(s) = Ls\,I(s)$



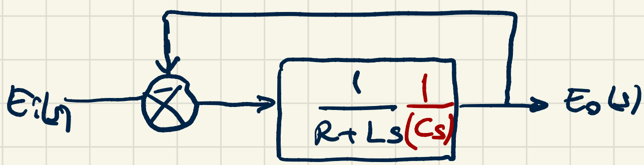

③ $i_c(t) = i(t) = C\,\frac{de_o(t)}{dt}$

$I(s) = Cs\,E_o(s)$





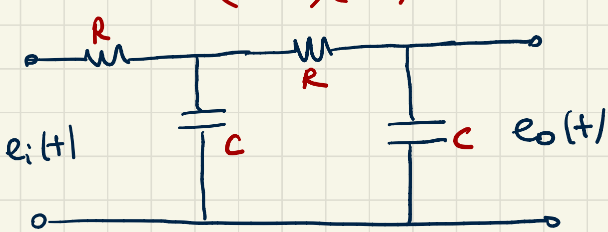

$$\frac{\frac{1}{R}}{1+\frac{1}{R}Ls} = \frac{1}{R+Ls}$$



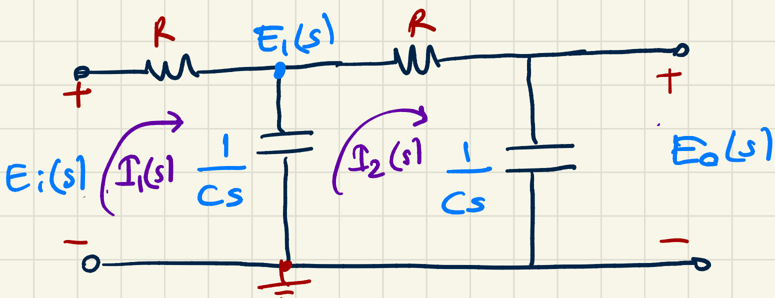

$$\frac{\frac{1}{(R+Ls)(Cs)}}{1+\frac{1}{(R+Ls)(Cs)}} = T_i$$

$$T(s) = \frac{1}{(R+Ls)(Cs)+1} = \frac{1}{LCs^2+RCs+1}$$

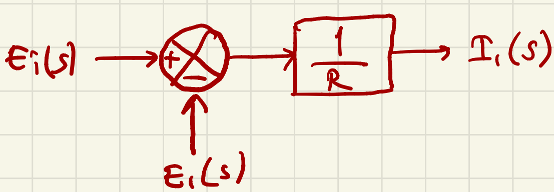
Ör:



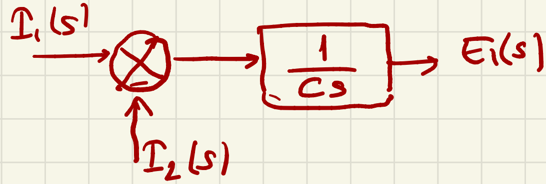

$$\frac{E_o(s)}{E_i(s)} = ?$$



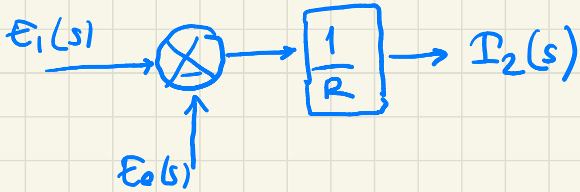

$$I_1(s) = \frac{E_i(s)-E_1(s)}{R}$$



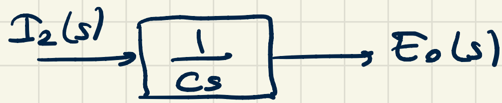
$$E_1(s) = \frac{1}{Cs}\left(I_1(s) - I_2(s)\right)$$

