27.10.2023

4. Hafta

1

ELEKTRİKSEL ELEMANLARIN MODELLENMESİ

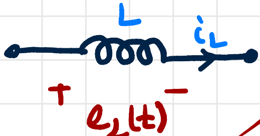
DİRENÇLER (RESISTORS)

$$\boxed{e_R(t) = i(t) \cdot R} \qquad i(t) = \frac{e_R(t)}{R}$$

$$\frac{e_R(t)}{i_R(t)} = R \quad \xrightarrow{\mathcal{L}\{\}} \quad \boxed{\frac{E_R(s)}{I_R(s)} = R} \rightarrow \text{impedance}$$

BOBİN (INDUCTOR)

$$\phi = L \cdot i \rightarrow \frac{d\phi}{dt} = L\frac{di}{dt} \rightarrow e = L\frac{di}{dt}$$



$$e_L(t) = L \cdot \frac{di_L(t)}{dt}$$

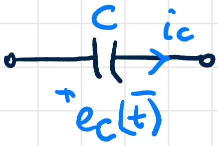
$$i_L(t) = \frac{1}{L}\int_0^t e_L(t)\,dt + i_L(0)$$

$\mathcal{L}\{\}$

$$E_L(s) = Ls\, I_L(s) \qquad \boxed{\frac{E_L(s)}{I_L(s)} = Ls} \rightarrow \text{impedance}$$

KAPASİTÖR (CAPACITOR)

$$q = C\cdot E \rightarrow \underbrace{\frac{dq}{dt}}_{=i} = C \cdot \frac{de}{dt}$$



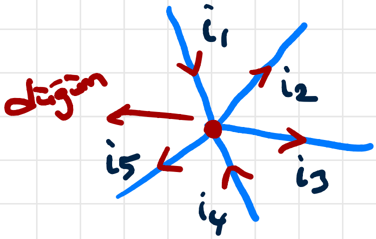
$$i_c(t) = C \cdot \frac{de_c(t)}{dt}$$

$$e_c(t) = \frac{1}{C}\int_0^t i_c(t)\,dt + e_c(0)$$

$\mathcal{L}\{\}$

$$I_c(s) = Cs\, E_c(s) \rightarrow \boxed{\frac{E_c(s)}{I_c(s)} = \frac{1}{Cs}} \rightarrow \text{impedance}$$

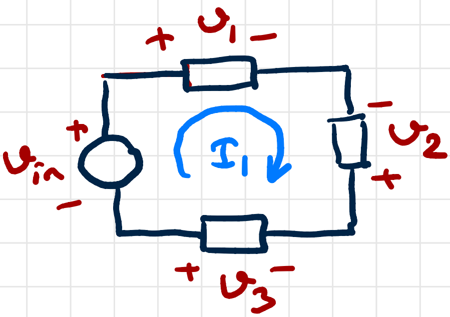
KIRCHHOFF'UN AKIM KANUNU



$$-i_1 + i_2 + i_3 - i_4 + i_5 = 0$$

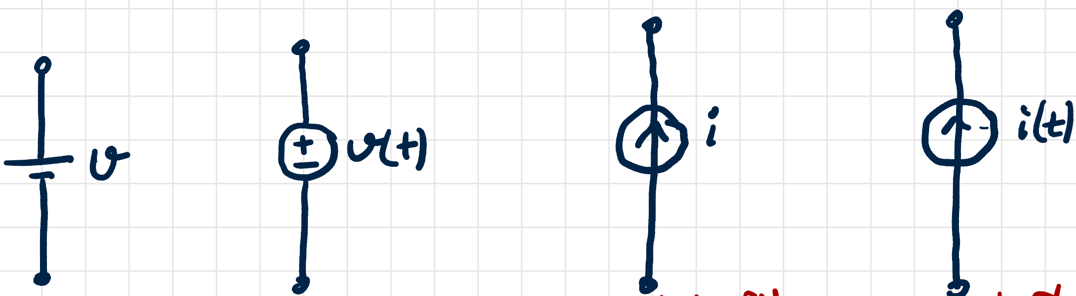
$$i_1 + i_4 = i_2 + i_3 + i_5$$

KIRCHHOFF'UN GERİLİM KANUNU



$$-v_{in} + v_1 - v_2 - v_3 = 0$$

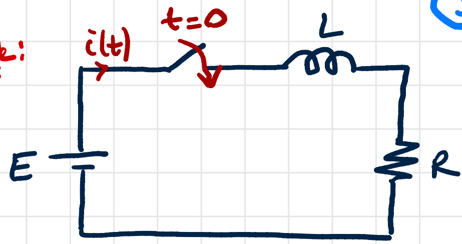
KAYNAKLAR



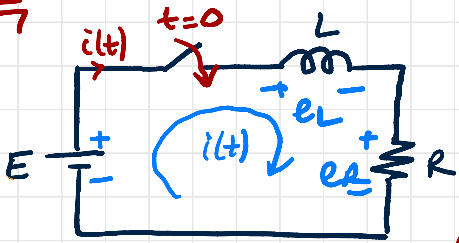
zamana bağlı değil (DC)	zamana bağlı !	zamana bağlı değil	zamana bağlı
$\mathcal{L} \downarrow \frac{v}{s}$	$\mathcal{L} \downarrow \; v(s)$	$\mathcal{L} \downarrow \; \frac{I}{s}$	$\mathcal{L} \downarrow \; I(s)$

Örnek: ③ $i(0)=0$

Anahtar $t=0$ anında kapatılırsa $i(t)=?$



Çözüm:



$$-E+e_L(t)+e_R(t)=0$$

$$-E+L\frac{di(t)}{dt}+i(t).R=0$$

$$\Big\downarrow \mathcal{L}$$

$$-\frac{E}{s}+L\left[sI(s)-\cancelto{0}{i(0)}\right]+I(s).R=0$$

$$LsI(s)+RI(s)=\frac{E}{s} \Rightarrow I(s)=\frac{E}{s}\cdot\frac{1}{Ls+R}$$

$$\mathcal{L}^{-1}\{I(s)\}=\mathcal{L}^{-1}\left\{\frac{E}{s}\cdot\frac{1}{Ls+R}\right\}$$

$$\frac{E}{s}\cdot\frac{1}{Ls+R}=\frac{A}{s}+\frac{B}{Ls+R}$$

$$A(Ls+R)+Bs=E$$

$$s=0 \Rightarrow A=\frac{E}{R}$$

$$s=\frac{-R}{L} \Rightarrow B=-\frac{EL}{R}$$

$$\mathcal{L}^{-1}\left\{\frac{E}{s}\cdot\frac{1}{Ls+R} = \frac{A}{s} + \frac{B}{Ls+R}\right\} = \mathcal{L}^{-1}\left\{\frac{E}{Rs} + \frac{-EL/R}{Ls+R}\right\}$$

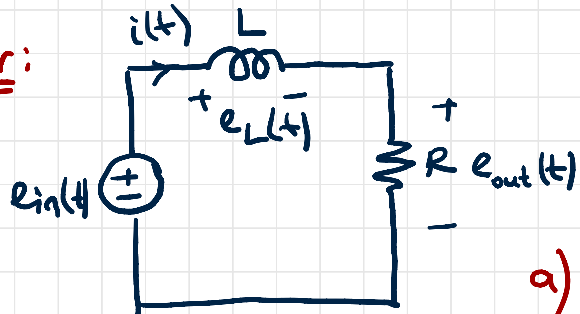
4

$$\mathcal{L}^{-1}\left\{\frac{E}{Rs}\right\} = \frac{E}{R}$$

$$\mathcal{L}^{-1}\left\{\frac{-EL/R}{Ls+R}\right\} = \mathcal{L}^{-1}\left\{-\frac{E}{R}\frac{L}{Ls+R} = -\frac{E}{R}\frac{1}{s+R/L}\right\} = -\frac{E}{R}e^{-R/L}$$

$$\mathcal{L}^{-1}\{I(s)\} = i(t) = \frac{E}{R} - \frac{E}{R}e^{-R/L}$$

Ör:

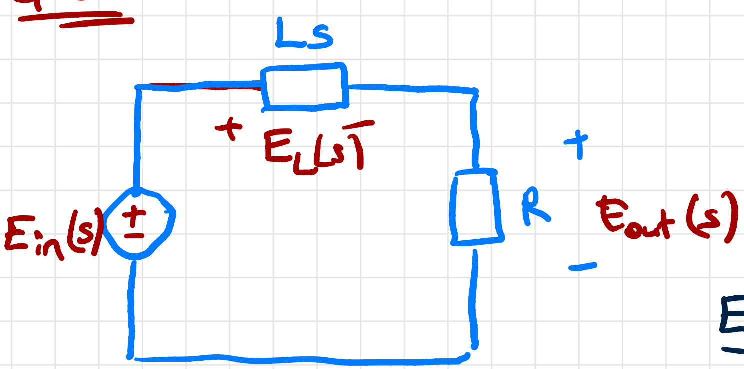


Transfer fonk. bulunuz:

a) $\frac{E_{out}(s)}{E_{in}(s)} = ?$

b) $\frac{E_{in}(s)}{I(s)} = ?$

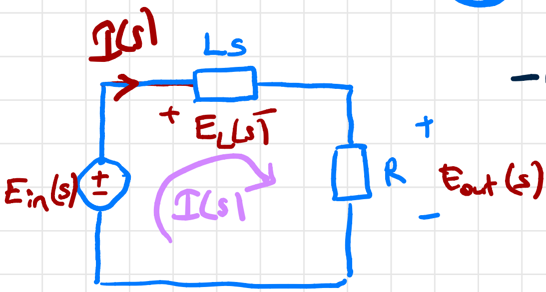
Çözüm:



$$\frac{E_{out}(s)}{E_{in}(s)} = \frac{R}{Ls+R}$$

$$\frac{E_L(s)}{E_{in}(s)} = \frac{Ls}{Ls+R}$$

5



$$-E_{in}(s) + I(s) \cdot Ls + R \cdot I(s) = 0$$

$$E_{in}(s) = I(s) \cdot Ls + R \cdot I(s)$$

$$E_{out}(s) = R \cdot I(s)$$

$$\frac{E_{out}(s)}{E_{in}(s)} = \frac{R \cdot \cancel{I(s)}}{\cancel{I(s)}(Ls+R)} = \frac{R}{Ls+R}$$

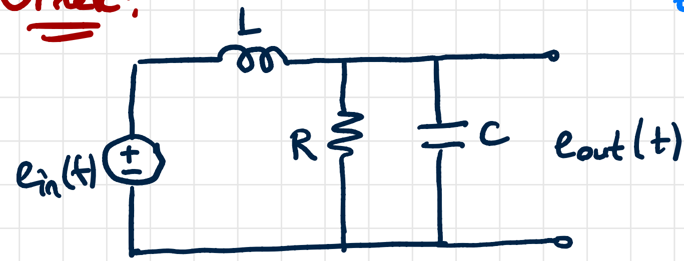
b) $$\frac{E_{in}(s)}{I(s)} = Ls+R$$

b) $$-E_{in}(s) + I(s) Ls + R \cdot I(s) = 0$$

$$E_{in}(s) = Ls\, I(s) + R I(s)$$

$$\frac{E_{in}(s)}{I(s)} = Ls + R$$

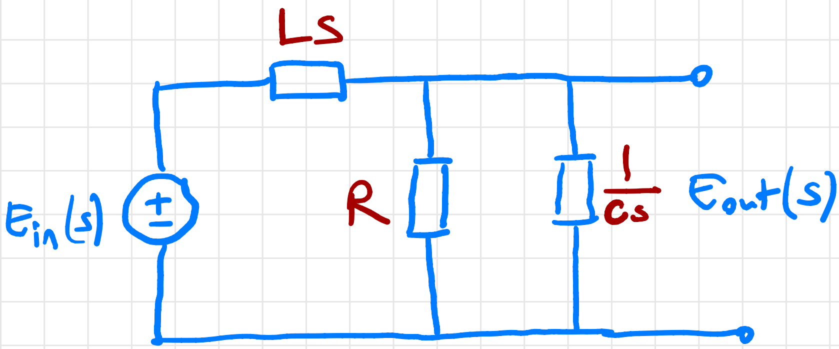
Örnek:



$i(0) = 0$

$e(0) = 0$

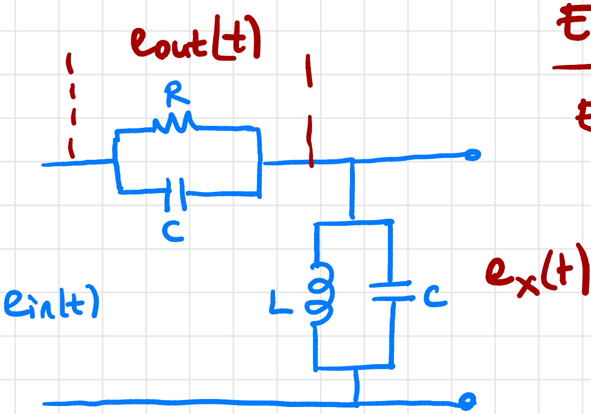
$$\frac{E_{out}(s)}{E_{in}(s)} = ?$$



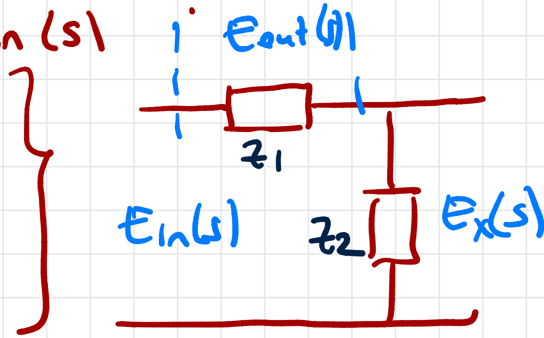
$$\frac{E_{out}(s)}{E_{in}(s)} = \frac{\frac{1}{Cs} /\!/ R}{Ls + \left(R /\!/ \frac{1}{Cs}\right)} = \frac{\frac{\frac{1}{Cs} \cdot R}{\frac{1}{Cs} + R}}{Ls + \left(\frac{\frac{1}{Cs} \cdot R}{\frac{1}{Cs} + R}\right)}$$

$$\frac{\frac{1}{Cs} \cdot R}{\frac{1}{Cs} + R} = \frac{R/Cs}{(1+RCs)/Cs} = \frac{R}{1+RCs}$$

$$= \frac{\frac{R}{1+RCs}}{Ls + \frac{R}{1+RCs}} = \frac{R}{Ls + LRCs^2 + R}$$



$$\frac{E_{out}(s)}{E_{in}(s)} = ?$$



$$Z_1 = R \parallel \frac{1}{Cs}$$

$$Z_2 = Ls \parallel \frac{1}{Cs}$$

$$\frac{E_{out}(s)}{E_{in}(s)} = \frac{Z_1}{Z_1 + Z_2}$$

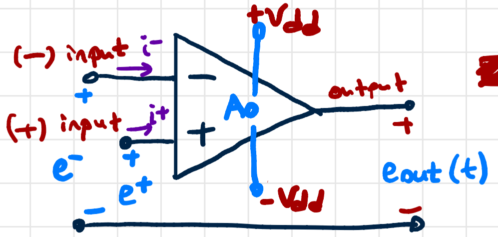
03. 11. 2023

5. HAFTA

* Op-Amp T.F

* Blok Diyagram İndirgeme

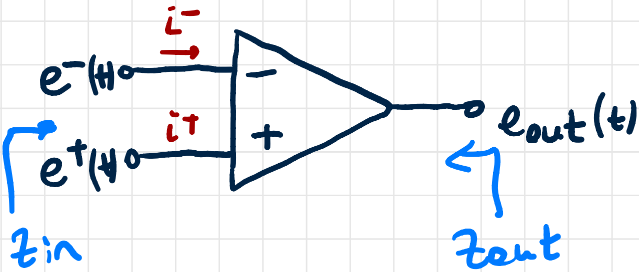
İK – (OP-AMP)
İŞLEMSEL KUVVETLENDİRİCİLERİN TRANSFER FON



■ $e_{out} = A_0(\underbrace{e^+ - e^-}_{=0})$

Kazanç

idealde $A_0 \to \infty$



■ (+) ve (−) girişlere (inputs) len akımlar sıfırdır.

$\left.\begin{array}{l}\text{(idealde)} \\ A_0 \to \infty\end{array}\right\} \quad i^- = i^+ = 0$

z_{in} : Giriş Empedansı $\left(z = R \pm jX \right)$

z → Empedans, R → Resistans, X → Reaktans

■ Op-Amp'ın Giriş Empedansı $z_{in} = \infty$

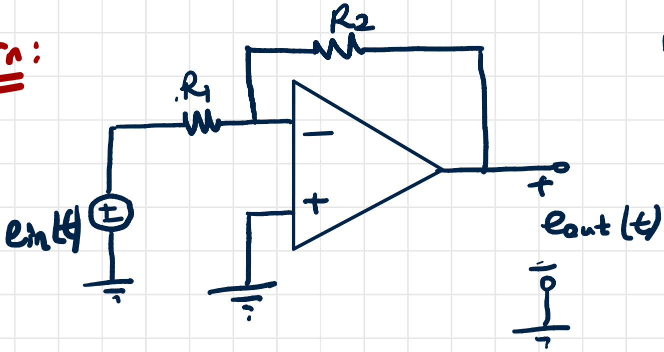
■ Op-Amp'ın Çıkış Empedansı $z_{out} = 0$

$$\left.\frac{E_{out}(s)}{E_{in}(s)} = \text{Kazanç (Gain)}\right\} \text{Transfer Fonksiyonu}$$

$$\frac{e_{in}(t)}{e_{out}(t)} = ? \quad \text{TF değil!} \quad \xrightarrow{\text{TF}} \quad \frac{E_{in}(s)}{E_{out}(s)}$$

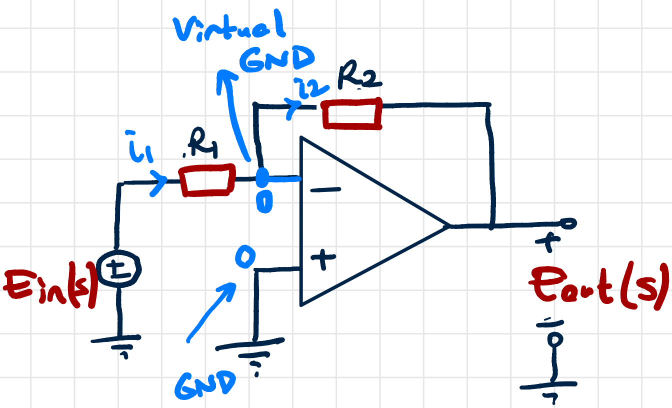
TF!

Örn:



Kazanç = ?

$$\frac{E_{out}(s)}{E_{in}(s)} = ?$$



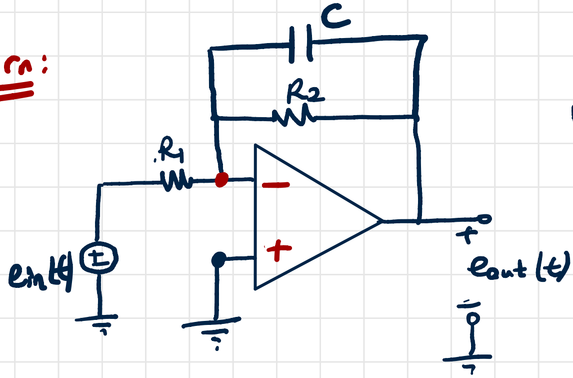
$Z_1 = R_1$

$Z_2 = R_2$

$$i_1 - i_2 = 0$$

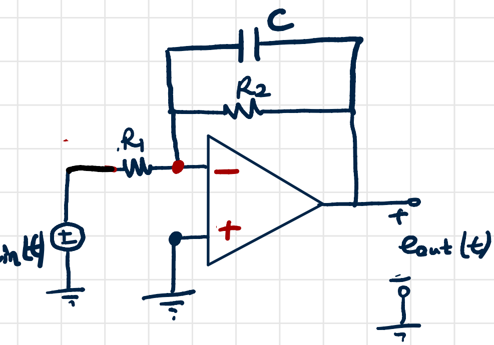
$$\frac{E_{in} - 0}{R_1} - \frac{0 - E_{out}}{R_2} = 0 \longrightarrow \frac{E_{in}}{R_1} + \frac{E_{out}}{R_2} = 0 \rightarrow \frac{E_{out}(s)}{E_{in}(s)} = \frac{-R_2}{R_1}$$

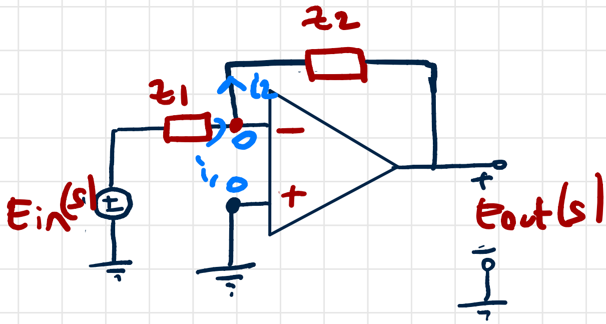
Örn:



Kazanç?

$$\frac{E_{out}(s)}{E_{in}(s)} = ?$$





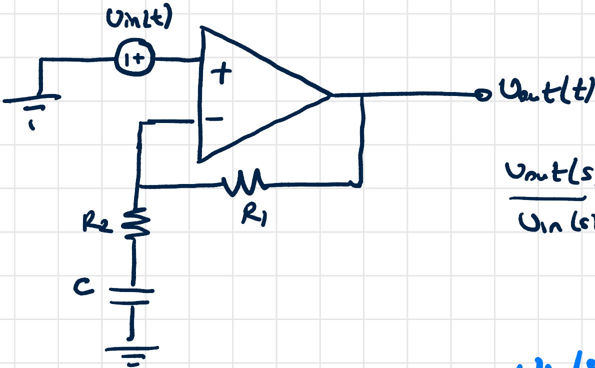
$$Z_1 = R_1$$

$$Z_2 = R_2 \,//\, \frac{1}{sC} = \frac{R_2 \cdot \frac{1}{sC}}{R_2 + \frac{1}{sC}} = \frac{R_2/sC}{(R_2 sC+1)/sC} = \frac{R_2}{R_2 sC+1}$$

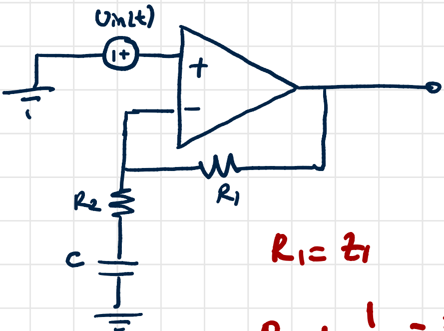
$$i_1 - i_2 = 0 \Rightarrow \frac{E_{in}-0}{Z_1} - \frac{0-E_{out}}{Z_2} = 0$$

$$\frac{E_{in}}{Z_1} = \frac{-E_{out}}{Z_2} \Rightarrow \frac{E_{out}}{E_{in}} = \frac{-Z_2}{Z_1} = \frac{-\frac{R_2}{R_2 sC+1}}{R_1} = -\frac{R_2}{R_1}\,\frac{1}{R_2 sC+1}$$

Ör:

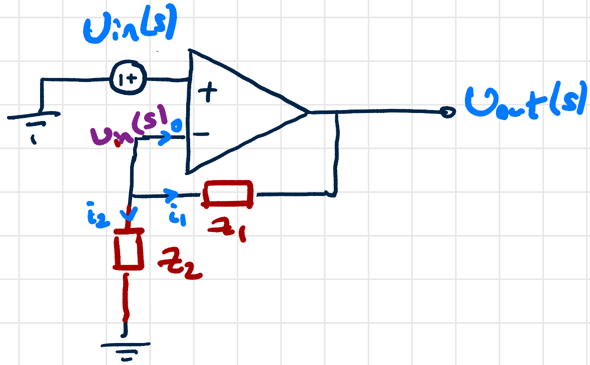


$$\frac{V_{out}(s)}{V_{in}(s)} = ?$$



Vout(t)

$$\frac{V_{out}(s)}{V_{in}(s)} = ?$$



$$R_1 = Z_1$$

$$R_2 + \frac{1}{sC} = Z_2$$

$$\left.\begin{array}{l} i_2 + i_1 = 0 \\ \dfrac{V_{in}}{Z_2} + \dfrac{V_{in} - V_{out}}{Z_1} = 0 \end{array}\right\} \quad \frac{V_{in}}{(R_2sC+1)/sC} + \frac{V_{in}}{R_1} = \frac{V_{out}}{R_1}$$

$$\frac{sC\,V_{in}\cdot R_1}{(R_2sC+1)\cancel{R_1}} + \frac{(R_2sC+1)\,V_{in}}{(R_2sC+1)\cancel{R_1}} = \frac{V_{out}}{\cancel{R_1}}$$

$$(R_1) \qquad (R_2sC+1)$$

$$\frac{V_{out}}{V_{in}} = \frac{R_2sC+1 + R_1sC}{R_2sC+1}$$

BLOK DİYAGRAMLARI (BLOCK DIAGRAMS)

işaret (signal)

$C(s) = R(s) \cdot G(s)$

$G(s) \rightarrow$ Transfer fonk

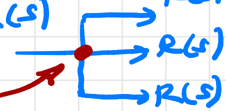
$$G(s) = \frac{C(s)}{R(s)}$$

Blok Diyagramlar aşağıdaki elemanlardan oluşur?