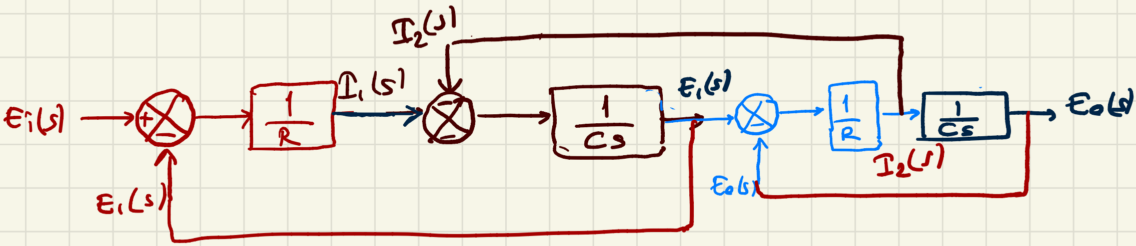


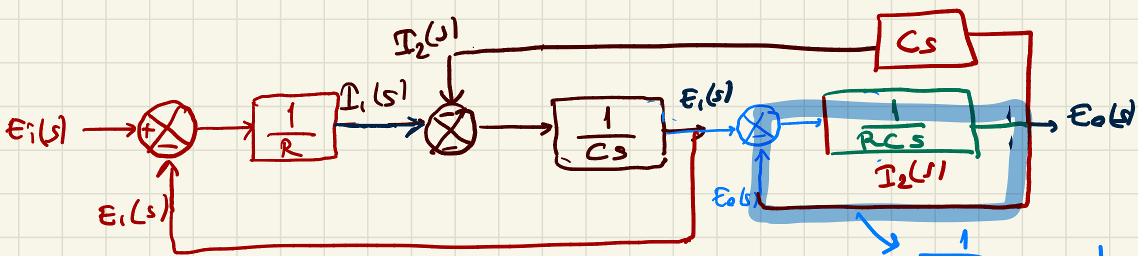

$$I_2(s) = \frac{E_1(s) - E_0(s)}{R}$$




$$E_0(s) = I_2(s) \cdot \frac{1}{Cs}$$

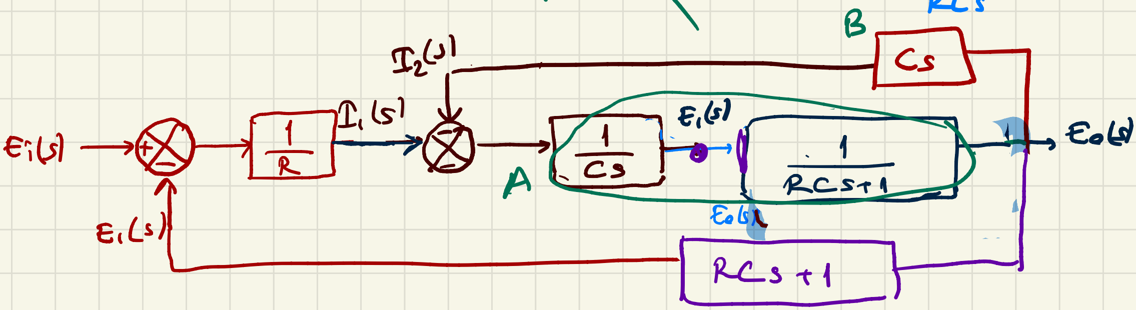



$$\frac{\frac{1}{RCs}}{1 + \frac{1}{RCs}} = \frac{1}{RCs+1}$$

$$\frac{A}{1+AB}$$



# DURUM DENKLEMLERİ (STATE EQUATIONS)

Temel Fizik Kuralları → R, L, C

Temel Fizik Kuralları → m, k, b

↓

Diferansiyel Denklem

Transfer Fonksiyonu $\left(\frac{\text{Çıkış}}{\text{Giriş}}\right)$

Tek Giriş Tek Çıkış

Durum Değişkenleri metodu

Tek Giriş Tek Çıkış
+
Çok Giriş Çok Çıkış

## Durum Vektörü

Vektör → $\begin{bmatrix} v \\ i \\ x \end{bmatrix}$

$$\boxed{\dot{x} = Ax + Bu}$$

$x$ → Durum Vektörü (State Vector)