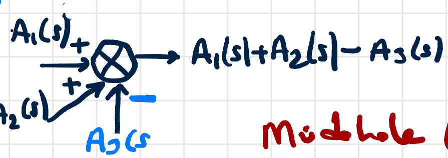
1) Bloklardan

2) Oklardan

3) Ayrım Noktası (Take off / Pick off points)



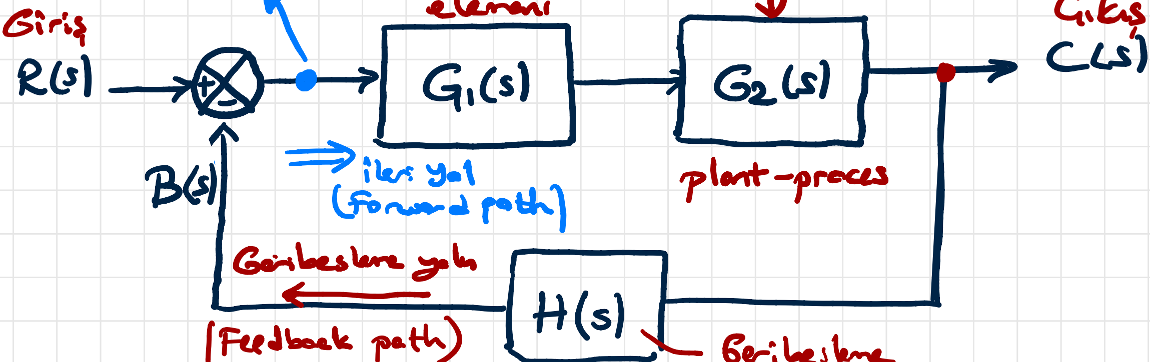
4) Toplam Noktası (summing point)



$E(s) = R(s) - B(s)$

Kontrol elemanı

Müdahale (Dist)



ileri yol (Forward path)

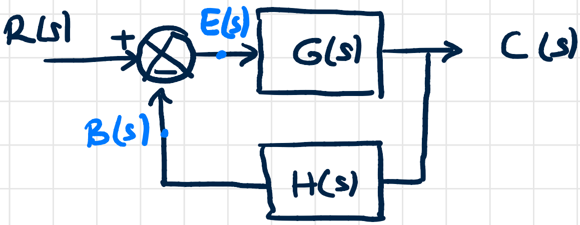
plant-proces

Geribesleme yolu (Feedback path)

Geribesleme elemanı

$B(s) = H(s) \cdot C(s)$

Kapalı Çevrim Negatif Geri Besleme



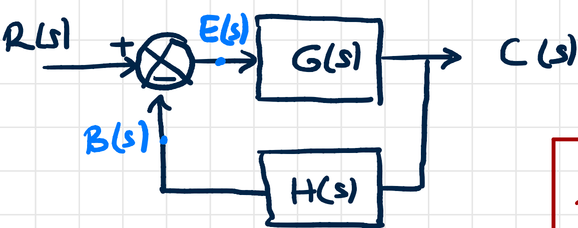
$G(s)$: Transfer Fonksiyonu = İleri yol Transfer Fonk.

$H(s)$: Geribesleme Transfer Fonksiyonu

$G(s).H(s)$: Açık-Çevrim Transfer Fonk.

$\dfrac{C(s)}{R(s)}$ = Kapalı-Çevrim Transfer Fonksiyonu

$\dfrac{C(s)}{E(s)}$ = İleri-yol Transfer fonksiyonu $= G(s)$



$$\frac{C(s)}{R(s)} = ?$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)}}$$

$C(s) = G(s).E(s)$ & $B(s) = C(s).H(s)$

$E(s) = R(s) - B(s) = \underbrace{R(s) - C(s).H(s)}$

$C(s) = G(s).\left[R(s) - C(s).H(s)\right]$

$C(s) = G(s).R(s) - G(s)C(s).H(s)$

$C(s) + G(s)C(s)H(s) = G(s).R(s)$

$C(s)\left(1+G(s)H(s)\right) = G(s)R(s) \Big\} \quad \dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$

$1+G(s)H(s) = 0$

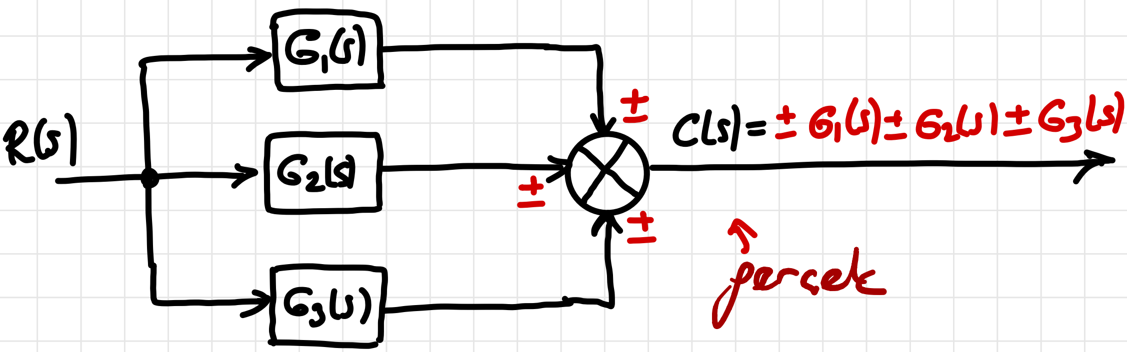
Karakteristik Denklem (Characteristic Equation)

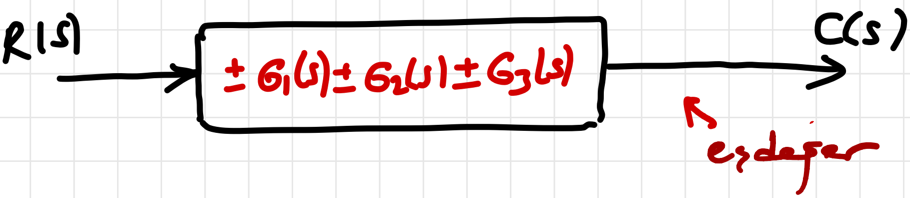
Blok Diyagramların Sadeleştirilmesi:

▲ Seri Bağlı

$$C(s) = R(s) \cdot G_1(s) \cdot G_2(s) \cdot G_3(s)$$

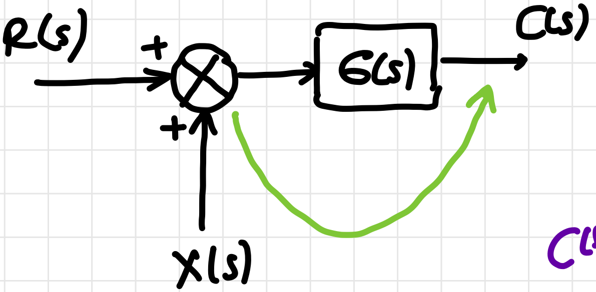
▲ Paralel Bağlı



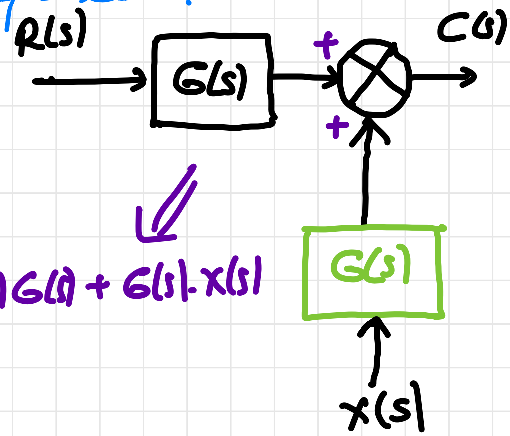


Toplama Noktası Aritmetiği

Aşağıdaki ifadeler eşdeğerler :



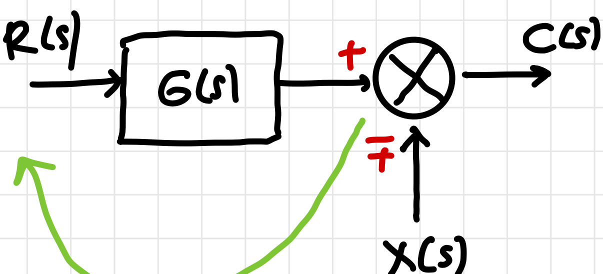
$\equiv$



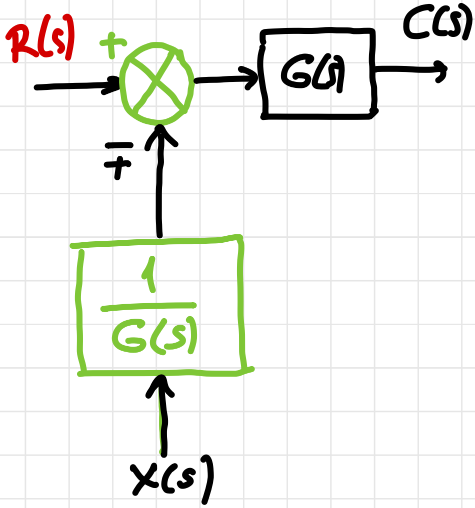
$C(s) = R(s)G(s) + G(s)\cdot X(s)$

$C(s) = \big(R(s) + X(s)\big)\,G(s)$

$= R(s)\cdot G(s) + X(s)\cdot G(s)$



$\equiv$

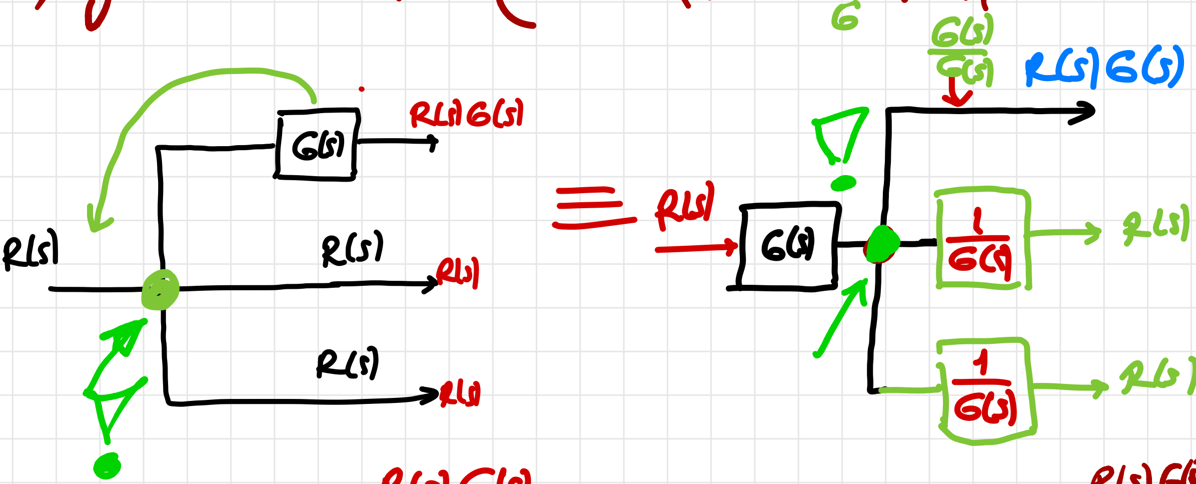


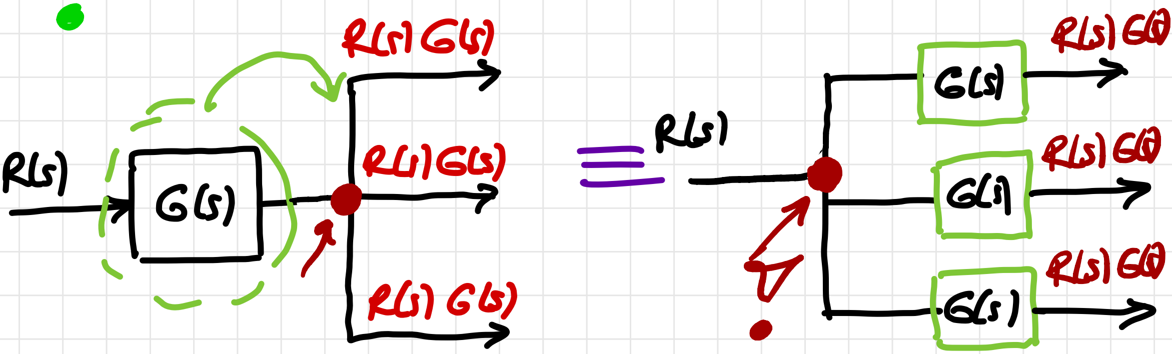
$C(s) = R(s)\cdot G(s) \mp X(s)$

$\left[R(s) \mp \frac{X(s)}{G(s)}\right] G(s) = C(s)$

$R(s)\cdot G(s) \mp X(s) = C(s)$

Ayrım Noktaları (Pick-off Points) işlemleri





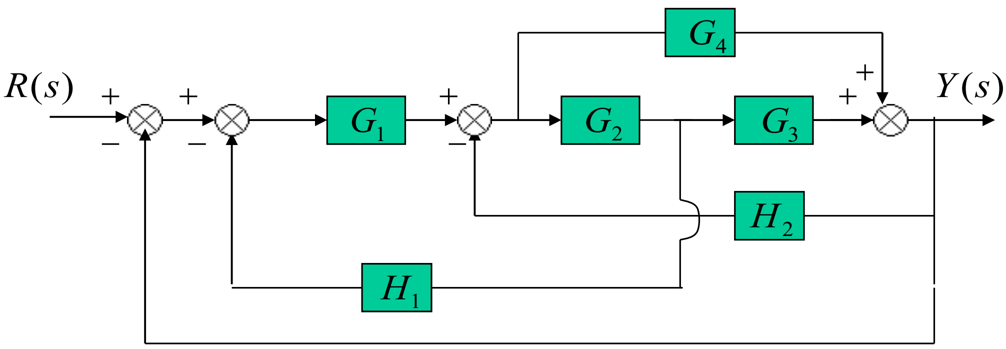
Görüldüğü gibi G(s) bloğu ayrım noktasının sağına getirilince her bir işarete G(s) eklenir.

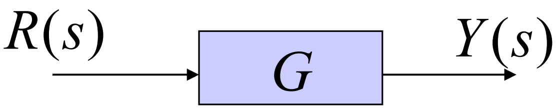
Block diagram

Transfer Function

Consists of Blocks

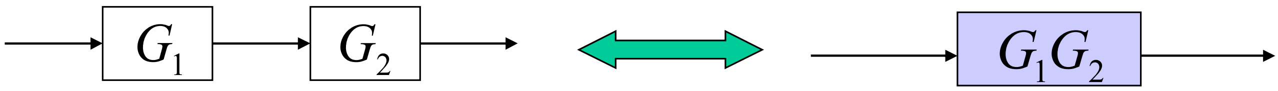
Can be reduced

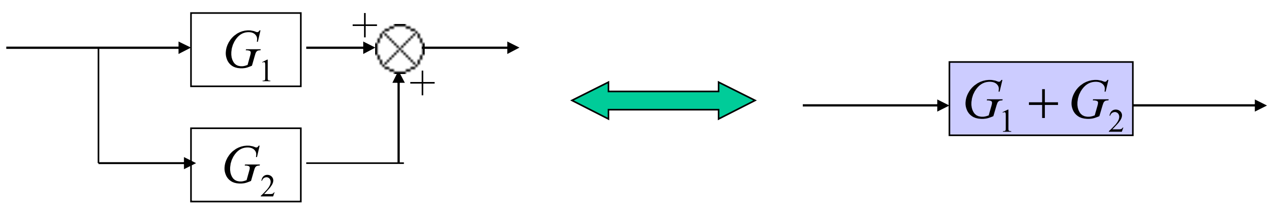




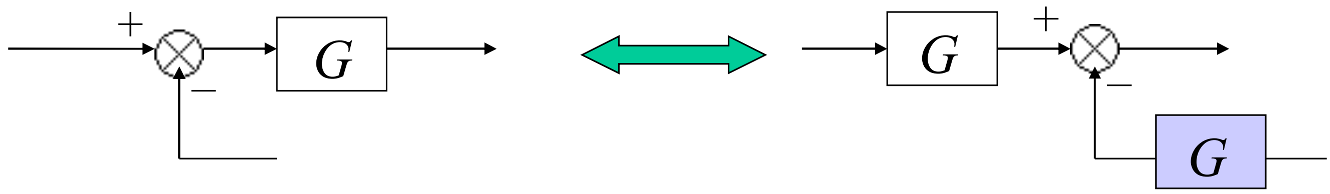
Reduction techniques

1. Combining blocks in cascade or in parallel

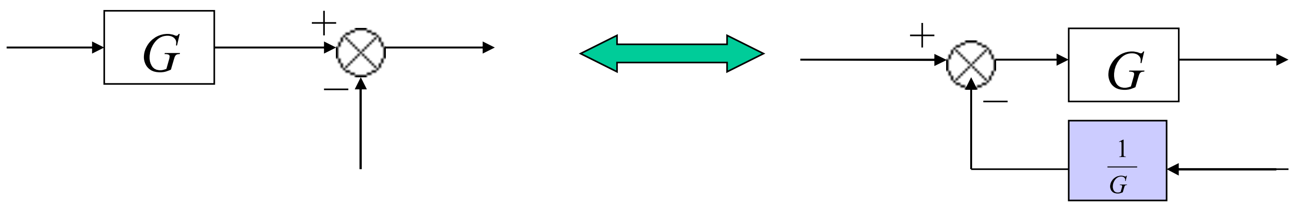




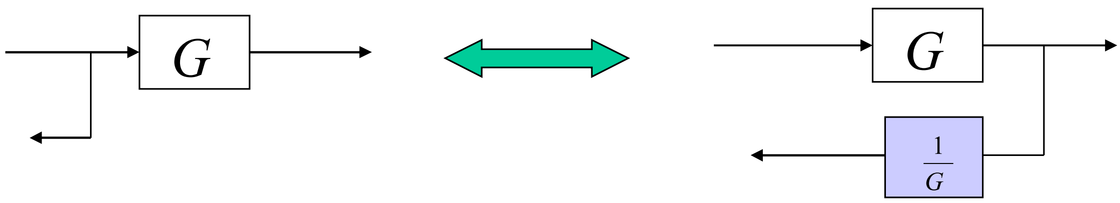
2. Moving a summing point behind a block



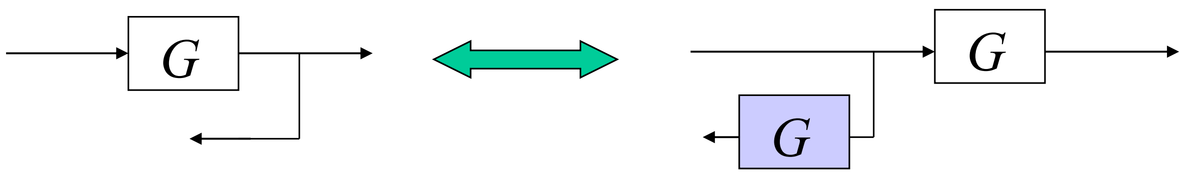
3. Moving a summing point ahead of a block



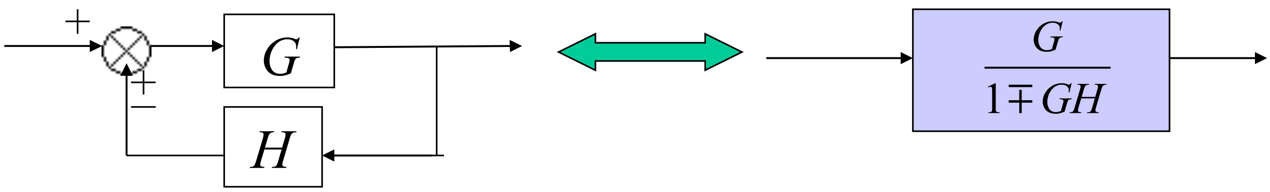
4. Moving a pickoff point behind a block

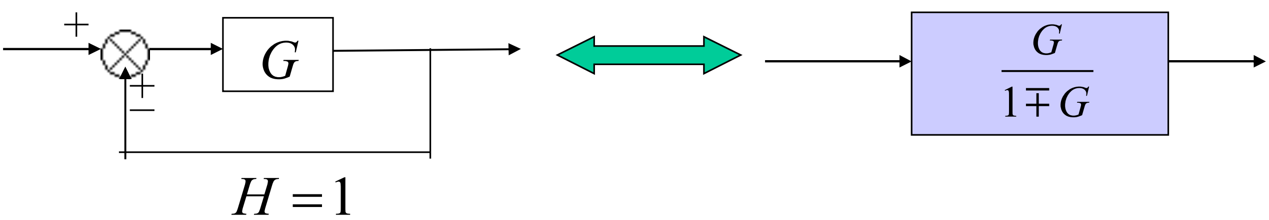


5. Moving a pickoff point ahead of a block

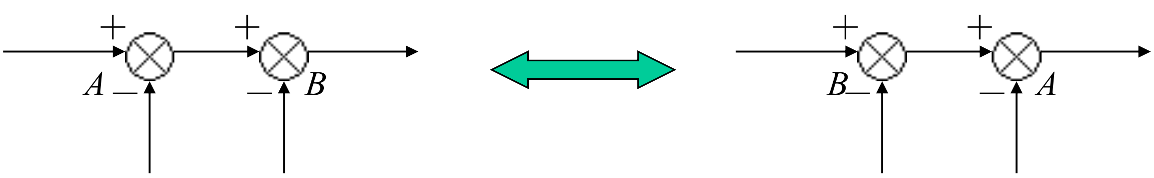


6. Eliminating a feedback loop





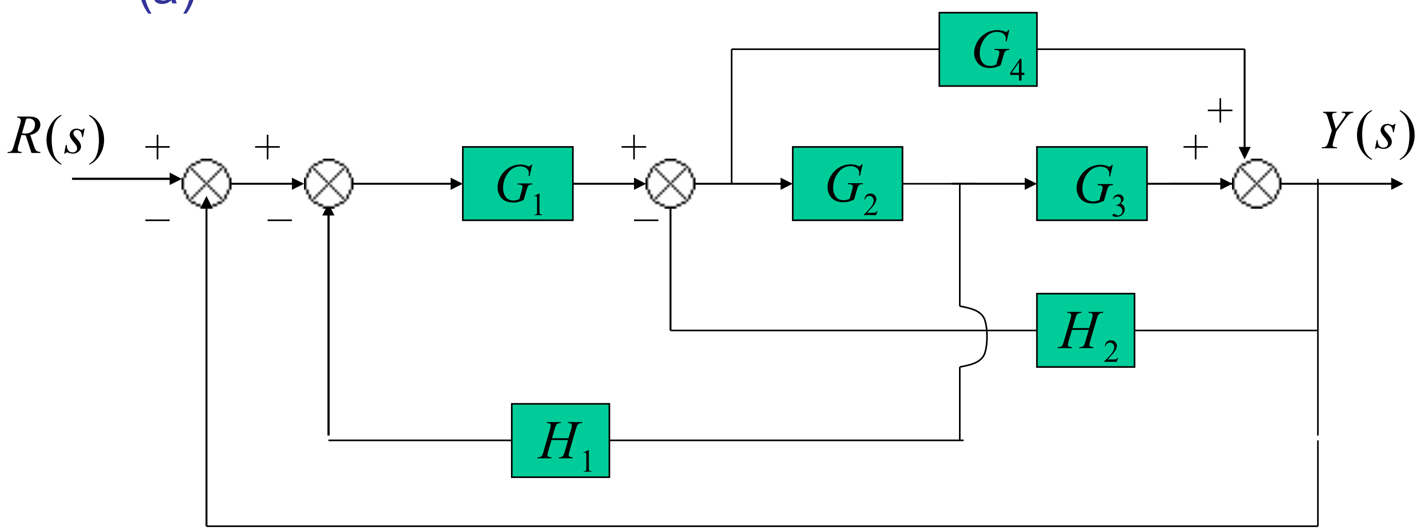
7. Swap with two neighboring summing points

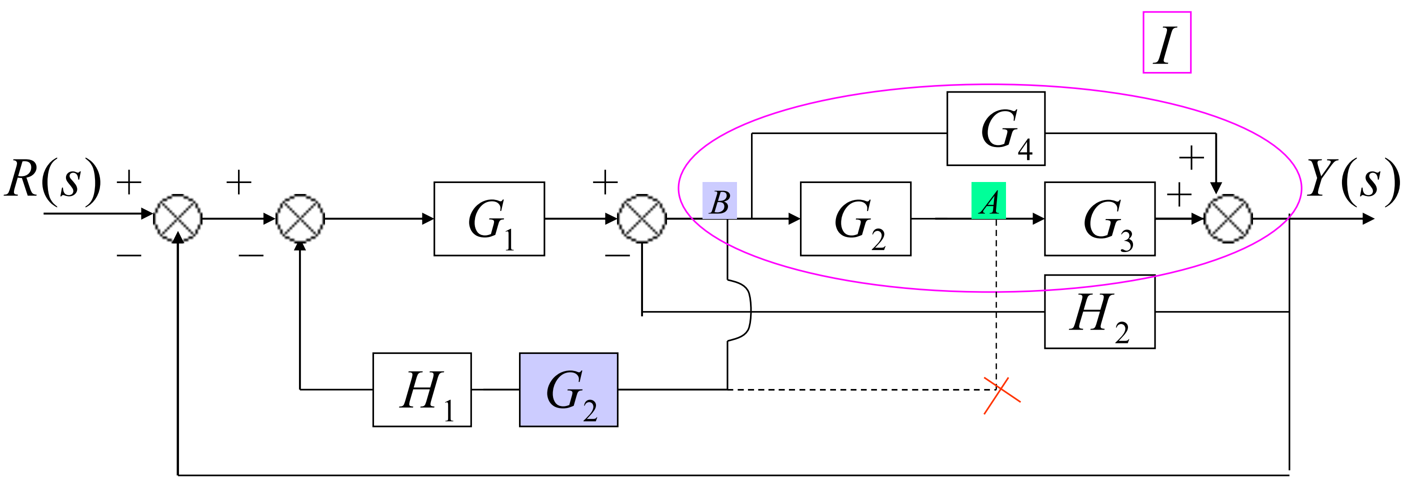


Example 1

Find the transfer function of the following block diagrams

(a)