$$\boxed{y = Cx + Du}$$

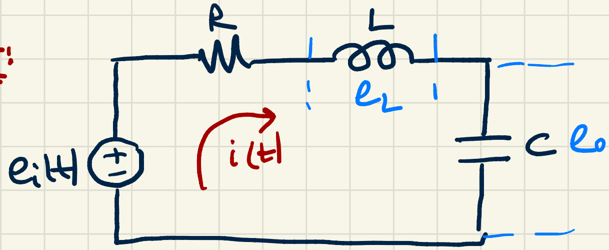
$y$ → Çıkış Vektörü

A, B, C ve D matrisler

$u$ → Giriş

$D$ → Sabit

**Ör:**




$$e_L = L\cdot\frac{di(t)}{dt}$$

$$i = C\cdot\frac{de_o(t)}{dt}$$

$$e_R = R\cdot i(t)$$

Durum değişkeni

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

Çıkış

$$x = \left.\begin{bmatrix} i(t) \\ e_o(t) \end{bmatrix}\right\} \text{Durum Değişkenleri}$$

$$v = L\cdot\frac{di}{dt}$$

$$\frac{d\phi}{dt} = L\cdot\frac{di}{dt}$$

$$\boxed{\phi = L\cdot i}$$

$$\boxed{Q = C\cdot v} \quad \frac{dq}{dt} = C\cdot\frac{dv}{dt}$$

$$i = C\cdot\frac{dv}{dt}$$

$$\frac{di(t)}{dt} = ? \qquad \frac{de_o(t)}{dt} = ?$$

$$-e_i(t) + R\cdot i(t) + e_L(t) + e_o(t) = 0$$

$$-e_i(t) + R\cdot i(t) + L\frac{di(t)}{dt} + e_o(t) = 0$$

$$\boxed{\cancel{L}\frac{di(t)}{dt} = -\frac{R}{L}\cdot i(t) - \frac{e_o(t)}{L} + \frac{e_i(t)}{L}}$$

$$i(t) = C\cdot\frac{de_o(t)}{dt} \longrightarrow \boxed{\frac{de_o(t)}{dt} = \frac{1}{C}\,i(t)}$$

$$\begin{bmatrix}\dot{i}(t)\\ \dot{e}_o(t)\end{bmatrix} = A\begin{bmatrix} i(t)\\ e_o(t)\end{bmatrix} + B\, \underbrace{e_i(t)}_{\text{giriş}}$$

$$\boxed{\cancel{L}\,\frac{di(t)}{dt} = -\frac{R}{L}.\, i(t) - \frac{e_o(t)}{L} + \frac{e_i(t)}{L}}$$

$$\boxed{\frac{de_o(t)}{dt} = \frac{1}{C}\, i(t)}$$

$$\begin{bmatrix}\dot{i}(t)\\ \dot{e}_o(t)\end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L}\\ \frac{1}{C} & 0\end{bmatrix}\begin{bmatrix} i(t)\\ e_o(t)\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix} e_i(t)$$

$e_o(t)$ çıkış ise çıkış denklemini yazınız.

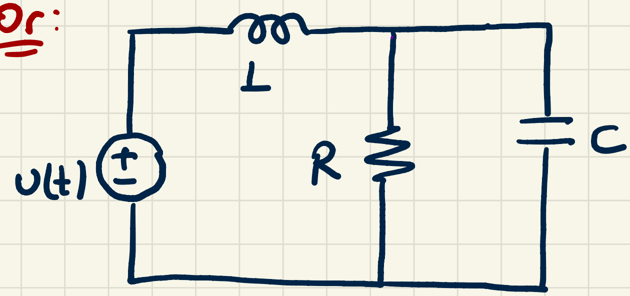
$$e_o(t) = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix} i(t)\\ e_o(t)\end{bmatrix} + \begin{bmatrix}0\end{bmatrix} e_i(t) \qquad e_R(t) = R.\, i(t)$$