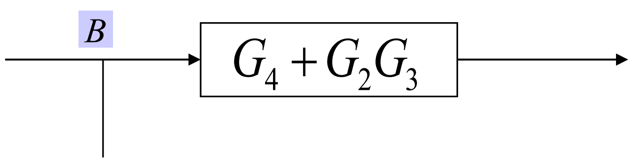


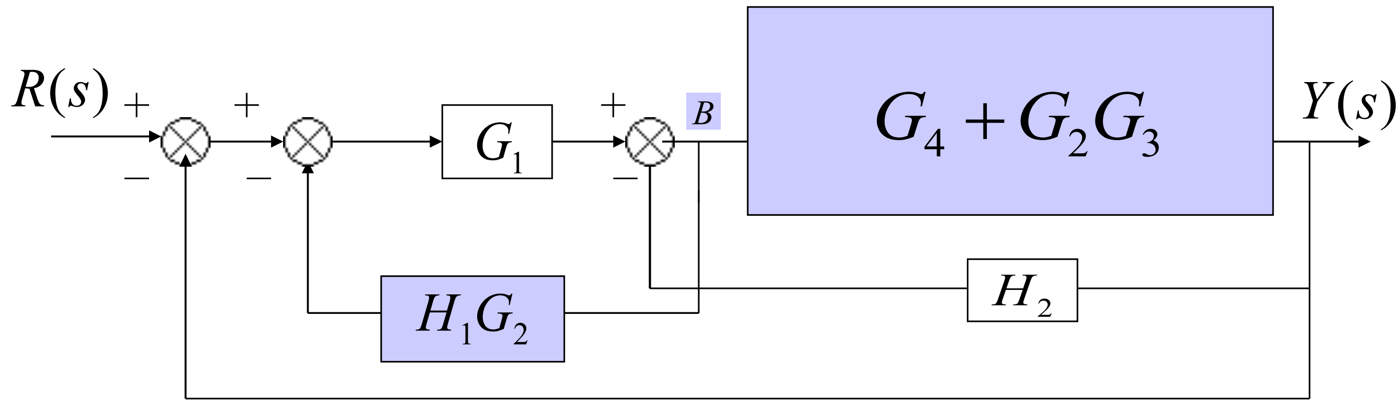


Solution:

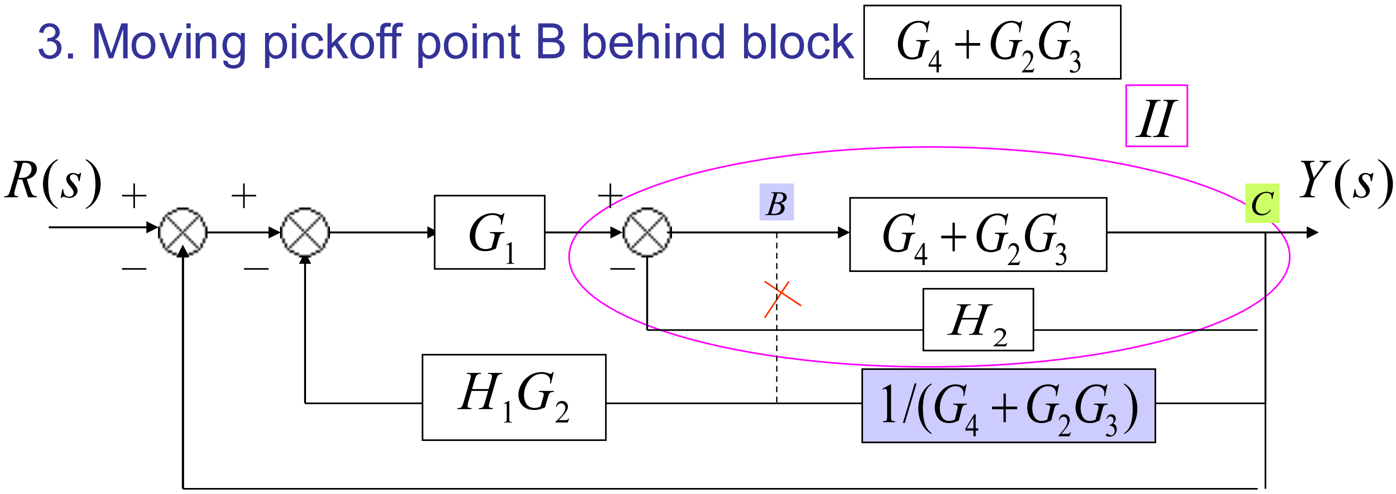
1. Moving pickoff point A ahead of block $\boxed{G_2}$

2. Eliminate loop I & simplify

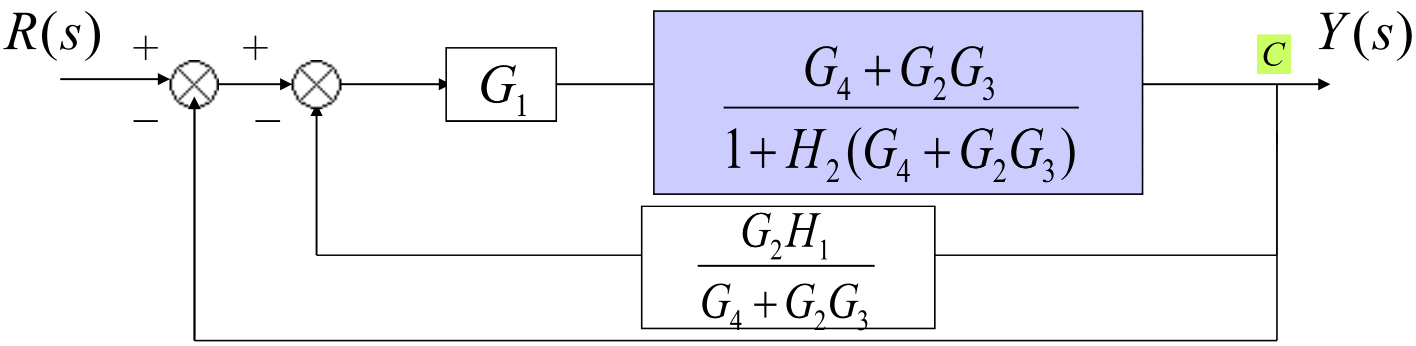




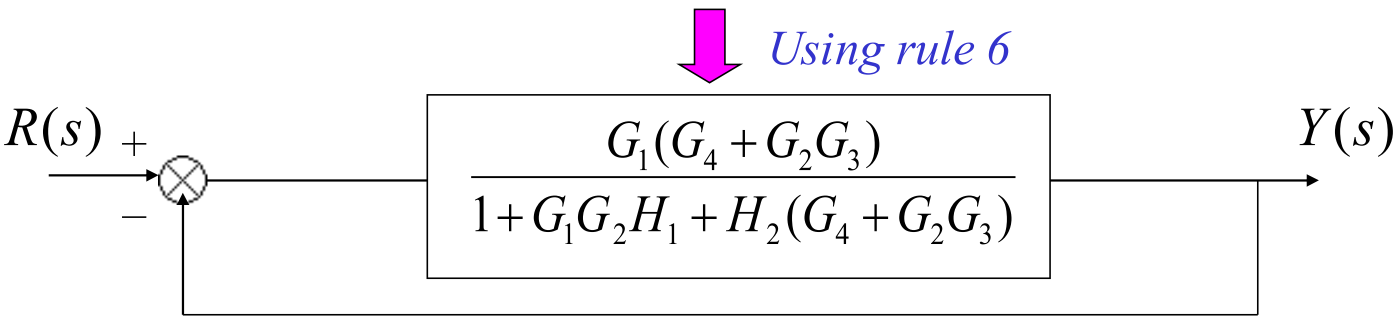
3. Moving pickoff point B behind block $G_4+G_2G_3$



4. Eliminate loop III

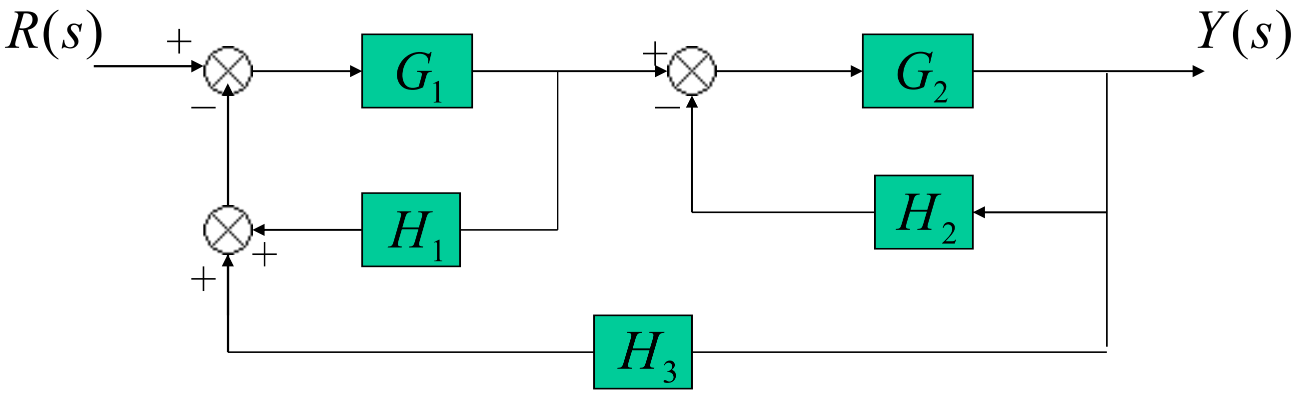


Using rule 6



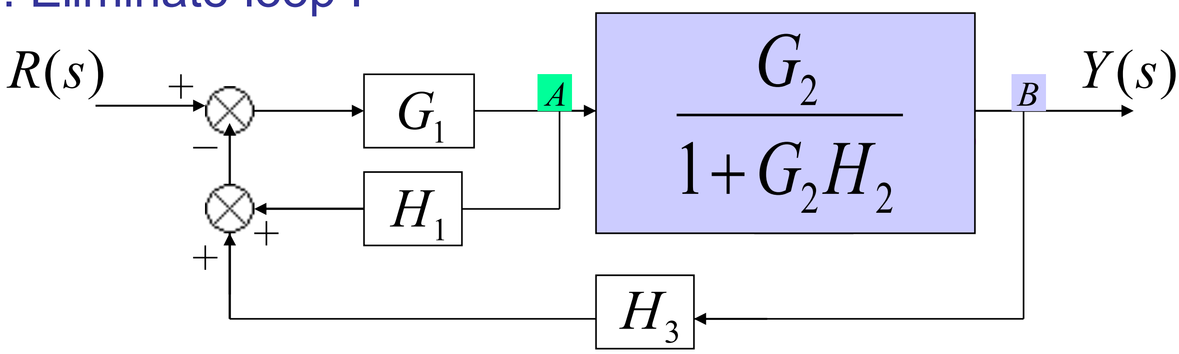
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1(G_4+G_2G_3)}{1+G_1G_2H_1+H_2(G_4+G_2G_3)+G_1(G_4+G_2G_3)}$$

(b)