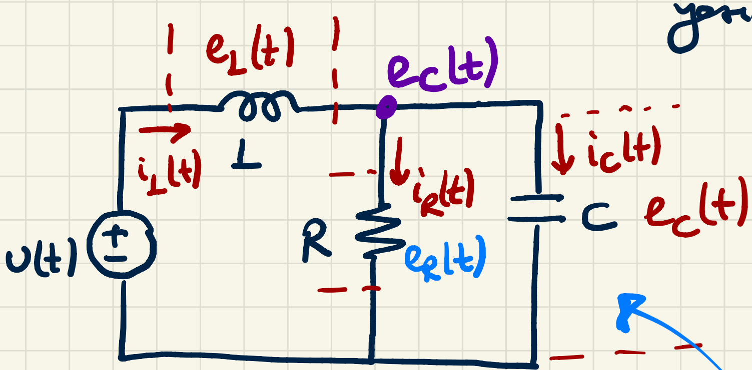
çıkış $e_R(t)$ ve $e_o(t)$ olsaydı çıkış denklemini yaz

$$\begin{bmatrix}e_R(t)\\ e_o(t)\end{bmatrix} = \begin{bmatrix} R & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix} i(t)\\ e_o(t)\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix} e_i(t)$$

**Ör:**

Durum denklemlerini çıkarınız. Çıkış R direnci üzerinden geçen akım ise çıkış denklemini yazınız.




$$e_L = L \cdot \frac{di_L(t)}{dt}$$

$$i_C = C \cdot \frac{de_C(t)}{dt}$$

$$\text{Durum Değişkenleri} = \begin{bmatrix} i_L(t) \\ e_C(t) \end{bmatrix}$$

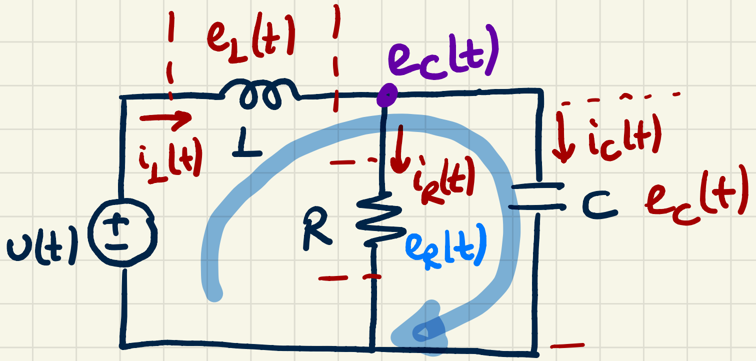
$$e_R(t) = e_C(t)$$

$$i_L(t) - i_R(t) - i_C(t) = 0$$

$$i_L(t) - \frac{e_R(t)}{R} - C\frac{de_C(t)}{dt} = 0$$

$$C\frac{de_C(t)}{dt} = i_L(t) - \frac{e_C(t)}{R}$$

(1) $$\boxed{\frac{de_C(t)}{dt} = \frac{1}{C} i_L(t) - \frac{1}{RC} e_C(t)}$$




Durum değişkeni

$\dot{x} = Ax + Bu$ giriş

$y = Cx + Du$

çıkış   Durum değişkeni   giriş

$$-u(t) + e_L(t) + e_c(t) = 0$$

$$-u(t) + L\cdot\frac{di_L(t)}{dt} + e_c(t) = 0$$

(2) $$\boxed{\frac{di_L(t)}{dt} = -\frac{1}{L}e_c(t) + \frac{1}{L}u(t)}$$

(1) $$\boxed{\frac{de_c(t)}{dt} = \frac{1}{C}i_L(t) - \frac{1}{RC}e_c(t)}$$

Durum denklemi (1) ve (2) ye göre:

$$\begin{bmatrix} \dot{i}_L(t) \\ \dot{e}_c(t) \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L(t) \\ e_c(t) \end{bmatrix}' + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t)$$

Çıkış için: $i_R(t) \longrightarrow i_R(t) = \frac{e_c(t)}{R}$

$$i_R(t) = \begin{bmatrix} 0 & \frac{1}{R} \end{bmatrix} \begin{bmatrix} i_L(t) \\ e_c(t) \end{bmatrix}' + \begin{bmatrix} 0 \end{bmatrix} u(t)$$

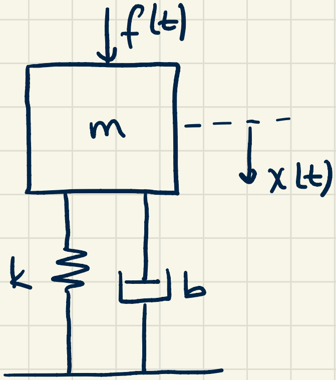
Çıkış $i_R(t)$, $e_L(t)$, $e_c(t)$ olsaydı nasıl yapardık?