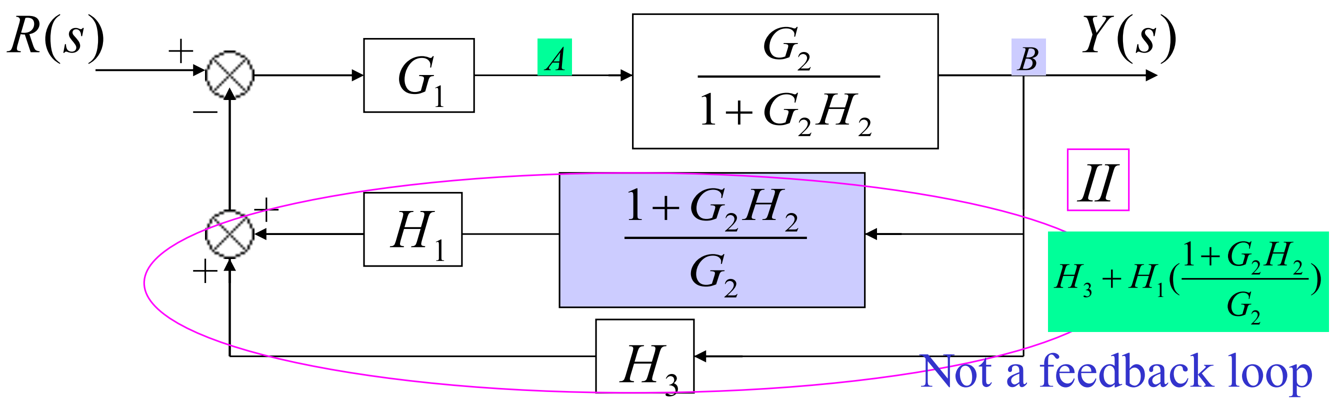


Solution:

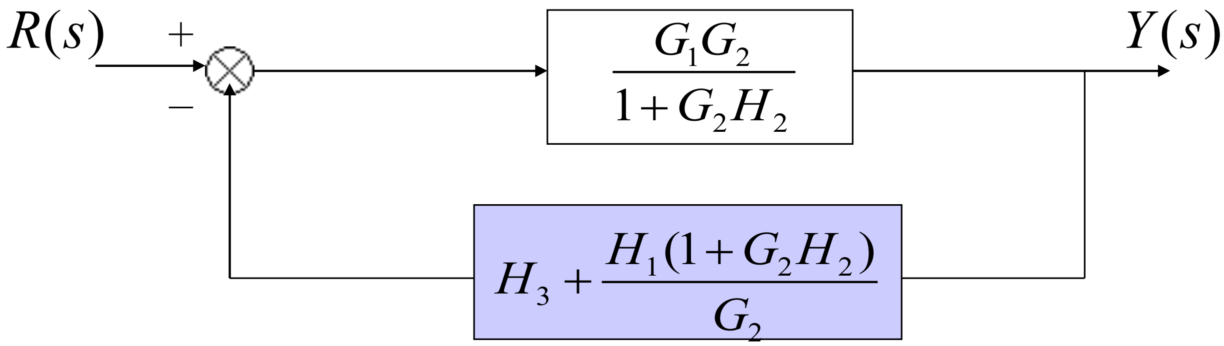
1. Eliminate loop I



2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



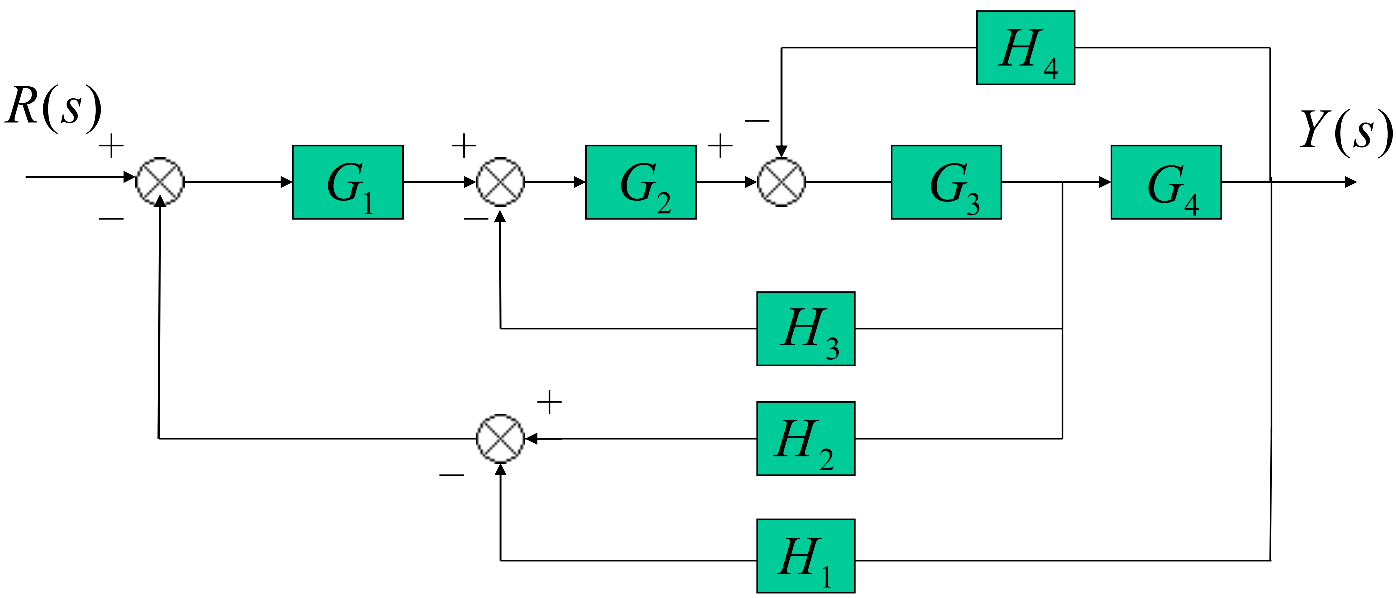
3. Eliminate loop II



Using rule 6

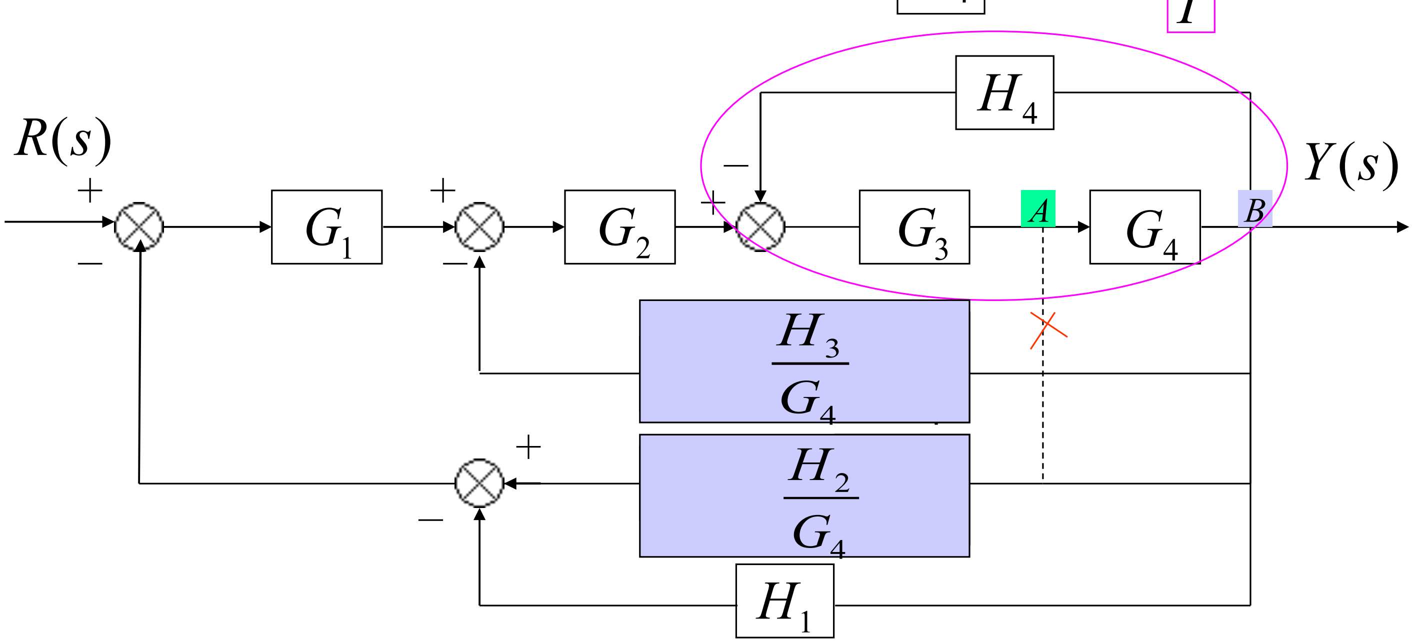
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2}{1+G_2H_2+G_1G_2H_3+G_1H_1+G_1G_2H_1H_2}$$

(c)

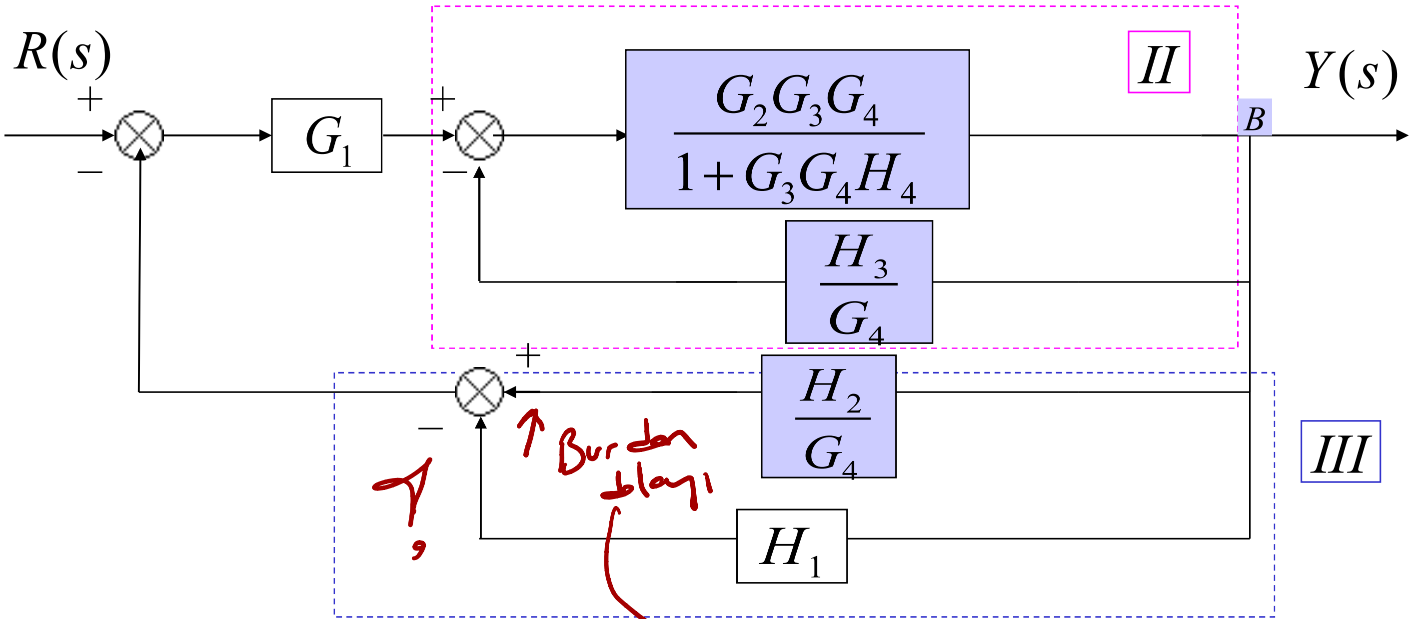


Solution:

1. Moving pickoff point A behind block G_4



2. Eliminate loop I and Simplify



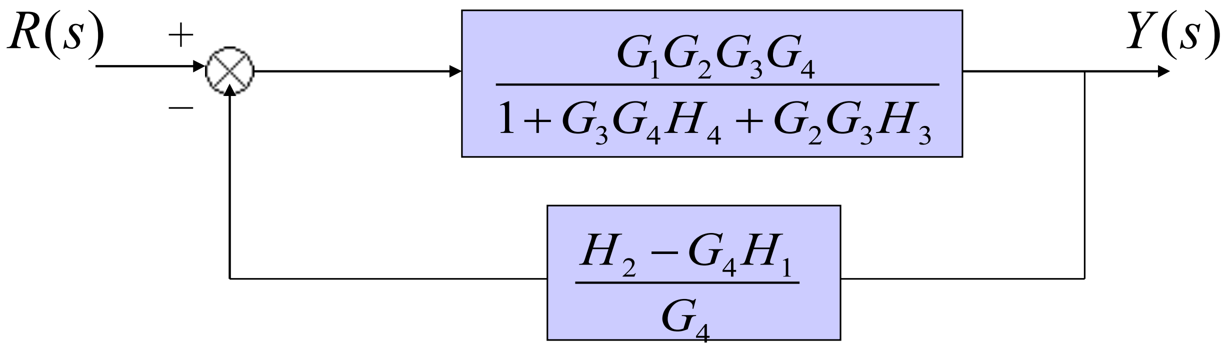
II feedback

$$\frac{G_2G_3G_4}{1+G_3G_4H_4+G_2G_3H_3}$$

III Not feedback

$$\frac{H_2-G_4H_1}{G_4}$$

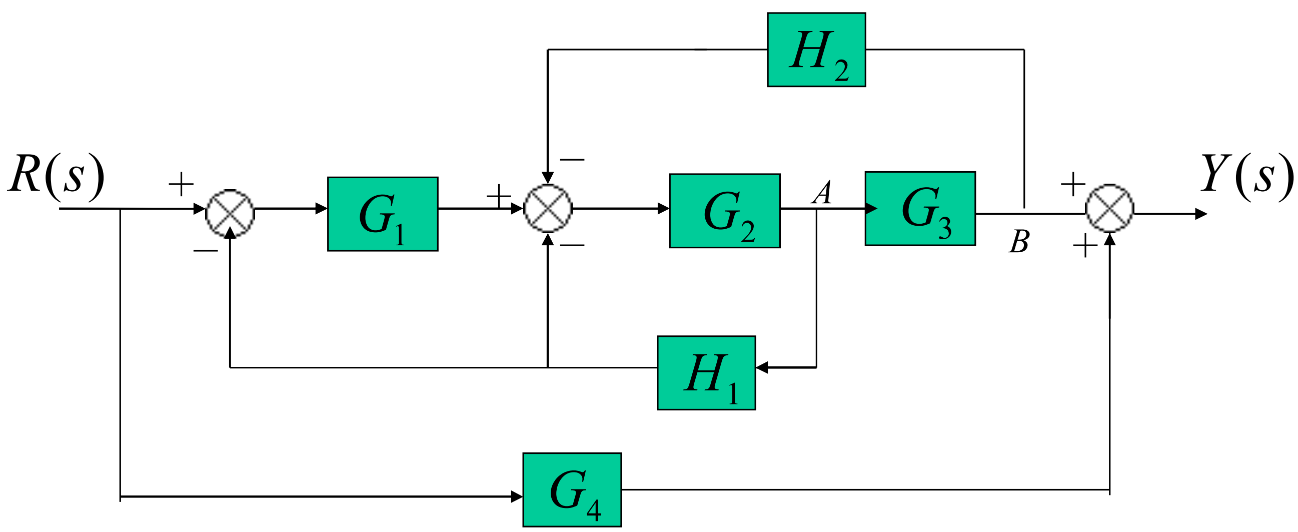
3. Eliminate loop II & IIII



Using rule 6 (Geribesleme!)

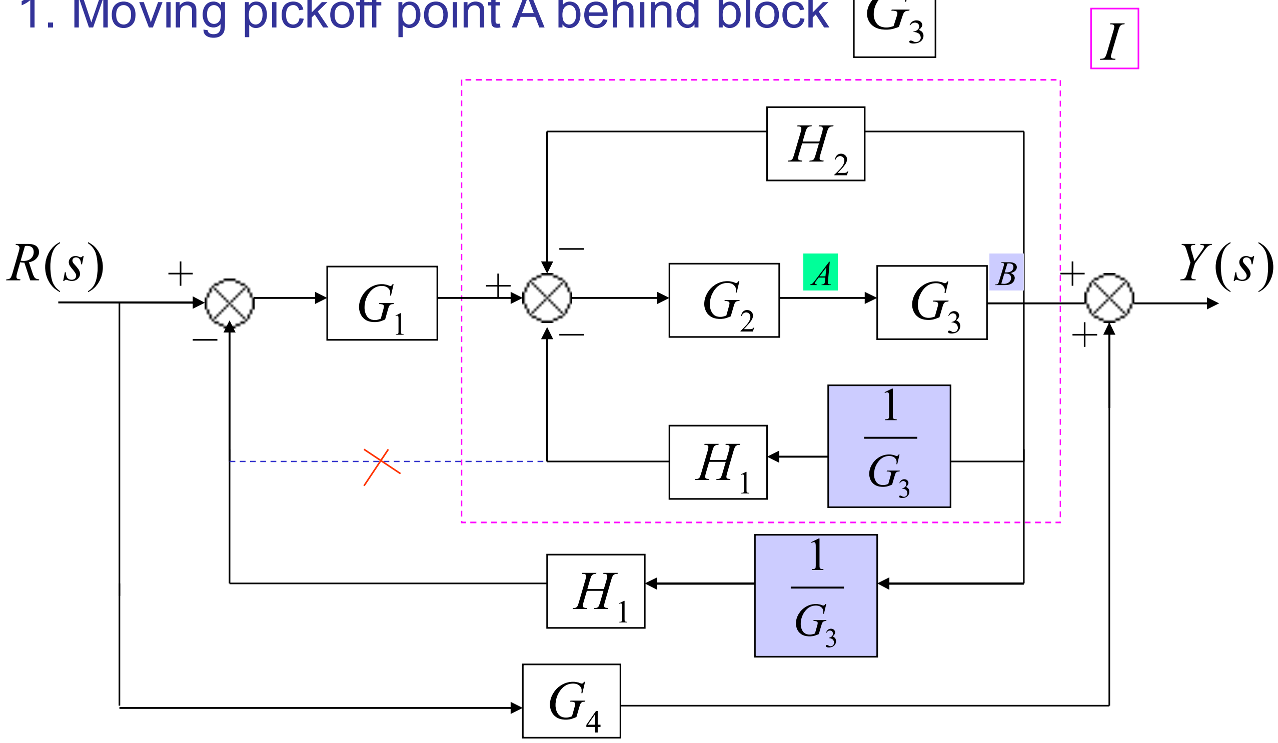
$$T(s)=\frac{Y(s)}{R(s)}=\frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_3G_4H_4+G_1G_2G_3H_2-G_1G_2G_3G_4H_1}$$

(d)

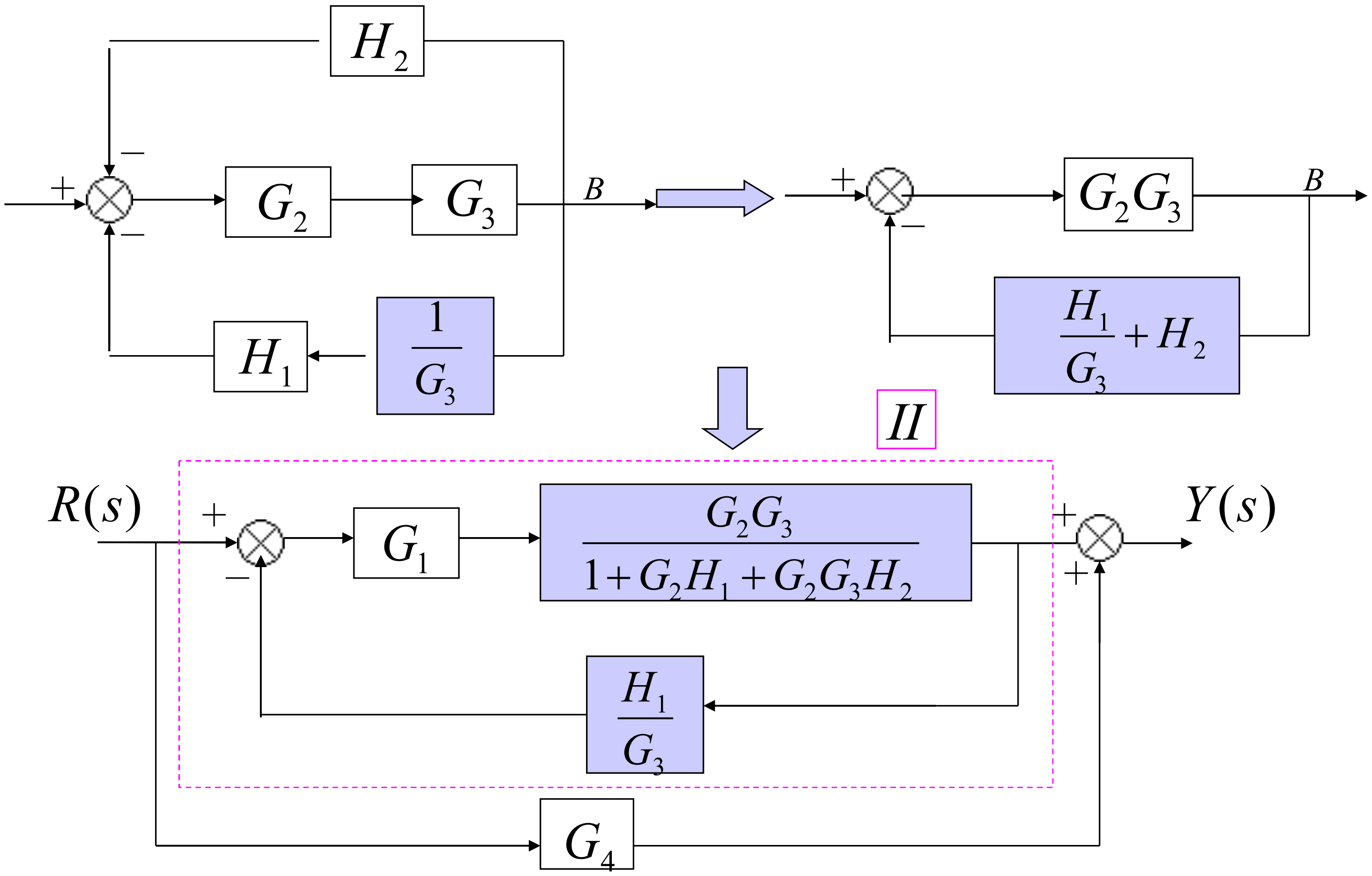


Solution:

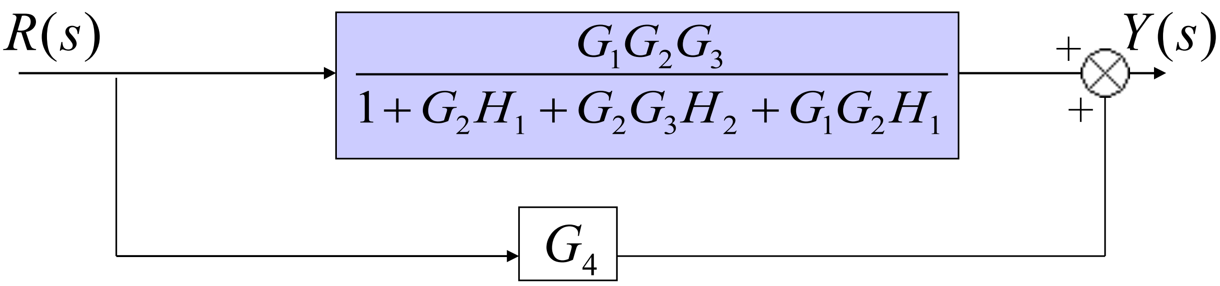
1. Moving pickoff point A behind block $\boxed{G_3}$



2. Eliminate loop I & Simplify



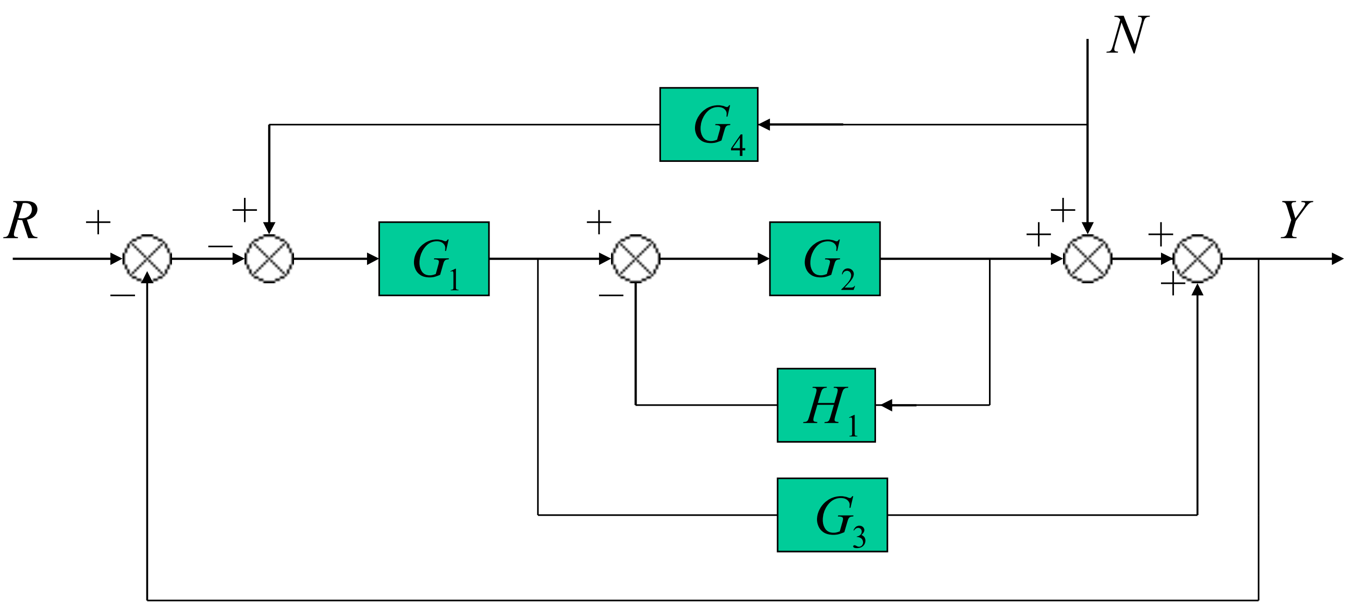
3. Eliminate loop II



$$T(s)=\frac{Y(s)}{R(s)}=G_4+\frac{G_1G_2G_3}{1+G_2H_1+G_2G_3H_2+G_1G_2H_1}$$

Example 2

Determine the effect of R and N on Y in the following diagram



In this linear system, the output Y contains two parts, one part is related to R and the other is caused by N:

$$Y = Y_1 + Y_2 = T_1 R + T_2 N$$

If we set N=0, then we can get Y1:

$$Y_1 = Y_{N=0} = T_1 R$$

The same, we set R=0 and Y2 is also obtained:

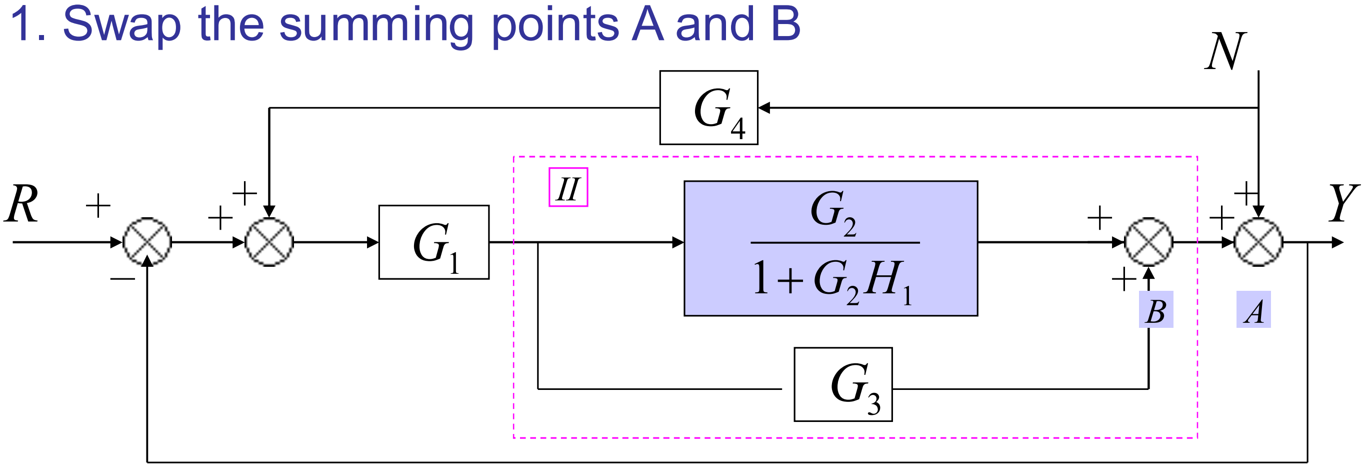
$$Y_2 = Y_{R=0} = T_2 N$$

Thus, the output Y is given as follows:

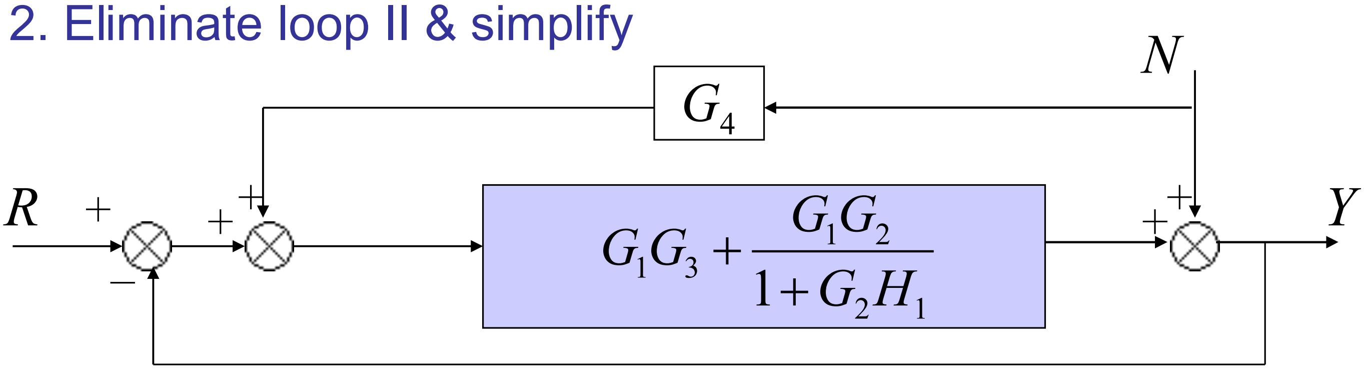
$$Y = Y_1 + Y_2 = Y_{N=0} + Y_{R=0}$$

Solution:

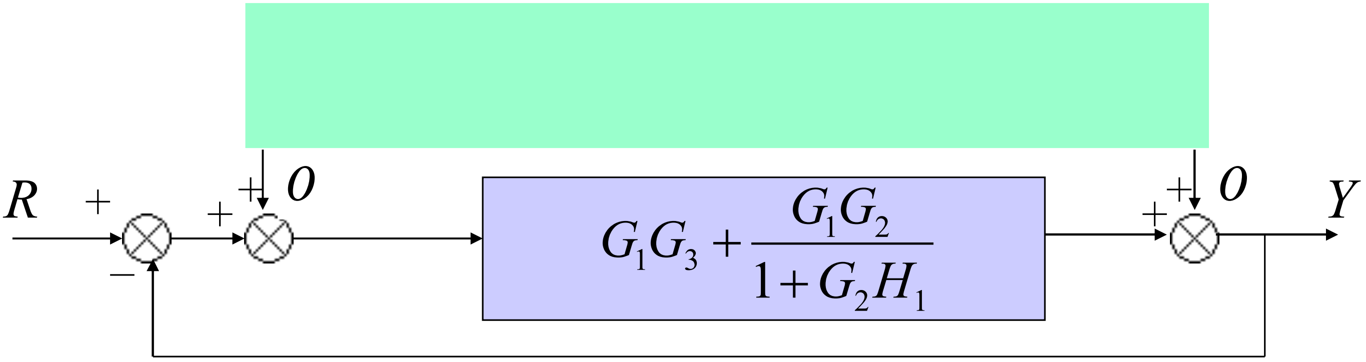
1. Swap the summing points A and B



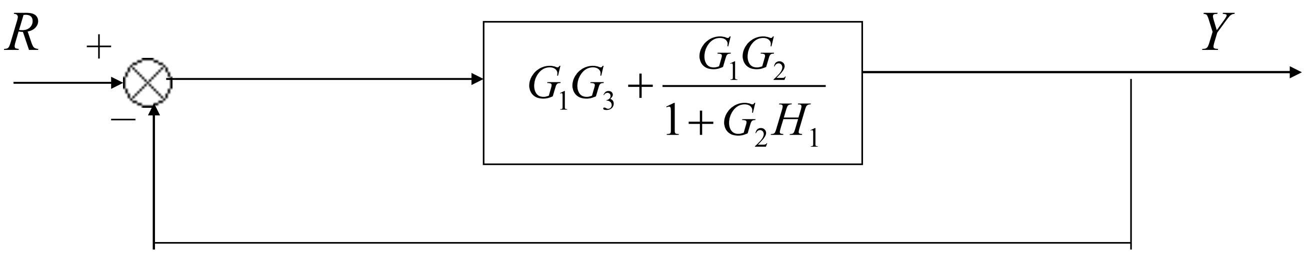
2. Eliminate loop II & simplify



Rewrite the diagram:



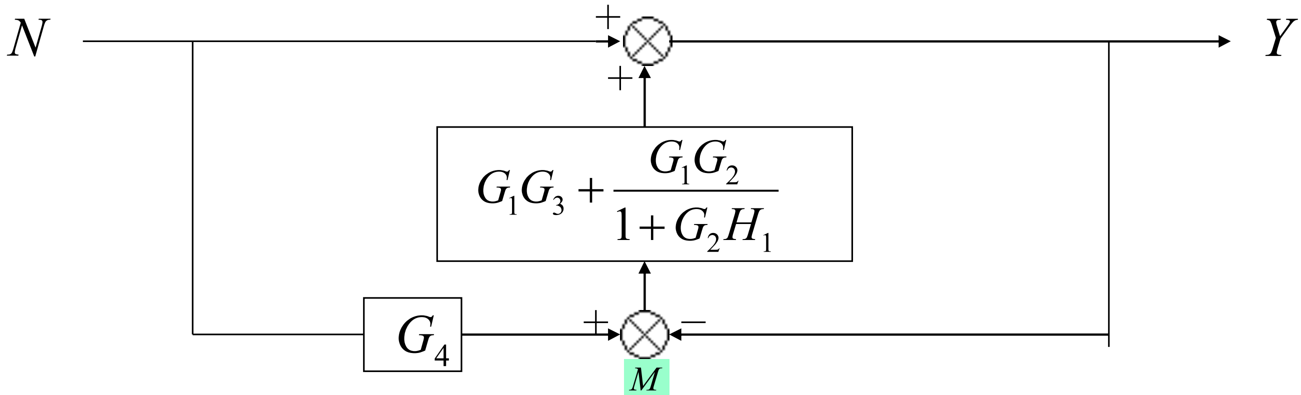
3. Let N=0



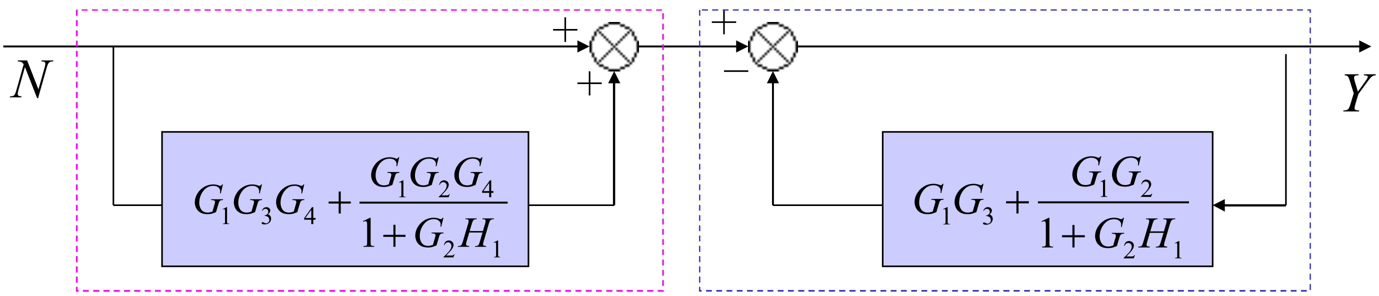
We can easily get Y_1

$$Y_1 = \frac{G_1G_2 + G_1G_3 + G_1G_2G_3H_1}{1 + G_2H_1 + G_1G_2 + G_1G_3 + G_1G_2G_3H_1} R$$

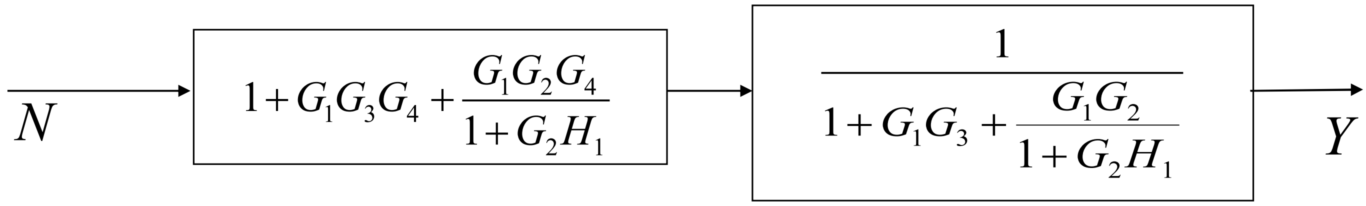
4. Let R=0, we can get:



5. Break down the summing point M:



6. Eliminate above loops:



$$Y_2=\frac{1+G_2H_1+G_1G_2G_4+G_1G_3G_4+G_1G_2G_3G_4H_1}{1+G_2H_1+G_1G_2+G_1G_3+G_1G_2G_3H_1}N$$

7. According to the principle of superposition, Y_1 and Y_2 can be combined together, So:

$$\begin{aligned}Y&=Y_1+Y_2\\&=\frac{1}{1+G_2H_1+G_1G_2+G_1G_3+G_1G_2G_3H_1}[(G_1G_2+G_1G_3+G_1G_2G_3H_1)R\\&+(1+G_2H_1+G_1G_2G_4+G_1G_3G_4+G_1G_2G_3G_4H_1)N]\end{aligned}$$