$i_R(t) = \frac{e_c(t)}{R}$   $-u(t) + e_L(t) + e_c(t) = 0$

$$e_L(t) = -e_c(t) + u(t)$$

$$\begin{bmatrix} i_R(t) \\ e_L(t) \\ e_c(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{R} & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ e_c(t) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t)$$

# MEKANİK SİSTEMLERİN DURUM DENKLEMİ




Sistemin durum değişkenlerini belirleyip, durum denklemlerini yazınız. Çıkış $f_k + f_b$ ise çıkış denklemini yazınız.

yer değiştirme

$$z = \begin{bmatrix} x(t) \\ v(t) \end{bmatrix}$$

hız.

$$\boxed{\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}}$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = A \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + B\,u(t)$$

$$\boxed{\frac{dx(t)}{dt} = v(t)}$$

$$f(t) - m\ddot{x}(t) - f_k - f_b = 0$$

$$f(t) - m\ddot{x}(t) - kx(t) - b\dot{x}(t) = 0$$

$$f(t) - m\dot{v}(t) - k\,x(t) - b\,v(t) = 0$$

$$\frac{m\dot{v}(t)}{m} = \frac{f(t)}{m} - \frac{k\,x(t)}{m} - \frac{b\,v(t)}{m}$$

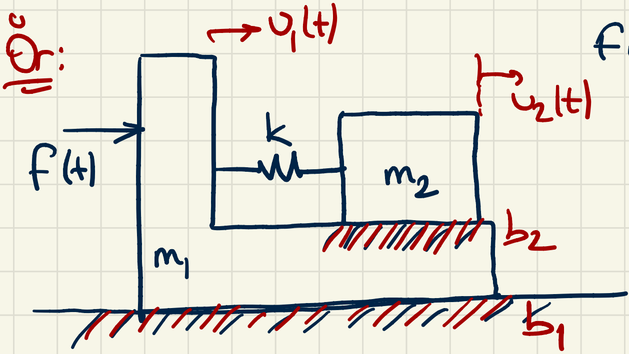
$$\boxed{\dot{v}(t) = \frac{-k}{m}x(t) - \frac{b}{m}v(t) + \frac{1}{m}f(t)}$$

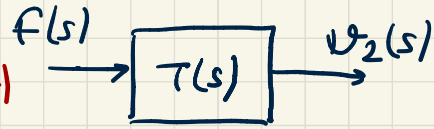
$$\boxed{\frac{dx(t)}{dt} = v(t)}$$

$$\boxed{\dot{v}(t) = \frac{-k}{m}x(t) - \frac{b}{m}v(t) + \frac{1}{m}f(t)}$$

$$\begin{bmatrix} \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} f(t)$$

Ör:




$$f_k = \frac{k}{s}(v_1 - v_2)$$

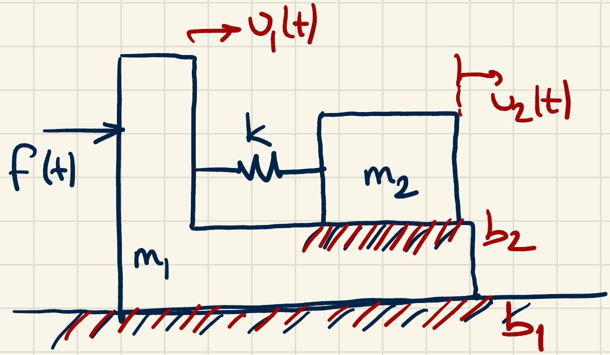
vektörün ismi

$$z = \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix}$$

$$\frac{dx_1}{dt} = v_1$$

$$\frac{dx_2}{dt} = v_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ - & & & \\ 0 & 0 & 0 & 1 \\ & & & \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \cdot \\ 0 \\ \ \end{bmatrix} f(t)$$




$m_1$

$$-m_1\dot{v}_1(t) - b_1 v_1(t) - k\left(x_1(t)-x_2(t)\right) - b_2\left(v_1(t)-v_2(t)\right) + f(t) = 0$$

$$\cancel{m_1}\dot{v}_1(t) = -\frac{k x_1(t)}{m_1}\ \frac{(b_1+b_2)}{m_1} v_1(t) + \frac{k\, x_2(t)}{m_1} + \frac{b_2 v_2(t)}{m_1} + \underline{f(t)}$$

$m_2$

$$m_2\dot{v}_2(t) + k(x_2-x_1) + b_2(v_2-v_1) = 0$$

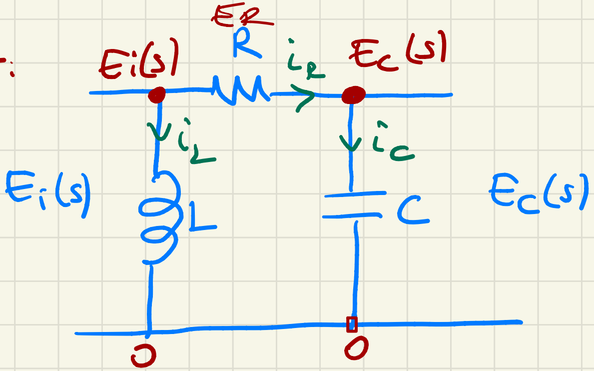
$$\dot{v}_2(t) = \frac{k}{m_2}x_1 + \frac{b_2}{m_2}v_1 - \frac{k}{m_2}x_2 - \frac{b_2}{m_2}v_2$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{v}_1\\ \dot{x}_2\\ \dot{v}_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ \frac{-k}{m_1} & \frac{-(b_1+b_2)}{m_1} & \frac{k}{m_1} & \frac{b_2}{m_1}\\ 0 & 0 & 0 & 1\\ \frac{k}{m_2} & \frac{b_2}{m_2} & \frac{-k}{m_2} & \frac{-b_2}{m_2}\end{bmatrix}\begin{bmatrix}x_1\\ v_1\\ x_2\\ v_2\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\\ 0\end{bmatrix} f(t)$$

Çıkış

$$\begin{bmatrix}v_1\\ v_2\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ v_1\\ x_2\\ v_2\end{bmatrix} + \begin{bmatrix}0\\ 0\end{bmatrix} f(t)$$

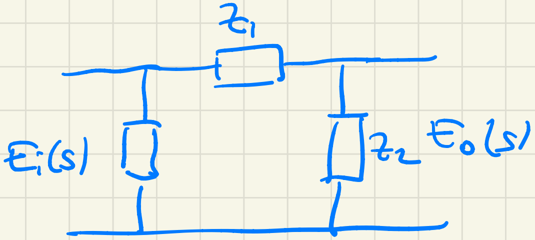
Ör:




$$\frac{E_c(s)}{E_i(s)} = ?$$

$$i_R = i_c$$

$$i_L = -i_R$$




$$\frac{E_i(s) - E_c(s)}{R} = \frac{E_c(s)}{\frac{1}{Cs}}$$

$$\frac{E_i(s)}{R} = E_c(s)\,Cs + \frac{E_c(s)}{R}$$

$$\frac{E_i(s)}{R} = E_c(s)\cdot\left(Cs + \frac{1}{R}\right)$$

$$\frac{Cs + \frac{1}{R}}{R} = \frac{E_c(s)}{E_i(s)}$$

$$\frac{CsR+1}{R} = \frac{E_c(s)}{E_i(s)}$$

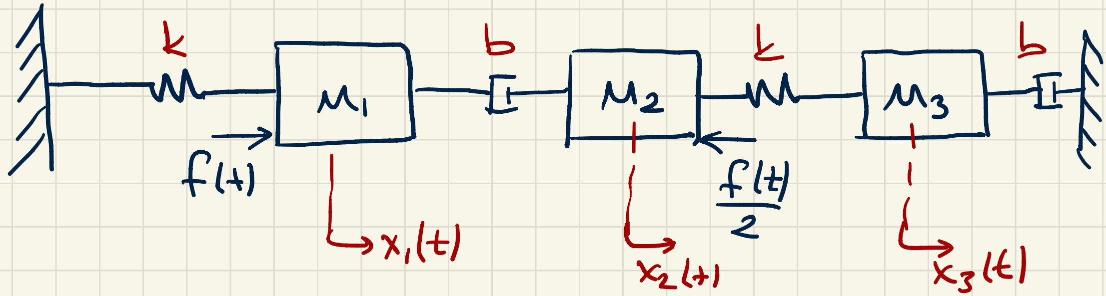
$$\frac{E_L(s)}{E_R(s)} = ?$$

$$i_L = -i_R$$

$$\frac{E_L(s)}{sL} = \frac{E_R(s)}{R}$$

$$\frac{E_L(s)}{E_R(s)} = -\frac{sL}{R}$$

## Ör:




$k = 1\ N/m$

$b = 1\ \frac{Ns}{m}$

$m_1 = m_2 = m_3 = 1\ kg$

Durum değişkenleri oluştur

Çıkış $x_3(t)$ ise çıkış denk.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \\ \dot{x}_3 \\ \dot{v}_3 \end{bmatrix} = \begin{bmatrix} \quad & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \\ x_3 \\ v_3 \end{bmatrix} + \begin{bmatrix} \quad \\ \\ \\ \\ \\ \\ \end{bmatrix} f(t)$$

$$\frac{dx_1}{dt} = v_1 \qquad \frac{dx_2}{dt} = v_2 \qquad \frac{dx_3}{dt} = v_3$$

$\underline{m_1}$: $-k x_1(t) + f(t) - b(v_1 - v_2) - m_1 \dot{v}_1 = 0$

$\underline{m_2}$: $+ b(v_2 - v_1) + \frac{f(t)}{2} + k(x_2 - x_3) + m_2 \dot{v}_2 = 0$

$\underline{m_3}$: $k(x_3 - x_2) + b v_3 + m_3 \dot{v}_3 = 0$

$\underline{\underline{m_1}}$ $-kx_1(t)+f(t)-b(v_1-v_2)-m_1\dot{v}_1=0$

$\dot{v}_1=-x_1-v_1+0x_2+1.v_2+f(t)$

$\underline{\underline{m_2}}$ $+b(v_2-v_1)+\frac{f(t)}{2}+k(x_2-x_3)+m_2\dot{v}_2=0$

$\dot{v}_2=0x_1+v_1-x_2-v_2+x_3+0v_3-\frac{f(t)}{2}$

$\underline{\underline{m_3}}$ $k(x_3-x_2)+bv_3+m_3\dot{v}_3=0$

$\dot{v}_3=0x_1+0v_1+x_2+0v_2-x_3-v_3$

$\frac{dx_1}{dt}=v_1$

$\frac{dx_2}{dt}=v_2$

$\frac{dx_3}{dt}=v_3$

$$\begin{bmatrix}\dot{x}_1\\ \dot{v}_1\\ \dot{x}_2\\ \dot{v}_2\\ \dot{x}_3\\ \dot{v}_3\end{bmatrix}=\begin{bmatrix}0&1&0&0&0&0\\ -1&-1&0&1&0&0\\ 0&0&0&1&0&0\\ 0&1&-1&-1&1&0\\ 0&0&0&0&0&1\\ 0&0&1&0&-1&-1\end{bmatrix}\begin{bmatrix}x_1\\ v_1\\ x_2\\ v_2\\ x_3\\ v_3\end{bmatrix}+\begin{bmatrix}0\\ 1\\ 0\\ -\frac{1}{2}\\ 0\\ 0\end{bmatrix}f(t)$$

Çıkış $v_3$ ise

$$\begin{bmatrix}x_3\\ v_3\\ f(t)\end{bmatrix}=\begin{bmatrix}0&0&0&0&1&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\end{bmatrix}\begin{bmatrix}x_1\\ v_1\\ x_2\\ v_2\\ x_3\\ v_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}f(t)$$